

NATURE OF ROOTS

→ for quadratics/parabolas

From the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

→ from form
 $ax^2 + bx + c = 0$

The content under the square root can be called the discriminant-

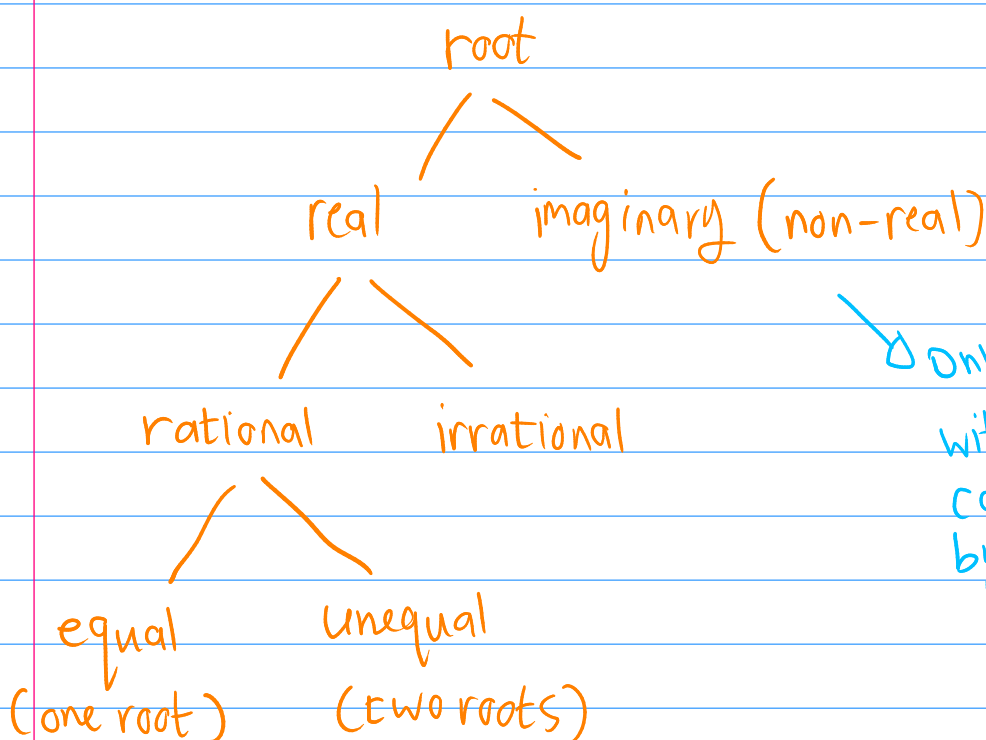
This is shown by the symbol δ or Δ → some textbooks will call this

$$\therefore \Delta = b^2 - 4ac$$

delta

The discriminant/delta tells us about the roots of a function in the following ways:

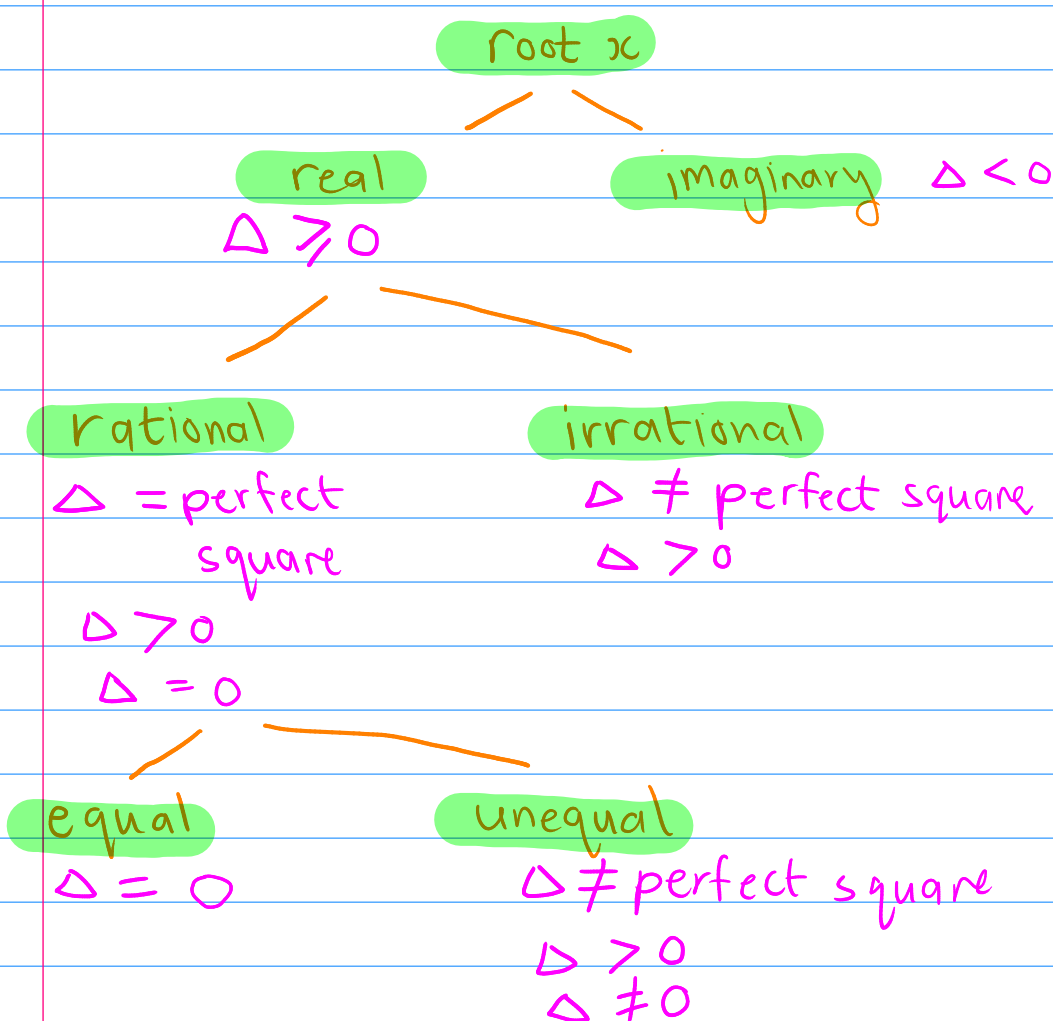
- ① are the roots real or imaginary (non-real)
- ② are the roots rational or irrational
- ③ are the roots equal (one root) or unequal (two roots)



→ Only equations with real roots can be solved by factorisation.

HOW TO USE THE DISCRIMINANT :

- ① $b^2 - 4ac < 0 \rightarrow$ non-real / imaginary roots
 \rightarrow because $\sqrt{-ve}$ is imaginary
- ② $b^2 - 4ac \geq 0 \rightarrow$ real roots
- ③ $b^2 - 4ac = \text{perfect square} \rightarrow$ rational and unequal
 \hookrightarrow not including 0
- ④ $b^2 - 4ac = 0 \rightarrow$ rational and equal
 \hookrightarrow zero is a perfect square.
- ⑤ $b^2 - 4ac > 0$; but not a perfect square
 \rightarrow irrational and unequal



EXAMPLES

1. Classify the roots of the following using the discriminant.

(a) $5x^2 - 2x + 1 = 0$

(b) $7x^2 - 3x - 4 = 0$

(c) $4x^2 - 12x + 9 = 0$

(d) $3x^2 + 5x + 1 = 0$

ANSWERS

$$\Delta = b^2 - 4ac$$

when $ax^2 + bx + c = 0$

CHECK: CALCULATOR

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(5)(1)}}{2(5)}$$

= error.

(a) $5x^2 - 2x + 1 = 0$

$$\Delta = (-2)^2 - 4(5)(1)$$

$$= -16 \quad \therefore \Delta < 0$$

\therefore roots are imaginary / non-real

CHECK: $(7x+4)(x-1)$

$$x = 1 \text{ or } x = -\frac{4}{7}$$

(b) $7x^2 - 3x - 4 = 0$

$$\Delta = (-3)^2 - 4(7)(-4)$$

$$= 121$$

$\therefore \Delta$ is positive perfect square

\therefore roots are real, rational and unequal

CHECK: $(2x-3)(2x-3)$

$$x = \frac{3}{2}$$

(c) $4x^2 - 12x + 9 = 0$

$$\Delta = (-12)^2 - 4(4)(9)$$

$$= 0$$

\therefore roots are real, rational & equal

CHECK: $\frac{-5 \pm \sqrt{5^2 - 4(3)(1)}}{2(3)}$

(d) $3x^2 + 5x + 1 = 0$

$$\Delta = 5^2 - 4(3)(1)$$

$$= 13$$

$\therefore \Delta$ is positive but not a perfect square $x = -0.23$ or $x = -1.43$

\therefore roots are real, irrational, unequal