

FUNCTIONS 1

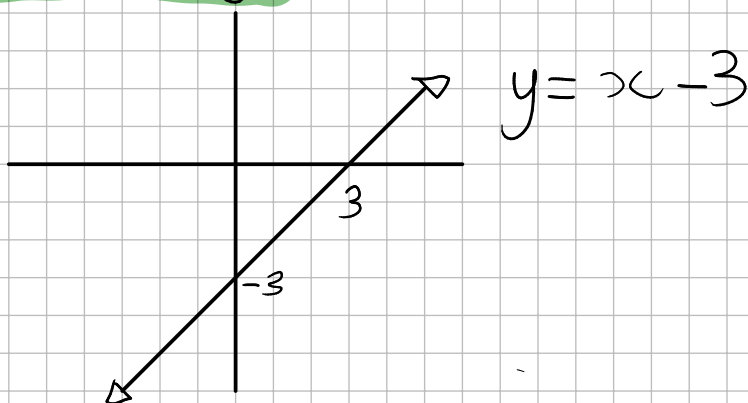
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LINEAR (STRAIGHT LINE) FUNCTIONS

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EXAMPLE ①



gradient/slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$m = f'(x)$$

STANDARD FORM

$$y = mx + c$$

gradient/slope

y coordinate of y-int.

QUESTION

① What is the gradient of the line in example 1?

ANSWER: $m = 1$

GENERAL NOTES

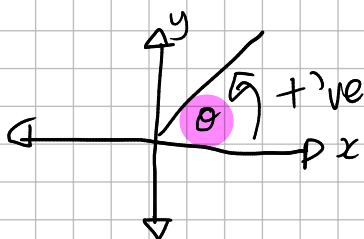
for y-int; let $x=0$

for x-int; let $y=0$

$$\tan \theta = m$$

$$\theta = \tan^{-1}(m)$$

for positive angle off the x-axis in



any other point on the function

Intercept form

$$y - y_1 = m(x - x_1)$$

gradient/slope

y-coordinate of y-intercept

x-coordinate of x-int.

WHAT IS A FUNCTION?

A function is a relationship with one output per INPUT.
ie, one y for every x .

TYPES OF FUNCTIONS RELATIONS

- ① one-to-one
- ② many-to-one

NOTE: one-to-many relation is NOT A FUNCTION

DOMAIN OF A FUNCTION

x -values of the function

RANGE OF A FUNCTION

y -values of the function

TYPES OF REFLECTIONS OF FUNCTIONS

- ① over y -axis is

$f(x)$ becomes

$f(-x)$

- ② over x -axis is

$f(x)$ becomes

$-[f(x)]$

- ③ over $y=x$

$f(x)$ becomes

$f^{-1}(x)$

→ inverse

↳ everything x
becomes
everything y

NOTE: for inverse of quadratic functions:

restrict the original function in order that
the inverted function can exist



QUADRATIC FUNCTIONS (PARABOLAS)

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General form:

$$f(x) = ax^2 + bx + c$$

to get the x-coordinate of the TP from general form
 $x = \frac{-b}{2a}$

ie: TP $(\frac{-b}{2a}; y)$

Turning point form:

$$f(x) = a(x-p)^2 + q$$

opp sign \swarrow same sign
TP (p, q)

to solve for the unknown a ; sub in a second point (x, y)

X-intercept form

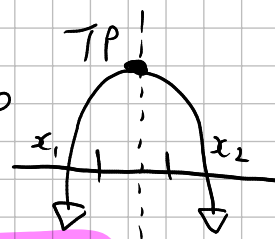
$$f(x) = a(x-x_1)(x-x_2)$$

x-coordinate of one x-intercept

x-coordinate of the other x-intercept

to solve for the unknown a ; sub in a third point (x, y) .

NOTE: x-intercepts are symmetrical around the TP



FINDING THE TP BY COMPLETING THE SQUARE:

EXAMPLE 2: find the TP by completing the square
 $f(x) = x^2 + 6x + 7$

ANSWER:

$$f(x) = x^2 + 6x + \left(\frac{b}{2}\right)^2 + 7 - \left(\frac{b}{2}\right)^2$$

$$f(x) = (x^2 + 6x + 9) + 7 - 9$$

$$f(x) = (x+3)^2 - 2$$

$$\therefore \text{TP}(-3, -2)$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$$

to construct the square: $(\sqrt{x^2} \text{ sign of middle term } \sqrt{9})^2$

THE DERIVATIVE

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NOTATION:

$$f'(x) ; \frac{d}{dx} ; \frac{dy}{dx} ; dx [\quad]$$

DEFINITION: The derivative of a function at a given point (x, y) is the slope/gradient of the tangent at that point

$$\text{ie: } f'(x) = m \text{ of } f(x)$$

POWER RULE: if $f(x) = ax^2 + bx + c$
then $f'(x) = 2ax + b$

$$\frac{d}{dx} ax^n = nax^{n-1}$$

NOTE: remember
 $\tan \theta = m$
as well.

TO FIND THE FORMULA OF A TANGENT TO A QUADRATIC

- ① find m
↳ either $f'(x)$ or $\tan \theta$
- ② sub in one additional point to
 $y = mx + c$ and your m
to find c
- ③ write your tangent in general form

THE INVERSE OF A FUNCTION

Steps to find $f^{-1}(x)$ of $f(x)$

- ① switch x/y
- ② make y the subject

eg. $y = x - 3$

① $x = y - 3$

② $y = x + 3$

NOTES:

- ① Points also change

if $f(x) : (x; y)$

then $f^{-1}(x) : (y; x)$

$f(x)$ y is now

$f^{-1}(x)$ x

$f(x)$ x is now

$f^{-1}(x)$ y

eg: $f(x)$ becomes $f^{-1}(x)$
 $(2; 1)$ becomes $(1; 2)$

- ② Domain of $f(x)$ is Range of $f^{-1}(x)$
 Range of $f(x)$ is Domain of $f^{-1}(x)$

- ③ Signs pos/neg stay the same

- ④ Asymptotes also switch

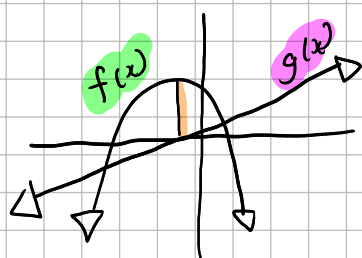
if: $f(x)$ asymptote: $y = p$

then $f^{-1}(x)$ asymptote: $x = p$

VERTICAL LENGTH BETWEEN TWO FUNCTIONS

To calculate verticle length:

$$f(x) - [g(x)]$$



Where $f(x)$ is the top function
 and $g(x)$ is the bottom
 function

y -value of top function - y -value of
 bottom function when VL is parallel
 to y -axis

AVERAGE GRADIENT

The straight line gradient between two points on a graph

$$\text{average gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

therefore

MAXIMUM / MINIMUM LENGTH (OPTIMISATION)

- Steps:
- ① find a formula for the length to optimise: $f(x)$
 - ② find the first derivative: $f'(x)$
 - ③ let $f'(x) = 0$ and solve for x
this gives us the x -value(s) that will give us max/min length
 - ④ sub x -values into $f(x)$ formula to find optimised max/min length

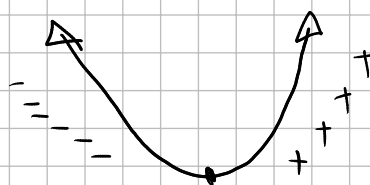
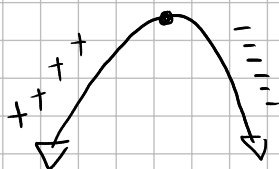
FIRST AND SECOND DERIVATIVE

gradient changes sign

$$f''(x) = \text{negative}$$

local max

$$f'(x) = 0$$



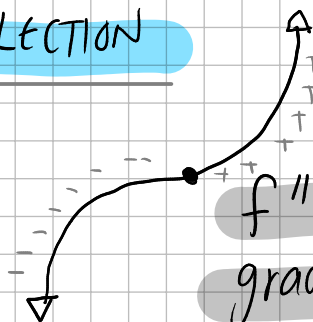
$$f'(x) = 0$$

local min

$$f''(x) = \text{positive}$$

gradient changes sign

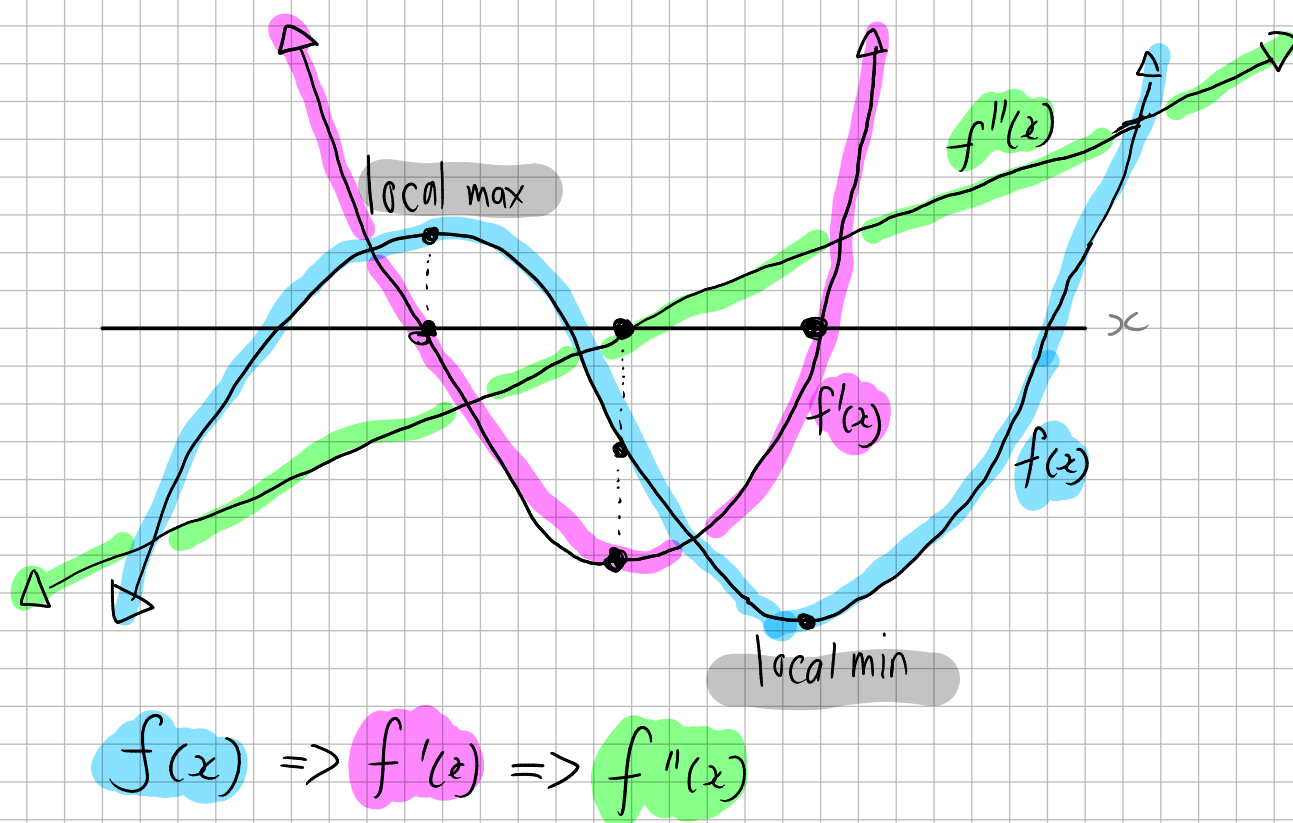
POINT OF INFLECTION



$$f''(x) = 0$$

gradient changes sign

RELATIONSHIP BETWEEN THE FUNCTION, 1st, 2nd DERIVATIVE



HYPERBOLIC FUNCTIONS

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STANDARD FORM

$$y = \frac{a}{x-p} + q$$

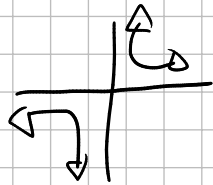
vertex shift
up/down



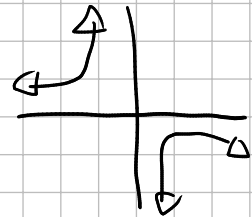
horizontal shift (counter intuitive)
left/right



$a > 0$:



$a < 0$:



Asymptotes

Horizontal: $y = q$ (same sign)

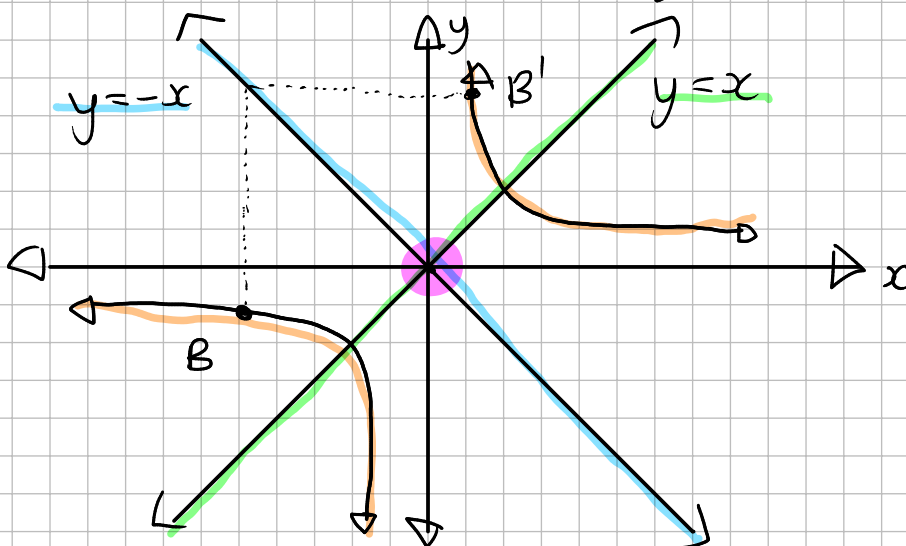
Vertical: $x = p$ (opposite sign)

Axes of symmetry:

over $y = x$

and over $y = -x$

$y = x$ meets $y = -x$ where the asymptotes meet



* This point of intersection can be used to find the asymptotes' equations

* The point of intersection of the asymptotes can also be used to find the axes of symmetry equations:

POI (a;b)

Steps:

① Write out general form for the two axes of symmetry

$$y = x + c$$

$$y = -x + c$$

② sub in POI (a;b) to find c

③ write out the two new equations

* Reflection across the POI of the axes of symmetry

Steps: ① How far from the point (x;y) to the POI?

② do that same distance, but switch x/y's because this is a reflection over $y = x$ (inverse)

e.g: if you went up two and Across three to get to the POI

you would go UP three and Across two

to get to the reflected point

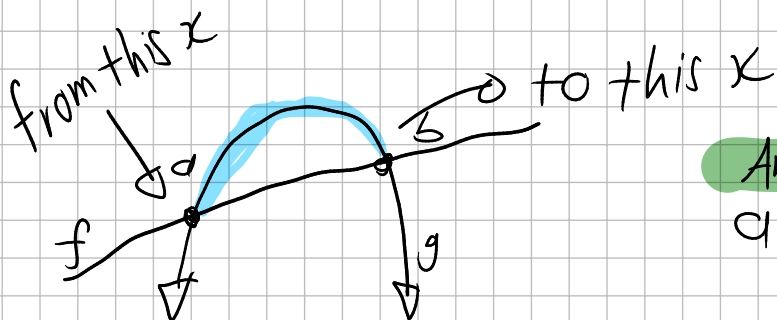
INTERPRETATION QUESTIONS

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① $g(x) > f(x)$

means

What x 's will give us where y_g is ABOVE y_f



Answer:

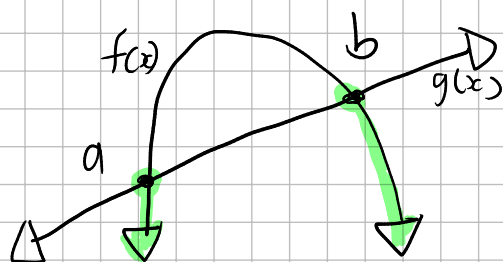
$$a < x < b$$

x -coordinates
at points a & b

② $f(x) - g(x) \leq 0$

Here first take $g(x)$ over

$$\therefore f(x) \leq g(x)$$



Answer

$$x \leq a \quad \text{or} \quad x \geq b$$

x -coordinates
at points a & b

NON-STANDARD FORM

EXAMPLE 3

change $f(x) = \frac{x-3}{x+1}$ to standard form

Answer:

Steps: ① find a way to SPLIT the denominator so that the $x+1$ cancels out in one of the terms

ie: make it $\frac{x+1}{x+1}$; which $= 1$

② do this as shown:

$$\begin{aligned} & \frac{x-3}{x+1} \\ &= \frac{(x+1)-4}{x+1} \\ &= \frac{x+1}{x+1} - \frac{4}{x+1} \\ &= -\frac{4}{x+1} + 1 \end{aligned}$$

- end -