

STT 3250 Practice Problems Set 1

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October 31, 2020

Question 1.

The time a postal clerk spends with his or her customer has an exponential distribution with a mean of four minutes. Suppose a customer has spent four minutes with a postal clerk. What is the probability that he or she will spend at least an additional three minutes with the postal clerk?

Question 2.

At a certain intersection, accidents occur with a Poisson process at an average of 4 per week. What is the probability that it will take at least 3 days for the first accident to occur?

Question 3.

At a certain intersection, accidents occur with a Poisson process at an average of 4 per week. What is the probability that it will take at least 3 days for the second accident to occur?

Question 4.

At a certain intersection, accidents occur with a Poisson process at an average of 3.5 per week. What is the probability that it will take at least 5 days for 5 accidents to occur?

Question 5.

If the moment-generating function of a random variable W is

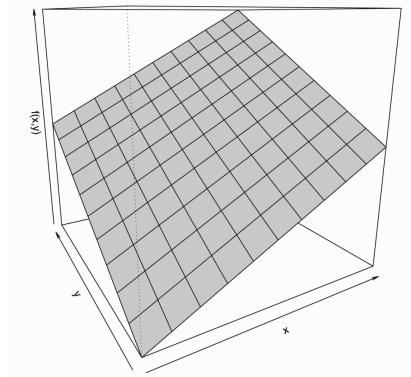
$$M(t) = (1 - 7t)^{-20},$$

- (i) Find the pdf of W .
- (ii) Find the mean of W .
- (iii) Find the variance of W .

Question 6.

Suppose (X, Y) has a probability density function $f_{X,Y}(x, y) = x + y$ for $0 < x < 1, 0 < y < 1$

(I made the following plot to help you visualize the joint pdf)



- i) Find $f_{Y|x}(y)$ ii) Show the $\int_{-\infty}^{\infty} f_{Y|x}(y)dy = 1$
 iii) What is the conditional mean of Y given $X = 0.5$?

Question 7.

The random variable X has a range of 0, 1, 2 and the random variable Y has a range of 1, 2. The joint distribution of X and Y is given by the following table:

x	y	$P(X = x, Y = y)$
0	1	0.2
0	2	0.1
1	1	0.0
1	2	0.2
2	1	0.3
2	2	0.2

- i) Sketch the joint pmf on (X, Y) plane
 ii) Write down tables for the marginal distributions of X and of Y , i.e. give the values of $P(X = x)$ for all x , and of $P(Y = y)$ for all y .
 iii) Write down a table for the conditional distribution of X given that $Y = 2$.
 iv) Compute $E(X)$ and $E(Y)$
 v) Compute $E(XY)$.
 vi) Are X and Y independent? Explain why or why not.

Question 8.

Let X and Y be two jointly continuous random variables with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

i) Sketch the support ii) Find $f_X(x)$ and $f_Y(y)$ iii) Are X and Y independent? iv) Find the conditional PDF of X given $Y = y$, $f_{X|Y}(x|y)$. v) Find $E[X|Y = y]$, for $0 \leq y \leq 1$. vi) Find $Var(X|Y = y)$, for $0 \leq y \leq 1$.

Question 9.

Let X and Y be two jointly continuous random variables with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{81}x^2y & 0 \leq x \leq K, 0 \leq y \leq K \\ 0 & \text{otherwise} \end{cases}$$

- i) Find the value of K
- ii) Find $P(X > 3Y)$.
- iii) Find $P(X + Y > 3)$.
- iv) Are X and Y independent? If not, find $Cov(X, Y)$.