Please check the examination details below before entering your candidate information

Candidate surname: Annotated by Tom.

Other names

Centre Number

Candidate Number

HastingsMath.com

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Pure Mathematics P2

June 2022

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

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1. Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2+\frac{3}{8}x\right)^{10}$$ $\to n$

r is position -1

Give each coefficient as an integer.

${}^{n}C_r \times (a)^{n-r} \times (b)^r$

(4)

1

$U_1 : {}^{10}C_0 \times (2)^{10-0} \times \left(+\frac{3}{8}x\right)^0 = 1 \times 2^{10} \times 1$

$U_2 : {}^{10}C_1 \times (2)^{10-1} \times \left(+\frac{3}{8}x\right)^1 = 10 \times 2^9 \times \frac{3}{8}x$

$U_3 : {}^{10}C_2 \times (2)^{10-2} \times \left(\frac{3}{8}x\right)^2 = 45 \times 2^8 \times \frac{9}{64}x^2$

$U_4 : {}^{10}C_3 \times (2)^{10-3} \times \left(\frac{3}{8}x\right)^3 = 120 \times 2^7 \times \frac{27}{512}x^3$

$\therefore 1024 + 1920x + 1620x^2 + 810x^3$ ✓

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Question 1 continued

(Total 4 marks)

Q1

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2.

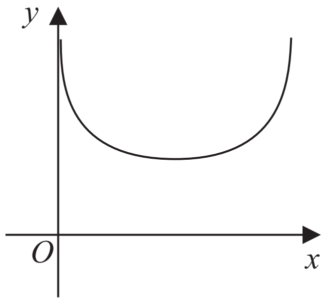

Figure 1

Figure 1 shows the graph of

$$y = 1 - \log_{10}(\sin x) \qquad 0 < x < \pi$$

where x is in radians.

The table below shows some values of x and y for this graph, with values of y given to 3 decimal places.

x	0.5	1	1.5	2	2.5	3
y	1.319	1.075	1.001	1.041	1.223	1.850

(a) Complete the table above, giving values of y to 3 decimal places. **(2)**

(b) Use the trapezium rule with all the y values in the completed table to find, to 2 decimal places, an estimate for

$$\int_{0.5}^{3} \left(1 - \log_{10}(\sin x)\right) dx$$

(3)

(c) Use your answer to part (b) to find an estimate for

$$\int_{0.5}^{3} \left(3 + \log_{10}(\sin x)\right) dx$$

(3)

trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\left[y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n\right]$

h = distance btw x-values

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Question 2 continued

(radian mode)

(a) where $x = 1$

$y = 1 - \log_{10}(\sin 1)$

$= 1 - \log(\sin 1)$

$= 1.074960855$

≈ 1.075 (3 dec. pl.)

where $x = 2$

$y = 1 - \log(\sin 2)$

$= 1.041294038$

≈ 1.041 (3 dec pl)

(b) $h = 0.5$ distance between x-values

$A = \frac{1}{2} \times 0.5 \left(1.319 + 2(1.075 + 1.001 + 1.041 + 1.223) + 1.850\right)$

$= 2.96225$

≈ 2.96 (2 dec pl) ✓

(c) $\int_{0.5}^{3} (1 - \log_{10}(\sin x))\,dx$ rearrange and solve

$= \int_{0.5}^{3} \left(4 - (3 + \log_{10}(\sin x))\right) dx$

$= \int_{0.5}^{3} (4) - \int_{0.5}^{3} (3 + \log_{10}(\sin x))\,dx$

+8801854238322, +8801716264292

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Question 2 continued

$$\therefore\ 2.96225 = \int_{0.5}^{3} (4) - \int_{0.5}^{3} \left(3+\log_{10}(\sin x)\right) dx$$

$$\therefore\ 2.96225 = \left[4x'\right]_{0.5}^{3} - \int_{0.5}^{3} \left(3+\log_{10}(\sin x)\right) dx$$

$$\therefore\ \int_{0.5}^{3} \left(3+\log_{10}(\sin x)\right) d = \left[4(3) - 4(0.5)\right] - 2.96225$$

$$= 7.03775$$

$$= 7.04 \quad (2\ \text{dec pl}) \checkmark$$

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Question 2 continued

(Total 8 marks)

Q2

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3. (i) Show that the following statement is **false**:

"$(n+1)^3 - n^3$ is prime for all $n \in \mathbb{N}$"

(2)

(ii) Given that the points $A(1, 0)$, $B(3, -10)$ and $C(7, -6)$ lie on a circle, prove that AB is a diameter of this circle.

(5)

3 (i) $n=1$ $(1+1)^3-(1)^3 = 8-1 = 7$ PRIME

$n=2$ $(2+1)^3-(2)^3 = 27-8 = 19$ PRIME

$n=3$ $(3+1)^3-(3)^3 = 64-27 = 37$ PRIME

$n=4$ $(4+1)^3-(4)^3 = 125-64 = 61$ PRIME

$n=5$ $(5+1)^3-(5)^3 = 216-125 = 91$ PRIME

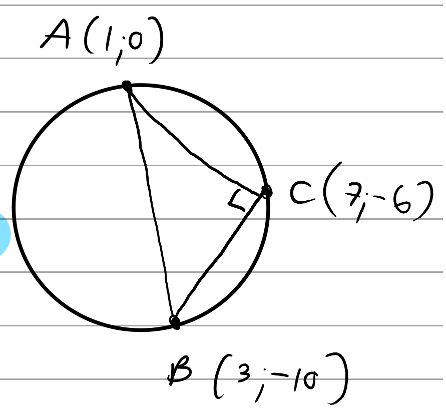
$n=6$ $(6+1)^3-(6)^3 = 343-216 = 127$ PRIME

$n=7$ $(7+1)^3-(7)^3 = 512-343 = 169$

↳ not prime

$\therefore$ false ✓

(ii) RTP: AB is diam.



if $M_{AC} \times M_{BC} = -1$ [<'s subtend by diam. = 90°]

$\frac{0-(-6)}{1-7} \times \frac{-6-(-10)}{7-3}$

$-\frac{6}{6} \times \frac{4}{4}$ ✓

$-1 \times 1 = -1$

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

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4. **In this question you must show all stages of your working.**

Give your answers in fully simplified surd form.

Given that a and b are positive constants, solve the simultaneous equations

$$a - b = 8$$
$$\log_4 a + \log_4 b = 3$$

(6)

4

$a = b+8 \quad \ldots (1)$

$\log_4 ab = 3 \quad \ldots (2)$

sub (1) into (2)

$\log_4 (b+8)b = 3$

$\log_4 (b^2+8b) = 3$

$a^b = c$

$\log_a c = b$

$4^3 = b^2 + 8b$

$b^2 + 8b - 64 = 0$

$b = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-64)}}{2(1)}$

$b = -4+4\sqrt{5}$ or $b = -4-4\sqrt{5}$

invalid

$b = -4+4\sqrt{5}$ ✓

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Question 4 continued

Q4

(Total 6 marks)

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5. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Solve, for $-180° < \theta \leqslant 180°$, the equation

$$3\tan(\theta + 43°) = 2\cos(\theta + 43°)$$

(6)

Identities to use

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

(5) $3\tan(\theta+43°) = 2\cos(\theta+43°)$

$\frac{3\sin(\theta+43)}{\cos(\theta+43)} = 2\cos(\theta+43)$

$3\sin(\theta+43) = 2\cos^2(\theta+43)$

$3\sin(\theta+43) = 2(1-\sin^2(\theta+43))$

$3\sin(\theta+43) = 2 - 2\sin^2(\theta+43)$

$2\sin^2(\theta+43) + 3\sin(\theta+43) - 2 = 0$

$(2\sin(\theta+43) - 1)(\sin(\theta+43) + 2) = 0$

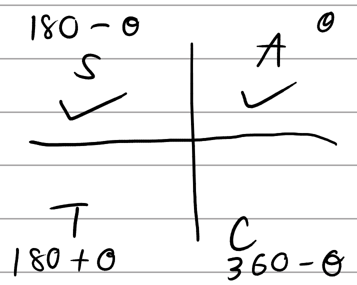
$\sin(\theta+43) = \frac{1}{2}$ or $\sin(\theta+43) = -2$ invalid

$\theta + 43 = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta + 43 = 30°$

$\theta = -13 + n360 \quad n \in \mathbb{Z}$

or



check quadrants

$\theta = (180 - 30) - 43 + n360 \quad n \in \mathbb{Z}$

$= 107$

check restrictions

but $-180 < \theta \leqslant 180$

$\therefore \theta = -13°$ or $107°$ only.

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Question 5 continued

Q5

(Total 6 marks)

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6. In a geometric sequence $u_1, u_2, u_3, \ldots$

- the common ratio is r
- $u_2 + u_3 = 6$
- $u_4 = 8$

(a) Show that r satisfies

$$3r^2 - 4r - 4 = 0$$

(3)

Given that the geometric sequence has a sum to infinity,

(b) find u_1

(3)

(c) find S_∞

(2)

(a) $U_2+U_3=6$ $U_4=8$ RTP : $3r^2-4r-4$

$ar^3=8$

$a=\frac{8}{r^3}$ ...1

$ar+ar^2=6$ ...2

Sub 1 into 2

$\frac{8}{r^3}(r)+\frac{8}{r^3}(r^2)=6$

$\frac{8}{r^2}+\frac{8}{r}=\frac{6}{1}$

$8+8r=6r^2$

$6r^2-8r-8=0$

$3r^2-4r-4=0$ ✓

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Question 6 continued

(b) $3r^2 - 4x - 4 = 0$

$(3r + 2)(r - 2) = 0$

$r = -\frac{2}{3}$ or $r = 2$ ✓

S_∞ exists $-1 < r < 1$ $r \neq 0$

$a = \frac{8}{\left(-\frac{2}{3}\right)^3} = u_1$

$\therefore u_1 = -27$ ✓

(c) $S_\infty = \frac{a}{1-r}$

$= \frac{-27}{1-\left(-\frac{2}{3}\right)}$

$= -\frac{81}{5}$ ✓

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Question 6 continued

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Question 6 continued

Q6

(Total 8 marks)

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7. $$f(x) = Ax^3 + 6x^2 - 4x + B$$

where A and B are constants.

Given that

- $(x + 2)$ is a factor of $f(x)$
- $\int_3^5 f(x)\,dx = 176$

find the value of A and the value of B.

(7)

7

$f(x) = ax^3 + 6x^2 - 4x + b$

$f(-2)$: $a(-2)^3 + 6(-2)^2 - 4(-2) + b$

$-8a + 24 + 8 + b = 0$

$-8a + 32 + b = 0$

$b = 8a - 32 \quad \dots (1)$

$$\left[\frac{ax^4}{4} + \frac{6x^3}{3} - \frac{4x^2}{2} + \frac{bx}{1}\right]_3^5 = 176$$

$$\left[\frac{a}{4}x^4 + 2x^3 - 2x^2 + bx\right]_3^5 = 176$$

$$\left(\frac{a}{4}(5)^4 + 2(5)^3 - 2(5)^2 + b(5)\right) - \left(\frac{a}{4}(3)^4 + 2(3)^3 - 2(3)^2 + 3b\right) = 176$$

$$\frac{625}{4}a + 250 - 50 + 5b - \frac{81}{4}a - 54 + 18 - 3b = 176$$

$136a + 164 + 2b = 176$

$136a + 2b = 12 \quad \dots (2)$

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Question 7 continued

sub 1 into 2

$136a + 2(8a - 32) = 12$

$136a + 16a - 64 - 12 = 0$

$152a = 76$

$a = \frac{76}{152} = \frac{1}{2}$ ✓

$\therefore b = 8\left(\frac{1}{2}\right) - 32 = -28$ ✓

Q7

(Total 7 marks)

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8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A curve has equation

$$y = 256x^4 - 304x - 35 + \frac{27}{x^2} \qquad x \neq 0$$

$27x^{-2}$

(a) Find $\frac{dy}{dx}$

(3)

(b) Hence find the coordinates of the stationary points of the curve.

(5)

8 (a) $\frac{dy}{dx} = 256(4)x^3 - 304 - 2 \times 27x^{-3}$

$= 1024x^3 - 304 - \frac{54}{x^3}$ ✓

(b) $1024x^3 - 304 - 54x^{-3} = 0$ SP $\frac{dy}{dx} = 0$

$\frac{1024x^6}{x^3} - \frac{304x^3}{x^3} - \frac{54}{x^3} = 0$

$1024x^6 - 304x^3 - 54 = 0$

$x^3 = \frac{-(-304) \pm \sqrt{(-304)^2 - 4(1024)(-54)}}{2(1024)}$

$x^3 = \frac{27}{64}$ or $-\frac{1}{8}$

$\therefore x = \sqrt[3]{\frac{27}{64}}$ or $x = \sqrt[3]{-\frac{1}{8}}$

$x = \frac{3}{4}$ or $x = -\frac{1}{2}$

$\therefore y = 256\left(\frac{3}{4}\right)^4 - 304\left(\frac{3}{4}\right) - 35 + \frac{27}{\left(\frac{3}{4}\right)^2} = -134$

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Question 8 continued

OR

$y = 256\left(-\frac{1}{2}\right)^4 - 304\left(-\frac{1}{2}\right) - 35 + \frac{27}{\left(-\frac{1}{2}\right)^2} = 241$

$\therefore$ SP's $\left(\frac{3}{4}; -134\right)$ & $\left(-\frac{1}{2}; 241\right)$ ✓

Q8

(Total 8 marks)

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9. A scientist is using carbon-14 dating to determine the age of some wooden items.

The equation for carbon-14 dating an item is given by

$$N = k\lambda^t$$

where

- N grams is the amount of carbon-14 **currently** present in the item
- k grams was the **initial** amount of carbon-14 present in the item
- t is the number of years since the item was made
- λ is a constant, with $0 < \lambda < 1$

(a) Sketch the graph of N against t for $k = 1$

(2)

Given that it takes 5700 years for the amount of carbon-14 to reduce to half its initial value,

(b) show that the value of the constant λ is 0.999878 to 6 decimal places.

(2)

Given that Item A

- is known to have had 15 grams of carbon-14 present initially
- is thought to be 3250 years old

(c) calculate, to 3 significant figures, how much carbon-14 the equation predicts is currently in Item A.

(2)

Item B is known to have initially had 25 grams of carbon-14 present, but only 18 grams now remain.

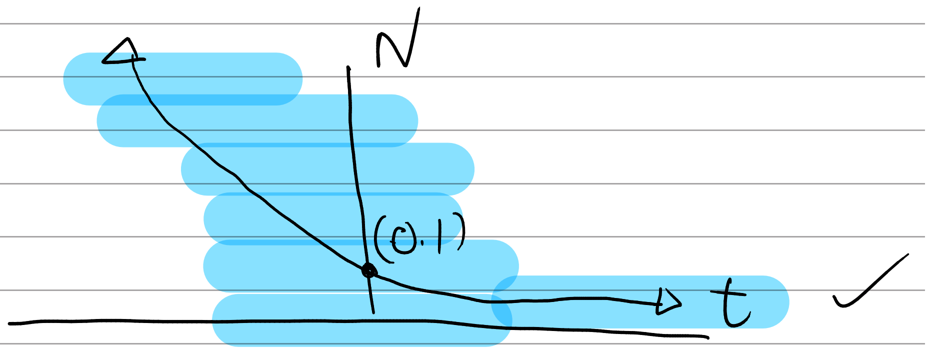
(d) Use algebra to calculate the age of Item B to the nearest 100 years.

(3)

9) $N = k\lambda^t$ $k=1$ $N = \lambda^t$ $0 < \lambda < 1$

let $t = 0$ $N = \lambda^0 = 1$ $\therefore$

$\therefore (0;1)$



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Question 9 continued

(b) $N = k\lambda^t$ ... innitial

$k\left(\frac{1}{2}\right) = k\lambda^{5700}$ ... 5700 yearto be half of innitial

$\frac{1}{2} = \lambda^{5700}$

$\left(\frac{1}{2}\right)^{\frac{1}{5700}} = \lambda$

0.99987784024

0.999878 (6 dec pl) ✓

(c) $k = 15$ $t = 3250$ ...currently

$N = 15.(0.99878...)^{3250}$

$= 10.1$ grams ✓

(d) $t = ?$ $k = 25$ grams

$N = 18$ .... currently present

$\therefore 18 = 25 \times (0.999878...)^t$

$\frac{18}{25} = (0.999878...)^t$

$a^b = c$

$\log_a c = b$

$t = \log_{(0.999878...)} \frac{18}{25}$

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Question 9 continued

$= 2701.402774$

$\therefore 2701.4$ years

$\therefore$ 2700 yrs (to nearest 100) ✓

Leave blank

Question 9 continued

Q9

(Total 9 marks)

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10. The circle C has centre $X(3, 5)$ and radius r

The line l has equation $y = 2x + k$, where k is a constant.

(a) Show that l and C intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that l is a tangent to C,

(b) show that $5r^2 = (k + p)^2$, where p is a constant to be found.

(3)

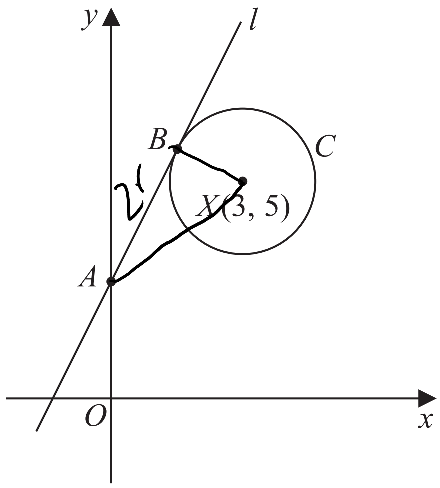

Figure 2

The line l

- cuts the y-axis at the point A
- touches the circle C at the point B

as shown in Figure 2.

Given that $AB = 2r$

(c) find the value of k

(6)

a $(x-3)^2 + (y-5)^2 = r^2$

centre - radius

$(x-a)^2 + (y-b)^2 = r^2$

centre (a,b)

sub $y = 2x+k$

$\therefore (x-3)^2 + (2x+k-5)^2 = r^2$

$x^2 - 6x + 9 + 4x^2 + 2(2x)(k-5) + (k-5)^2 - r^2 = 0$

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Question 10 continued

$x^2 - 6x + 9 + 4x^2 + 4x(k-5) + (k-5)^2 = r^2$

$5x^2 - 6x + 4(k-5)x + k^2 - 10k + 34 - r^2 = 0$

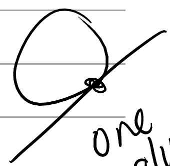
$5x^2 - 6x + 4kx - 20x + k^2 - 10k + 34 - r^2 = 0$

$5x^2 + (4k-26)x + k^2 - 10k + 34 - r^2 = 0$ ✓

(b) RTP: $5r^2 = (k+p)^2$

$m_{BX} = -\frac{1}{2}$ .... because l is tangent

$b^2 - 4ac = 0$ .... because $\Delta = 0$



one solution / equal roots

$\therefore (4k-26)^2 - 4(5)(k^2 - 10k + 34 - r^2) = 0$

$16k^2 - 208k + 676 - 20k^2 + 200k - 680 + 20r^2 = 0$

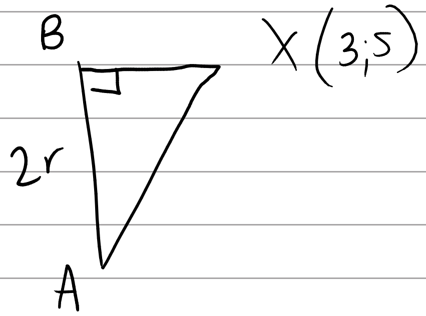
$\frac{-4k^2 - 8k - 4}{-4} = \frac{-20r^2}{-4}$

$k^2 + 2k + 1 = 5r^2$

$(k+1)^2 = 5r^2$ ✓

$\therefore p$ is constant

(c) $d_{AB} = 2r$



$AB^2 + BX^2 = AX^2$

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Question 10 continued

$$BX^2 = r^2$$

$$(k+1)^2 = 5r^2$$

$$\frac{(k+1)^2}{5} = r^2$$

$$(2r)^2 + \frac{(k+1)^2}{5} = (AX)^2$$

point A is y-int of l

$\therefore A(0,k)$

$$d_{AX} = \sqrt{(3-0)^2+(5-k)^2}$$

$$d_{AX} = \sqrt{3^2+(5-k)^2}$$

$$d_{AX}^2 = 9+(5-k)^2$$

$$\therefore (2r)^2 + \frac{(k+1)^2}{5} = 9+(5-k)^2$$

$$4r^2 + \frac{k^2+2k+1}{5} = 9+25-10k+k^2$$

but $4r^2 + r^2 = 5r^2$

$$\therefore 5r^2 = 9+25-10k+k^2$$

from (B)

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Question 10 continued

$\therefore (k+1)^2 = 9+25-10k+k^2$] from (B)

$\cancel{k^2} + 2k + 1 = 9+25-10k+\cancel{k^2}$

$-33 = -12k$

$k = \frac{33}{12} = \frac{11}{4}$ ✓

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Question 10 continued

Q10

(Total 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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