HASTINGS MATH

EUCLIDEAN GEOMETRY

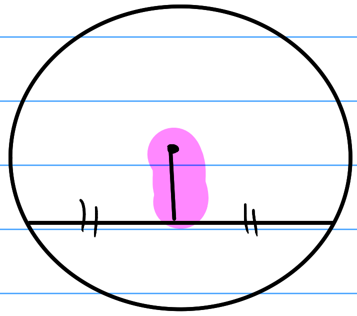
CIRCLE GEOMETRY

Including the following:

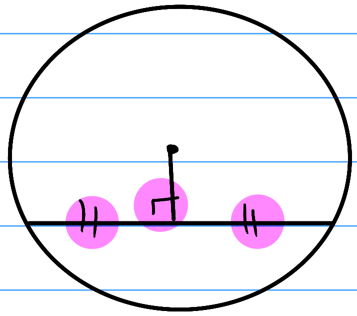
SUMMARY OF CIRCLE THEOREMS

Theorem 1

[line from centre to mdpt chord]



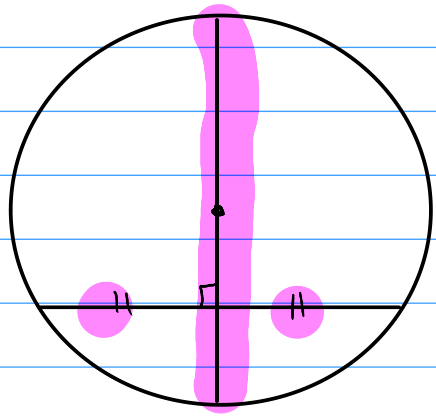
if line from centre
cuts the chord in half
then the line is a radius



if radius meets the
chord at 90°
then the radius
cuts the chord in half

Theorem 2

[line from centre to midpoint chord]



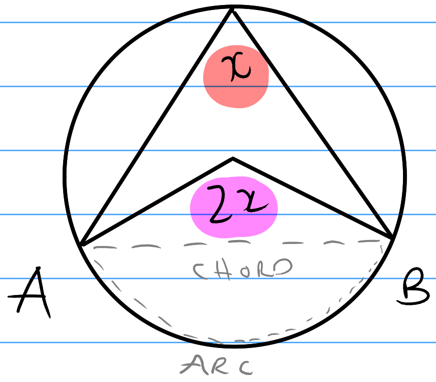
if line from centre cuts the chord in half
then the line is a diameter

and

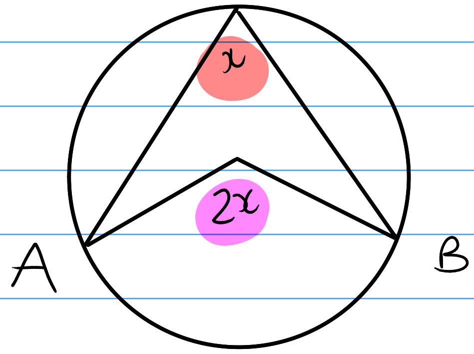
if the diameter meets the chord at 90°
then the radius cuts the chord in half

Theorem 3

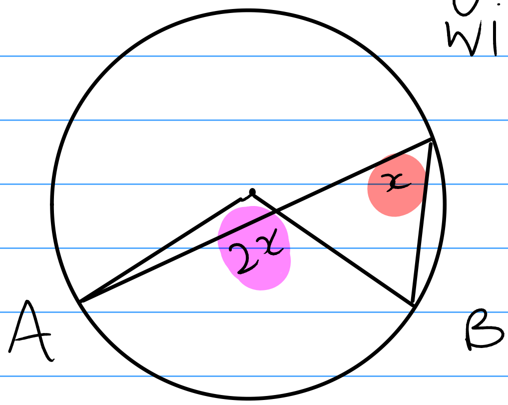
[< at centre = 2x < at circ.]



If the angle at the centre subtended by arc / chord AB then the angle subtended to the circumference of the circle by the same arc / chord AB will be half the angle at the centre



If the angle at the circumference is subtended by the chord / arc AB then the angle subtended to the centre of the circle by the same chord / arc AB will be double the angle at the circumference of the circle

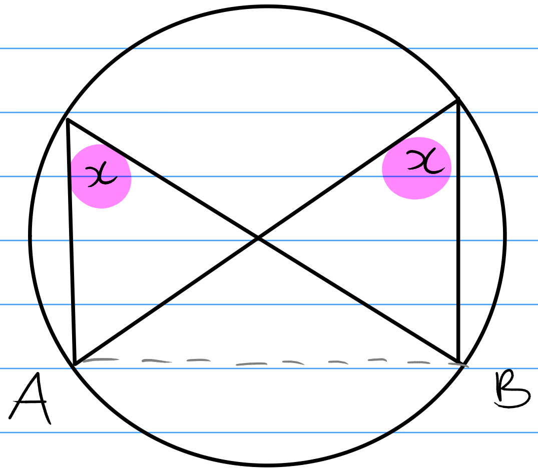


can look like this.

Theorem 4

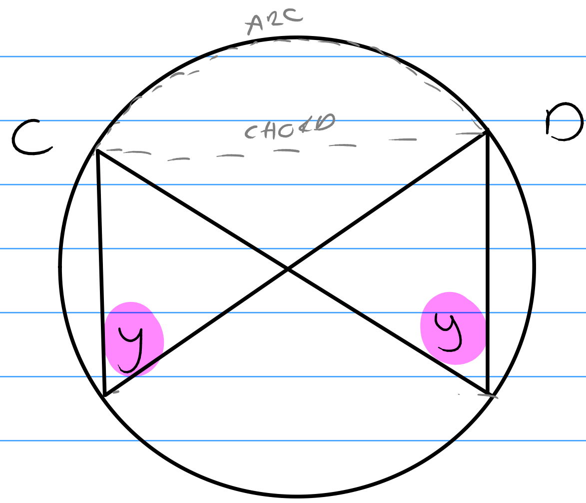
[<'s subtended by = chord]

NOTE: thrm 4 and thrm 5 are proof of cyclic quads.



if an angle is subtended by the same chord (AB or CD)

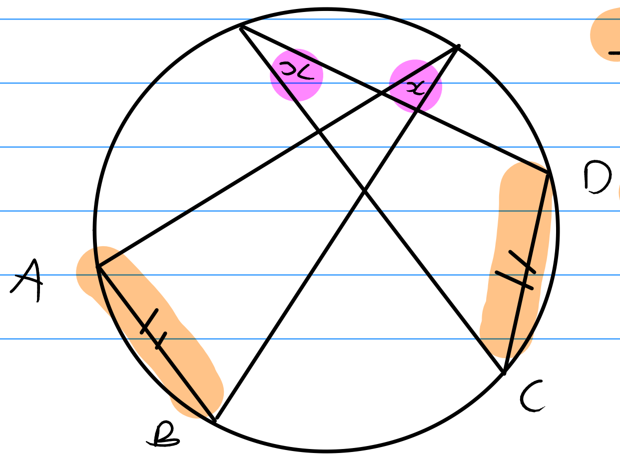
then any other angle subtended by the same chord will be equal to that angle



NOTE: Angles can be subtended by a chord or an arc

another application:

[= chords, subtend = <'s]

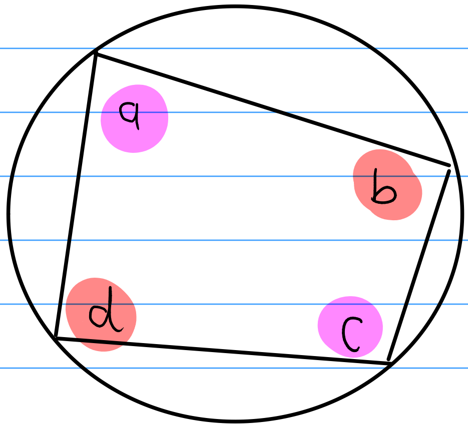


If: the chords are equal in length

then: the angles subtended by the chords (in the same circle) will be equal.

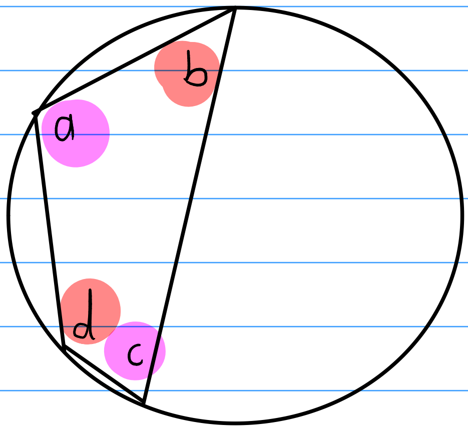
Theorem 5

[opp. <'s cyclic quad. supp.]



if: Opposite angles are part of a cyclic quadrilateral (all four points in the same circle)

then: the angles will be supplementary ($= 180°$)



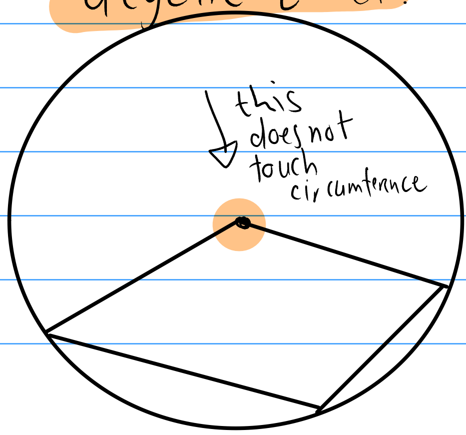
if: opposite angles add up to $180°$ (supplementary)

then: the shape is a cyclic quad

$a + c = 180°$
$b + d = 180°$
$a + b + c + d = 360°$

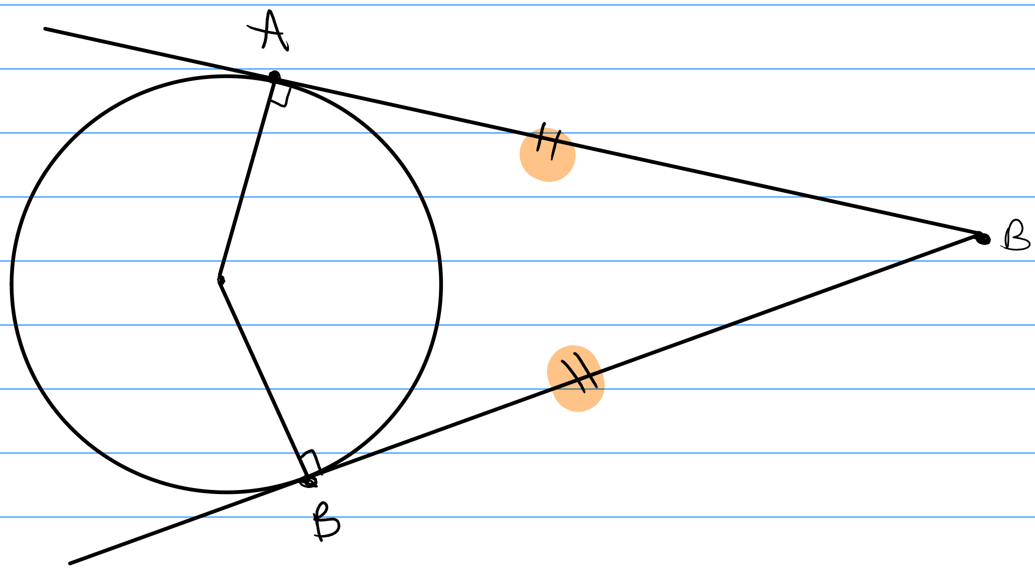
NOTE:

This is NOT a cyclic quad.



Theorem 6

[lines of tangent that meet at single point = length]

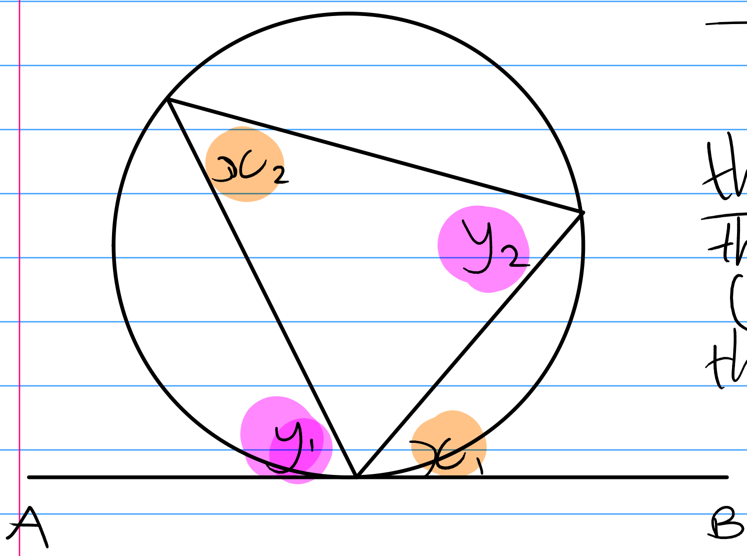


if: AB and CB are tangents to the circle and they meet at point B

then: AB and CB are equal in length

Theorem 7

[tan - chord thrm]



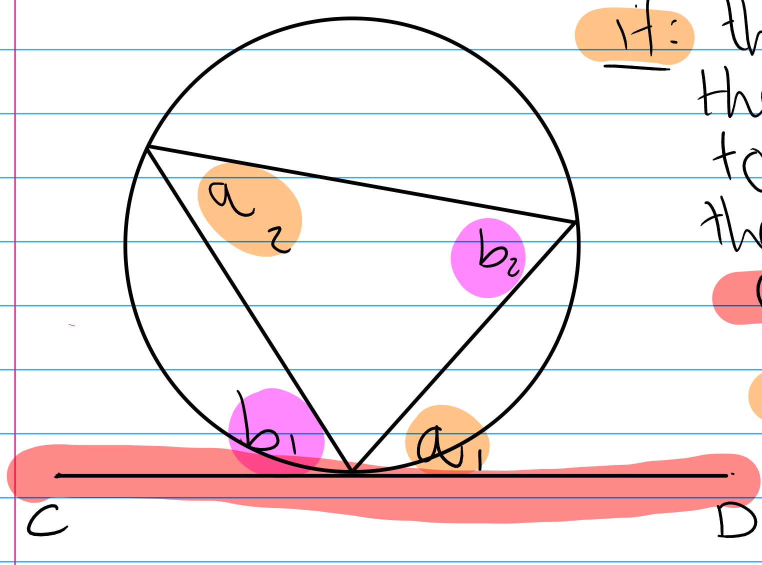
<u>if</u>: AB is a tangent to the circle

<u>then</u>: the angle between the chord and the tangent (x_1) will be equal to the angle opposite that same chord (x_2)

NOTE: y_1 is also $= y_2$

[converse tan - chord thrm]

→ this is the reverse of the tan-chord thrm.

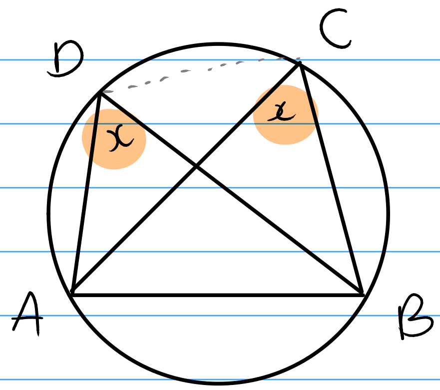


<u>if</u>: the angle opposite the chord is equal to the angle between the chord and the line CD

then: CD is a tangent to the circle

3 WAYS TO PROVE CYCLIC QUADRILATERALS

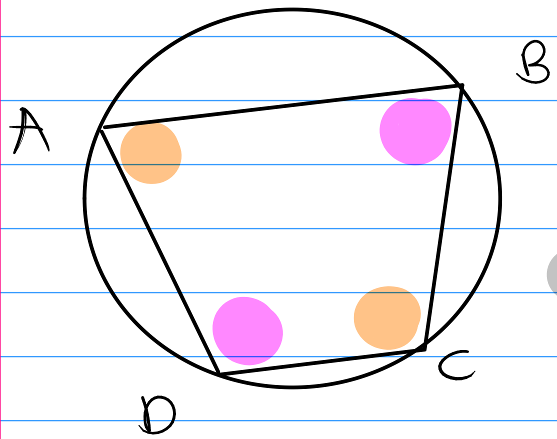
1. [<'s subtended by = chord]



if: $\hat{D} = \hat{C}$

then: ABCD is a cyclic quadrilateral

2. [opp <'s of cyclic quad. supp]



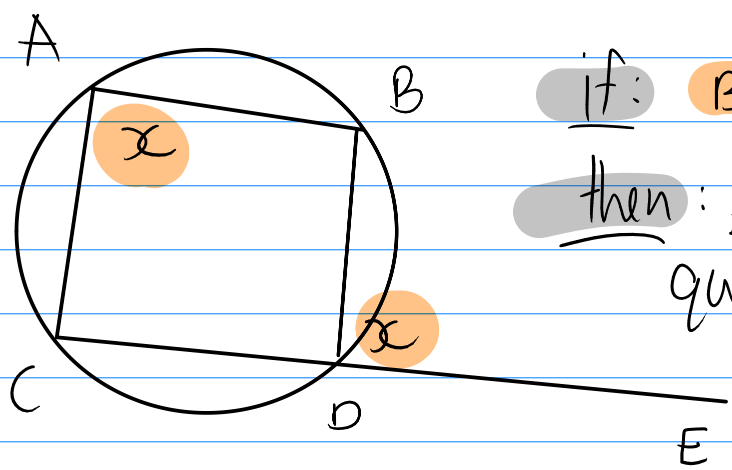
if $\hat{A} + \hat{C} = 180$

or

$\hat{B} + \hat{D} = 180$

then: ABCD is a cyclic quadrilateral

3. [ext. <'s of cyclic quad = opp. int. <]

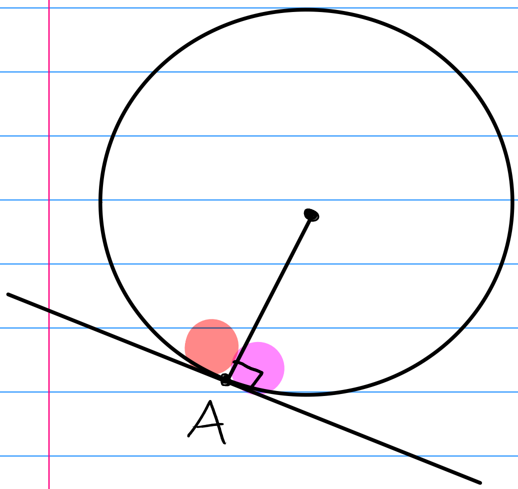


if: $B\hat{D}E = \hat{A}$

then: ABCD is a cyclic quadrilateral

OTHER

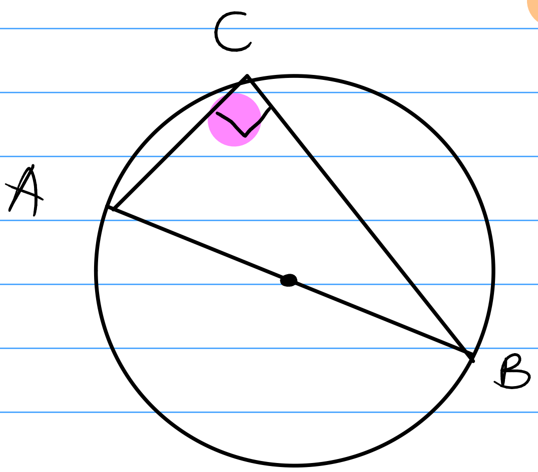
1. [radius ⊥ tangent]



if: a tangent meets a radius

then: the angle between the tangent and the radius (or diameter) is 90°

2. [< on diameter]



if: AB is the diameter

then: $\hat{C}$ will be 90°

this is because [< at centre = 2x < at circ]
∵ diameter = 180°