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Candidate surname
Annotated by Tam.

Other names

Pearson Edexcel
International
Advanced Level

Centre Number

Candidate Number

Time 1 hour 30 minutes

Paper reference **WST01/01**

Mathematics
International Advanced Subsidiary/Advanced Level
Statistics S1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical
formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Values from the statistical tables should be quoted in full. If a calculator is used instead of the tables, the value should be given to an equivalent degree of accuracy.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 6 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶

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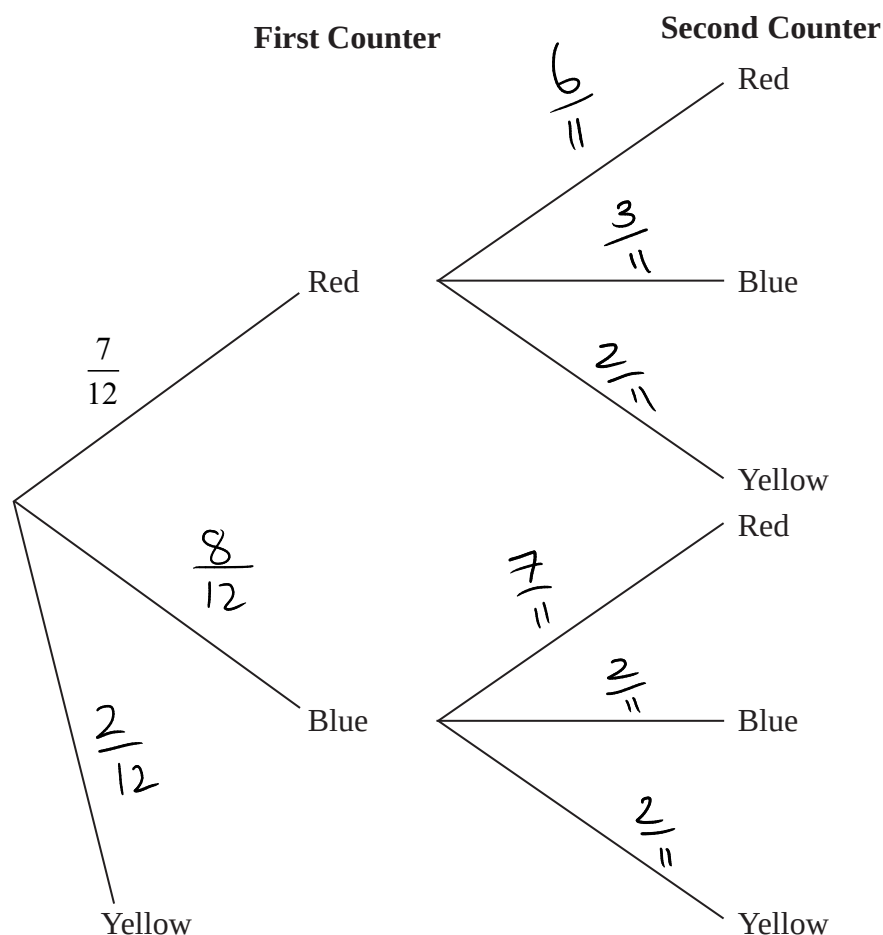©2021 Pearson Education Ltd.
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Pearson

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1. There are 7 red counters, 3 blue counters and 2 yellow counters in a bag. Gina selects a counter at random from the bag and keeps it. If the counter is yellow she does not select any more counters. If the counter is not yellow she randomly selects a second counter from the bag.

   (a) Complete the tree diagram.

   First Counter   **Second Counter**

   $\frac{6}{11}$ Red

   $\frac{3}{11}$ Blue

   Red

   $\frac{7}{12}$

   $\frac{2}{11}$ Yellow

   Red

   $\frac{8}{12}$ $\frac{7}{11}$

   $\frac{2}{11}$ Blue

   Blue

   $\frac{2}{12}$

   $\frac{2}{11}$

   Yellow   Yellow

   (2)

   Given that Gina has selected a yellow counter,   conditional probability

   (b) find the probability that she has 2 counters.

   (3)

   $$P(2\,\text{counters} \mid \text{Yellow}) = \frac{P(2\,\text{counters} \cap Y)}{P(Y)}$$

   $$P(Y) = \frac{2}{12} + \left(\frac{7}{12} \times \frac{2}{11}\right) + \left(\frac{3}{12} \times \frac{2}{11}\right) = \frac{7}{22}$$

   $$P(2\,\text{counters} \cap Y) = \left(\frac{7}{12} \times \frac{2}{11}\right) + \left(\frac{3}{12} \times \frac{2}{11}\right) = \frac{5}{33}$$

Question 1 continued

$$\therefore P(2\text{ counters} \mid Y) = \frac{\frac{5}{33}}{\frac{7}{22}} = \frac{10}{21} \checkmark$$

(Total 5 marks)

Conditional Probability

2. In the Venn diagram below, *A*, *B* and *C* are events and *p*, *q*, *r* and *s* are probabilities.

The events *A* and *C* are independent and P(*A*) = 0.65

A = 0.65, B, *p*, *q*, 0.13, C, *r* = 0.07, *s* = 0.28
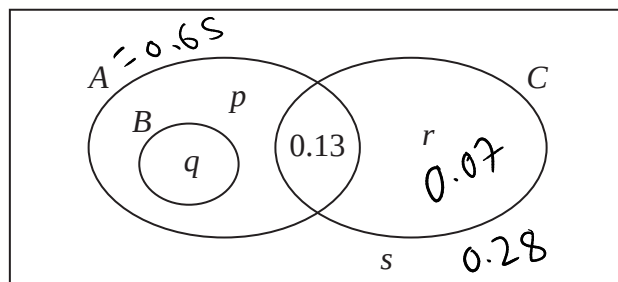
(a) State which two of the events *A*, *B* and *C* are mutually exclusive.

(1)

(b) Find the value of *r* and the value of *s*. r=? s=?

(5)

The events $(A \cap C')$ and $(B \cup C)$ are also independent.

(c) Find the exact value of *p* and the exact value of *q*. Give your answers as fractions.

(6)

(a) mutually exclusive : no intersection
→ don't touch on the Venn diagram

$P(A \cap B) = 0$

by inspection   B and C ✓

(b) for independent events A and C

$P(A) \times P(C) = P(A \cap C)$

$0.65 \times P(C) = 0.13$

$P(C) = 0.13 \div 0.65 = 0.2$

$\therefore r = 0.2 - 0.13$
$= 0.07$ ✓

$s = 1 - P(A \cup C)$ $= 1 - P(A \cup C)$
$= 1 - (0.65 + 0.07)$

$s = 0.28$ ✓

Leave
blank

Question 2 continued

(c) $P(A \cap C') \times P(B \cup C)$
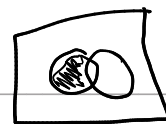
$= P[(A \cap C') \cap (B \cup C)]$
(independent events)

$\therefore (p+q) \times (q + P(C))$

$P(A \cap C')$

$P(B \cup C)$
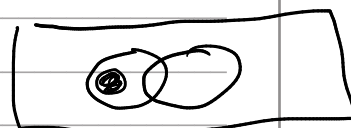

$P(C) = 0.13 + 0.07$
$= 0.2$

$\therefore (p+q) \times (q + 0.2) = q$

$P[(A \cap C') \cap (B \cap C)]$
$= q$

$(p+q) = P(A) - 0.13$
$= \dfrac{52}{100}$

⤷common
(union)

$\therefore \dfrac{52}{100} \times (q + 0.2) = q$

$\dfrac{52}{100} q + \dfrac{13}{125} = q$

$\dfrac{13}{125} = q - \dfrac{52}{100} q$

$q = \dfrac{\frac{13}{125}}{\frac{12}{25}} = \dfrac{13}{60} \checkmark$

(Total 12 marks)

Q2

Histograms / Interpolation (handwritten)

3. A random sample of 100 carrots is taken from a farm and their lengths, L cm, recorded. The data are summarised in the following table.

Cl width (handwritten)

Length, L cm	Frequency, f	Class mid point, x cm
$5 \leqslant L < 8$	5	6.5
$8 \leqslant L < 10$	13	9
$10 \leqslant L < 12$	16	11
$12 \leqslant L < 15$	25	13.5
$15 \leqslant L < 20$	30	17.5
$20 \leqslant L < 28$	11	24

(handwritten class widths beside table: 3, 2, 2, 3, 5, 8)

(handwritten cumulative frequency: 5, 18, 34, 59, 89, 100)

(handwritten: half)

(handwritten: $n = 100$)

(handwritten right margin: Fr den. = $\frac{Fr}{Class\ width}$; $\frac{5}{3}$; $^{30}/_5 = 6$)

A histogram is drawn to represent these data.
The bar representing the class $5 \leqslant L < 8$ is 1.5 cm wide and 1 cm high.

(handwritten: 1.5 w   1 cm high)

(a) Find the width and height of the bar representing the class $15 \leqslant L < 20$

(3)

(b) Use linear interpolation to estimate the median length of these carrots.

(handwritten: Q_2)

(2)

(c) Estimate

(i) the mean length of these carrots,   (handwritten: $\overline{x} = ?$)

(2)

(ii) the standard deviation of the lengths of these carrots.   (handwritten: $\sigma = ?$)

(3)

A supermarket will only buy carrots with length between 9 cm and 22 cm.

(d) Estimate the proportion of carrots from the farm that the supermarket will buy.

(2)

Any carrots that the supermarket does not buy are sold as animal feed.

The farm makes a profit of 2.2 pence on each carrot sold to the supermarket, a profit of 0.8 pence on each carrot longer than 22 cm and a loss of 1.2 pence on each carrot shorter than 9 cm.

(e) Find an estimate of the mean profit per carrot made by the farm.

(2)

(handwritten answer at bottom:)

(a) Use given information $5 \leqslant l < 8$

W = 3 units

∴ 3

use ratios: ∴ $\frac{3 : 1)5}{3}$ $\frac{}{3}$ ∴ 1 unit = $\frac{1}{2}$ cm

Leave blank

Question 3 continued

$\therefore$ $\underline{1 \text{ unit}} = 0.5 \text{ cm}$

for $15 \le \ell < 20$ width $= 5$ units

$\therefore \quad 0.5 \times 5 = 2,5 \text{ cm}$ ✓

for height: $fr \cdot$ density $= \dfrac{fr}{\text{class width}}$

use given information $5 \le \ell < 8$

$fr \cdot$ density $= \dfrac{5}{3}$ units

use ratio: $\dfrac{\frac{5}{3}}{\frac{5}{3}}$ $: 1\text{cm} = = \dfrac{3}{5} \text{ cm} = 1 \text{ unit}$

$\therefore$ for $15 \le \ell < 20$

$\dfrac{3}{5} \times fr \cdot$ density

$= \dfrac{3}{5} \times \dfrac{20}{5} = \dfrac{3}{5} \times 6$ $= 3.6 \text{ cm}$ ✓

(b) Q_2 position $= \dfrac{n}{2} = 50^{th}$ position

$\therefore$ class $12 \le \ell < 15$

$\overset{3}{\underset{12 \qquad Q_2 \qquad 15}{\longrightarrow}}$

$\overset{16}{\underset{34 \qquad \boxed{50} \qquad 59}{\longrightarrow}}$

$\underset{25}{\longrightarrow}$

$Q_2 = 12 + \dfrac{16}{25} \times 3 \quad = 13.92$ ✓

(c) (i)   mean length $= ?$

$\bar{x} = \dfrac{(\text{mdpt} \times fr) + (\text{mdpt} \times fr) + \ldots + (\text{mdpt} \times fr)}{\text{cum. fr.}}$

$= \dfrac{(5 \times 6.5) + (13 \times 9) + \ldots + (11 \times 24)}{100} \quad = \dfrac{363}{25} = 14.52 \text{ cm}$ ✓

Question 3 continued

(ii) $S.D = ?$

$$S.D = \sqrt{M_{1an} \text{of Squares} - \text{Square of mean}}$$

$$\sigma = \sqrt{\left(\frac{5\times(6.5)^2 + 13\times(9)^2 + ... + 11\times(24)^2}{100}\right) - \left(\frac{36^3}{25}\right)^2}$$

$$= \boxed{4.69 \text{ cm}} \checkmark$$

(d) $9 < \ell < 22$ will buy ?

$8 \leq \ell < 10$ ↙   ↘ $20 \leq \ell < 28$

half | quarter

middle classes

$\therefore 16 + 25 + 30$

$\frac{1}{2} \times 13$ $\frac{1}{4} \times 11$

$\therefore \frac{13}{2} + 16 + 25 + 30 + \frac{11}{4}$

$= 80.25$ of 100 carrots

$\therefore \boxed{80.25\%} \checkmark$

(e) Carrots sold to supermarket : 80.25   2.2p ea profit

longer than 22 : $\left(11 - \frac{11}{4}\right) = 8.25$ 0.8p ea profit

↳ up to 22 is a quarter of width

Shorter than 9 : $\left(13 - \frac{13}{2}\right) + 5 = 11.5$ 1.2p loss

half of class width

Leave blank

Question 3 continued

$$\therefore \quad \frac{80.25 \times 2.2 + 8.25 \times 0.8 - 11.5 \times 1.2}{100 \text{ carrots}}$$

Profit $=$ 1.69p per carrot ✓ $\quad (3sf)$

(Total 14 marks)

Q3

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Normal distribution & Probability

4. Kris works in the mailroom of a large company and is responsible for all the letters sent by the company. The weights of letters sent by the company, *W* grams, have a normal distribution with mean 165 g and standard deviation 35 g.

(a) Estimate the proportion of letters sent by the company that weigh less than 120 g.

(3)

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

Kris splits the letters to be sent into 3 categories: heavy, medium and light, with $\frac{1}{3}$ of the letters in each category.

(b) Find the weight limits that determine medium letters.

(4)

A heavy letter is chosen at random.

Conditional probability.

(c) Find the probability that this letter weighs less than 200 g.

(3)

Kris chooses a random sample of 3 letters from those in the mailroom one day.

(d) Find the probability that there is one letter in each of the 3 categories.

(3)

(a)

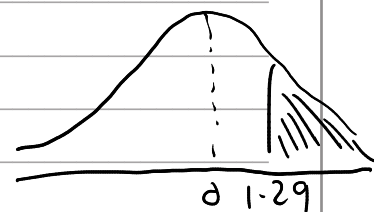$W \sim N(165, 35^2)$

$P(W < 120)$

$z = \dfrac{x - \mu}{\sigma}$

$= P\left(z < \dfrac{120 - 165}{35}\right)$

W   120   165
z   ?   0

$= P(z < -1.29)$ use symmetry

$\therefore \; 1 - P(z < 1.29)$ → areas go to the left

$\sigma \; 1.29$

$\therefore \; 1 - 0.9015$ → from tables

$= 0.0985$

(b)

$W \sim N(165, 35)^2$

weight limits = ?

(Medium)

SPLIT into 3

$\frac{1}{3}$

$\frac{1}{3}$ L   M   H $\frac{1}{3}$

W   W'   165   W
z   z'   0   z

P 6 3 1 5 0 A 0 1 0 2 0

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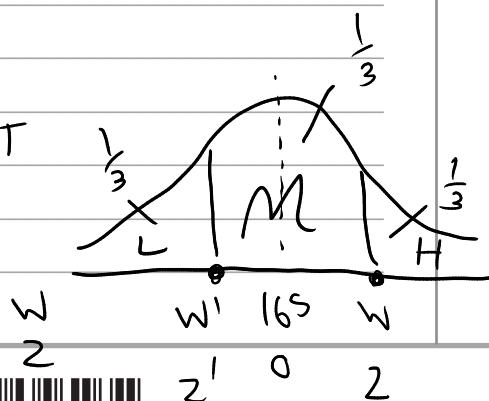
Question 4 continued

$$z' = z \qquad \text{(by symmetry)}$$

$$P(Z < z) = \frac{2}{3} = 0.6666 \quad \text{in area}$$

$$\therefore z = 0.43 \qquad z' = -0.43$$

Convert to w $\qquad z = \dfrac{w - \mu}{6}$

$$0.43 = \frac{w - 165}{35}$$
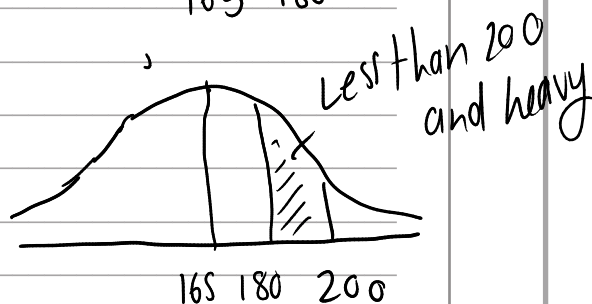
$$w = 180.05 \qquad = 180g \, (3sf)$$

$$z' = -0.43$$
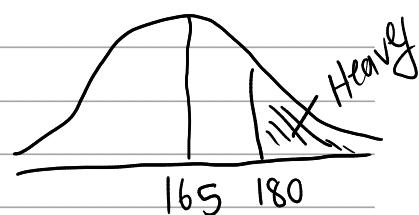$$\therefore w' = 149.85 \qquad = 150g \quad (3sf)$$

$$\boxed{150g < \text{Medium} < 180g}$$

(c) $P(\text{less than } 200 \mid \text{Heavy})$

$$P(\text{Heavy}) = \frac{1}{3}$$

$$\therefore \quad \frac{P((w < 200) \cap H)}{\frac{1}{3}}$$

Heavy

165 180

Less than 200 and heavy

165 180 200

$$P((w<200) \cap H)$$

$$= P(w < 200) - P(w < 180)$$

$$= P(w < 200) - \frac{2}{3}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Question 4 continued

$$z = \frac{W - \mu}{6}$$

$$P(W < 200)$$

$$= P\left(z < \frac{200 - 165}{35}\right) = P(z < 1) = 0.8413$$

$$\llcorner_0 \text{ from tables}$$

$$\therefore \quad \frac{0.8413 - \frac{2}{3}}{\frac{1}{3}}$$

$$= 0.5239$$

$$= 0.524 \qquad (3sf)$$

Leave blank

Question 4 continued

Q4

(Total 13 marks)

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Discrete random variables

5. The discrete random variable X has the following probability distribution

x	-2	-1	0	1	4
$P(X = x)$	0.25 a	0.13 b	0.24 c	0.13 b	a 0.25

Given that $E(X) = 0.5$ Mean $x = 0.5$

(a) find the value of a.

(2)

Given also that $Var(X) = 5.01$

(b) find the value of b and the value of c.

(5)

The random variable $Y = 5 - 8X$ → Coding

(c) Find   (i) $E(Y)$

(ii) $Var(Y)$

(3)

(d) Find $P(4X^2 > Y)$

(5)

(a) $E(X) = \sum(x \times p)$ all x values × their probability added together

$0.5 = -2 \times a + -1 \times b + \dots + 4 \times a$

$a = 0.25$ ✓

(b) $\sqrt{ar}(X) = 5.01$ $Var(X) = MS - SM$

$5.01 = (4 \times 0.25 + 1 \times b + \dots + 16 \times 0.25) - (0.5)^2$

$b = 0.13$ ✓

$c = 1 - (0.25)2 - (0.13)2$

$= 0.24$ ✓

Leave blank

Question 5 continued

(c)(i) $\quad E(Y) = -8\,E(x) + 5$

$$= 1 \quad \checkmark$$

mean is affected
by $x \div$ and $+ -$

(ii) $\text{Var}(y) = (-8)^2 \times \text{Var}(x)$

$$= 320.64 \quad \checkmark$$

variance is
$s.D^2$
and variance
is only
affected by
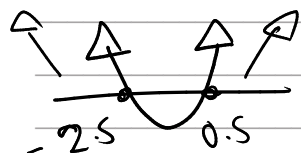$x \div$

(d) $\quad P(4x^2 > Y) = ? \qquad y = 5 - 8x$

sub Y

$\therefore P(4x^2 > 5 - 8x)$

isolate x

this is quadratic inequality

Cr Val: $4x^2 + 8x - 5 = 0$

$\qquad (\quad)(\quad) = 0$

$\qquad x = 0.5 \quad \text{or} \quad x = -2.5$

$-2.5 \qquad 0.5$

$\therefore P(X < -2.5) \quad \text{or} \quad P(X > 0.5) \checkmark$

invalid

$\big(\text{out of range of } x\big)$

Leave blank

Question 5 continued

$$P(X > 0.5) = P(X = 1) + P(X = 4)$$
$$= 0.13 \quad + \quad 0.25$$
$$= 0.38$$

Question 5 continued

CORRELATION & REGRESSION *(handwritten)*

6. Two economics students, <u>Andi</u> and <u>Behrouz</u>, are studying some data relating to unemployment, $x\%$, and increase in wages, $y\%$, for a European country. The least squares regression line of y on x has equation

$$y = 3.684 - 0.3242x$$

wages (y)   unemployment (x%) *(handwritten)*

and $\qquad \sum y = 23.7 \qquad \sum y^2 = 42.63 \qquad \sum x^2 = 756.81 \qquad n = 16$

(a) Show that $S_{yy} = 7.524\,375$ *formula book (handwritten)*

(1)

(b) Find S_{xx} *use regression line in formula book* $\bar{x} = \frac{\sum x}{n}$ *(handwritten)*

$a = \bar{y} - b\bar{x}$ *(handwritten)*

(4)

(c) Find the product moment correlation coefficient between x and y.

(3)

Behrouz claims that, assuming the model is valid, the data show that when unemployment is 2% wages increase at over 3%

(d) Explain how Behrouz could have come to this conclusion.

(1)

Andi uses the formula

$$\text{range} = \text{mean} \pm 3 \times \text{standard deviation}$$

to estimate the range of values for x.

(e) Find estimates of the minimum value and the maximum value of x in these data using Andi's formula.

(3)

(f) Comment, giving a reason, on the reliability of Behrouz's claim.

(2)

Andi suggests using the regression line with equation $y = 3.684 - 0.3242x$ to estimate unemployment when wages are increasing at 2%

(g) Comment, giving a reason, on Andi's suggestion.

(2)

(handwritten working)

(a) $S_{yy} = \sum(y - \bar{y})^2 = \sum y^2 - \frac{(\sum y)^2}{n}$

$= 42.63 - \frac{(23.7)^2}{16}$

$= 7.524375$ ✓

Question 6 continued

(b) $S_{xx} = ?$

$\Sigma x^2 = 756.81$ (given)

$\Sigma x = ?$ $n = 16$

use regression line to find Σx

$a = \bar{y} - b\bar{x}$ from (formula book)

$a = \dfrac{\Sigma y}{n} - b\dfrac{\Sigma x}{n}$ $\bar{x} = \dfrac{\Sigma x}{n}$

$\therefore 3.674 = \dfrac{237}{16} - (-0.3242) \times \dfrac{\Sigma x}{16}$

$\dfrac{\Sigma x}{16} = 6.79441\ldots$

$\Sigma x = 6.79441\ldots \times 16$

$= 108.71067\ldots$

$\therefore S_{xx} = 756.71 - \dfrac{(108.71067)^2}{16}$

$= 18.2$ ✓

(c) $r = ?$ $S_{xx} = 18.18435\ldots$ $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

          $S_{yy} = 7.524375$

$S_{xy} = ?$ $b = \dfrac{S_{xy}}{S_{xx}}$ $\longrightarrow$ from regression line

$\therefore S_{xy} = b \times S_{xx}$

$= -0.3242 \times 18.18435$

$= -5.8953$

$\therefore r = \dfrac{-5.8953}{\sqrt{18.18435 \times 7.524375}} = -0.5039957 = -0.504$ ✓ (3sf)

Leave blank

Question 6 continued

(d) regression line (he could have used)

$y = 3.684 - 0.3242x$

Sub $x = 2$ (2%)

$y = 3.684 - 0.3242(2)$
$= 3.0356$

∴ more than 3%.

(e) range $= \dfrac{\text{mean}}{\bar{x}} \pm \dfrac{3 \times \text{S.D}}{6}$ max = ?   min = ?

$\Sigma x = 108.71067 \quad \Sigma x^2 = 756.81 \quad n = 16$

$\bar{x} = 6.79441$

$\text{Var}(x) = MS - SM$

$SD = \sqrt{\dfrac{\Sigma x^2}{n} - \left(\Sigma x\right)^2}$

$SD = \sqrt{\dfrac{756.81}{16} - (6.79441)^2}$

$= 1.0661$

∴ range $= \bar{x} \pm 3\,6 = 6.79441 \pm 3(1.0661)$

min $= 3.596...$
$= 3.6$ ✓ (3sf)

max $= 9.992$
$= 9.99$ (3sf)

(f) min → max of x (range)

3.6 to 9.99

∴ $x = 2$ is invalid (out of range)
∴ extrapolation ∴ unreliable ✓

(g) not good
y should be unemployment
and x should be wages
to see how wages affect
unemployment (reversed)

Q6

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

END

20

P 6 3 1 5 0 A 0 2 0 2 0