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Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes | Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4
OCTOBER 2022

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

PARAMETRIC EQUATIONS

1. A curve C has parametric equations

$$x = \frac{t}{t-3} \qquad y = \frac{1}{t} + 2 \qquad t \in \mathbb{R} \qquad t > 3$$

Show that all points on C lie on the curve with Cartesian equation

$$y = \frac{ax-1}{bx}$$

where a and b are constants to be found. **(3)**

1.(a) $t = xt - 3x$ — make t subject
$t - xt = -3x$
$t(1-x) = -3x$
$t = \frac{-3x}{1-x}$
$t = \frac{3x}{x-1}$

$y = \frac{1}{t} + 2$ — sub t into y
$\therefore y = \frac{1}{\frac{3x}{x-1}} + 2$
$y = \frac{x-1}{3x} + 2$
$y = \frac{x-1+6x}{3x}$
$y = \frac{7x-1}{3x}$ ✓

PARTIAL FRACTIONS, INTEGRATION

2. (a) Express $\frac{3x}{(2x-1)(x-2)}$ in partial fraction form. **(3)**

(b) Hence show that

$$\int_5^{25} \frac{3x}{(2x-1)(x-2)}\,dx = \ln k$$

where k is a fully simplified fraction to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*) **(4)**

2(a) $\frac{3x}{(2x-1)(x-2)} = \frac{A}{(2x-1)} + \frac{B}{(x-2)}$

$3x = A(x-2) + B(2x-1)$
sub $x = 2$
$3(2) = 0 + B(2(2)-1)$
$6 = 3B$
$B = 2$
sub $x = \frac{1}{2}$
$3(\frac{1}{2}) = A(\frac{1}{2}-2) + 0$
$\frac{3}{2} = -\frac{3}{2}A$
$A = -1$
$\therefore \frac{-1}{2x-1} + \frac{2}{x-2}$ ✓

Question 2 continued

(b) $\frac{-1}{2x-1} + \frac{2}{x-2}$ — if $\frac{f'(x)}{f(x)}$ then $\ln|f(x)|$ adjust k

$\int_5^{25} \frac{-1}{2x-1} + \frac{2}{x-2}\,dx$

$= \left[\frac{-\ln|2x-1|}{2} + \frac{2\ln|x-2|}{1}\right]_5^{25}$

$= \left[-\frac{1}{2}\ln(2x-1) + 2\ln(x-2)\right]_5^{25}$

$= \left(-\frac{1}{2}\ln(2(25)-1) + 2\ln(25-2)\right) - \left(-\frac{1}{2}\ln(2(5)-1) + 2\ln(5-2)\right)$

$= \left(-\frac{1}{2}\ln 49 + 2\ln 23\right) - \left(-\frac{1}{2}\ln 9 + 2\ln 3\right)$

$= -\ln 49^{\frac{1}{2}} + 2\ln 23 + \ln 9^{\frac{1}{2}} - 2\ln 3$

$= -\ln 7 + 2\ln 23 + \ln 3 - 2\ln 3$

$= -\ln 7 + \ln 23^2 + \ln 3 - \ln 3^2$

$= \ln \frac{23^2 \times 3}{7 \times 3^2}$

$= \ln \frac{529}{21}$ ✓

(Total for Question 2 is 7 marks)

VECTORS, SCALAR PRODUCT

3.

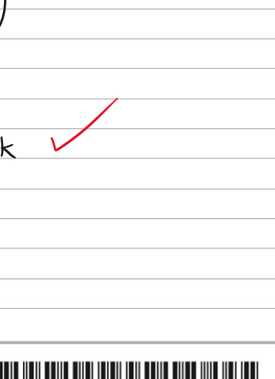


Figure 1

Figure 1 shows a sketch of triangle PQR.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
- $\overrightarrow{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

(a) Find $\overrightarrow{RQ}$ **(2)**

(b) Find the size of angle PQR, in degrees, to three significant figures. **(3)**

3(a) $\overrightarrow{RQ} = \overrightarrow{PQ} - \overrightarrow{RP}$
$= \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 8 \\ -5 \\ 3 \end{pmatrix}$
$= \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix}$
$= -6i + 2j + k$ ✓

Question 3 continued

SCALAR PRODUCT $a \cdot b = |a||b|\cos\theta$

$-\overrightarrow{PQ} \times -\overrightarrow{RQ}$
$= |PQ| \times |RQ| \cos P\hat{Q}R$

$\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \times \begin{pmatrix} -6 \\ 2 \\ 1 \end{pmatrix} = \sqrt{2^2+3^2+4^2} \times \sqrt{6^2+2^2+1^2} \times \cos P\hat{Q}R$

$-12 - 6 + 4 = \sqrt{29} \times \sqrt{41} \times \cos P\hat{Q}R$

$\cos^{-1}\left(\frac{-14}{\sqrt{29}\sqrt{41}}\right) = P\hat{Q}R$

$P\hat{Q}R = 113.9544\ldots$

$P\hat{Q}R = 114°$ (3sf) ✓

(Total for Question 3 is 5 marks)

BINOMIAL EXPANSION, APPROXIMATIONS

4.

$$g(x) = \frac{1}{\sqrt{4-x^2}}$$

(a) Find, in ascending powers of x, the first four non-zero terms of the binomial expansion of $g(x)$. Give each coefficient in simplest form. **(5)**

(b) State the range of values of x for which this expansion is valid. **(1)**

(c) Use the expansion from part (a) to find a fully simplified rational approximation for $\sqrt{3}$

Show your working and make your method clear. **(2)**

4(a) $(4-x^2)^{-\frac{1}{2}}$
$= \left[4\left(1-\frac{1}{4}x^2\right)\right]^{-\frac{1}{2}}$
$= 4^{-\frac{1}{2}}\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}}$

$(1-ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$

$\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}}$
$= 1 + \left(-\frac{1}{2}\right)\left(-\frac{1}{4}x^2\right)^1 + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(\frac{1}{4}x^2\right)^2 + \frac{(-\frac{1}{2})\times(-\frac{3}{2})\times(\frac{5}{2})}{3!}\left(-\frac{1}{4}x^2\right)^3$

$= 1 + \frac{1}{8}x^2 + \frac{3}{128}x^4 + \frac{5}{1024}x^6$

$\therefore \frac{1}{2}\left(1-\frac{1}{4}x^2\right)^{-\frac{1}{2}} = \frac{1}{\sqrt{4-x^2}}$
$= \frac{1}{2}\left(1 + \frac{1}{8}x^2 + \frac{3}{128}x^4 + \frac{5}{1024}x^6\right)$
$= \frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256} + \frac{5}{2048}x^6$ ✓

Question 4 continued

(b) $-1 < -\frac{1}{4}x^2 < 1$
$\therefore 4 > x^2 > -4$
$\sqrt{-2^2} < x^2 < \sqrt{2^2}$
$-2 < x < 2$
$|x| < 2$ ✓

(c) let $x = 1$ $(|x| < 2)$ → valid — what x gives LHS $\sqrt{3}$?

LHS RHS
$\frac{1}{\sqrt{4-x^2}} = \frac{1}{2} + \frac{1}{16}x^2 + \frac{3}{256}x^4 + \frac{5}{2048}x^6$

$\frac{1}{\sqrt{4-1^2}} = \frac{1}{2} + \frac{1}{16}(1)^2 + \frac{3}{256}(1)^4 + \frac{5}{2048}(1)^6$

$\frac{1}{\sqrt{3}} = \frac{1181}{2048}$

$\sqrt{3} = \frac{2048}{1181}$ ✓

(Total for Question 4 is 8 marks)

INTEGRATION, VOLUMES OF REVOLUTION

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

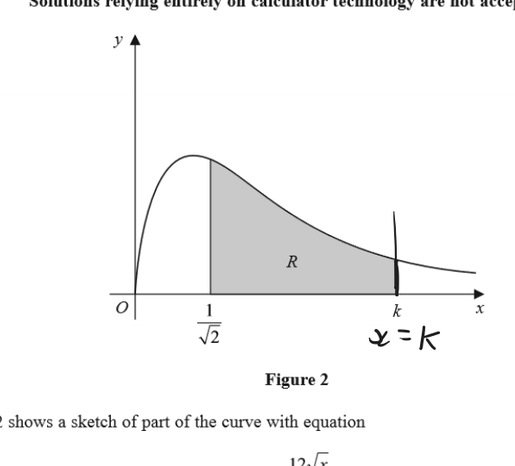


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2+3)^{1.5}}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{1}{\sqrt{2}}$, the x-axis and the line with equation $x = k$.

This region is rotated through 360° about the x-axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the exact value of the constant k. **(6)**

5. $V = \frac{713}{648}\pi$ $k = ?$

$V = \int_{\frac{1}{\sqrt{2}}}^{k} \pi\left(\frac{12\sqrt{x}}{(2x^2+3)^{1.5}}\right)^2 dx$

Question 5 continued

$V = \pi\int_{\frac{1}{\sqrt{2}}}^{k}\left(\frac{144x}{(2x^2+3)^3}\right)dx$

$= \pi\int_{\frac{1}{\sqrt{2}}}^{k} 144x(2x^2+3)^{-3}\,dx$

$= 144\pi\int_{\frac{1}{\sqrt{2}}}^{k} x(2x^2+3)^{-3}\,dx$

$= 144\pi\left[\frac{x(2x^2+3)^{-2}}{-2\times 4x}\right]_{\frac{1}{\sqrt{2}}}^{k}$

$= \left[\frac{144\pi}{-8}(2x^2+3)^{-2}\right]_{\frac{1}{\sqrt{2}}}^{k}$

$\left[-18\pi(2x^2+3)^{-2}\right]_{\frac{1}{\sqrt{2}}}^{k}$

$\left[-18\pi(2k^2+3)^{-2} - \left(-18\pi\left(2\left(\frac{1}{\sqrt{2}}\right)^2+3\right)^{-2}\right)\right.$

$\therefore \frac{-18\pi}{(2k^2+3)^2} + \frac{18\pi}{16} = \frac{713}{648}\pi$

$\therefore \frac{-18\pi}{(2k^2+3)^2} = \frac{713}{648}\pi - \frac{18\pi}{16}$

$\frac{-18}{(2k^2+3)^2} = -\frac{2}{81}$

Question 5 continued

$\frac{(2k^2+3)^2}{18} = \frac{81}{2} \times 18$

$(2k^2+3)^2 = 729$

$k = \sqrt{\frac{\sqrt{729} - 3}{2}}$

$k = 2\sqrt{3}$ ✓

Question 5 continued

(Total for Question 5 is 6 marks)

PARAMETRIC EQUATIONS, DIFFERENCIATION DOMAIN & RANGE

6.

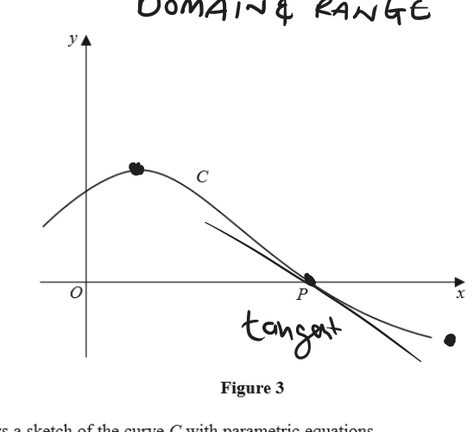


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 1 + 3\tan t \qquad y = 2\cos 2t \qquad -\frac{\pi}{6} \leqslant t \leqslant \frac{\pi}{3}$$

The curve crosses the x-axis at point P, as shown in Figure 3.

(a) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found. **(5)**

The curve C has equation $y = f(x)$, where f is a function with domain $[k, 1 + 3\sqrt{3}]$

(b) Find the exact value of the constant k. **(1)**

(c) Find the range of f. **(2)**

6(a) tangent $m = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

Question 6 continued

$\frac{dy}{dt} = -2\sin 2t \times 2 \qquad \frac{dx}{dt} = 3\sec^2 t$
$= -4\sin 2t \qquad \therefore \frac{dt}{dx} = \frac{1}{3\sec^2 t}$

$\therefore \frac{dy}{dt} = \frac{-4\sin 2t}{3\sec^2 t}$

at P; $y = 0$
$0 = 2\cos 2t$
$2t = \frac{\cos^{-1}(0)}{2}$
$t = \frac{\pi}{4}$

$\therefore x = 1 + 3\tan\frac{\pi}{4}$
$= 4$

$\therefore \frac{dy}{dt} = \frac{-4\sin 2(\frac{\pi}{4})}{\frac{3}{(\cos\frac{\pi}{4})^2}}$
$= -\frac{2}{3}$

Question 6 continued

$\therefore$ tangent:
$y - y_1 = m(x - x_1)$
$y - 0 = -\frac{2}{3}(x - 4)$
$y = -\frac{2}{3}x + \frac{8}{3}$ ✓

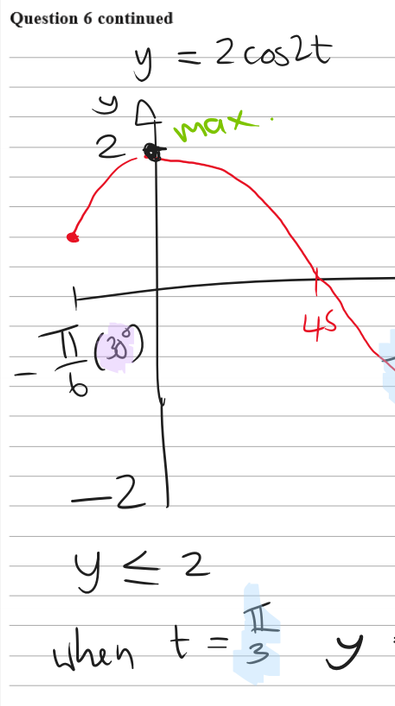
(b) k is x-value of c max/min
$-\frac{\pi}{6} \leq t \leq \frac{\pi}{3}$

$\therefore x = 1 + 3\tan\left(-\frac{\pi}{6}\right)$
$= 1 - \sqrt{3}$ ✓

(c) y's where
$-\frac{\pi}{6} \leq t \leq \frac{\pi}{3}$

Question 6 continued

$y = 2\cos 2t$



$y \leq 2$
when $t = \frac{\pi}{3}$ $y = 2\cos 2(\frac{\pi}{3})$
$\therefore y = -1$

$\therefore$ range: $-1 \leq y \leq 2$ ✓

(Total for Question 6 is 8 marks)

INTEGRATION, SUBSTITUTION

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Use the substitution $u = e^x - 3$ to show that

$$\int_{\ln 5}^{\ln 7} \frac{4e^{3x}}{e^x - 3}\,dx = a + b\ln 2$$

where a and b are constants to be found. **(7)**

(ii) Show, by integration, that

$$\int 3e^x \cos 2x\,dx = pe^x \sin 2x + qe^x \cos 2x + c$$

where p and q are constants to be found and c is an arbitrary constant. **(5)**

7(i) $u = e^x - 3$
$\frac{du}{dx} = e^x$
$\frac{dx}{du} = \frac{1}{e^x}$

in u: e^{3x}: $(u+3)^3 = (e^x)^3$
$e^x - 3 = u$, $e^x = u + 3$, $\frac{1}{e^x} = \frac{1}{u+3}$

$= \int_{u_2}^{u_1}\left(\frac{4(u+3)^3}{u}\right)\frac{dx}{du}\,du$

$= \int_{u_2}^{u_1}\left(\frac{4(u+3)^3}{u}\right)\left(\frac{1}{u+3}\right)du$

$= \int_{u_2}^{u_1}\left(\frac{4(u+3)^3}{u(u+3)}\right)du$

Question 7 continued

$= \int_{u_2}^{u_1} \frac{4(u+3)^2}{u}\,du$

$= \int_{u_2}^{u_1} \frac{4u^2 + 24u + 36}{u}\,du$

$= \int_{u_2}^{u_1} \left(4u + 24 + 36u^{-1}\right)du$

$= \left[\frac{4u^2}{2} + 24u + 36\ln u\right]_{u_2=2}^{u_1=4}$

$u_2 = e^{\ln 5} - 3 = 2 \qquad u_1 = e^{\ln 7} - 3 = 4$

$= \left(2(4)^2 + 24(4) + 36\ln 4\right) - \left(2(2)^2 + 24(2) + 36\ln 2\right)$

$= 32 + 96 + 36\ln 4 - 8 - 48 - 36\ln 2$

$= 72 + 36\ln 2^2 - 36\ln 2$

$= 72 + 72\ln 2 - 36\ln 2$

$= 72 + 36\ln 2$ ✓

Question 7 continued

(ii) $3\int e^x \cos 2x\,dx$

INTEGRATION BY PARTS
neither e^x nor $\cos 2x$
To avoid going in circles ..
(special case)
(rare)
O.I. method won't work.

$3\int e^x \cos 2x\,dx$
$\int u\,dv = uv - \int v\,du$

diff: $u = e^x$, $\frac{du}{dx} = e^x$ — int: $\frac{dv}{dx} = \cos 2x$, $v = \frac{\sin 2x}{2}$

Question 7 continued

$3\int e^x \cos 2x\,dx = 3\left[\frac{1}{2}e^x \sin 2x - \frac{1}{2}\int e^x \sin 2x\,dx\right]$

integrate by parts again

$\int 3e^x \cos 2x\,dx = \frac{3}{2}e^x \sin 2x - \frac{3}{2}\int e^x \sin 2x\,dx$

$u = e^x \qquad \frac{dv}{dx} = \sin 2x$
$u' = e^x \qquad v = -\frac{1}{2}\cos 2x$

$\int 3e^x \cos 2x\,dx = \frac{3}{2}e^x \sin 2x - \frac{3}{2}\left[-\frac{1}{2}e^x \cos 2x + \frac{1}{2}\int e^x \cos 2x\,dx\right]$

Keep LHS!

$\int 3e^x \cos 2x\,dx = \frac{3}{2}e^x \sin 2x + \frac{3}{4}e^x \cos 2x - \frac{3}{4}\int e^x \cos 2x\,dx$

and like terms

$\frac{15}{4}\int e^x \cos 2x\,dx = \frac{3}{2}e^x \sin 2x + \frac{3}{4}e^x \cos 2x$

make it look like original integral $\div \frac{15}{4} \times 3$ $\frac{3}{4} \times 3$

$\frac{5}{4}\int 3e^x \cos 2x\,dx = \frac{3}{2}e^x \sin 2x + \frac{3}{4}e^x \cos 2x$

$\int 3e^x \cos 2x\,dx = \frac{4}{5}\left(\frac{3}{2}e^x \sin 2x + \frac{3}{4}e^x \cos 2x\right) + C$

$= \frac{6}{5}e^x \sin 2x + \frac{3}{5}e^x \cos 2x + c$

$p = \frac{6}{5}$ $q = \frac{3}{5}$ ✓

(Total for Question 7 is 12 marks)

PROOF BY CONTRADICTION

8. A student was asked to prove by contradiction that

"there are no positive integers x and y such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the box below.

> Assume that integers x and y exist such that $3x^2 + 2xy - y^2 = 25$
>
> $\Rightarrow (3x - y)(x + y) = 25$
>
> If $(3x - y) = 1$ and $(x + y) = 25$
>
> $\left.\begin{array}{l} 3x - y = 1 \\ x + y = 25 \end{array}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5$ Not integers

Show the calculations and statements that are needed to complete the proof. **(4)**

If $3x - y = 25$ and $x + y = 1$ (switch)

$3x - y = 25$
$+ \; x + y = 1$
$4x = 26$
$x = \frac{26}{4} = 6.5$
$y = 1 - 6.5 = -5.5$
Not integers

If $3x - y = 5$ and $x + y = 5$
$3x - y = 5$
$+ \; x + y = 5$
$4x = 10$
$x = 2.5$
$y = 5 - 2.5 = 2.5$
Not integers

In each case x and y are not integers
$\therefore$ there are no positive integers x & y
Such that $3x^2 + 2xy - y^2 = 25$ ✓

Question 8 continued

(Total for Question 8 is 4 marks)

VECTORS, SKEW

9. With respect to a fixed origin O, the equations of lines l_1 and l_2 are given by

vector equations — r = position vector + λ(direction vector)

$$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2: \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$$

where λ and μ are scalar parameters.

Prove that lines l_1 and l_2 are skew. **(5)**

(→ not parallel, → do not intersect)

if ∥ direction vectors will have common factor

$\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \neq k\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$ $\therefore l_1$ & l_2 are not parallel

if l_1 & l_2 intersect there will be a value of λ & μ that bring position vectors to touch

$\begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu\begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$

Solve for λ & μ

Question 9 continued

① $2 - \lambda = -4 + 5\mu$
② $8 + 2\lambda = -1 + 4\mu$
③ $10 + 3\lambda = 2 + 8\mu$

solve for λ and μ of 2 of the equation
if those values satisfy ③ then they intersect

①×2 (elimination method)
$4 - 2\lambda = -8 + 10\mu$
②+ $8 + 2\lambda = -1 + 4\mu$
$12 = -9 + 14\mu$
$\mu = \frac{21}{14} = \frac{3}{2}$
$2 - \lambda = -4 + 5(\frac{3}{2})$
$\lambda = -\frac{3}{2}$

CHECK ③
$10 + 3\lambda = 2 + 8\mu$
$10 + 3(-\frac{3}{2}) \neq 2 + 8(+\frac{3}{2})$
$5.5 \neq 14$
$\lambda = -\frac{3}{2}$ & $\mu = \frac{3}{2}$ do not satisfy ③
$\therefore l_1$ & l_2 do not intersect ✓
$\therefore$ skew

(Total for Question 9 is 5 marks)

RELATED RATES OF CHANGE, DIFFENCIAL EQUATIONS

10. A spherical ball of ice of radius 12 cm is placed in a bucket of water. ($t = 0$; $r = 12$)

In a model of the situation,

- the ball remains spherical as it melts
- t minutes after the ball of ice is placed in the bucket, its radius is r cm
- the rate of decrease of the radius of the ball of ice is inversely proportional to the square of the radius ($\frac{dr}{dt} = -\frac{k}{r^2}$)
- the radius of the ball of ice is 6 cm after 15 minutes (when $t = 15$, $r = 6$)

Using the model and the information given,

(a) find an equation linking r and t, **(5)**

(b) find the time taken for the ball of ice to melt completely. **(2)**

(c) On Diagram 1 on page 27, sketch a graph of r against t. **(1)**

10.(a) $\frac{dr}{dt} = -\frac{k}{r^2}$ → want r and t only

$r^2\frac{dr}{dt} = -k$

integrate both sides

$\int r^2 \frac{dr}{dt}\,dt = -k\int dt$

$\int_{r_2}^{r_1} r^2\,dr = -k\int_{t_2}^{t_1} dt$

$k = ?$

insert known limits
when $r = 12$, $t = 0$ when $r = 6$, $t = 15$

Question 10 continued

$\int_6^{12} r^2\,dr = -k\int_{15}^{0} dt$

$\left[\frac{r^3}{3}\right]_6^{12} = -k\left[t\right]_{15}^{0}$ (Could use +c method instead of limits sub in r and t etc)

$\frac{12^3 - 6^3}{3} = -k(0 - 15)$

$\frac{504}{15} = k$

$k = \frac{168}{5}$ $k = 33.6$

$\therefore \left[\frac{r^3}{3}\right]_6^{r} = -33.6\left[t\right]_{15}^{t}$ to get r in terms of t

$\frac{r^3 - 6^3}{3} = -33.6(t - 15)$

$r^3 - 6^3 = -100.8t + 1512$

$r^3 = -100.8t + 1720$ ✓

(c)

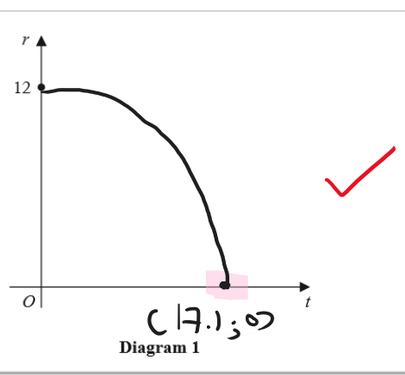


Diagram 1

Question 10 continued

(b) Melt completely $r = 0$ $t = ?$
$0^3 = -100.8t + 1728$
$t = \frac{1728}{100.8}$
$= \frac{120}{7} = 17.1428..$
$\therefore t = 17.1$
$\therefore$ It takes 17.1 minutes to melt completely

(c) working → what happens to gradient as r increases

$\frac{dr}{dt} = -\frac{33.6}{r^2}$

when $r = 0$ $t = 17.1$

as r decreases, $\frac{dr}{dt}$ gets steeper slope

Question 10 continued

$r = 10 \qquad \frac{dr}{dt} = \frac{-33.6}{10}$

$r = 2$
$r = 1$

(to work out slope of slope)

r = 10	r = 2	r = 1
−3.36	−16.8	−33.6

ie: gradient more negatively sloped

(Total for Question 10 is 8 marks)

IMPLICIT DIFFENCIATION STATIONARY POINTS

11.

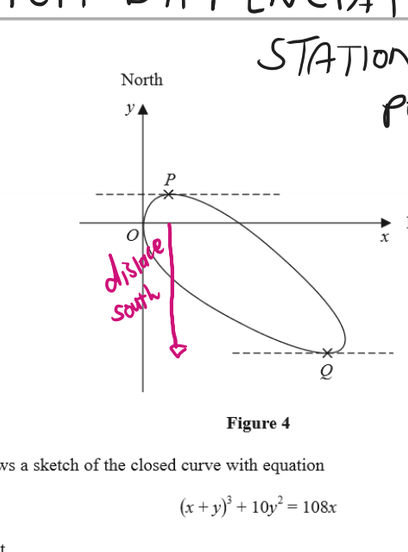


Figure 4

Figure 4 shows a sketch of the closed curve with equation

$$(x + y)^3 + 10y^2 = 108x$$

(a) Show that

$$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2}$$

(5)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest north and furthest south of the origin O, as shown in Figure 4.

Using the result given in part (a),

(b) find how far the point Q is south of O. Give your answer to the nearest 100 m. **(4)**

11 (a) $\frac{dy}{dx} = ?$

Question 11 continued

$(x+y)^3 + 10y^2 = 108x$

$\frac{d}{dx}\left[(x+y)^3 + 10y^2\right] = \frac{d}{dx}\left[108x\right]$

$\frac{d}{dx}(x+y)^3 + \frac{d}{dx}10y^2 = \frac{d}{dx}108x$

$3(x+y)^2 \times \left(1 + 1\frac{dy}{dx}\right) + 20y\frac{dy}{dx} = 108$

$3(x+y)^2\left(1 + \frac{dy}{dx}\right) + 20y\frac{dy}{dx} = 108$

$3(x+y)^2 + 3(x+y)^2\frac{dy}{dx} + 20y\frac{dy}{dx} = 108$

$\left(3(x+y)^2 + 20y\right)\frac{dy}{dx} = 108 - 3(x+y)^2$

$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{3(x+y)^2 + 20y}$

$\frac{dy}{dx} = \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2}$ ✓

Question 11 continued

(b) P & Q are stationary points
$\therefore \frac{dy}{dx} = 0$
$\therefore \frac{108 - 3(x+y)^2}{20y + 3(x+y)^2} = 0$
$108 - 3(x+y)^2 = 0$
$108 = 3(x+y)^2$
$36 = (x+y)^2$
$x + y = \pm 6$
① $x + y = 6$ or $x + y = -6$ ②
$x = 6 - y$
sub ① into $(x+y)^3 + 10y^2 = 108x$
$6^3 + 10y^2 = 108(6 - y)$
$216 + 10y^2 = 648 - 108y$
$10y^2 + 108y - 432 = 0$
$5y^2 + 54y - 216 = 0$
$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$= \frac{-54 \pm \sqrt{54^2 - 4(5)(-216)}}{2(5)}$
$y = 3.106\ldots$ (P) or $y = -13.906$ (Q)
$\therefore y = 13.906$ km
$\therefore$ 13 900 m (to nearest 100m) below zero ✓

(Total for Question 11 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS