CHAPTER 2 · DIFFERENTIATION 9

2.3 Rules for differentiation

A level C3 Differentiation

The sum rule

$$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The product rule

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

The chain rule

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

The quotient rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

2.4 Polar co-ordinates

The chain rule can be rearranged as given:

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Example:

$$y = x^2 + 4, u = x$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$= 2x \times 1$$

$$= 2x$$

CHAPTER 3 · EXPONENTIALS AND LOGARITHMS 11

3.2 Logarithms

A level C3 Exponentials and Logarithms

A level C3 Exponentials and Logarithms

Definition
The inverse of a^x is $\log_a x$

Laws of Logs

1. $\log_a(MN) = \log_a M + \log_a N$
2. $\log_a(M^k) = k\log_a M$

Example:
Find x given $3^x = 5$

$$\ln(3^x) = \ln 5$$
$$x\ln 3 = \ln 5$$
$$x = \frac{\ln 5}{\ln 3}$$
$$x = 1.46$$

3.3 Differentiation of e to the power x

A level C3 Exponentials and Logarithms

Property

$$\frac{d}{dx}(e^x) = e^x$$

3.3.2 Differentiation of other logarithms

A level C3 Exponentials and Logarithms

CHAPTER 4 · INTEGRATION 14

Multiplication by a constant

$$\int kf(x)\,dx = k\int f(x)\,dx \qquad (4.0)$$

A special case

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

Example: Integration by Substitution

A level C3 Integration

$$\int (ax+b)^n\,dx$$
$$u = (ax+b)$$
$$\frac{du}{dx} = a$$
$$dx = \frac{1}{a}\,du$$

$$\int (ax+b)^n\,dx = \int u^n \frac{1}{a}\,du$$
$$= \frac{1}{a}\int u^n\,du$$
$$= \frac{1}{a}\frac{u^{n+1}}{n+1} + c$$
$$= \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + c$$

Integration by Parts

A level C3 Integration

$$\int \frac{dv}{dx}u\,dx = uv - \int v\frac{du}{dx}\,dx$$

4.1.1 Partial fractions

A level C3 Integration

CHAPTER 5 · EXPONENTIALS AND LOGARITHMS 12

Property: Change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Property

$$\frac{d}{dx}\log_e(x) = \frac{1}{x}$$

5.3 Differentiation of other exponentials

A level C3 Exponentials and Logarithms

In general, for any positive constant a

$$\frac{d}{dx}(a^x) = a^x\ln a$$

Chapter 4

Integration

Integration – Finding the area under the curve.

Theorem: Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = g(b) - g(b)$$

4.1 Finding integrals

A level C3 Integration
A level C3 Integration
A level C3 Integration

$\int x^n\,dx$	$\frac{1}{n+1}x^{n+1} + c$
$\int \frac{1}{x}\,dx$	$\ln\lvert x\rvert + c$
$\int e^x\,dx$	$e^x + c$
$\int a^x\,dx$	$\frac{1}{\ln a}a^x + c$
$\int \ln x\,dx$	$x\ln x - x + c$

4.2 Rules for integration

Sum Rule

$$\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx \qquad (4.2)$$

Chain Rule
(Exercise 12B.2 – page 294)

Example 1: $\sqrt{x^2+4x}$

Example 2: $\frac{1}{(2x-1)^3}$

- All these expressions can be written in the form:

$[f(x)]^n$ where $n \neq 1,0$

- The chain rule says the derivative of these types of functions are:

$n[f(x)]^{n-1}f'(x)$

If $y = (ax+b)^n$
$y' = n(ax+b)^{n-1} \times (a)$
If $y' = (ax+b)^n$
$y = \frac{(ax+b)^{n+1}}{a(n+1)} + C$

$y' = (\sin 3x)^2$

$y = \frac{(-\cos 3x)}{3}$

$a = 3$

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International Advanced Level

Centre Number | Candidate Number

Thursday 08 October 2020

Afternoon Time: hours 30 minutes | Paper Reference **WMA11/01**

Mathematics
International Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables 2 (blue), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have stored in them any mathematical formulae stored in them.

Instructions

- Use black ink or ballpoint pen.
- **Fill in the boxes at the top of this page with your name, centre number and candidate number.**
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show the steps in your working, so that your methods (for. Answers without working may not gain full marks.
- Unless otherwise stated, when a value of π is required, use $\pi = 3.141$ figures unless otherwise stated.

Information

- A formulae Mathematical Formulae and Statistical Tables is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- Each question is worth 4 marks unless otherwise stated.
- Use this as a guide as how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to leave answers if you have time at the end.
- Check your answers if you have time at the end.

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Hard to Mark on

6. Solve, for $0 \le x < 360°$, the equation

$$\cos(x+30°) = \cos(x+60°)$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\cos x\cos 30 - \sin x\sin 30 = \cos x\cos 60 - \sin x\sin 60$$

$$\cos x\left(\frac{\sqrt{3}}{2}\right) - \sin x\left(\frac{1}{2}\right) = \cos x\left(\frac{1}{2}\right) - \sin x\left(\frac{\sqrt{3}}{2}\right)$$

$$\cos x\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) = \sin x\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)$$

$$\frac{\sin x}{\cos x} = \frac{\frac{\sqrt{3}-1}{2}}{\frac{1-\sqrt{3}}{2}}$$

$$\tan x = -1$$

$$x = 135, 315$$

$$\therefore x = 180 - 35.5224937\ldots$$

$$x = 180 + 35.5224937\ldots$$

$$\therefore x = 144.4775\ldots, 215.5224\ldots$$

$$x = 144.4, 215.5$$

Question 2 continued

2. A scientist monitored the growth of bacteria on a dish over a 10-day period.

The area, N mm^2, of the dish covered by bacteria, t days after monitoring began, is modelled by the equation

$$\log_{10} N = 0.06t + 1.478 \qquad 0 \le t \le 10$$

(a) Show that this equation may be written in the form

$$N = ab^t$$

where a and b are constants to be found. Give the value of a to the nearest integer and give the value of b to 3 significant figures.

(3)

(b) Use the model to find the area of the dish covered by bacteria 30 days after monitoring began. Give your answer, in mm^2, to 2 significant figures.

(2)

(a) $\log_{10} N = 0.06t + 1.478$

$10^{0.06t+1.478} = N$ $\quad \log_b a = c \Leftrightarrow b^c = a$

$N = 10^{0.06t} \times 10^{1.478}$

$N = 10^{1.478} \times 10^{0.06t}$

$N = a \times b^t$

$a = 10^{1.478} \quad b = 10^{0.06}$

$a = 30 \quad b = 1.15$

$\therefore N = 30 \times 1.15^t$ ✓

Question 2 continued

(b) $N = ab^t \quad t$ in days

$N = 30 \times 1.15^t \quad t = 30$

$\therefore N = 30 \times 1.15^{30}$

$N = 2595.4996$

$= 2600 \text{ mm}^2$ (2sf) ✓

(Total 4 marks)

3.

Figure 1

Figure 1 shows a sketch of a curve with equation $y = f(x)$ where

$$f(x) = \frac{2x+3}{\sqrt{4x-1}} \qquad x > \frac{1}{4}$$

(a) Find, in simplest form, $f'(x)$

(4)

(b) Hence find the range of f

(2)

(a) $f(x) = \frac{2x+3}{\sqrt{4x-1}}$

$u = 2x+3 \quad v = (4x-1)^{\frac{1}{2}}$

$u' = 2 \quad v' = \frac{1}{2}(4x-1)^{-\frac{1}{2}} \times 4$

Question 4 continued

$u \cdot v' + v \cdot u'$

$2 \times (4x-1)^{-\frac{1}{2}} - (2x+3) \times 2(4x-1)^{-\frac{3}{2}}$

$((4x-1)^{\frac{1}{2}})^2$

HCF $2(4x-1)^{-\frac{1}{2}}$ → take lowest from each power

$= 2(4x-1)^{-\frac{1}{2}}\{(4x-1) - (2x+3)\}$

$(4x-1)$

$= 2(4x-1)^{-\frac{1}{2}}(2x-4)$

$(4x-1)$

$= \frac{2(2x-4)}{(4x-1)^{\frac{1}{2}}} = \frac{4x-8}{(4x-1)^{\frac{1}{2}}}$ ✓

$\therefore f'(x) = \frac{4x-8}{(4x-1)^{\frac{1}{2}}}$

(b) Range of f?

what y's?

$f'(x) = \frac{4x-8}{(4x-1)^{\frac{1}{2}}}$

$\Rightarrow$ at $f'(x) = 0$

$m = 0$

$\Rightarrow$ TP in lowest y

$\therefore f'(x) = 0$

$f'(x) = \frac{4x-8}{(4x-1)^{\frac{1}{2}}}$

$4x - 8 = 0$

$(4x-1)^{\frac{1}{2}} = 0$

$4x - 8 = 0$

$x = 2$

$f(2) = \frac{2(2)+3}{\sqrt{4(2)-1}}$

$= \frac{7}{\sqrt{7}} = \frac{7\sqrt{7}}{7} = \sqrt{7}$

Question 3 continued

$\therefore$ Range of $f(x)$

$y \ge \sqrt{7}$ ✓

4.

Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = f(x)$ where

$$f(x) = 2 - |2x - a| \qquad x \ge 0$$

(a) Find $f(0)$

(1)

(b) Solve the equation $f(x) = 5x$

(3)

Given that the equation $f(x) = k$, where k is a constant, has exactly two roots,

(c) state the set of possible values of k

(3)

The graph with equation $y = f(x)$ is transformed into the graph with equation $y = af(x - b)$

The vertex of the graph with equation $y = af(x - b)$ is $(6, 5)$.

Given that a and b are constants,

(d) find the value of a and the value of b

(3)

(a) $f(0) = 2 - |2 \times 0 - a|$

$f(0) = 2 - |-a|$ Modulus value always +ve

$= 2 - |+a|$

$= 2 - a + 2 = 4$

$\therefore f(0) = 4$

(b) $f(x) = 5x$

$y = 2 - 2|x - 2|$ $\quad y = 2 - 2(-2x+4)$

$y = 2 - 4|x - 2|$ $\quad y = 2 + 4 - 2x$

$y = 2x + 8$ $\quad y = -2x + 6$

x^2 $\quad x^2$

$y = 2x + 8$ $\quad y = -2x + 6$

$y = 5x$ $\quad y = 5x$

$2x + 8 = 5x$ $\quad -2x + 6 = 5x$

$-3x = 8$ $\quad -7x = -6$

$x = \frac{8}{3}$ $\quad x = \frac{6}{7}$

$\therefore 5x = -2x + 6$

$-3x = 2.5$

$x = \frac{5}{6}$

Question 4 continued

(c) $f(x) = k$

exactly 3 roots

between y-int and point of intersection of negative and positive parts of the graph

$y = 2x + 13$ and $y = -2x + 25$

y-int $(0, 13)$

$2x + 13 = -2x + 25$

$4x = 8$

$x = 2$

$\therefore f(2) = 13 + 2 = 15 = 21$

$\therefore 13 \le k < 21$ ✓

Question 4 continued

$f(x) = 2 - 2|2 - 2|$

$= 2 - 2(2)$

$= 2 - 2 = 0$

(b) $f(x) = 5x$

$f(x) = 2 - 2|2 - x|$

$y = 2 - 2(2 - x)$ $\quad y = 2 - 2(-2x + 4)$

$y = 2 - 4 + 2x$ $\quad y = 2 + 4 - 2x$

$y = 2x + 2$ $\quad y = -2x + 6$

x^2 $\quad x^2$

$y = 2x + 2$ $\quad y = -2x + 6$

$y = 5x$ $\quad y = 5x$

$2x + 2 = 5x$ $\quad -2x + 6 = 5x$

$-3x = 2$ $\quad -7x = -6$

$x = \frac{2}{3}$ $\quad x = \frac{6}{7}$

$\therefore 5x = -2x + 6$

$-3x = 2.5$

$x = \frac{5}{6}$

Question 5 continued

$e^{x-1} = 3$ $\quad \ln(e^x) = x$

$\ln(e^{x-1}) = \ln 3$ $\quad \ln e = 1$

$x - 1 = \ln 3$ $\quad \ln(e^x) = \log_e e^x = x\log_e e = x$

$x = \ln 3 + 1$

$x = \ln 3 + \ln e$

$x = \ln 3e$

$\therefore k = 3e$ ✓

(b) prove $\alpha = 1.34$

α is root of equation between C, & C2

$\therefore 5x^{x-1} + 3 = 10 - x^2$

$5x^{x-1} + x^2 - 7 = 0$

find upper and lower bound α:

UB: 1.1345

LB: 1.1335

$f(x) = 5e^{x-1} + x^2 - 7$

Show that the root is between the upper and lower bounds:

$f(1.1345) = 5e^{1.1345-1} + 1.1345^2 - 7 = 0.0000\ldots$

$f(1.1335) = 5e^{1.1335-1} + 1.1335^2 - 7 = -1.0000\ldots$

Change in sign $\therefore \alpha$ is a root (3dp)

$1.1335 < \alpha < 1.1345$

Question 6 continued

(c) $\int_0^{\frac{\pi}{4}} \sin^2 x\,dx$

$\int_0^{\frac{\pi}{4}} 3e^{2x}\sec^2 2x + 2e^{3x}\sec^2 2x\tan 2x = 0$

$e^{3x}\sec^2 2x(3 + 2\tan 2x) = 0$

$\therefore e^{3x} = 0$ or $\sec^2 2x = 0$ or $3 + 2\tan 2x = 0$

$\ln(e^{3x})$ not defined on reals

$\therefore \frac{-\pi}{4} < 2x < \frac{\pi}{4}$

$\therefore \frac{-\pi}{2} < 2x < \frac{\pi}{2}$

$2x = -0.982793$

$\therefore x = -0.491396$

$= -0.491$ (3dp)

(no other solutions)

NOTE: Always use radians for trig functions

add 2 causes problems

Question 6 continued

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\int_0^{\frac{\pi}{4}} \frac{1}{2}(1 - \cos 2x)\,dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{\sin\frac{\pi}{2}}{2}\right]$

$= \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{8} - \frac{1}{4}$

(d) $\int_0^{\frac{\pi}{4}} 2\sin^2 x\,dx$

$4\sin^2 x = 2(1 - \cos 2x)$

$\therefore \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\therefore \int_0^{\frac{\pi}{4}} \frac{1}{2}(1 - \cos 2x)\,dx$

TYPO CHANGED TO $\cos 2x$ in MS

Question 7 continued

$\int_0^{\frac{\pi}{4}} (3\sin x - \sin 2x)\,dx$

$= \frac{1}{4}\int_0^{\frac{\pi}{4}} \left[-3\cos x - \left(-\frac{\cos 2x}{2}\right)\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{4}\left[-3\cos x + \frac{\cos 2x}{2}\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{4}\left[-3\cos\frac{\pi}{4} + \frac{\cos\frac{\pi}{2}}{2} - \left(-3\cos 0 + \frac{\cos 0}{2}\right)\right]$

$= \frac{1}{4}\left(-3\cdot\frac{1}{\sqrt{2}} + \frac{1}{2} + 3 - \frac{1}{2}\right)$

$= \frac{1}{4}\left(-\frac{3}{\sqrt{2}} + 3\right)$

$= \frac{3}{4}\left(\frac{1}{\sqrt{2}} - 1\right)$

$= \frac{3}{4}\left(\frac{\sqrt{2}}{2} - 1\right)$

$= \frac{3}{8}(\sqrt{2} - 2)$ ✓

Question 7 continued

$\int_0^{\frac{\pi}{4}} 2\sin^2 x\,dx$

$= \int_0^{\frac{\pi}{4}} (1 - \cos 2x)\,dx$

$= \left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$

$= \left(\frac{\pi}{4} - \frac{\sin\frac{\pi}{2}}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)$

$= \frac{\pi}{4} - \frac{1}{2}$

$= \frac{\pi}{4} - \frac{1}{2}$

$= \frac{\pi}{8} - \frac{1}{4}$

6.

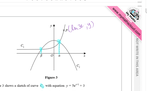

Figure 3

Figure 3 shows a sketch of curve C_1 with equation $y = 5e^{x-1} + 3$

and curve C_2 with equation $y = 10 - x^2$

The point P lies on C_1 and has y coordinate 18

(a) Find the x coordinate of P, writing your answer in the form $\ln k$, where k is a constant to be found.

(5)

The curves C_1 meets the curve C_2 at $x = \alpha$ and at $x = \beta$, as shown in Figure 3.

(b) Using a suitable interval and a suitable function that should be stated, show that $\alpha = 1.34$ to 3 decimal places, correct to 3dp.

(5)

The method of iteration

$x_{n+1} = \sqrt{10 - 5e^{x_n - 1}}$

is used to find an approximation to β — gives root

Using this iterative formula with $x_0 = 2.5$

(c) Find the value of x_1 and the value of x_2, giving each answer to 4 decimal places.

(3)

$C_1: y = 5e^{x-1} + 3$

$y = 18 \quad 18 = 5e^{x-1} + 3$

$\frac{15}{5} = e^{x-1}$

Question 7 continued

(a) $\ln(e^{x-1}) = \ln 3$

$\ln(e^x) = x$

$x - 1 = \ln 3$

$x = \ln 3 + 1$

$x = \ln 3 + \ln e$

$x = \ln 3e$

$\therefore k = 3e$ ✓

(b) prove $\alpha = 1.34$

α is point of intersection between C_1 & C_2

$\therefore 5e^{x-1} + 3 = 10 - x^2$

$5e^{x-1} + x^2 - 7 = 0$

find upper and lower bound α:

UB: 1.1345

LB: 1.1335

$f(x) = 5e^{x-1} + x^2 - 7$

Show that the root is between the upper and lower bounds:

$f(1.1345) = 5e^{1.1345-1} + 1.1345^2 - 7 = 0.0000\ldots$

$f(1.1335) = 5e^{1.1335-1} + 1.1335^2 - 7 = -1.0000\ldots$

Change in sign $\therefore \alpha$ is a root (3dp)

$1.1335 < \alpha < 1.1345$

Question 7 continued

(c) $x_1 = -\sqrt{7 - 5e^{x_0 - 1}}$

$x_2 = ? \quad \beta = ?$

$x_0 = 2.5$

$x_1 = -\sqrt{7 - 5e^{2.5-1}}$

$x = -2.43398$

(keep going ...)

$\beta = -2.620330$ ✓

$x_1 = -3$

$x_2 = -\sqrt{7 - 5e^{-3-1}}$

$x_2 = -2.6203$

$\beta = -2.620330$

β doesn't stop changing, you have your answer

Question 8 continued

5. (a) Express $\cos x + 2\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians, to 3 decimal places.

(3)

A scientist is studying the behaviour of seabirds on a cliffop.

The models the height above sea level of seabirds, w metres, of one of the birds in the colony by the equation

$$w = 7\cos(x - \alpha) + 25 \qquad x \in \mathbb{R}$$

where x seconds is the time after it leaves the nest.

Find, according to the model,

(b) the minimum height of the seabird above sea level, giving your answer to the 1 decimal place.

(2)

(c) the value of x, to 3 decimal places, when $w = 30$

(3)

(a) $\cos x + 4\sin x = R\cos(x - \alpha)$

$R\cos(x - \alpha)$

$= R(\cos x\cos\alpha + \sin x\sin\alpha)$

$= R\cos x\cos\alpha + R\sin x\sin\alpha$

$\therefore$ In the form $1\cos x + 4\sin x$

$1 = R\cos\alpha$ and $4 = R\sin\alpha$

Question 8 continued

$\cos\alpha = \frac{1}{R} \quad \sin\alpha = \frac{4}{R}$

$\therefore R = \sqrt{4^2 + 1^2}$

$R = \sqrt{17}$

$\tan\alpha = \frac{o}{a} = \frac{4}{1} = 4$

$\alpha = \tan^{-1}(4)$

$\alpha = 1.326$ (radians)

$\therefore \cos x + 4\sin x = \sqrt{17}\cos(x - 1.326)$

(b) $H = \frac{24}{3 + \cos\left(\frac{1}{3}t\right) + 4\sin\left(\frac{1}{3}t\right)}$ $\quad 0 \le t \le 6s$

t in seconds

min height = ?

$\cos\left(\frac{1}{3}t\right) + 4\sin\frac{1}{3}t$

$= \sqrt{17}\cos\left(\frac{1}{3}t - 1.326\right)$

$\therefore H = \frac{24}{3 + \sqrt{17}\cos\left(\frac{1}{3}t - 1.326\right)}$

NOTE: Affect too ms

for ms $\Rightarrow t = 0$

$\therefore H = \frac{24}{3 + \sqrt{17} \times \cos\left(\frac{1}{3}(0) - 1.326\right)}$

$\cos\left(\frac{1}{3}(0) - 1.326\right) \approx 2$

$\therefore \frac{24}{3 + \sqrt{17}(2)} = \frac{24}{3 + \sqrt{17}} = 3.74\text{m (2dp)}$

Question 7 continued

(c) $H = \frac{24}{3 + \sqrt{17}\cos\left(\frac{1}{3}t - 1.326\right)}$

$10 = \frac{24}{3 + \sqrt{17}\cos\left(\frac{1}{3}t - 1.326\right)}$

$10\left(3 + \sqrt{17}\cos\left(\frac{1}{3}t - 1.326\right)\right) = 24$

$30 + 10\sqrt{17}\cos\left(\frac{1}{3}t - 1.326\right) = 24$

$\cos\left(\frac{1}{3}t - 1.326\right) = \frac{-6}{10\sqrt{17}}$

$\frac{1}{3}t - 1.326 = \cos^{-1}\left(-\frac{6}{10\sqrt{17}}\right)$

$\frac{1}{3}t = \cos^{-1}\left(-\frac{6}{10\sqrt{17}}\right) + 1.326$

$t = \cos^{-1}\left(-\frac{6}{10\sqrt{17}}\right) + 1.326$

$t = 5.97$ answer different from MS

Question 8 continued

(a) $g(x) = e^{3x}\sec 2x \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$

$g'(x) = ?$

PRODUCT RULE

$u = e^{3x} \quad v = \sec 2x$

$uv + v'u$

$u' = e^{3x} \times 3 \quad v' = \sec 2x\tan 2x$

$g'(x) = 3e^{3x}\sec 2x + \sec 2x\tan 2x e^{3x}$

$= 3e^{3x}\sec 2x + 2e^{3x}\sin 2x\tan 2x$ ✓

Differentiation

$f(x)$	$f'(x)$
$\sec 2x$	$2\sec 2x\tan 2x$
$\sin^2 x$	$\sin 2x$
$\cos^2 x$	$-\sin 2x$
$\cos(x - \alpha)$	$-\sin(x - \alpha)$

Question 8 continued

(b) $g'(x) = 0$

$-\frac{\pi}{4} < x < \frac{\pi}{4}$

$\therefore 3e^{3x}\sec 2x + 2e^{3x}\sec 2x\tan 2x = 0$

$e^{3x}\sec 2x(3 + 2\tan 2x) = 0$

$\therefore e^{3x} = 0$ or $\sec 2x = 0$ or $3 + 2\tan 2x = 0$

$\ln(e^{3x})$ not defined on reals

$\cos 2x = \frac{1}{\sec 2x}$

$\tan 2x = -\frac{3}{2}$

$2x = \tan^{-1}\left(-\frac{3}{2}\right)$

$\therefore -\frac{\pi}{4} < 2x < \frac{\pi}{4}$

$2x = -0.982793$

$\therefore x = -0.491396$

$= -0.491$ (3dp)

(no other solutions)

NOTE: Always use radians for trig functions

add 2 causes problems

Question 8 continued

(c) $x = \ln(\sin y) \quad 0 < y < \frac{\pi}{2}$

$\sin y = e^x$

$y = \ln(f(x))$

$y = f(f(x))$

$\frac{dx}{dy} = \frac{1}{\sin y} \times \cos y$

$= \frac{\cos y}{\sin y}$

$\therefore \frac{dy}{dx} = \frac{\sin y}{\cos y} = \tan y$

$x = \ln\sin y$ $\quad \ln x$

$x = \log_e \sin y$ $\quad \log_e$

$e^x = \sin y$ $\quad \log_e e^x = x$

$\therefore \frac{dy}{dx} = \frac{e^x}{\cos y}$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1 - \sin^2 y}$

$\therefore \frac{dy}{dx} = \frac{e^x}{\sqrt{1 - \sin^2 y}}$

$= \frac{e^x}{\sqrt{1 - (e^x)^2}}$

$\frac{dy}{dx} = \frac{e^x}{\sqrt{1 - e^{2x}}}$

$\therefore f(x) = \sqrt{1 - e^{2x}}$ ✓

Question 9 continued

$\int_0^2 \left(x^2 + 2 + \frac{5}{x-5}\right)dx$

$\left[\frac{x^3}{3} + 2x + 5\ln|x-5|\right]_0^2$

$= \frac{2^3}{3} + 2(2) + 5\ln|2-5|$

$- (5\ln|0-5|)$

then $y' = \frac{5\ln|x-5|}{1}$

$= \frac{8}{3} + 4 + 5\ln|-3| - (5\ln|-5|)$

$= \frac{8}{3} + 4 + 5\ln 3 - 5\ln 5$

$= \frac{20}{3} + 5\ln 3 - 5\ln 5^2$

$= \frac{20}{3} + 5\ln 3 - 10\ln 5$

$= \frac{20}{3} - 5\ln 3$ ✓

$\therefore a = \frac{20}{3} \quad b = -5$

$y = \frac{1}{3}x - 2$

(c) $\int_0^2 \left(x^2 + 2 + \frac{5}{x-5}\right)dx$

f $\quad y = 5$ → constant

$x - 5$ → x' in den m

$\left[\frac{x^3}{3} + 2x + 5\ln|x-5|\right]_0^2$

$= \frac{2^3}{3} + 2(2) + 5\ln|2-5|$

$- (5\ln|0-5|)$

$= \frac{8}{3} + 4 + 5\ln|-3| - (5\ln|-5|)$

$= \frac{8}{3} + 4 + 5\ln 3 - 5\ln 5$

$= \frac{20}{3} + 5\ln 3 - 5\ln 5^2$

$= \frac{20}{3} + 5\ln 3 - 10\ln 5$

$= \frac{20}{3} - 5\ln 3$ ✓

$\therefore a = \frac{20}{3} \quad b = -5$

Question 9 continued

(a) Simplify $\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - 2x - 12}$ $\quad x > -3$

$a = ? \quad a = 5$

$x^2 - 2x - 12$

factorise

$x^4 - x^3 - 10x^2 + 3x - 9$

$- x^3 - x^2 - 2x^2$

$2x^2 + 2x - 9$

$- 2x^2 - 2x - 24$

$5x + 15$

$x^2 + 2 + \frac{5}{x-6}$

$\frac{5x + 15}{x^2 - 2x - 12}$

$\therefore \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - 2x - 12} = x^2 + 2 + \frac{5x + 15}{x^2 - 2x - 12}$

$x^2 + 2$ + $\frac{5x + 15}{x^2 - 2x - 12}$ quotient + remainder

$= x^2 + 2 + \frac{5}{x-6}$

$= x^2 + 2 + 5(x^{-6})$

$= x^2 + 2 + \frac{5}{x-6}$

$\therefore Q = 5$

$p = 2$

(b) $x^2 + 2 + \frac{5}{x-6} = g(x)$

m of tangent $= g'(x)$

$g(x) = x^2 + 2 + 5(x-6)^{-1}$

$g'(x) = 2x + -5(x-6)^{-2} \times 1$

$= 2x - 5(x-6)^{-2}$

Question 9 continued

at $x = 7$

$g'(x) = 2(2) - 5(2-6)^{-2}$

$= 4 - 5(-2)^{-2}$

$= 4 - \frac{5}{4}$

$= \frac{16 - 5}{4}$

$= \frac{11}{4}$ = m of tangent

$f(2) = 2^2 + 2 + \frac{5}{2-6} = 6 + \left(-\frac{5}{2}\right)$

$= \frac{12 - 5}{2} = \frac{7}{2}$

$\therefore m = \frac{11}{4}$ & sub $(2, \frac{7}{2})$

$\frac{7}{2} = \frac{11}{4}(2) + c$

$c = -2$