

Math 668 Homework 4

Due at class Monday, November 21, 2022

1. Find an asymptotic expansion as $|x| \rightarrow \infty$ for

$$\int_{\gamma} ds e^{sx} e^{-s^3/3}$$

where $x = Re^{i\theta}$, and γ is the path associated with the *Ai* solution of the equation (terminating in appropriate bullseye regime). Note, the answer depends on θ , and exhibits the Stoke's phenomena.

2. Find large x asymptotics ($x \in \text{Reals}$) for solutions to

$$\frac{d^3 y}{dx^3} = xy$$

Find integral representations for the three linearly independent solutions using the generalized Laplace method, and justify all steps in computing the asymptotics.

3. Consider the ODE we studied in class

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = y$$

First, find large x asymptotics for both independent solutions of the ODE. Second, find small x asymptotics for both independent solutions of the ODE working directly on the two different contours (one encircling the essential singularity at the origin in the complex s plane, the other an appropriately chosen contour we discussed in class). Note, recall the method of Frobenius, it calls for a log term in this problem (confirm that), and try to find it as an asymptotic expansion of the integral of the second contour (the first contour i worked out in class via residues).