

FINANCIAL MATH

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SIMPLE GROWTH / DECAY

2

↗ final (accrued) amount

$$A = P(1 \pm in)$$

↘ principle value

↘ number of years

↘ interest as a decimal

(straight line method)

COMPOUND GROWTH / DECAY

3

$$A = P(1 \pm i)^n$$

Diagram labels for $A = P(1 \pm i)^n$:

- A : final amount
- P : Principle amount
- i : interest in decimal
- n : number of years

COMPOUND GROWTH / DECAY \rightarrow COMPOUNDED
QUARTERLY / MONTHLY

$$A = P \left(1 \pm \frac{i}{m} \right)^{n \times m}$$

Diagram labels for $A = P \left(1 \pm \frac{i}{m} \right)^{n \times m}$:

- A : Accrued amount (end)
- P : start (principle) amount
- i : interest as a decimal over the number of months / compounding periods
- $n \times m$: number of years times number of months / compounding periods

* When find P for a known A (going back in time) use:

$$A = P(1 \pm i)^{-n}$$

FUTURE VALUE (SAVING FOR THE FUTURE)

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$$FV = x \left[\frac{\left(1 + \frac{i}{m}\right)^n - 1}{\frac{i}{m}} \right]$$

total future value

fixed/monthly payments

number of payments made

interest in decimal divided by amount of months

PRESENT VALUE (LOANS)

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$$P_v = x \left[\frac{1 - \left(1 + \frac{i}{m}\right)^{-n}}{\frac{i}{m}} \right]$$

Diagram illustrating the Present Value formula for loans, with variables defined by arrows:

- P_v → Present value available
- x → fixed regular/monthly payments
- $\frac{i}{m}$ → rate of interest in decimals divided by the number of months/compounding period
- n → number of payments made

EFFECTIVE INTEREST

$$i_{\text{eff}} = \left(1 + \frac{i_{\text{nom}}}{m}\right)^m - 1$$

starting (nominal) interest rate compounded m times a year
 effective interest (actual rate of interest that is obtained)
 compounding times

TO CONVERT FROM ONE COMPOUNDING PERIOD TO ANOTHER:

Use: $i_{\text{eff}(1)} = i_{\text{eff}(2)}$

$$\therefore \left(1 + \frac{i^{(m)}}{m}\right)^m = \left(1 + \frac{i^{(n)}}{n}\right)^n$$

OTHER NOTES:

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- * **Once off investments**: use compound growth formula

$$A = P(1+i)^n$$

- * **Annuity**: Future value | Present value
with regular payments (x)

Fv
↓
What amount
will I have
in the future?

vs

Pv
↓
What amount do I
have right now
(loans)

- * **'One month after'**: that is how questions are typically asked. It does not change the formula

- * **'Rate of depreciation'** means $i = ?$

- * **Reducing balance method**:

use compound depreciation

$$A = P(1-i)^n$$

* Population: compound interest formula

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$$A = P(1 \pm i)^n$$

* Sinking fund: Saving up for needing something paid for in the future

ie: Fv

* Quarterly rate: $\frac{\text{percentage } i}{\text{compounding periods}}$

* How many withdrawals / deposits?

this means 'what is n?'

* Fv: 'one month after purchase' \rightarrow normal way of asking

'immediately' \rightarrow + one month

* Fv: On the time line, change all years to months

Eg 3 yrs

$T_0 \quad T_1 \quad \dots \quad T_{36}$ $\rightarrow 12 \times 3$
 $x_1 \quad T_n$

* If payments end early: use compound interest for the remaining months

BALANCE OUTSTANDING

9

* PV - full loan

* What is the balance?

Compound interest - FV



of the full loan
for the years already
paid



monthly
payments
already
paid

= loan amount still to pay.

DELAYED START QUESTIONS (Pv)

10

$$P = x \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

but

P has compound interest for the first missed months due to the delay

$$\therefore P(1+i)^n = x \left[\frac{1 - (1+i)^{-n}}{i} \right]$$



Principal
amount with its
interest for
the months of
the delay