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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

Annotated by Tam.

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2022

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x-6)(x+2)=0$ (2)

1.1.2 $2x^2-6x+1=0$ (correct to TWO decimal places) (3)

1.1.3 $x^2-90>x$ (4)

1.1.4 $x-7\sqrt{x}=-12$ (4)

1.2 Solve for x and y simultaneously:

$2x-y=2$
$xy=4$ (5)

1.3 Show that $2.5^n-5^{n+1}+5^{n+2}$ is even for all positive integer values of n. (3)

1.4 Determine the values of x and y if: $\dfrac{3^{y+1}}{32}=\sqrt{96^x}$ (4)

[25]

QUESTION 2

2.1 The first term of a geometric series is 14 and the 6^{th} term is 448.

2.1.1 Calculate the value of the constant ratio, r. (2)

2.1.2 Determine the number of consecutive terms that must be added to the first 6 terms of the series in order to obtain a sum of 114 674. (4)

2.1.3 If the first term of another series is 448 and the 6^{th} term is 14, calculate the sum to infinity of the new series. (3)

2.2 If $\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$, determine the value of k. (5)

[14]

Question ①

1.1.1 $(3x-6)(x+2)=0$

$3x-6=0$ or $x+2=0$

$x=\frac{6}{3}$ $\qquad$ $x=-2$ ✓

$x=2$ ✓

1.1.2 $2x^2-6x+1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(1)}}{2(2)}$$

$x=2,82$ or $x=0.18$ ✓ $\qquad$ (2dec pl)

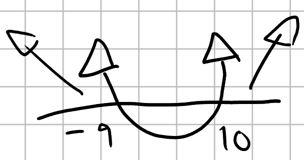
1.1.3 $x^2-90>x$

$x^2-x-90>0$

$(x+9)(x-10)>0$

Cr.Val: $x+9=0$ or $x-10=0$

$x=-9$ $\qquad$ $x=10$



$\therefore$ $x<-9$ OR $x>10$ ✓

1.1.4 $x-7\sqrt{x}=-12$

$x^1-7x^{\frac{1}{2}}+12=0$

$(x^{\frac{1}{2}}-3)(x^{\frac{1}{2}}-4)=0$

$\therefore x^{\frac{1}{2}} - 3 = 0$ or $x^{\frac{1}{2}} - 4 = 0$

$\left(x^{\frac{1}{2}}\right)^{\frac{2}{1}} = (3)^{\frac{2}{1}}$ $\quad\quad$ $\left(x^{\frac{1}{2}}\right)^{\frac{2}{1}} = (4)^{\frac{2}{1}}$

$x = 3^2 = 9$ $\quad\quad$ $x = 4^2 = 16$ ✓

1.2 $2x - y = 2$ | $xy = 4$

$y = 2x - 2 \quad \ldots (1)$ | $y = \frac{4}{x} \quad \ldots (2)$

let (1) = (2)

$\frac{4}{x} = 2x - 2$

$0 = 2x^2 - 2x - 4$

$0 = 2(x+1)(x-2)$

$(x+1) = 0$ $\quad\quad$ $x - 2 = 0$

$x = -1$ $\quad\quad$ $x = 2$

sub into (2)

$y = \frac{4}{-1}$ or $y = \frac{4}{2}$

$y = -4$ $\quad\quad$ $y = 2$ ✓

1.3 $2.5^n - 5^{n+1} + 5^{n+2}$

$= 5^n(2^1 - 5^1 + 5^2)$

$= 5^n(22)$

RTP even for all
→ pos/neg whole numbers that n can be

let $n=1$

$S^{1}(22) = 110$ even

let $n=-1$

$S^{-1}(22) = 4.4$ evem

$\therefore$ Multiply by any integer and the answer will be even.

1.4 $\dfrac{3^{y+1}}{32} = \sqrt{96^{x}}$

$\dfrac{3^{y+1}}{2^{5}} = \sqrt{(2^{5}\times 3)^{x}}$

$3^{y+1} \times 2^{-5} = 2^{5\times\frac{1}{2}\times x} \times 3^{\frac{1}{2}\times x}$

$3^{y+1} \times 2^{-5} = 3^{\frac{x}{2}} \times 2^{\frac{5x}{2}}$ same base 3.2

base 2 base 3

$\therefore -5 = \frac{5x}{2}$ or $y+1 = \frac{x}{2}$

$\frac{-10}{5} = \frac{5x}{5}$ $\to$ sub $x=-2$

$y+1 = -\frac{2}{2}$

$x=-2$

$y=-2$

QUESTION 2

2.1.1

$T_1 = a = 14$

$T_6 = 448 = ar^5$

$\frac{448}{14} = \frac{14r^5}{14}$

$r = \sqrt[5]{32}$

$r = 2$ ✓

... geometric

$\therefore T_n = ar^{n-1}$

2.1.2

$S_6 + a = 114674$

$S_6 = \frac{a(r^n - 1)}{r-1}$

... S_n formula geometric

$= \frac{14(2^6 - 1)}{2-1}$

$= 882$

$S_6 + S_x = 114674$

$882 + S_x = 114674$

$S_x = 114674 - 882$

$S_x = 113792$

$S_x = \frac{a(r^n - 1)}{r-1}$

$113792 = \frac{14(2^x - 1)}{2-1}$

$8129 = 2^x$

$x = \log_2 8129$

$= 12.98886217$

$x \approx 13$

$13 - 6 = 7$

$\therefore$ 7 more terms must be added ✓

2.1.3 $T_1 = a = 448$ $\quad\quad$ $T_6 = 14$

$$S_\infty = \frac{a}{1-r}$$

$T_6 = ar^5$

$\therefore 14 = 448r^5$

$$\sqrt[5]{\frac{14}{448}} = r$$

$r = \frac{1}{2}$

$$\therefore S_\infty = \frac{448}{1-\frac{1}{2}}$$

$= 896$ ✓

2.2 $\sum_{p=0}^{k} \left(\frac{1}{3}p + \frac{1}{6}\right) = 20\frac{1}{6}$ $\quad\quad$ $k = ?$

k is final p

$a = T_1 : \frac{1}{3}(0) + \frac{1}{6} = \frac{1}{6}$

$T_2 : \frac{1}{3}(1) + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6}$ $+\frac{2}{6}$

$T_3 : \frac{1}{3}(2) + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$ $+\frac{2}{6}$ $\therefore$ arithmetic

$\therefore d = \frac{2}{6} = \frac{1}{3}$

$S_n = 20\frac{1}{6}$ ...given

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ ... arithmetic S_n

$\therefore 20\frac{1}{6} = \frac{n}{2}\left(2\left(\frac{1}{6}\right) + (n-1)\left(\frac{1}{3}\right)\right)$

$\frac{121}{6} \times 2 = n\left[\left(\frac{2}{6}\right) + \frac{1}{3}n - \frac{1}{3}\right]$

$\frac{121}{3} = \frac{1}{3}n + \frac{1}{3}n^2 - \frac{1}{3}n$

$\frac{121}{3} = \frac{1}{3}n^2$

$\frac{121}{3} \div \frac{1}{3} = n^2$

$n = \pm\sqrt{121}$

$= \pm 11$

n is pos $\therefore n = 11$

but p starts at 0 ; $\therefore k = 10$ ✓

QUESTION 3

It is given that the general term of a quadratic number pattern is $T_n = n^2 + bn + 9$ and the first term of the first differences is 7.

3.1 Show that $b = 4$. (2)

3.2 Determine the value of the 60^{th} term of this number pattern. (2)

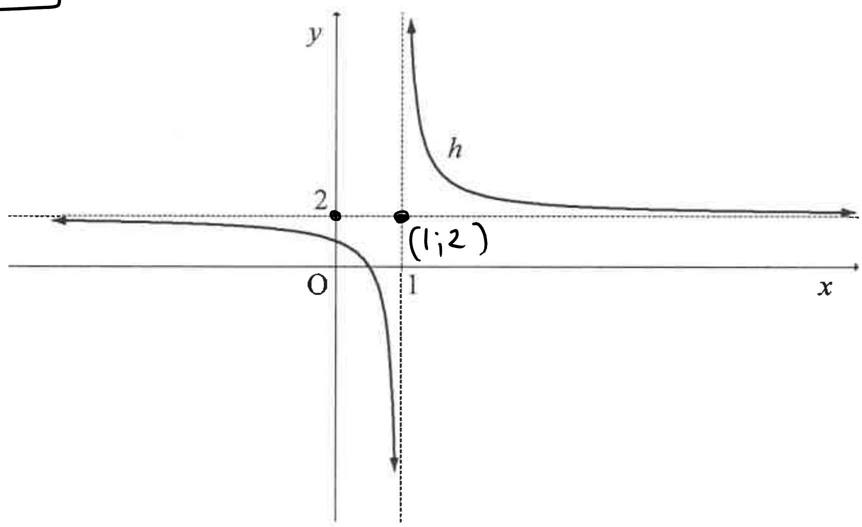
3.3 Determine the general term for the sequence of first differences of the quadratic number pattern. Write your answer in the form $T_p = mp + q$. T first diff (3)

3.4 Which TWO consecutive terms in the quadratic number pattern have a first difference of 157? (3)

[10]

QUESTION 4

4.1 Sketched below is the graph of $h(x) = \dfrac{1}{x+p} + q$. The asymptotes of h intersect at (1 ; 2).



4.1.1 Write down the values of p and q. (2)

4.1.2 Calculate the coordinates of the x-intercept of h. (2)

4.1.3 Write down the x-coordinate of the x-intercept of g if $g(x) = h(x+3)$. (2)

4.1.4 The equation of an axis of symmetry of h is $y = x + t$. Determine the value of t. (2)

4.1.5 Determine the values of x for which $-2 \leq \dfrac{1}{x-1}$. (3)

QUESTION 3

3.1 $T_n = n^2 + 4n + 9$ ... quadratic

1st diff : 7; ....

RTP: $b = 4$

2 T_2 T_3

7 ○ ○

○ = ○

$c = 9$ ...given

$a = 1$ given

$2(1) = 2$nd diff

don't need

$3(1) + b = 7$

$b = 4$ ✓

$a + b + c = T_1$

$a + b + 9 =$

don't need.

3.2 $T_{60} = (60)^2 + 4(60) + 9$

$= 3849$ ✓

3.3 $T_p = mp + q$ $T_n = n^2 + 4n + 9$

$2(1) = 2$nd diff

$= 2$

$3a + b =$

$b = 4$

$(1) + (4) + 9 = T_1$

$14 = T_1$

$\therefore$ 14; 21; 30; 41

7 + 9 + 11 + 13 ... T_p

2 + 2 + 2

$T_p: 7; 9; 11; 13$

$\underbrace{7;9}_{+2}\underbrace{;11}_{+2}\underbrace{;13}_{+2}$ $\therefore$ linear

$T_n = a + (n-1)d$

$= 7 + (n-1)2$

$= 7 + 2n - 2$

$= 2n + 5$ ✓

3.4 Consecutive terms

$\underbrace{T_a \quad T_b}_{157}$ ... 1st diff

RTP which n?

$T_p = 2n+5$

$\therefore 157 = 2n+5$

$\frac{157-5}{2} = n$

$n = 76$

$\therefore$ between the 76th and 77th term ✓

QUESTION 4

4.1.1 $p=-1\ ;\ q=2$ ✓ ... by inspection of asymptotes

4.1.2 $h(x)=\frac{1}{x-1}+2$

x int let $y=0$

$0=\frac{1}{x-1}+2$

$-2(x-1)=1$

$-2x+2=1$

$x=\frac{-1}{-2}$

$x=\frac{1}{2}$

$\therefore\left(\frac{1}{2};0\right)$ ✓

4.1.3 $g(x)=h(x+3)$

$=\frac{1}{(x+3)-1}+2$

$h(x)=\frac{1}{x+2}+2$

let $y=0$

$-2(x+2)=1$

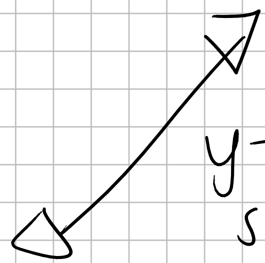
$-2x=5$ → $x=-\frac{5}{2}$ ✓

4.1.4 $y = x + t$

axis of symmetry

sub point of intersection of asymptotes of h

$(1;2)$



$y = x$ shifted

$2 = 1 + t$

$t = 1$ ✓

4.1.5

$$-2 \leq \frac{1}{x-1}$$

$$0 \leq \frac{1}{x-1} + 2$$

$$\frac{0}{(x-1)} \leq \frac{1}{x-1} + \frac{2(x-1)}{(x-1)}$$

$x - 1 \neq 0$

$x \neq 1$

$$0 \leq \frac{1 + 2x - 2}{x-1}$$

$$0 \leq \frac{2x-1}{x-1}$$

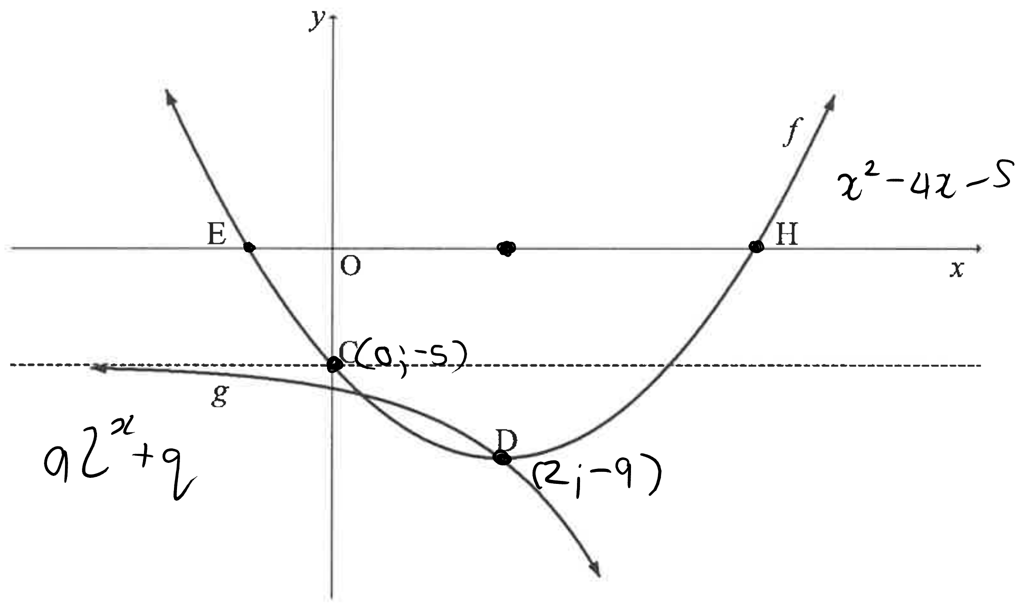
$0 \leq 2x - 1$ or $0 \leq x - 1$

$\frac{1}{2} \leq x$ $\qquad$ $1 < x$

✓

4.2 The graphs of $f(x) = x^2 - 4x - 5$ and $g(x) = a.2^x + q$ are sketched below.

- E and H are the x-intercepts of f.
- C is the y-intercept of f and lies on the asymptote of g.
- The two graphs intersect at D, the turning point of f.



4.2.1	Write down the y-coordinate of C.	(1)
4.2.2	Determine the coordinates of D.	(2)
4.2.3	Determine the values of a and q.	(3)
4.2.4	Write down the range of g.	(1)
4.2.5	Determine the values of k for which the value of $f(x) - k$ will always be positive.	(2)
		[20]

4.2.1 C: let $x=0$; y-int

$f(0) = (0)^2 - 4(0) - 5$

$\therefore C(0;-5)$

$\therefore y = -5$

4.2.2 D: turning point of f

$\therefore$ x is given by $\frac{-b}{2a}$

$x = \frac{-(-4)}{2(1)} = 2$

$f(2) = 2^2 - 4(2) - 5$

$= 4 - 8 - 5$

$= -9$

$\therefore D(2;-9)$

4.2.3 g: asymptote is $y = -5$

$\therefore q = -5$

sub $D(2;-9)$ int $a2^2 - 5$

$-9 = a2^{(2)} - 5$

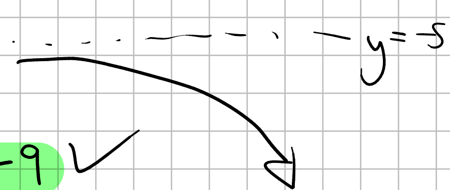
$\frac{-9+5}{4} = a$

$-\frac{4}{4} = a$

$a = -1$

$\therefore g(x) = -2^x - 5$ ✓

4.2.4 $y < -5$; $y \in R$ ✓

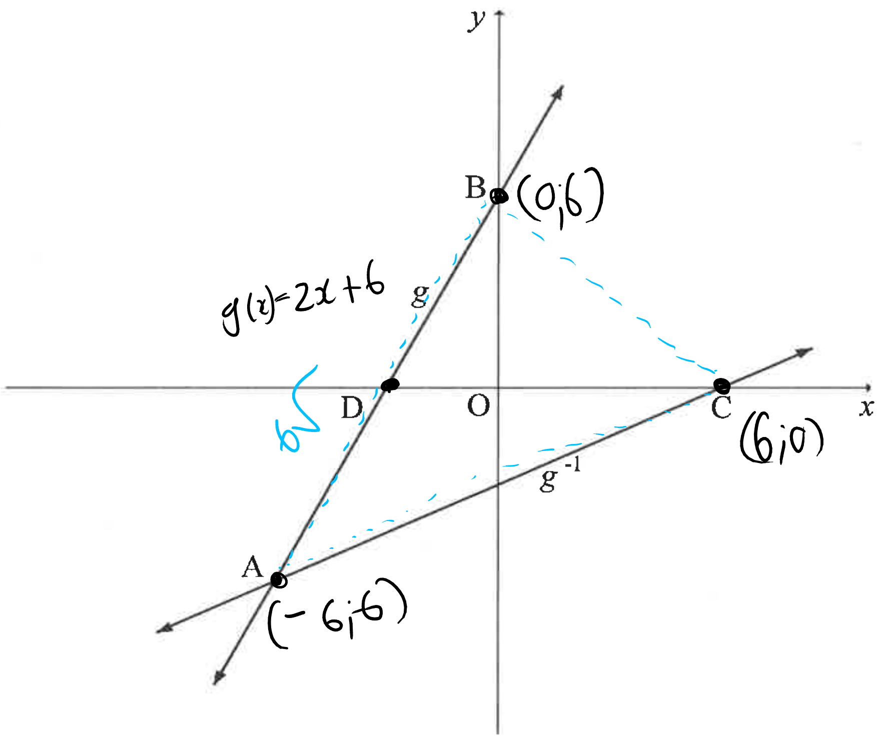


4.2.5 $f(x) - k$ $\therefore$ vertical shift $k < -9$ ✓

QUESTION 5

The graphs of $g(x) = 2x + 6$ and g^{-1}, the inverse of g, are shown in the diagram below.

- D and B are the x- and y-intercepts respectively of g.
- C is the x-intercept of g^{-1}.
- The graphs of g and g^{-1} intersect at A.



5.1 Write down the y-coordinate of B. (1)

5.2 Determine the equation of g^{-1} in the form $g^{-1}(x) = mx + n$. (2)

5.3 Determine the coordinates of A. (3)

5.4 Calculate the length of AB. (2)

5.5 Calculate the area of ΔABC. (5)
[13]

QUESTION 5

5.1 y-int; let $x=0$

$$g(0) = 2(0)+6$$

$\therefore B(0;6)$

$\therefore y = 6$ ✓

5.2 $g(x) = 2x+6$

$g^{-1}(x)$: $x = 2y+6$ ... switch x/y

$$x-6 = 2y$$

$$\frac{x}{2} - \frac{6}{2} = y$$ ... make y subject

$$y = \frac{1}{2}x - 3$$ ✓

5.3 A: $\frac{1}{2}x - 3 = 2x+6$

$$-3-6 = 2x - \frac{1}{2}x$$

$$-9 = \frac{3}{2}x$$

$$-9 \times \frac{2}{3} = x$$

$$x = -6$$

$$y = 2(-6)+6$$

$$= -6$$

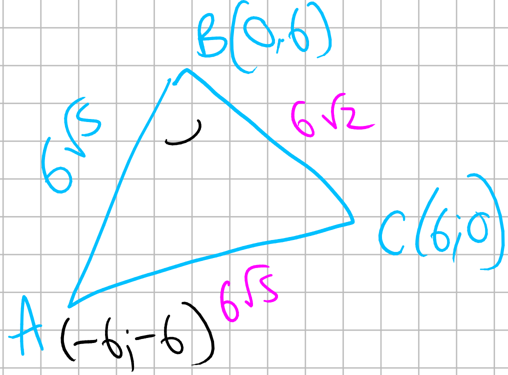
$\therefore A(-6;-6)$ ✓

5.4 $d_{AB} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$= \sqrt{(-6-0)^2+(-6-6)^2}$$

$= 6\sqrt{5} = 13{,}42$ ✓ (2 dec pl)

5.5 $A = \frac{1}{2}(a)(b)\sin\hat{C}$



$$A = \frac{1}{2}(AB)(BC)\sin\hat{B}$$

$$A = \frac{1}{2}(6\sqrt{5})(BC)\sin\hat{B}$$

$$d_{BC} = \sqrt{(0-6)^2+(6-0)^2}$$
$$= \sqrt{(-6)^2+(6)^2}$$
$$= 6\sqrt{2}$$

$$d_{AC} = \sqrt{(-6-6)^2+(-6-0)^2}$$
$$= 6\sqrt{5}$$

$$a^2 = b^2 + c^2 - 2(b)(c)\cos A$$

$$(6\sqrt{5})^2 = (6\sqrt{5})^2 + (6\sqrt{2})^2 - 2(6\sqrt{5})(6\sqrt{2})\cos\hat{B}$$

$$\frac{(6\sqrt{5})^2 - (6\sqrt{5})^2 - (6\sqrt{2})^2}{-2(6\sqrt{5})(6\sqrt{2})} = \cos\hat{B}$$

$$\cos\hat{B} = \frac{\sqrt{10}}{10}$$

$$\hat{B} = \cos^{-1}\left(\frac{\sqrt{10}}{10}\right)$$
$$= 71,56505118$$

$$\therefore \text{Area} = \frac{1}{2}(6\sqrt{5})(6\sqrt{2})\sin 71.56505118$$
$$= 54 \text{ units}^2$$

QUESTION 6

6.1 R12 000 was invested in a fund that paid interest at $m\%$ p.a., compounded quarterly. After 24 months, the value of the investment was R13 459.

Determine the value of m. (4)

6.2 On 31 January 2022, Tino deposited R1 000 in an account that paid interest at 7,5% p.a., compounded monthly. He continued depositing R1 000 on the last day of every month. He will make the last deposit on 31 December 2022.

Will Tino have sufficient funds in the account on 1 January 2023 to buy a computer that costs R13 000? Justify your answer by means of an appropriate calculation. (4)

6.3 Thabo plans to buy a car that costs R250 000. He will pay a deposit of 15% and take out a loan for the balance. The interest on the loan is 13% p.a., compounded monthly.

6.3.1 Calculate the value of the loan. (1)

6.3.2 The first repayment will be made 6 months after the loan has been granted. The loan will be repaid over a period of 6 years after it has been granted. Calculate the MONTHLY instalment. (5)

[14]

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x)=x^2+x$. (5)

7.2 Determine $f'(x)$ if $f(x)=2x^5-3x^4+8x$. (3)

7.3 The tangent to $g(x)=ax^3+3x^2+bx+c$ has a minimum gradient at the point $(-1\,;-7)$. For which values of x will g be concave up? (4)

[12]

QUESTION 6

6.1 $P = 12000$ $\quad m\%$ p.a. c.q

24 months later $\quad A = 13459$

$$A = P\left(1+\frac{i}{m}\right)^{n \times m}$$

$$13459 = 12000\left(1+\frac{m\%}{4}\right)^{2\times 4}$$

$$\frac{13459}{12000} = \left(1+\frac{m\%}{4}\right)^{8}$$

$$\left[\sqrt[8]{\frac{13459}{12000}} - 1\right] \times 4 = m\%$$

$$\therefore m\% = 0.057784\ldots$$

$$\therefore m = 5.78\%$$ ✓ (2 dec. pl)

6.2

31 Jan '22

+ 1000 monthly

7.5% p.a. c.m

12 months

31 Dec '22

fv (savings)

1 Jan 2023 $\quad$ 13000 or more?

$$Fv = \frac{x\left[\left(1+\frac{i}{m}\right)^{n\times m} - 1\right]}{\frac{i}{m}}$$

$$Fv = \frac{1000\left[\left(1+\frac{0.075}{12}\right)^{12}-1\right]}{\frac{0.075}{12}}$$

$= R \quad 12421,22$

$\therefore$ not enough $12421.22 < 13000$

6.3.1 R250000

deposit $15\% \times 250000 = 37500$

balance $250000 - 375000$

loan amount = R212500 ✓

6.3.2 $x = ?$ (monthly instalment

loan $\therefore Pv$ (6 years − 5 months) $= (12\times 6) - 5$

6 months after .. $\therefore$ delay start ?

interest for first 6 months: compound interest 13% c.m.

$A = 212500\left(1+\frac{0.13}{12}\right)^{5}$ → 5 months before first payment

$= R224262.5254\ldots$

$$\therefore 224262.5254 = \frac{x\left(1-\left(1+\frac{0.13}{12}\right)^{-((12\times 6)-5)}\right)}{\frac{0.13}{12}}$$

$$\frac{224262.5254 \times \frac{0.13}{12}}{\left(1-\left(1+\frac{0.13}{12}\right)^{-67}\right)} = x$$

$x = 4724.962642$

$\therefore R4724.96$ ✓

QUESTION 7

7.1

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - x^2 - x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + x + h - \cancel{x^2} - x}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h+1)}{\cancel{h}}$$

$\therefore f'(x) = 2x+1$ ✓ ... (sub $h=0$)

7.2 $f'(x) = 10x^4 - 12x^3 + 8$ ✓ ... use power rule

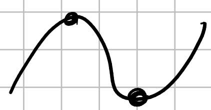
$f(x) = a^n$

$f'(x) = na^{n-1}$

7.3 $g(x) = ax^3 + 3x^2 + bx + c$

min $(-1;-7)$ concave up where?

* min / max : $f'(x) = 0$



(not this)

* min / max of tangent
* the gradient of tangent $= f'(x) = 3ax^2 + 6x + b$
* find min / max of first derivative of gradient of tangent

$\therefore f''(x) = 0$ ... 1st derivative of 1st derivative is 2nd derivative

$f''(x)$: $6ax + 6 = 0$

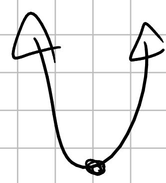
at $x = -1$: $f''(-1)$: $6a(-1) + 6 = 0$

$-6a = -6$

$\therefore a = 1$

$\therefore \quad f''(x) = 6(1)x + 6$

$\qquad\qquad = 6x + 6$



for concave up

$f''(x) > 0$

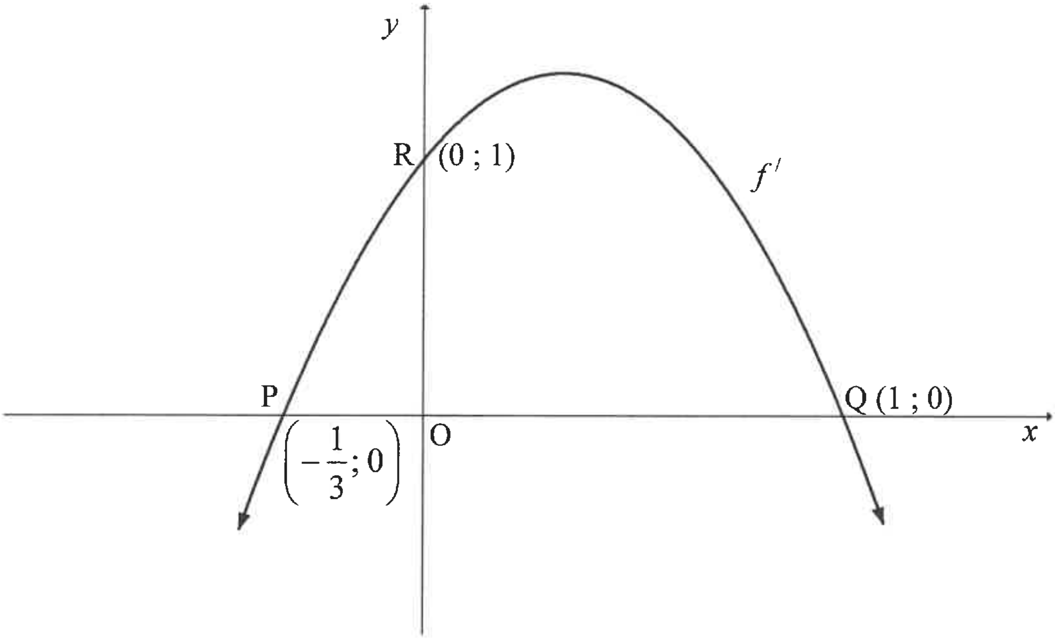
$\therefore \quad 6x + 6 > 0$

$\therefore \quad x > -1$

QUESTION 8

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.

The graph passes the points $P\left(-\frac{1}{3}\,;\,0\right)$, $Q(1\,;\,0)$ and $R(0\,;\,1)$.



8.1 Determine the values of m, n and k. (6)

8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:

8.2.1 Determine the coordinates of the turning points of f. (3)

8.2.2 Draw the graph of f. Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)

8.3 Points E and W are two variable points on f' and are on the same horizontal line.

- h is a tangent to f' at E.
- g is a tangent to f' at W.
- h and g intersect at $D(a\,;\,b)$.

8.3.1 Write down the value of a. (1)

8.3.2 Determine the value(s) of b for which h and g will no longer be tangents to f'. (2)

[17]

QUESTION 8

$y = f'(x) = mx^2 + nx + k$

8.1 $f'(x) = a(x+\frac{1}{3})(x-1)$ ...x-int form

sub $R(0;1)$

$1 = a(0+\frac{1}{3})(0-1)$

$1 = -\frac{1}{3}a$

$a = -3$

$\therefore f'(x) = -3(x+\frac{1}{3})(x-1)$

$= -3(x^2+\frac{1}{3}x - x - \frac{1}{3})$

$= -3(x^2 - \frac{2}{3}x - \frac{1}{3})$

$= -3x^2 + 2x + 1$

$\therefore m = -3 \ ; \ n = 2 \ ; \ k = 1$ ✓

8.2 $f(x) = -x^3 + x^2 + x + 2$

TP's $f'(x) = 0$

$\therefore -3x^2 + 2x + 1 = 0$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

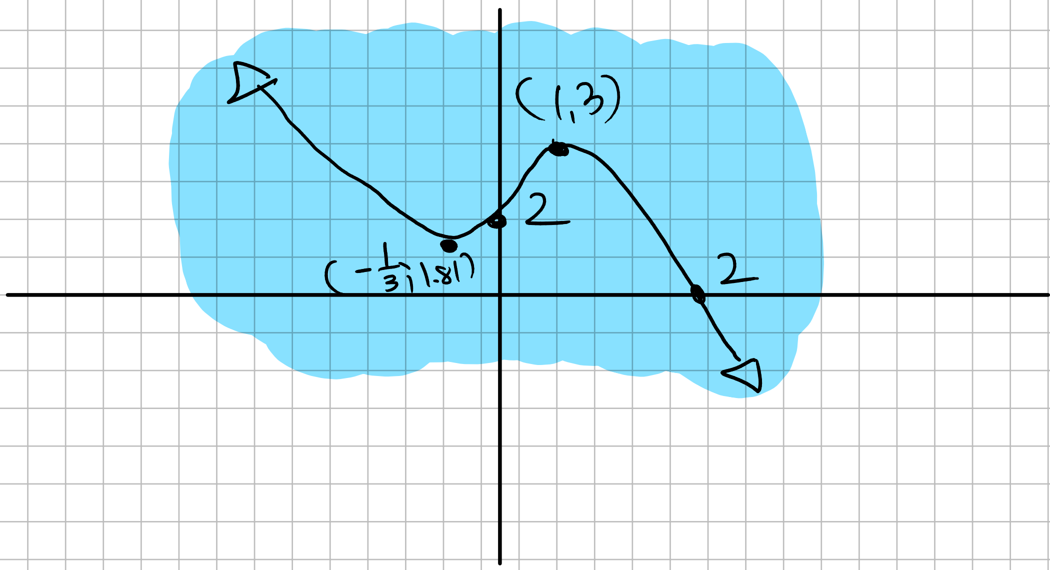
$x = -\frac{1}{3}$ or $x = 1$

y's: $f(-\frac{1}{3}) = -(-\frac{1}{3})^3 + (-\frac{1}{3})^2 + (-\frac{1}{3}) + 2 = \frac{49}{27}$

$f(1) = -(1)^3 + (1)^2 + (1) + 2 = 3$

$\therefore$ TP's $(-\frac{1}{3}; \frac{49}{27})$ & $(1;3)$

8.2.2



x-int ; let $y=0$

$$0 = -x^3 + x^2 + x + 2$$

$$0 = (x-2)(-x^2 - x - 1)$$ ... use tables on calculator to get first factor

for 2nd bracket:

middle term × ✓ $2x^2 \pm \_\_ = x^2$

first term $x \times -x^2 = -x^3$

3rd term $-2 \times -1 = +2$

$\therefore (x-2)(-x^2 - x - 1)$

$x = 2$ or $\quad x \neq \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(-1)(-1)}}{2(-1)}$

invalid / no solution

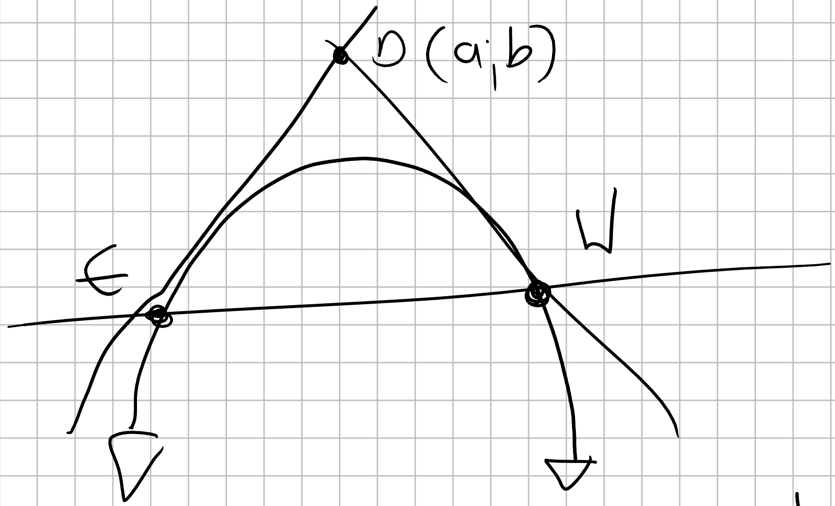
$\therefore x = 2$ only.

8.3 E & W on f'

h is tangent

g is tangent

intesect at D(a;b)



8.3.1

a: halfway between E & W : $\dfrac{-\frac{1}{3}+1}{2}$

$\therefore a = \dfrac{1}{3}$ ✓

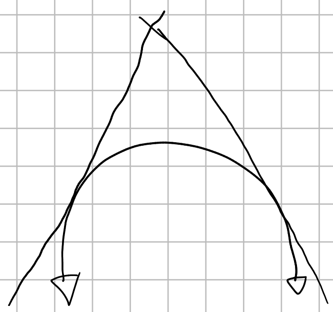
8.3.2 b when h and g are no longer tangents to f'

value of b now

$$f'\left(\tfrac{1}{3}\right) = -3\left(\tfrac{1}{3}\right)^2 + 2\left(\tfrac{1}{3}\right) + 1$$

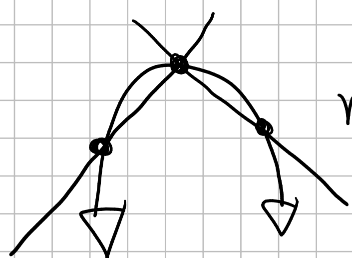
$$= \frac{4}{3}$$

$$b > \frac{4}{3}$$



Still tangent (only touch f' once each)

$\therefore b < \dfrac{4}{3}$ ✓



not tangents anymore

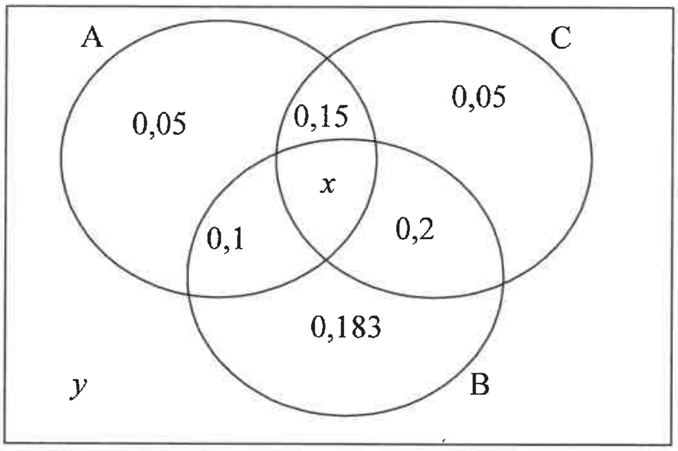
QUESTION 9

Given $f(x) = x^2$.

Determine the minimum distance between the point $(10 ; 2)$ and a point on f. **[8]**

QUESTION 10

10.1 A, B and C are three events. The probabilities of these events (or any combination of them) occurring is given in the Venn-diagram below



10.1.1 If it is given that the probability that at least one of the events will occur is 0,893, calculate the value of:

(a) y, the probability that none of the events will occur. (1)

(b) x, the probability that all three events will occur. (1)

10.1.2 Determine the probability that at least two of the events will take place. (2)

10.1.3 Are events B and C independent? Justify your answer. (5)

10.2 A four-digit code is required to open a combination lock. The code must be even-numbered and may not contain the digits 0 or 1. Digits may not be repeated.

10.2.1 How many possible 4-digit combinations are there to open the lock? (3)

10.2.2 Calculate the probability that you will open the lock at the first attempt if it is given that the code is greater than 5 000 and the third digit is 2. (5)

[17]

TOTAL: 150

QUESTION 9

$f(x) = x^2$

max distance $(x;y)$ & $(10;2)$

$$d = \sqrt{(10-x)^2 + (2-y)^2}$$

$$d^2 = (10-x)^2 + (2-y)^2$$

$$= 100 - 20x + x^2 + 4 - 4y + y^2$$

but $y = x^2$

$$\therefore 100 - 20x + x^2 + 4 - 4x^2 + x^4$$

$$= x^4 - 3x^2 - 20x + 104$$

max/min: $\frac{d^2}{dx} = 0$

$$\therefore \frac{d^2}{dx} : 4x^3 - 6x - 20 = 0$$

$$(x-2)(4x^2 + 8x + 10)$$

... use calc. tables for first factor

2nd bracket: ✗ ✓

middle term $-8x^2 + \underline{8x^2} = 0x^2$

first term $x \times 4x^2 = 4x^3$

3rd term $-2 \times 10 = -20$

$x = 2$ or $\quad x \neq \dfrac{-8 \pm \sqrt{8^2 - 4(4)(10)}}{2(4)}$

invalid / no solution

$\therefore x = 2$ only

sub $x=2$ into d^2

$d^2 = 2^4 - 3(2)^2 - 20(2) + 104$

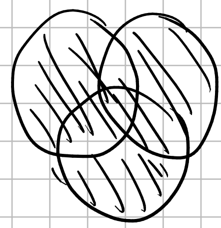
$= 68$

$\therefore d = \sqrt{68} = 2\sqrt{17} = 8.25$ units ✓

QUESTION 10

10.1.1 P(at least one) $= 0.893$

P(at least one) $= P(A \cup B \cup C)$



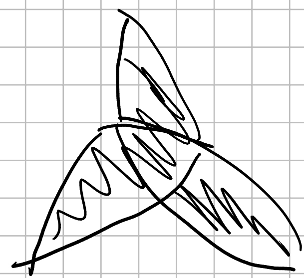
$\therefore$ (a) $y = 1 - 0.893$

$= 0.107$ ✓

(b) P(all 3 events) $= x$

$= 0.893 - 0.2 - 0.1 - 0.15 - 0.183 - 0.05 - 0.05$

$x = 0.16$ ✓



10.1.2 P(at least two)

$= 0.1 + 0.15 + 0.2 + 0.16$

$= 0.61$ ✓

10.1.3 independent events

$P(A) \times P(B) = P(A \cap B)$

$P(B) = 0.183 + 0.1 + 0.2 + 0.16$

$= 0.643$

$P(C) = 0.05 + 0.15 + 0.2 + 0.16$

$= 0.56$

$P(B \cap C) = 0.2 + 0.16 = 0.36$

$P(B) \times P(C) = 0.56 \times 0.643 = 0.36$

$\therefore$ B and C are independent events

$\therefore P(B \cap C) = P(B) \times P(C)$ ✓

10.2.1 4 digit even ; no 0 ; no 1 ; no repeat

refer to marksceme.

10.2.2 refer to marksceme

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r - 1}\ ; r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ; -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m\left(x - x_1\right)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$