INVERSE CONSISTENCY BY CONSTRUCTION FOR MULTISTEP DEEP REGISTRATION

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TRADITIONAL DEEP REGISTRATION

Fixed Image (IB)

Moving image (IA)

Transformation model : $\varphi(x) = kx$

$k = NN_\theta(I^A, I^B)$

Inverse consistent if $NN_\theta\ (I^A, I^B) * NN_\theta(I^B, I^A) = 1$

OUR APPROACH

Fixed Image (IB)

Moving image (IA)

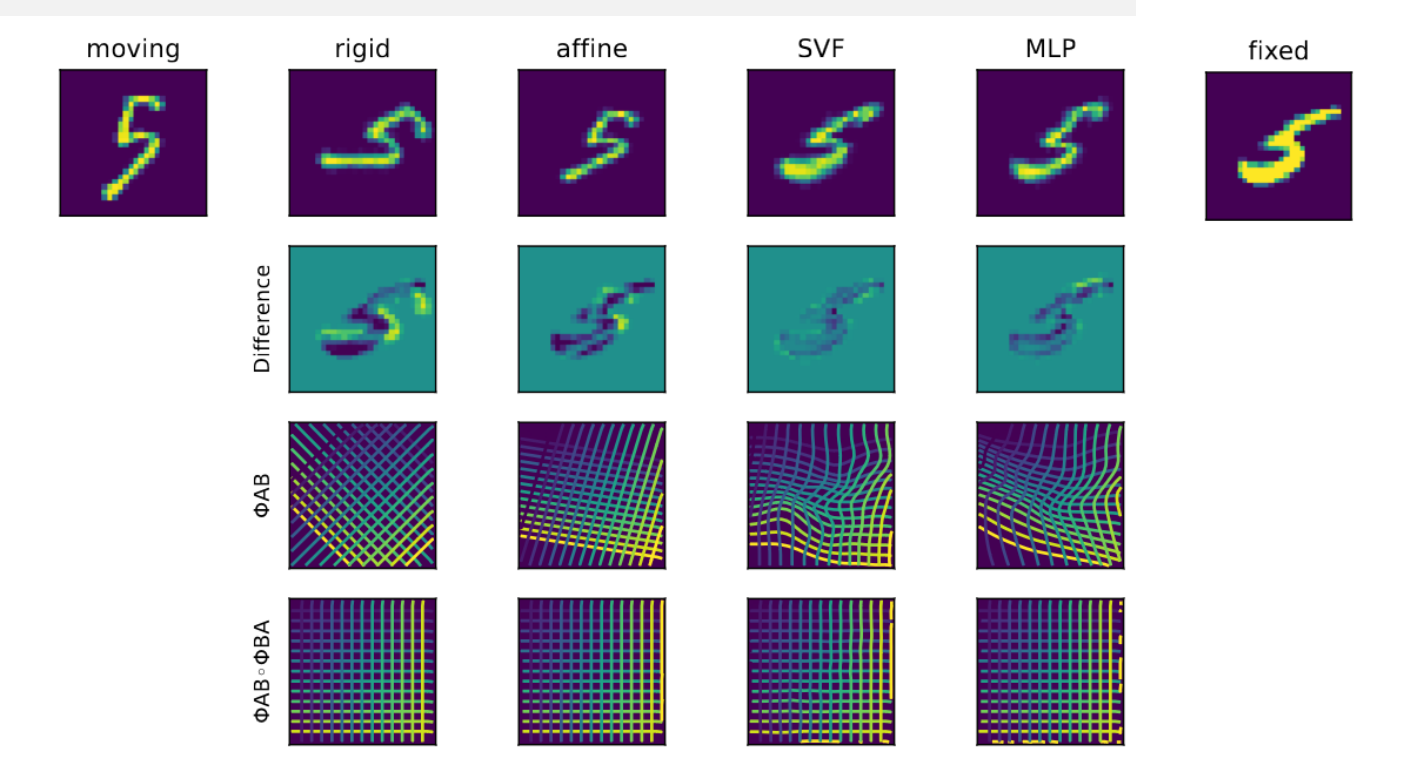
Transformation model : $\phi(x) = kx$

$$k = \exp(NN_\theta(I^A, I^B) - NN_\theta(I^B, I^A))$$

Inverse consistent if

$$\exp\left(NN_\theta(I^A, I^B) - NN_\theta(I^B, I^A)\right) * \exp\left(NN_\theta(I^B, I^A) - NN_\theta(I^A, I^B)\right) = 1$$

K DOESN'T HAVE TO BE A NUMBER



Transformation model : $\phi(x) = kx$

$$k = \exp(NN_\theta(I^A, I^B) - NN_\theta(I^B, I^A))$$

A NOTATION FOR FINDING PROPERTIES OF REGISTRATION ALGORITHMS

- I^A, I^B, I^M, I^F
- \varphi, \varpsi

- Operators are functions from registration algorithms to registration algorithms

- Images are functions from coordinates to intensities
- Transforms are functions from coordinates to coordinates
- (it's usually clearer to write a registration algorithm called on inputs instead of a naked transform)
- Registration algorithms are functionals from image pairs to transforms
- Operators are functions from registration algorithms to registration algorithms

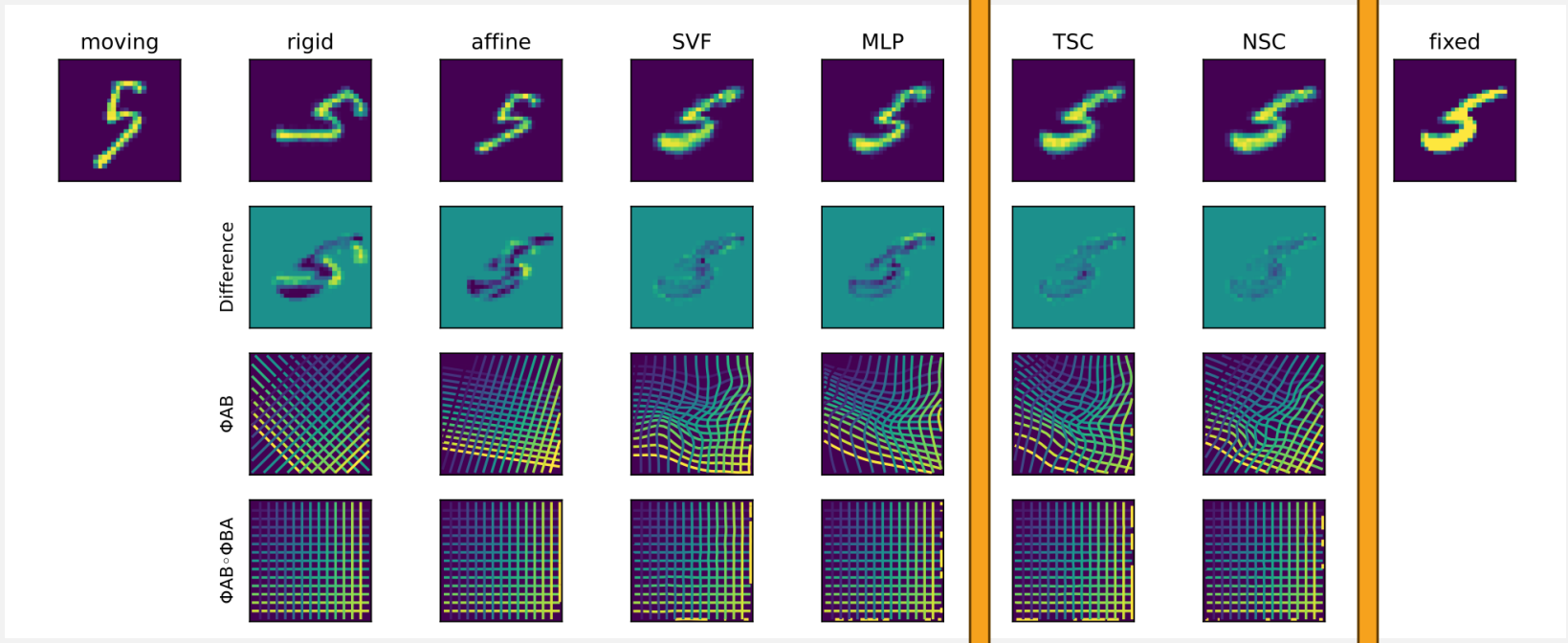
TWO STEP REGISTRATION

Given two Neural Networks Φ_θ, Ψ_ρ that are inverse consistent,
$TSC\{\Phi_\theta, \Psi_\rho\}[I^A, I^B] =$

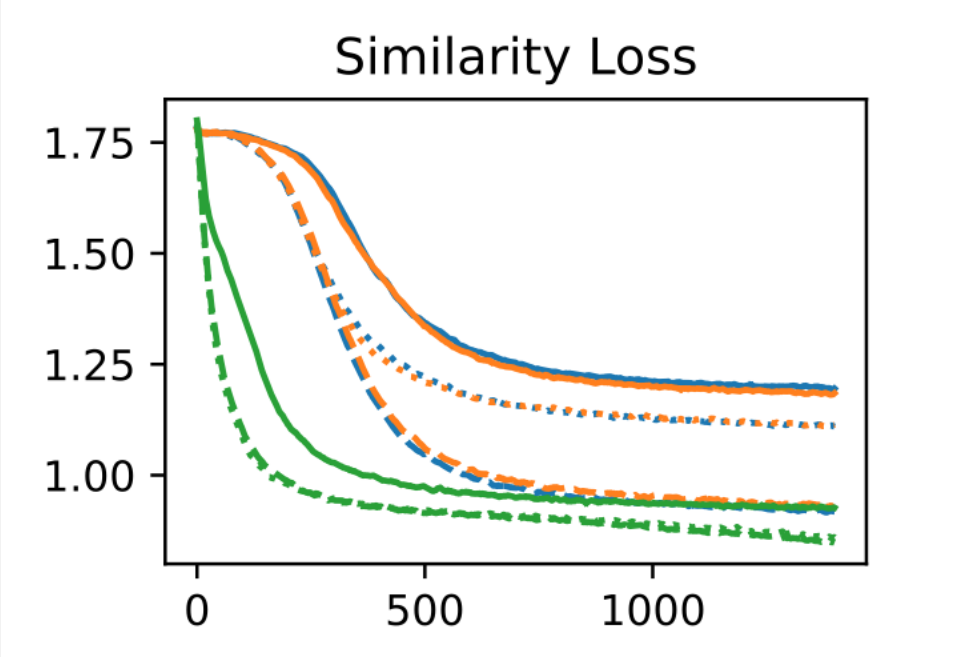
$$\sqrt{\Phi_\theta[I^A, I^B]} \circ \Psi_\rho[I_A \circ \sqrt{\Phi_\theta[I^A, I^B]}, I^B \circ \sqrt{\Phi_\theta[I^B, I^A]}] \circ \sqrt{\Phi_\theta[I^A, I^B]}$$

is also inverse consistent. Proof via symbol munging.

MULTI STEP REGISTRATION



ACCELERATED CONVERGENCE



REAL WORLD PERFORMANCE

HCP			
Approach	DICE	%$\lvert J\rvert$	$\lVert\Phi^{AB} \circ \Phi^{BA} - \mathrm{id}\rVert$
ANTs SyNOnly [2]	75.8	0	0.0350
ANTs SyN	77.2	0	1.30
ConstrICON	79.3	3.81e-6	0.000386
ConstrICON + io	80.1	0	0.00345
GradICON [23]	78.6	0.00120	0.309
GradICON + io	80.2	0.000352	0.123
SynthMorph [8] brain	78.4	0.364	–
SynthMorph shape	79.7	0.298	–
OASIS			
Approach	DICE	%$\lvert J\rvert$	$\lVert\Phi^{AB} \circ \Phi^{BA} - \mathrm{id}\rVert$
ConstrICON	79.7	9.73e-5	0.00776
SymNet [14]	79.1	0.00487	0.0595
EasyReg [10]	77.2	–	0.181