COORDINATE GEOMETRY

QUADRATIC FUNCTIONS

Forms of quadratic functions (Parabolas)

General form: $f(x) = ax^2 + bx + c$

Turning point form: $f(x) = a(x-p)^2 + q$

TURNING POINT $(p; q)$

- p: opposite sign
- q: same sign.

X-intercept form: $f(x) = a(x-x_1)(x-x_2)$

x-intercepts: x_1 & x_2

Using the different forms: Always use either the x-intercept form or turning point form. Then simplify to get general form.

X-intercept form:

To find the general form of a quadratic when given the x-intercept(s) and another point on the quadratic.

STEPS:

1. sub the x-values of your x-intercepts into the formula. x_1 is one x-value and x_2 is the other.
2. sub another point on the graph $(x; f(x))$ into the quadratic formula to solve for the unknown 'a'.
3. sub 'a', x_1, x_2 into quadratic formula and simplify to general form $f(x) = ax^2 + bx + c$.

Turning point form:

To find the general form of a quadratic when given the turning point and one other point on the quadratic.

STEPS: ① sub in your turning point $(p;q)$. Opposite sign for p, same sign for q.
② sub in another point on the quadratic to solve for the unknown 'a'.
③ sub $p;q$ and 'a' into the turning point form and expand and simplify into general form.

Finding the turning point from the general form

If $f(x) = ax^2+bx+c$

the x-value of the turning point is found by $\frac{-b}{2a}$

sub this x-value into $f(x)$ to find the corresponding y-value of the turning point.

General notes:

① Convert to turning point form from general form by completing the square → see p.3
② x-intercepts of a quadratic are equidistant from the turning point
③ the turning point can also be found by $f'(x)=0$
The x-value where the derivative of $f(x) = 0$ will be the x-value of the turning point. Sub this x-value into $f(x)$ to find the corresponding y-value.

Finding the turning point by completing the square

Example: $x^2+6x+7=0$

STEPS: ① Convert to turning point form
② read off turning point $(p;q)$

p: opposite sign
q: same sign

① Convert to turning point form

$x^2+6x+7=0$ → minus $\left(\frac{b}{2}\right)^2$ to balance the equation

$x^2+6x+\_\_\_+7-\_\_\_=0$

$\left(\frac{b}{2}\right)^2=\left(\frac{6}{2}\right)^2=9$ → add $\left(\frac{b}{2}\right)^2$ to complete the square

$(x^2+6x+9)+7-9=0$

$(x+3)^2-2=0$

$7-9$

→ contract the square
($\sqrt{x^2}$ then 'same sign' then $\sqrt{9}$)^2

$\therefore (-3;-2)$