Please check the examination details below before entering your candidate information

Candidate surname	Other names
Annotated by Tam	

Centre Number | Candidate Number

HastingsMath.com

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Pure Mathematics P1

June 2022

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Q:1/1/1/1/

P 6 9 4 5 8 A 0 1 3 2

Pearson

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1. Find

$$\int \left(10x^5 + 6x^3 - \frac{3}{x^2} \right) dx$$

giving your answer in simplest form.

(4)

$$= \frac{10x^6}{6} + \frac{6x^4}{4} - \frac{3x^{-1}}{-1} + c$$

$$= \frac{5}{3}x^6 + \frac{3}{2}x^4 + 3x^{-1} + c$$

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Question 1 continued

Q1

(Total 4 marks)

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2. In the triangle ABC,

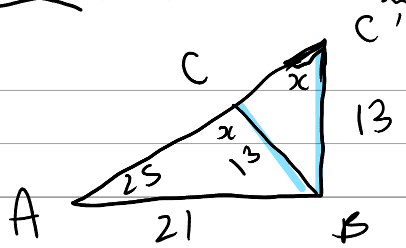
- $AB = 21$ cm
- $BC = 13$ cm
- angle $BAC = 25°$
- angle $ACB = x°$

(a) Use the sine rule to find the value of $\sin x°$, giving your answer to 4 decimal places. **(2)**

ambiguous case.

Given also that AB is the longest side of the triangle,

(b) find the value of x, giving your answer to 2 decimal places. **(3)**



(a)

for ambiguous case

either x or $180 - x$ are possible

$$\frac{\sin x}{21} = \frac{\sin 25}{13}$$

$\sin x = 0.6827$ (4 dec pl.)

(b)

$\therefore$ if AB is longest we are looking for x not x'

$x' = \sin^{-1}(0.6827)$

$= 43.054$

$x = 180 - 43.054$

$= 136.95°$ (2 dec pl.)

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Question 2 continued

Q2

(Total 5 marks)

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3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Show that $\dfrac{\sqrt{180}-\sqrt{80}}{\sqrt{5}}$ is an integer and find its value.

(2)

(ii) Simplify

$$\frac{4\sqrt{5}-5}{7-3\sqrt{5}}$$

giving your answer in the form $a+b\sqrt{5}$ where a and b are rational numbers.

(3)

3(i) $\dfrac{\sqrt{180}}{\sqrt{5}} - \dfrac{\sqrt{80}}{\sqrt{5}}$ (split denominator)

$= \sqrt{\dfrac{180}{5}} - \sqrt{\dfrac{80}{5}}$ $\qquad \dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

$= \sqrt{36} - \sqrt{16} = 6 - 4 = 2$ ✓

(ii) $\dfrac{(4\sqrt{5}-5)(7+3\sqrt{5})}{(7-3\sqrt{5})(7+3\sqrt{5})}$ rationalise denominator

$= \dfrac{28\sqrt{5}+12(5)-35}{49-9(5)} = \dfrac{25+13\sqrt{5}}{4}$

$= \dfrac{25}{4} + \dfrac{13\sqrt{5}}{4}$

$= \dfrac{25}{4} + \dfrac{13}{4}\sqrt{5}$ ✓

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Question 3 continued

Q3

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4.

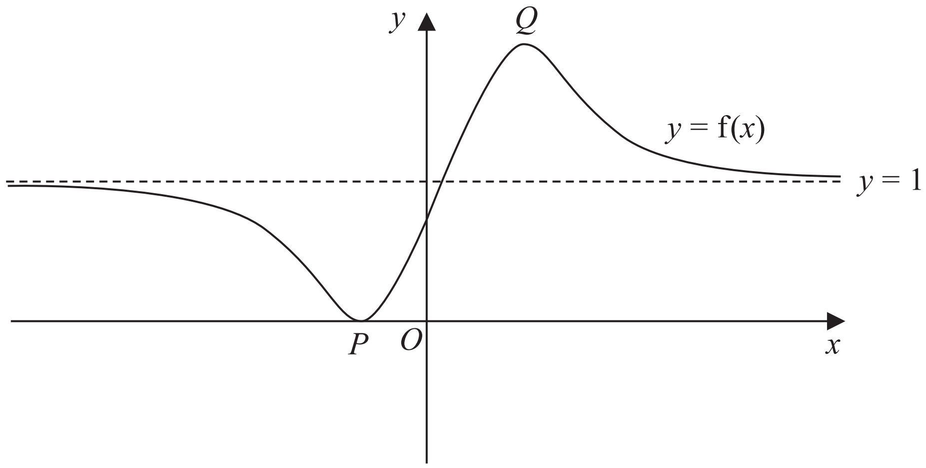


Figure 1

Figure 1 shows a sketch of a curve with equation $y = \mathrm{f}(x)$

The curve has a minimum at $P(-1, 0)$ and a maximum at $Q\left(\frac{3}{2}, 2\right)$

The line with equation $y = 1$ is the only asymptote to the curve.

On separate diagrams sketch the curves with equation

(i) $y = \mathrm{f}(x) - 2$ **(3)**

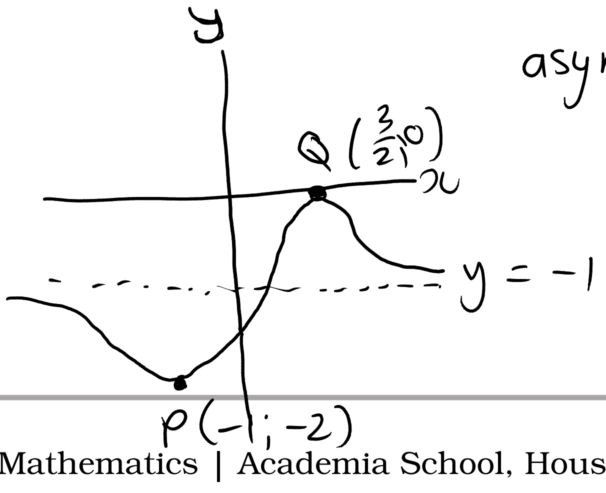
(ii) $y = \mathrm{f}(-x)$ **(3)**

On each sketch you must clearly state

- the coordinates of the maximum and minimum points
- the equation of the asymptote

(i) $f(x) - 2$ $P(-1; -2)$ $Q(\frac{3}{2}; 0)$

asymp. $y = -1$



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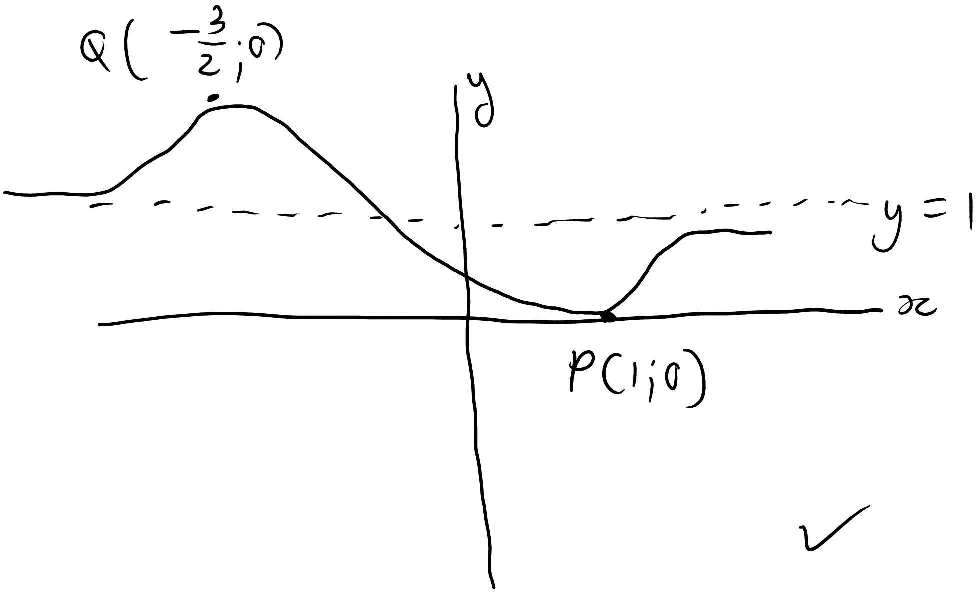
Question 4 continued

(ii) $f(-x)$ $P(1;0)$ $Q(-\frac{3}{2};2)$

$y=1$



Q4

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5. The curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}(x)$ is a quadratic expression
- the maximum turning point on C has coordinates $(-2, 12)$
- C cuts the negative x-axis at -5

(a) find $\mathrm{f}(x)$

(4)

The line l_1 has equation $y = \dfrac{4}{5}x$

Given that the line l_2 is perpendicular to l_1 and passes through $(-5, 0)$

(b) find an equation for l_2, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

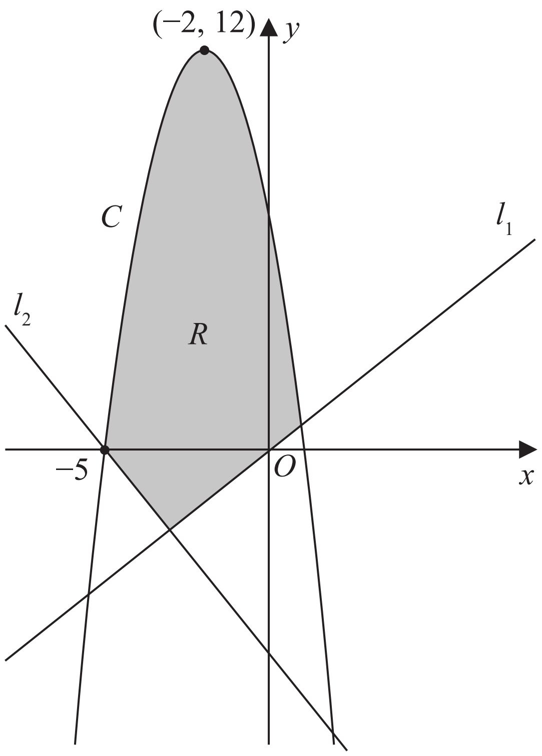

Figure 2

Figure 2 shows a sketch of the curve C and the lines l_1 and l_2

(c) Define the region R, shown shaded in Figure 2, using inequalities.

(2)

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Question 5 continued

a $ax^2+bx+c = f(x)$ ... quadratic

max TP $(-2;12)$ $f'(x)=0$ ... max/min

$(-5;0)$ ... x-int

$\therefore f'(x) = 2ax+b$

$f'(-2) = 0$

$2a(-2)+b=0$

$-4a+b=0$ ...1

$f(-5)=0$

$a(-5)^2+b(-5)+c=0$

$25a-5b+c=0$ ...2

$f(-2)=12$

$\therefore a(-2)^2+b(-2)+c=12$

$4a-2b+c=12$ ...3

$b=4a$ ...4 rearrange 1

$c=5b-25a$ ...5 rearrange 2

$c=2b-4a+12$ ...6 rearrange 3

let 5 = 6

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Question 5 continued

$5b - 25a = 2b - 4a + 12$

$3b - 21a = 12$

$3b = \frac{12 + 21a}{3}$

$b = 4 + 7a \quad \ldots (7)$

let $(7) = (4)$

$4 + 7a = 4a$

$3a = -4$

$a = -\frac{4}{3}$ ✓

sub $a = -\frac{4}{3}$ into (7)

$b = 4 + 7\left(-\frac{4}{3}\right)$

$= -\frac{16}{3}$ ✓

sub a & b into (5)

$c = 5b -$

$= 5\left(-\frac{16}{3}\right) - 25\left(-\frac{4}{3}\right)$

$= \frac{20}{3}$ ✓

$\therefore f(x) = -\frac{4}{3}x^2 - \frac{16}{3}x + \frac{20}{3}$ ✓

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Question 5 continued

(b) $l_1 = \frac{4}{5}x$ $\quad l_1 \perp l_2$ $\quad (-5;0)$ on l_2

$l_2 : y = mx + c$

$M_{l_1} \times M_{l_2} = -1$

$\therefore M_{l_2} = -\frac{5}{4}$

$y = -\frac{5}{4}x + c$

sub $(-5;0)$

$0 = -\frac{5}{4}(-5) + c$

$c = -\frac{25}{4}$

$\therefore l_2 : y = -\frac{5}{4}x - \frac{25}{4}$ ✓

(c) $y \leq \frac{4}{5}x^2 - \frac{16}{3}x + \frac{20}{3}$ ✓ ... For below the curve $y <$ curve ... solid line $[\leq]$

$y \geq \frac{4}{3}x$ ✓ ... for above line $y >$ line ... solid line $[\geq]$

$y \geq -\frac{5}{4}x - \frac{25}{4}$ ✓ "

Q5

(Total 9 marks)

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6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Given that

$$2xy - 3x^2 = 50$$

and

$$y - x^3 + 6x = 0$$

show that

$$2x^4 - 15x^2 - 50 = 0$$

(2)

(b) Hence solve the simultaneous equations

$$2xy - 3x^2 = 50$$
$$y - x^3 + 6x = 0$$

Give your answers in fully simplified surd form.

(5)

(a) $y = x^3 - 6x$ ... (1)

$2xy - 3x^2 = 50$ .. (2)

sub (2) into (1)

$2x(x^3 - 6x) - 3x^2 - 50 = 0$

$2x^4 - 15x^2 - 50 = 0$ ✓

(b) $(2x^2 + 5)(x^2 - 10) = 0$

$+5x^2$

$-20x^2$

So:

10

5

$\therefore\ 2x^2 + 5 = 0$ or $x^2 - 10 = 0$

$x^2 \neq -\frac{5}{2}$ $x = \pm\sqrt{10}$ ✓

invalid

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Question 6 continued

sub x into ①

$\therefore y = (-\sqrt{10})^3 - 6(-\sqrt{10})$ or $y = (\sqrt{10})^3 - 6(\sqrt{10})$

$= -4\sqrt{10}$ ✓ $= 4\sqrt{10}$ ✓

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Question 6 continued

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Question 6 continued

Q6

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7. The curve C has equation $y = \mathrm{f}(x),\ x > 0$

Given that

- $\mathrm{f}'(x) = \frac{2}{\sqrt{x}} + \frac{A}{x^2} + 3$, where A is a constant
- $\mathrm{f}''(x) = 0$ when $x = 4$

(a) find the value of A.

(4)

Given also that

- $\mathrm{f}(x) = 8\sqrt{3}$, when $x = 12$

(b) find $\mathrm{f}(x)$, giving each term in simplest form.

(5)

(a) $f'(x) = 2x^{-\frac{1}{2}} + ax^{-2} + 3$

$f''(4) = 0$

$f''(x) = x^{-\frac{3}{2}} - 2ax^{-3}$

$f''(4): \quad (4)^{-\frac{3}{2}} - 2a(4)^{-3} = 0$

$\frac{1}{\sqrt{4^3}} - \frac{2}{64}a = 0$

$a = \frac{-\frac{1}{8}}{\frac{1}{32}}$

$a = -4$ ✓

(b) $f(12) = 8\sqrt{3}$

$f'(x) = 2x^{-\frac{1}{2}} + 4x^{-2} + 3$

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Question 7 continued

$$f(x) = \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{4x^{-1}}{-1} + \frac{3x^{1}}{1} + C$$

$$= 4x^{\frac{1}{2}} - 4x^{-1} + 3x + C$$

$$f(12) : 4(12)^{\frac{1}{2}} - 4(12)^{-1} + 3(12) + c = 8\sqrt{3}$$

$$C = -\frac{108}{3}$$

$$\therefore f(x) = 4x^{\frac{1}{2}} - 4x^{-1} + 3x - \frac{108}{3}$$

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Question 7 continued

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Question 7 continued

Q7

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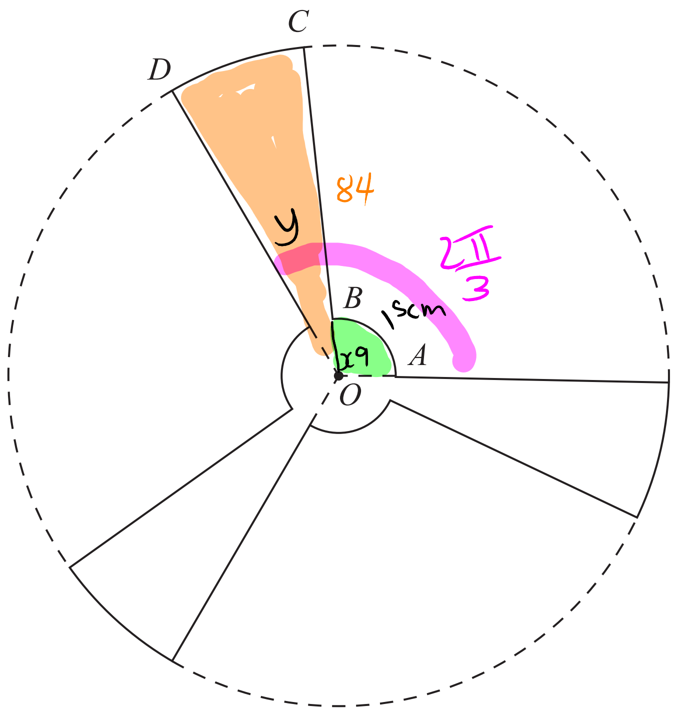

$y = AOD - x$

$l = r\theta$

B 15 x A 9

$x = \frac{l}{r}$

$= \frac{15}{9}$

$= \frac{5}{3}$ radians

Figure 3

Figure 3 shows a sketch of the outline of the face of a ceiling fan viewed from below.

The fan consists of three identical sections congruent to *OABCDO*, shown in Figure 3, where

- *OABO* is a sector of a circle with centre *O* and radius 9 cm
- *OBCDO* is a sector of a circle with centre *O* and radius 84 cm
- angle $AOD = \frac{2\pi}{3}$ radians

Given that the length of the arc *AB* is 15 cm,

(a) show that the length of the arc *CD* is 35.9 cm to one decimal place. **(3)**

The face of the fan is modelled to be a flat surface.

Find, according to the model,

(b) the perimeter of the face of the fan, giving your answer to the nearest cm, **(2)**

(c) the surface area of the face of the fan.

Give your answer to 3 significant figures and make your units clear. **(5)**

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Question 8 continued

(a) l_{CD}: Let $BOA = x = \frac{l}{r} = \frac{15}{9} = \frac{5}{3}$

Let $DOA - x = y$

$$y = \frac{2\pi}{3} - \frac{5}{3}$$

$$y = \frac{2\pi - 5}{3}$$

$$l_{CD} = r\theta$$

$$= 84 \times \frac{2\pi - 5}{3} = 34.929$$

$= 35.9\text{cm}$ ✓ (rounded)

(b) $P = ?$ $3(\ ) + 6(\ ) + 3(\ )$

$$P = 3(34.929\ldots) + 3(84 - 9) + 3(15)$$

$$= 602.787\ldots$$

$= 603\text{cm}$ ✓

(c) TSA = ? $3(\ ) + 3(\ )$

$$A = \frac{1}{2}r^2\theta$$

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Question 8 continued

$$TSA = 3\left(\frac{1}{2}(9^2)\left(\frac{5}{3}\right)\right) + 3\left(\frac{1}{2}(84^2)\left(\frac{2\pi-5}{3}\right)\right)$$

$$= 4729.577\ldots$$

$$= \boxed{4730\ \text{cm}^2} \quad (3sf)$$

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Question 8 continued

Q8

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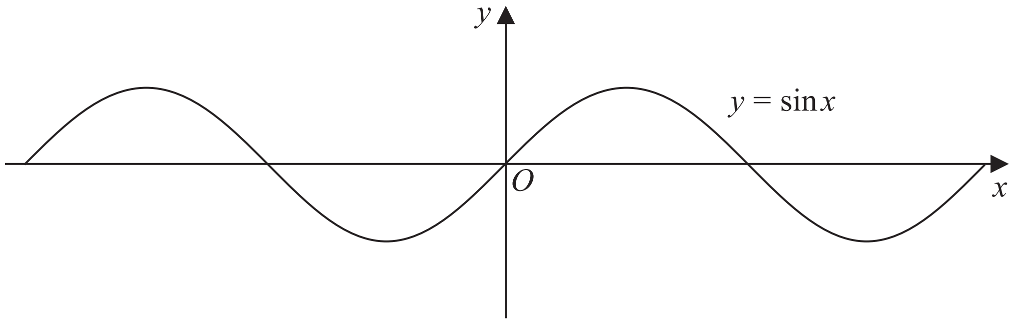


Figure 4

Figure 4 shows part of the graph of the curve with equation $y = \sin x$

Given that $\sin \alpha = p$, where $0 < \alpha < 90°$

(a) state, in terms of p, the value of

(i) $2\sin(180° - \alpha)$

(ii) $\sin(\alpha - 180°)$

(iii) $3 + \sin(180° + \alpha)$

(3)

A copy of Figure 4, labelled Diagram 1, is shown on page 27.

On Diagram 1,

(b) sketch the graph of $y = \sin 2x$

(2)

(c) Hence find, in terms of α, the x coordinates of any points in the interval $0 < x < 180°$ where

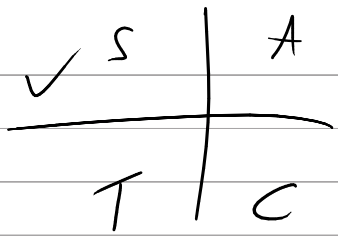
$$\sin 2x = p$$

(3)

a (i) 2 sin(180 − a)

∴ 2 sin a

∴ 2p ✓



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Question 9 continued

(b)

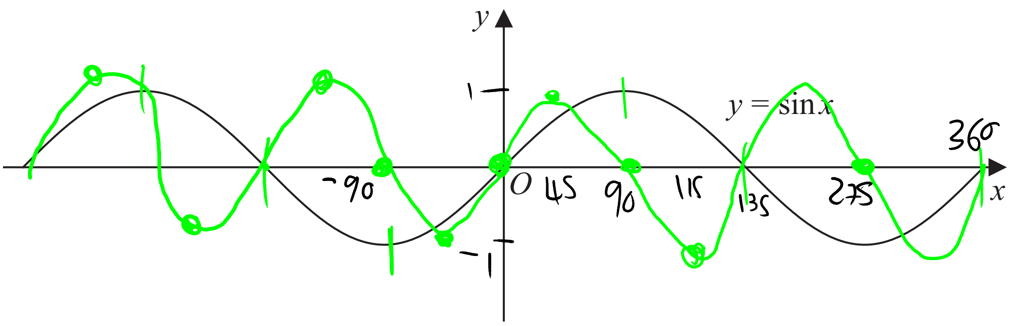
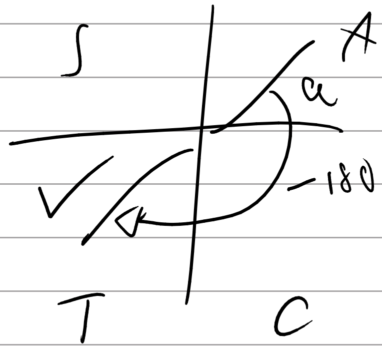


Diagram 1

(ii) $\sin(a - 180)$

$\therefore -\sin a$

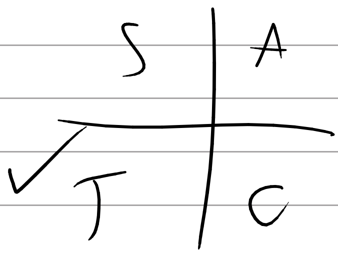
$\therefore -p$ ✓



(iii) $3 + \sin(180 + a)$

$\therefore 3 - \sin a$

$\therefore 3 - p$ ✓



(b) $y = \sin 2x$

(c) $\sin a = p$ $\qquad$ $\sin 2x = p$

$\therefore a = 2x$

$\therefore \frac{a}{2} = x$ or $x = \frac{180}{2} - \frac{a}{2}$

$x = \frac{a}{2}$ ✓ $\qquad$ $x = \frac{180 - a}{2}$ ✓

Q9

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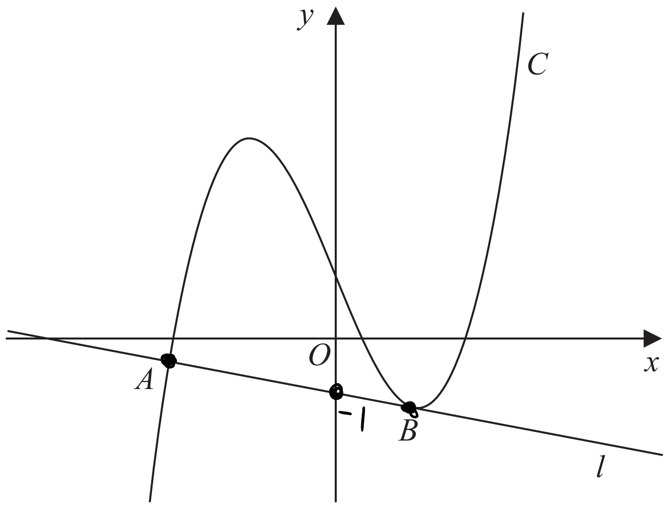

Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$$

where k is a constant.

(a) Find $\dfrac{dy}{dx}$

(2)

The line l, shown in Figure 5, is the normal to C at the point A with x coordinate $-\dfrac{7}{2}$

Given that l is also a tangent to C at the point B,

(b) show that the x coordinate of the point B is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

(4)

(c) Hence find the x coordinate of B, justifying your answer.

(2)

Given that the y intercept of l is -1

(d) find the value of k.

(4)

a) $\dfrac{dy}{dx} = 3\times\dfrac{2}{7}x^2 + 2\times\dfrac{1}{7}x - \dfrac{5}{2}$

$= \dfrac{6}{7}x^2 + \dfrac{2}{7}x - \dfrac{5}{2}$ ✓

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Question 10 continued

(b) normal: $m \times m = -1$

$\therefore \frac{dy}{dx} \times m = -1$

$\therefore \left(\frac{6}{7}\left(-\frac{7}{2}\right)^2 + \frac{2}{7}\left(-\frac{7}{2}\right) - \frac{5}{2}\right) \times m = -1$

$7m = -1$

$m = -\frac{1}{7}$ (normal to curve)

At B; $m = m$ $\therefore -\frac{1}{7} = \frac{dy}{dx}$

$\frac{6}{7}x^2 + \frac{2}{7}x - \frac{5}{2} + \frac{1}{7} = 0$ LCD 14

$12x^2 + 4x - 35 + 2 = 0$

$12x^2 + 4x - 33 = 0$ ✓

(c) $(2x-3)(6x+11) = 0$

$2x - 3 = 0$ or $6x + 11 = 0$

$x = \frac{3}{2}$ or $x = -\frac{11}{6}$

✓

B has positive x

$\therefore \frac{3}{2}$

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Question 10 continued

(d) $k = ?$

line AB $= -\frac{1}{7}x - 1$ ... given & proven

Curve C meets line AB at A & B

at A: $x = -\frac{7}{2}$

at B: $x = \frac{3}{2}$

let line AB = curve C

$$\frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k = -\frac{1}{7}x - 1$$

sub $x = -\frac{7}{2}$ and solve for k.

$k = \frac{5}{4}$ ✓

CHECK: sub $x = \frac{3}{2}$ (also works)

$k = \frac{5}{4}$

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Question 10 continued

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Question 10 continued

Q10

(Total 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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