CHAIN RULE - EXAMPLES

CHAIN RULE:

If $g(x) = (f(x))^n$

then $g'(x) = n(f(x))^{n-1} \times f'(x)$

→ With high n and two or more terms in the bracket

REVERSE CHAIN RULE:

If $f(x) = (f(x))^n$

then $F(x) = \dfrac{(f(x))^{n+1}}{(n+1) \times (f'(x))}$

↙ Capital F for integral of f

EXAMPLES $f(x)$ = the following

What is $f'(x)$ in each case?

A
1. $(x+2)^2$
2. $(2x+3)^4$
3. $(x^3+4)^5$
4. $(3x+2)^{-1}$
5. $\frac{1}{(x^2-3)^3}$
6. $(x^2-1)^{\frac{3}{2}}$
7. $\left(\frac{1}{x}+x\right)^3$
8. $4(x+2)^2$

$f(x)$ = the following

What is $F(x)$ in each case (integrate the following)

B
1. $2(x+2)$
2. $8(2x+3)^3$
3. $15x^2(x^3+4)^4$
4. $\frac{-3}{(3x+2)^2}$
5. $\frac{-6x}{(x^2-3)^4}$
6. $3x(x^2-1)^{\frac{1}{2}}$
7. $3\left(\frac{1}{x}+x\right)^2\left(-\frac{1}{x^2}+1\right)$
8. $8(x+2)$

ANSWERS

A ① $f(x) = (x+2)^2$

$f'(x) = 2(x+2) \times 1$

$= 2(x+2)$

② $f(x) = (2x+3)^4$

$f'(x) = 4(2x+3)^3 \times 2$

$= 8(2x+3)^3$

③ $f(x) = (x^3+4)^5$

$f'(x) = 5(x^3+4)^4 \times 3x^2$

$= 15x^2 (x^3+4)^4$

④ $f(x) = (3x+2)^{-1}$

$f'(x) = -1(3x+2)^{-2} \times 3$

$= -3(3x+2)^{-2}$

$= \frac{-3}{(3x+2)^2}$

⑤ $f(x) = \frac{1}{(x^2-3)^3}$

$f'(x) = -3(x^2-3)^{-4} \times 2x$

$= -6x(x^2-3)^{-4}$

$= \frac{-6x}{(x^2-3)^4}$

(6) $f(x) = (x^2-1)^{\frac{3}{2}}$

$f'(x) = \frac{3}{2}(x^2-1)^{\frac{1}{2}} \times 2x$

$= 3x\,(x^2-1)^{\frac{1}{2}}$

(7) $f(x) = \left(\frac{1}{x}+x\right)^3$

$f'(x) = 3\left(\frac{1}{x}+x\right)^2 \times (-x^{-2}+1)$

$= 3\left(\frac{1}{x}+x\right)^2\left(-\frac{1}{x^2}+1\right)$

(8) $f(x) = 4(x+2)^2$

$f'(x) = 8(x+2) \times \underline{1}$

$= 8(x+2)$

B (1) $f(x) = 2(x+2)$

$F(x) = \dfrac{2(x+2)^2}{2 \times 1}$

$= \dfrac{2(x+2)^2}{2}$

$= (x+2)^2$

(2) $f(x) = 8(2x+3)^3$

$$F(x) = \frac{8(2x+3)^4}{4 \times 2}$$

$$= \frac{8(2x+3)^4}{8}$$

$$= (2x+3)^4$$

(3) $f(x) = 15x^2(x^3+4)^4$

$$F(x) = \frac{15x^2(x^3+4)^5}{5 \times 3x^2}$$

$$= \frac{15x^2(x^3+4)^5}{15x^2}$$

$$= (x^3+4)^5$$

(4) $f(x) = \frac{-3}{(3x+2)^2}$

$$F(x) = \frac{-3(3x+2)^{-1}}{-1 \times 3}$$

$$= (3x+2)^{-1}$$

$$= \frac{1}{3x+2}$$

(5) $f(x) = \dfrac{-6x}{(x^2-3)^4}$

$F(x) = \dfrac{-6x\,(x^2-3)^{-3}}{-3 \times 2x}$

$= (x^2-3)^{-3}$

$= \dfrac{1}{(x^2-3)^3}$

(6) $f(x) = 3x\,(x^2-1)^{\frac{1}{2}}$

$F(x) = \dfrac{3x\,(x^2-1)^{\frac{3}{2}}}{\frac{3}{2} \times 2x}$

$= \dfrac{3x\,(x^2-1)^{\frac{3}{2}}}{3x}$

$= (x^2-1)^{\frac{3}{2}}$

(7) $f(x) = 3\left(\frac{1}{x}+x\right)^2\left(-\frac{1}{x^2}+1\right)$

$F(x) = \dfrac{3 \times \left(-\frac{1}{x^2}+1\right) \times \left(\frac{1}{x}+x\right)^3}{3 \times \left(-\frac{1}{x^2}+1\right)}$

$F(x) = \left(\frac{1}{x}+x\right)^3$

(8) $f(x) = 8(x+2)$

$F(x) = \dfrac{8(x+2)^2}{2 \times 1}$

$= 4(x+2)^2$