We then let:

Sets

Let $\overline{L}$ be the set of machines.

Let $\overline{T}$ be the set of weeks.

Let $\overline{I}$ be the set of products.

Let $\overline{I_l}$ be the set of products that can be produced on line l.

Parameters

$c_{ijl} \triangleq$ The given changeover time(hours) from product i to j on line l, where $i \in \overline{I}$ and $j \in \overline{I}$ and $l \in \overline{L}$.

$d_{it} \triangleq$ The given demand per product i in week t, where $i \in \overline{I}$ and $t \in \overline{T}$.

$r_{il} \triangleq$ The given production rates of product i on line l, where $i \in \overline{I}$ and $l \in \overline{L}$.

$s_i \triangleq$ The given safety stock on hand per product i, where $i \in \overline{I}$.

$a \triangleq$ The available time for production in a week for production excluding weekends.

$z \triangleq$ The available time for production in a week for production including weekends..

Variables

$p_{it} \triangleq$ The amount of product i produced during time period t, where $i \in \overline{I}$ and $t \in \overline{T}$.

$o_{ilt} \triangleq$ The order in which product i is manufactured on production l during time period t, where $i \in \overline{I}$ and $l \in \overline{L}$ and $t \in \overline{T}$.

$v_{it} \triangleq$ The inventory level of product i for time period t, where $i \in \overline{I}$ and $t \in \overline{T}$.

$pt_{ilt} \triangleq$ The production time of product i on prodution line l during time period t, where $\in \overline{I}$ and $l \in \overline{L}$ and $t \in \overline{T}$.

$$x_{ilt} \triangleq \begin{cases} 1 \text{ if product } i \text{ is produced on production line } l \text{ during time period } t, \text{ where } i \in \overline{I} \\ \text{and } l \in \overline{L} \text{ and } t \in \overline{T}. \\ 0 \text{ Otherwise} \end{cases}$$

$$lp_{ilt} \triangleq \begin{cases} 1 \text{ if product } i \text{ is the last product produced on production line } l \text{ during time} \\ \text{period } t, \text{ where } i \in \overline{I} \text{ and } l \in \overline{L} \text{ and } t \in \overline{T}. \\ 0 \text{ Otherwise} \end{cases}$$

$$fp_{ilt} \triangleq \begin{cases} 1 \text{ if product } i \text{ is the first product produced on production line } l \text{ during time} \\ \text{period } t, \text{ where } i \in \overline{I} \text{ and } l \in \overline{L} \text{ and } t \in \overline{T}. \\ 0 \text{ Otherwise} \end{cases}$$

$$ct_{ijlt} \triangleq \begin{cases} 1 \text{ if product } i \text{ is manufactured before product } j \text{ on production line } l \text{ during} \\ \text{time period } t\text{, where } i \in \overline{I} \text{ and } j \in \overline{I} \text{ and } l \in \overline{L} \text{ and } t \in \overline{T}. \\ 0 \text{ Otherwise} \end{cases}$$

$$bt_{ijlt} \triangleq \begin{cases} 1 \text{ if there is a changeover from product } i \text{ in week } t\text{-1 to product } j \text{ in week } t \text{ on} \\ \text{production line } l \\ \text{time period } t\text{, where } i \in \overline{I} \text{ and } j \in \overline{I} \text{ and } l \in \overline{L} \text{ and } t \in \overline{T}. \\ 0 \text{ Otherwise} \end{cases}$$

Objective Function

$$\boldsymbol{min\ z} = \sum_{i\in I_l}\sum_{\substack{j\in I_l\\ i\neq j}}\sum_{l\in L}\sum_{t\in T} ct_{ijlt} + bt_{ijlt} \tag{1}$$

Subject to:

$$\sum_{i\in I_l} fp_{ilt} = 1 \qquad \forall\, l \in \bar{L}; t \in \bar{T} \tag{2}$$

$$\sum_{i\in I_l} lp_{ilt} = 1 \qquad \forall\, l \in \bar{L}; t \in \bar{T} \tag{3}$$

$$fp_{ilt} \leq x_{ilt} \qquad \forall\, i \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{4}$$

$$lp_{ilt} \leq x_{ilt} \qquad \forall\, i \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{5}$$

$$o_{jlt} - (o_{ilt} + 1) \geq -M(1 - ct_{ijlt}) \qquad \forall i \in \bar{I_l}; j \in \bar{I_l}; j \neq i; l \in \bar{L}; t \in \bar{T} \tag{6}$$

$$\sum_{\substack{i\in I_l\\ i\neq j}} ct_{ijlt} = x_{jlt} - fp_{jlt} \qquad \forall\, j \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{7}$$

$$\sum_{\substack{j\in I_l\\ i\neq j}} ct_{ijlt} = x_{ilt} - lp_{ilt} \qquad \forall\, i \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{8}$$

$$\sum_{i\in I_l} bt_{ijlt} = fp_{jlt} \qquad \forall\, j \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} - \{1\} \tag{9}$$

$$\sum_{j\in I_l} bt_{ijlt} = lp_{il(t-1)} \qquad \forall\, i \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} - \{1\} \tag{10}$$

$$o_{ilt} \leq Mx_{ilt} \qquad \forall\, i \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{11}$$

$$fp_{ilt} \leq o_{ilt} \leq \sum_{j\in I_l} x_{jlt} \qquad \forall\, i \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{12}$$

$$ax_{ilt} \leq pt_{ilt} \leq zx_{ilt} \qquad \forall\, i \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{13}$$

$$\sum_{i\in I_l} pt_{ilt} + \sum_{i\in I_l}\sum_{j\in I_l}(ct_{ijlt} + bt_{ijlt})\, c_{ijl} \leq z \qquad \forall\, l \in \bar{L}; t \in \bar{T} - \{1\} \tag{14}$$

$$\sum_{i\in I_l} pt_{ilt} + \sum_{i\in I_l}\sum_{j\in I_l}(ct_{ijlt})\, c_{ijl} \leq z \qquad \forall\, l \in \bar{L}; t \in \bar{T} \in \{1\} \tag{15}$$

$$p_{it} = r_{il} pt_{ilt} \qquad \forall\, i \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{16}$$

$$v_{it} = v_{i(t-1)} + p_{it} - d_{it} \qquad \forall\, i \in \bar{I}; t \in \bar{T} \tag{17}$$

$$v_{it} \geq s_i \qquad \forall\, i \in \bar{I}; t \in \bar{T} \tag{18}$$

$$p_{it}, v_{it} \geq 0 \qquad \forall\, i \in \bar{I}; t \in \bar{T} \tag{19}$$

$$o_{ilt}, pt_{ilt} \geq 0 \qquad \forall\, i \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{20}$$

$$x_{ilt}, lp_{ilt}, fp_{ilt}, ct_{ijlt}, bt_{ijlt} \in \{0,1\} \qquad \forall i \in \bar{I_l}; j \in \bar{I_l}; l \in \bar{L}; t \in \bar{T} \tag{21}$$

(1) is the objective function that minimises the number of changeovers. Constraints (2)-(3) ensures that at least one product runs on the production line per shift. Where constraints (4)-(5) allocates the first and last products during each shift. Constraint (6) looks at the production order within the time period. In constraint (7)-(8) all the changeovers within the time period are assigned. The (9)-(10) is to allocate changeovers between time periods with constraint (11) being used to eliminate subtours by forcing it to 0 if it is not produced. (12) ensures that the order index follows a numerical order. Constraint (13) is used to restrict the production time for a week. (14)-(15) is used to restrict the summation of the production time and changeover times that it does not exceed the available time. (16) calculates the amount of products produced with (17) calculating the inventory level for the time period (18) is used to ensure that there is always two weeks worth of safety stock. Constraint (19)-(20) is the non negativity constraints and (20) is the binary constraint.