FUNCTIONS 1

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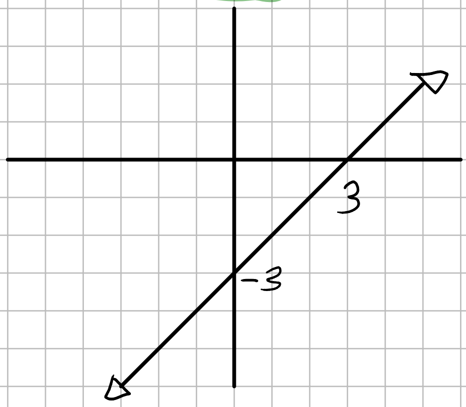
Including the following:

CONTENT	PG #

LINEAR (STRAIGHT LINE) FUNCTIONS

EXAMPLE ①



$y = x - 3$

gradient/slope:

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \tan\theta$

$m = f'(x)$

STANDARD FORM

$y = mx + c$

gradient/slope

y coordinate of y-int.

QUESTION

① What is the gradient of the line in example 1?

ANSWER: $m = 1$

GENERAL NOTES

for y-int; let $x=0$

for x-int; let $y=0$

Intercept form

any other point on the function

$y - y_1 = m(x - x_1)$

y-coordinate of y-int

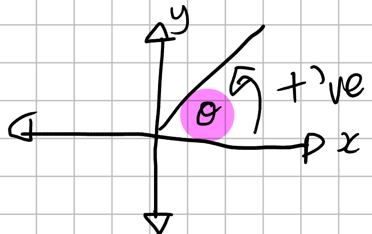
gradient/slope

x-coordinate of x-int

$\tan\theta = m$

$\theta = \tan^{-1}(m)$

for positive angle off the x-axis



WHAT IS A FUNCTION?

A function is a relationship with one OUTPUT per INPUT.
ie, one y for every x.

TYPES of FUNCTIONS RELATIONS

1. one-to-one
2. many-to-one

NOTE: one-to-many relation is NOT A FUNTION

DOMAIN of A FUNCTION

x-values of the function

RANGE OF A FUNCTION

y-values of the function

TYPES of REFLECTIONS of FUNCTIONS

1. over y-axis

$f(x)$ becomes $f(-x)$

2. Over x-axis

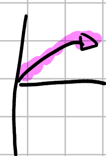
$f(x)$ becomes $-[f(x)]$

3. over $y=x$

$f(x)$ becomes $f^{-1}(x)$ → inverse

↳ everything x becomes everything y

NOTE: for inverse of quadratic functions:
restrict the original function in order that the inverted function can exist

if:  then:  or 

QUADRATIC FUNCTIONS (PARABOLAS)

General form:

$f(x) = ax^2 + bx + c$

to get the x-coordinate of the TP from general form
$$x = \frac{-b}{2a}$$
ie: TP $\left(-\frac{b}{2a}; y\right)$

Turning point form:

$f(x) = a(x-p)^2 + q$

opp sign, same sign

TP $(p; q)$

to solve for the unknown a; sub in a second point $(x; y)$

X-intercept form

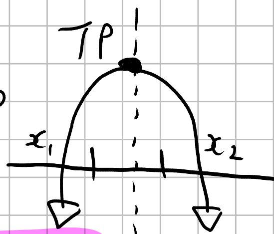
$f(x) = a(x - x_1)(x - x_2)$

x-coordinate of one x-intercept

x-coordinate of the other x-intercept

to solve for the unknown a; sub in a third point $(x; y)$.

NOTE: x-intercepts are symmetical around the TP



FINDING THE TP BY COMPLETING THE SQUARE:

EXAMPLE 2: find the TP by completing the square
$$f(x) = x^2 + 6x + 7$$

ANSWER:

$$f(x) = x^2 + 6x + \underline{\left(\frac{b}{2}\right)^2} + 7 - \underline{\left(\frac{b}{2}\right)^2}$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$$

$$f(x) = (x^2 + 6x + 9) + 7 - 9$$

$$f(x) = (x+3)^2 - 2$$

$\therefore$ TP$(-3; -2)$

to contract the square: $(\sqrt{x^2}$ sign of middle term $\sqrt{9})^2$

THE DERIVATIVE

NOTATION: $f'(x)$; $\frac{d}{dx}$; $\frac{dy}{dx}$; $dx[\quad]$

DEFINITION: The derivative of a function at a given point $(x;y)$ is the slope/gradient of the tangent at that point

ie: $f'(x) = m$ of $f(x)$

POWER RULE: if $f(x) = ax^2 + bx + c$

then $f'(x) = 2ax + b$

$$\boxed{\frac{d}{dx} ax^n = nax^{n-1}}$$

NOTE: remember $\tan\theta = m$ as well.

TO FIND THE FORMULA OF A TANGENT TO A QUADRATIC

1. find m
   ↳ either $f'(x)$ or $\tan\theta$
2. sub in one additional point to $y = mx + c$ and your m to find c
3. write your tangent in general form

THE INVERSE OF A FUNCTION

Steps to find $f^{-1}(x)$ of $f(x)$

1. switch x/y
2. make y the subject

eg. $y = x - 3$

1. $x = y - 3$
2. $y = x - 3$

NOTES:

1. Points also change

   if $f(x)$: $(x;y)$
   then $f^{-1}(x)$: $(y;x)$

   $f(x)$ y is now $f^{-1}(x)$ x

   $f(x)$ x is now $f^{-1}(x)$ y

   eg: $f(x)$ $(2;1)$ becomes $f^{-1}(x)$ $(1;2)$

2. Domain of $f(x)$ is Range of $f^{-1}(x)$
   Range of $f(x)$ is Domain of $f^{-1}(x)$

3. Signs pos/neg stay the same

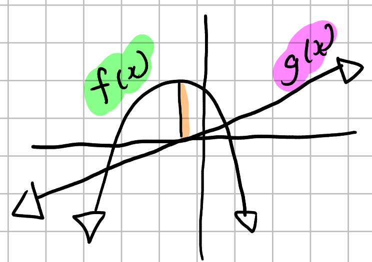
4. Asymptotes also switch

   if: $f(x)$ asymptote: $y = p$
   then $f^{-1}(x)$ asymptote: $x = p$

VERTICAL LENGTH BETWEEN TWO FUNCTIONS

To calculate verticle length: $f(x) - [g(x)]$

where $f(x)$ is the top function and $g(x)$ is the bottom function



→ y-value of top function − y-value of bottom function when VL is parallel to y-axis

AVERAGE GRADIENT

The straight line gradient between two points on a graph

average gradient = $\dfrac{y_2 - y_1}{x_2 - x_1}$
therefore

MAXIMUM / MINIMUM LENGTH (OPTIMISATION)

Steps:
1. find a formula for the length to optimise : $f(x)$
2. find the first derivative : $f'(x)$
3. let $f'(x) = 0$ and solve for x
   this gives us the x-value (s) that will give us max/min length
4. sub x-values into $f(x)$ formula to find optimised max/min length

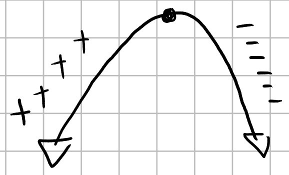
FIRST AND SECOND DERIVATIVE

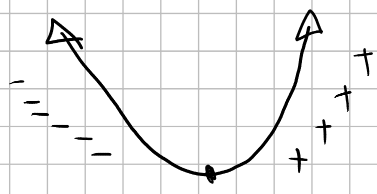
gradient changes sign

$f''(x)$ = negative

local max

$f'(x) = 0$





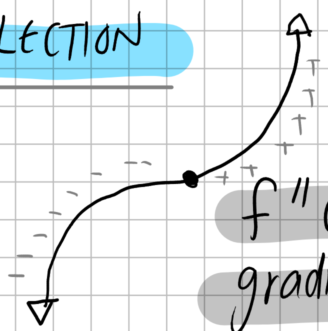
$f'(x) = 0$

local min

$f''(x)$ = positive

gradient changes sign

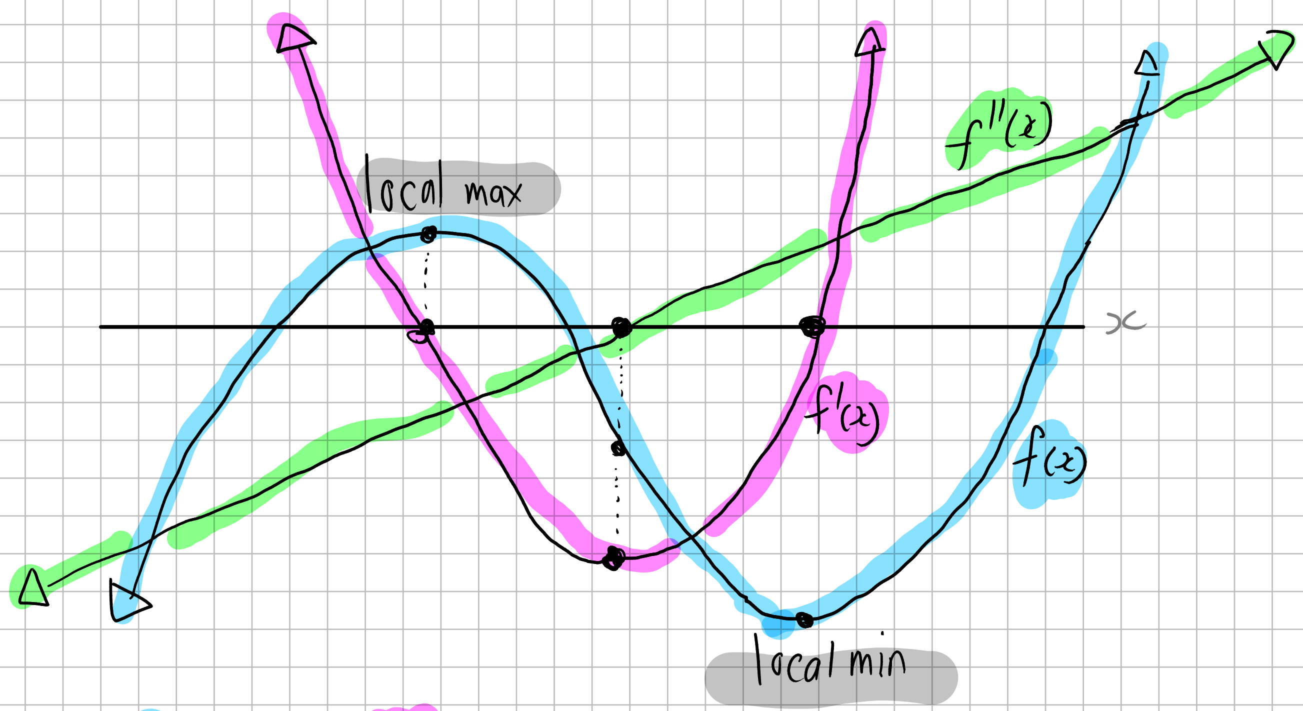
POINT of INFLECTION



$f''(x) = 0$

gradient changes sign

RELATIONSHIP BETWEEN THE FUNCTION, 1st, 2nd DERIVATIVE



$f(x) \Rightarrow f'(x) \Rightarrow f''(x)$

HYPERBOLIC FUNCTIONS

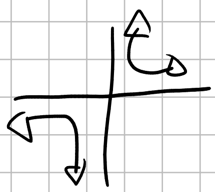
STANDARD FORM

$$y = \frac{a}{x-p} + q$$

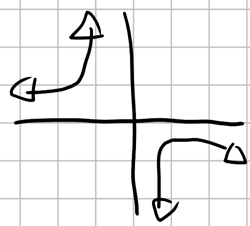
$+q$ → Verticle shift up/down (+ up, − down)

p → horizontal shift (counter intuitive) left/right (+ left, − right)

$a>0$:



$a<0$:



Asymptotes

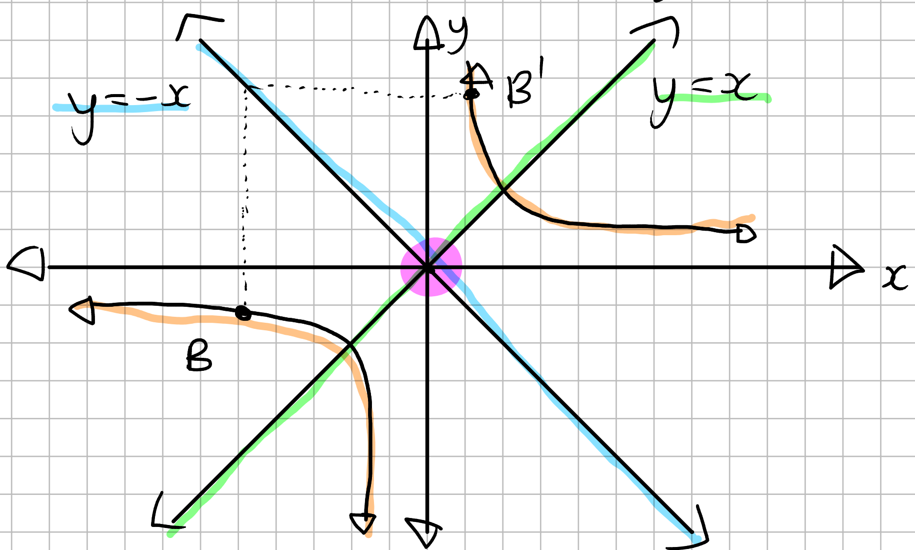
Horizontal : $y = q$ (same sign)

Verticle : $x = p$ (opposite sign)

Axes of symmetry :

- over $y=x$
- and over $y=-x$

- $y=x$ meets $y=-x$ where the asymptotes meet



* This point of intersection can be used to find the asymptotes' equations
* The point of intersection of the asymptotes can also be used to find the axes of symmetry equations:

POI $(a;b)$

Steps:

① Write out general form for the two axes of symmetry

$y = x + c$ $\qquad\qquad$ $y = -x + c$

② Sub in POI $(a;b)$ to find c

③ Write out the two new equations

* Reflection across the POI of the axes of symmetry

Steps: ① How far from the point $(x;y)$ to the POI?

② do that same distance, but switch x/y's because this is a reflection over $y = x$ (inverse)

e.g: if you went up two and Across three to get to the POI

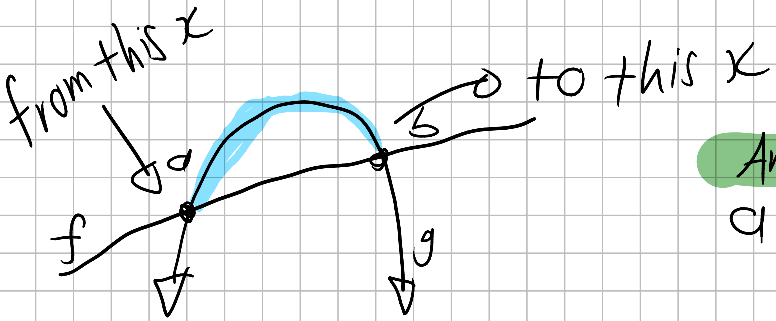
you would go Up three and Across two to get to the reflected point

INTERPRETATION QUESTIONS

① $g(x) > f(x)$

means

what x's will give us where y_g is ABOVE y_f



Answer:

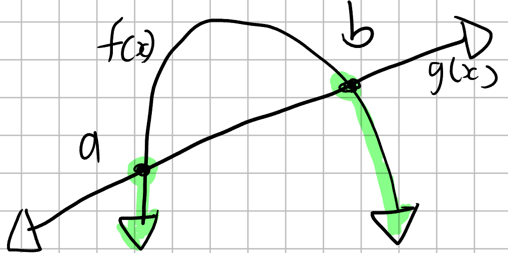
$a < x < b$

x-coordinates at points a & b

② $f(x) - g(x) \leq 0$

Here first take $g(x)$ over

$\therefore \; f(x) \leq g(x)$



Answer

$x \leq a$ or $x \geq b$

x-coordinates at points a & b

NON-STANDARD FORM

EXAMPLE 3

change $f(x) = \dfrac{x-3}{x+1}$ to standard form

Answer:

Steps: 1 Find a way to SPLIT the denominator so that the $x+1$ cancels out in one of the terms

ie : make it $\dfrac{x+1}{x+1}$; which $=1$

2 do this as shown:

$$\frac{x-3}{x+1}$$

$$= \frac{(x+1)-4}{x+1}$$

$$= \frac{x+1}{x+1} - \frac{4}{x+1}$$

$$= -\frac{4}{x+1} + 1$$

– end –