Math 668 Homework 3

Due at class Wednesday, October 19, 2022

1. Consider the complex functions

$$\begin{aligned} f(z) &= \sqrt{(x-i)(x+i)(x-2i)(x+2i)} \\ g(z) &= \sqrt{(x-i)(x+i)(x-2i)} \end{aligned}$$

   Can the functions, $f(z)$ and $g(z)$ be defined to have only finite length branch cuts? Evaluate the integral of $\frac{z^j}{f(z)}$ around around a circle of radius 10 about the origin for any integer, j.

2. Evaluate

$$\int_0^\infty dx \frac{\log x}{x^2+1}$$

3. Evaluate

$$\int_0^\infty dx \frac{\log x}{(x^2+1)^2}$$

4. Evaluate

$$\int_C f(z)dz$$

5. Evaluate

$$\int_0^\infty dx \frac{x^{-a}}{(1+x+i)^2}$$

   for $a \in (0,1)$

6. Evaluate

$$\int_0^\infty dx \frac{1}{1+x^N} dx$$

   for any integer, $N > 1$.

7. Find convergent Taylor and Laurent Expansions about $z = -1$ for $f(z) = \frac{1}{z(z+1)}$ for any radius not equal to one or two.