

FUNCTIONS

EXPONENTIAL AND LOG FUNCTIONS

general exponential: $f(x) = ab^{x+p} + q$

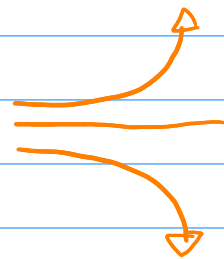
a reflects over x -axis / the greater a is the STEEPER the curve is
 $b > 0$ $b \neq 1$
 q : $y = q$ gives horizontal asymptote
 shift left/right \leftrightarrow
 shift up/down \uparrow \downarrow

* ab^{x+p} can be expanded to $ab^x \times ab^p$

TRANSFORMATIONS:

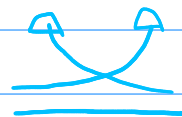
① reflection over the x -axis

$f(x)$ reflects to $-f(x)$



② reflect over the y -axis

$f(x)$ reflects to $f(-x)$



③ reflect over $y = x$ (inverse)

$f(x)$ inverts to $f^{-1}(x)$



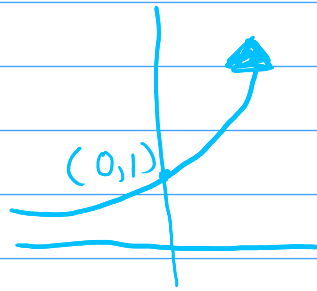
log functions are the inverse of exponential functions

method: swap x and y and make y the subject

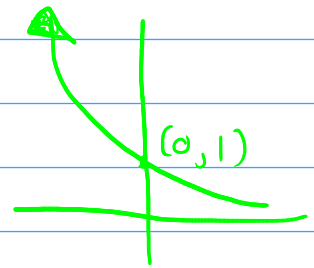
EXPONENTIAL SHAPES

2

$$y = 2^x$$



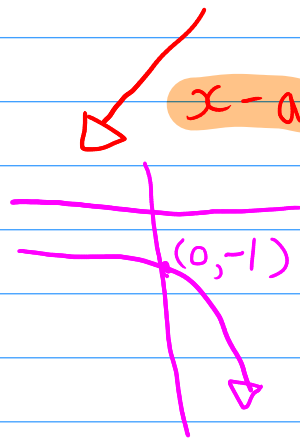
$$y = \frac{1}{2}^x$$



y-axis
 $y = 2^{-x}$

reflections

$$y = -2^x$$

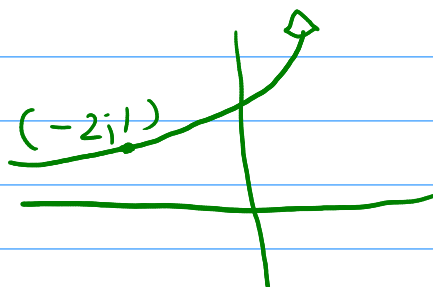


x-axis

$$y = 2^{x+2}$$

shift
two to the
left

(counter intuitive)



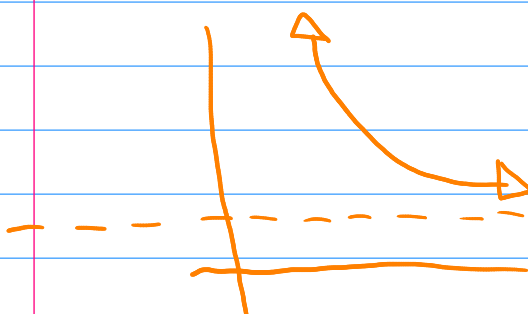
Horizontal

SHIFTS

$$y = 2\left(\frac{1}{3}\right)^{x-3} + 2$$

shifted up two (intuitive)
shifted right three (counter
intuitive)

Vertical

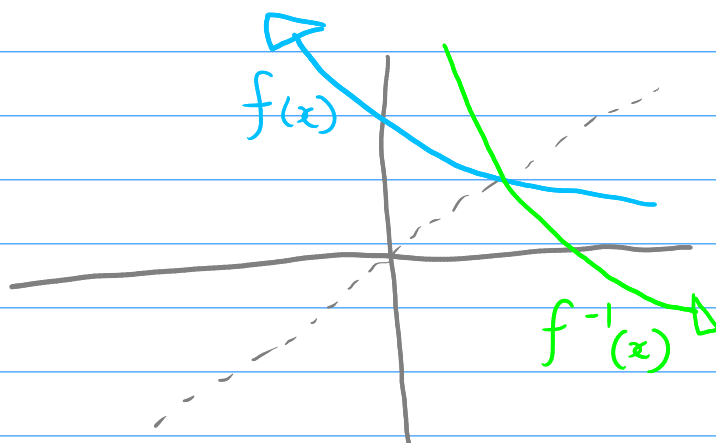


$y = 2$ horizontal
asymptote

CONVERTING TO LOG FUNCTIONS

STEPS TO GRAPH THE INVERSE FUNCTION (LOG)

- ① switch x/y
- ② let $x=1$ to find a point on the graph
- ③ find y -intercept of original function (let $x=0$) then switch x/y to find x -intercept of inverse



If $f(x) = \frac{1}{2}x$ then $f^{-1}(x) = \log_{\frac{1}{2}} x$
 asymptote: $y=0$ asymptote: $x=0$

CONVERTING EXPONENTIAL TO LOG

STEPS:

- ① switch x/y
- ② make y the subject using log law

If $a^b = c$
 then $\log_a c = b$

EG: If $f(x) = \frac{1}{3}x$
 $y = \frac{1}{3}x$
 $x = \frac{1}{3}y \therefore \log_{\frac{1}{3}} x = y$

LOG LAWS FOR SIMPLIFYING/REARRANGING

- ① if $a^b = c$
then $b = \log_a c$
- ② $\log_a c = \frac{\log_b a}{\log_b c}$
- ③ $\log_a a = 1$ ($a > 0 ; a \neq 1$)
- ④ $\log_a 1 = 0$ ($a > 0 ; a \neq 1$)
- ⑤ $\log_a x + \log_a y = \log_a xy$
- ⑥ $\log_a x - \log_a y = \log_a \frac{x}{y}$
- ⑦ $\log_a b^c = c \log_a b$
- ⑧ $\log_a b = \frac{1}{\log_b a}$

EXPONENT LAWS FOR SIMPLIFYING/REARRANGING

$$\textcircled{1} \quad b^x \cdot b^a = b^{x+a}$$

$$\textcircled{2} \quad \frac{b^x}{b^a} = b^{x-a}$$

$$\textcircled{3} \quad (b^a)^x = b^{a \cdot x}$$

$$\textcircled{4} \quad (a \cdot b)^x = a^x \cdot b^x$$

$$\textcircled{5} \quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$\textcircled{6} \quad \sqrt[x]{b^y} = b^{\frac{y}{x}}$$