STT3250 HW2 Solutions

September 2020

Section 1.3

1. (a) $P(A) = 5000/1000000$
   (b) $P(A_1) = 78515/1000000$
   (c) $P(A_1|B_2) = P(A_1 \cap B_2)/P(B_2) = 73630/995000$
   (d) $P(B_1|A_2) = P(B_1 \cap A_2)/P(A_2) = 4885/78515$
   (e) (1) Probability of a positive test when it is known that the person do not carry AIDS.
   (2) Probability of a person carrying AIDS, when a positive test result is present.

3. (a) $P(A_1 \cap B_1) = 5/35$
   (b) $P(A_1 \cup B_1)/P(B_1) = P(A_1) + P(B_1) - P(A_1 \cap B_1) = 12/35 + 19/35 - 5/35 = 26/35$
   (c) $P(A_1|B_1) = P(A_1 \cap B_1)/P(B_1) = 5/19$
   (d) $P(B_2|A_2) = P(B_2 \cap A_2)/P(A_2) = 9/23$

5. (a) $S = \{RR, RW, WR, WW\}$
   (b) $P(ww|whiteeyes) = 1/3$

7. Let R - be orange and B - be blue

   Find the probability of getting at least one R. This can happen either by getting 1) RR in the row (probability $= 2/4 \cdot 1.3 = 1/6$), 2) RB or BR (probability $= 2/4 \cdot 2.3 + 2/4 \cdot 2.3 = 2/3$).

   Therefore the probability at least one $R = 1/6 + 2/3)$

   Now find,

   $P(RR|atleastoneR) = \frac{1/6}{1/6+2/3} = 1/5$

11. (a) These r students in this class can have birthdays in any given day so,

    $365 \cdot 365 \cdot 365 \cdot \cdots \cdot 365$, r times as there are r students. Which is simply 356^r

(b) None of the students having a same birthday.
$365 \cdot 364 \cdot 363 \cdot \cdots \cdot 365 - (r+1)$, Which is simply ${}^{365}P_r$.

(c) P(at least one birthday is shared) =
$1 - P$(None of the students having a same birthday) =
$1 - \frac{{}^{365}P_r}{356^r}$

(d) need a computer to calculate.

15. P(2 same color) = 151/300 - given

let x be the number of red in the second urn.

Then, the $P(RR) = \frac{8}{15} \cdot \frac{x}{x+9}$

$P(BB) = \frac{7}{15} \cdot \frac{9}{x+9}$

Therefore, $\frac{8}{15} \cdot \frac{x}{x+9} + \frac{7}{15} \cdot \frac{9}{x+9} = 151/300$ solve for x.

$x = 11$.

Section 1.4

1. A and B independent $\implies P(A \cap B) = P(A) \cdot P(B)$

   (a) $P(A \cap B) = P(A) \cdot P(B) = 0.14$

   (b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.2 - 0.14 = 0.76$

   (c) $P(A' \cup B') = P((A \cap B)') = 1 - P(A \cap B) = 0.86$, due to DeMorgan's law and compliment rule.

3. (a) A and B independent $\implies P(A \cap B) = P(A) \cdot P(B) = 1/6$

   (b) $P(A) = P(A \cap B') + P(A \cap B)$, then $1/4 = P(A \cap B') + 1/6$, Therefore, $P(A \cap B') = 1/12$

   (c) $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B)$, find $P(A \cup B)$ and substitute. Answer is 1/4.

   (d) $P(A' \cup B') = P((A \cap B)') = 1/4$ from c) above

   (e) Using same logic as in b), answer is 1/2

5. Check if $P(A \cap B) = P(A) \cdot P(B)$

   $L.H.S = 0.4$ and $R.H.S = 0.8 \times 0.5 = 0.4$, YES.

7. Done in class

9. (a) $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = 0.36$ (because A, B and C are mutually independent)

   (b) $P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C) =$
   $P(A) \cdot P(B) \cdot P(C') + P(A) \cdot P(B') \cdot P(C) + P(A') \cdot P(B) \cdot P(C)$
   $= 0.49$

   (c) $P(A' \cap B' \cap C') = P(A') \cdot P(B') \cdot P(C') = 0.01$

17. (a) $P(\text{at least one match}) = 1 - P(nomatch) = 1 - (\frac{11}{12})^{12} = 0.648$

    (b) $P(\text{at least one student matches you}) = 1 - P(nomatchtoyou) = 1 - (\frac{11}{12})^{11} = 0.616$