HASTINGS MATH.

COORDINATE GEOMETRY (ANALYTICAL)

MAIN FORMULAS:

Gradient formula: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Angles of inclination: $m = \tan\theta$

Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint formula: $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

Perpendicular lines: $m_1 \times m_2 = -1$

use for normal to the curve, etc.

Parallel lines: $m_1 = m_2$

Straight line:

General form: $y = mx + c$

Intercept form: $y - y_1 = m(x - x_1)$

COORDINATE GEOMETRY

CIRCLES

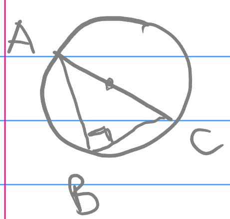
General form: $x^2+y^2+ax+by+c=0$

Centre radius form: $(x-p)^2+(y-q)^2=r^2$

where $(p;q)$ is the centre of the circle → opposite signs

and r is the radius of the circle

OTHER NOTES CONCERNING CIRCLES

① 

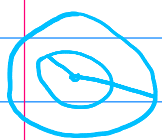
Diameter AC subtends a 90° angle at the circle → $\therefore m_{AB} \times m_{BC} = -1$

② r of a circle $=$ diameter $\div 2$

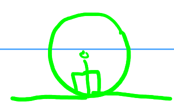
③ Pythagorus: $AB^2+BC^2=AC^2$

④ TERMINOLOGY:

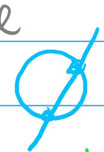
ⓐ Consecucircles: circles that have DIFFERENT radii, but the same midpoint



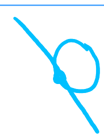
$r_1 \neq r_2$



ⓑ SECANT to the circle: a line segment that intersects the circle twice



ⓒ TANGENT to the circle: a line that intesects the circle once.



Note: the tangent meets the radius at 90°

HOW TO CONVERT A CIRCLE INTO CENTER RADIUS FORM:

Using the example $x^2+2x+y^2+6y+2=0$

To convert a circle to center radius form, we complete the square twice.

STEPS:

① Write your circle in this order:

$x^2+2x+y^2+6y+2=0$

Keep x terms together
Keep y terms together
Constant term far right
Make sure it's $=0$

② Complete the square for x and for y

$x+2x+\_\_\_ +y^2+6y+\_\_\_ +2-\_\_\_-\_\_\_=0$

$\left(\frac{b}{2}\right)^2=\left(\frac{2}{2}\right)^2=1$ $\qquad$ $\left(\frac{b}{2}\right)^2=\left(\frac{6}{2}\right)^2=9$

balance the equation by minusing what was added

$\therefore x^2+2x+1+y^2+6y+9+2-1-9=0$

③ Contract your squares and add constants

to contract:
x^2+2x+1
$\sqrt{x^2}$ $\quad$ sign of middle term $\quad$ $\sqrt{1}$
$\therefore (x+1)^2$

$(x+1)^2+(y+3)^2-8=0$

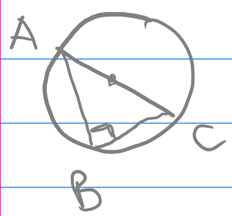
④ Take constant term over =

$\therefore (x+1)^2+(y+3)^2=8$

⑤ read off centre $(-1;-3)$ and $r=\sqrt{8}$

opposite signs.

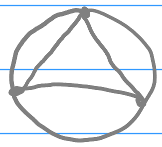
NOTES ON CIRCLES AND TRIANGLES

① 

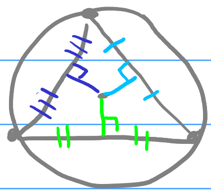
Diameter AC subtends a 90° angle at the circle $\rightarrow$ $\therefore m_{AB} \times m_{BC} = -1$

② **CIRCUMCIRCLE:**

If it is possible to make a circle around the three vertices of a triangle, this circle is called the circumcircle of the triangle.

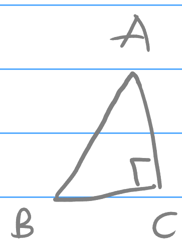


The perpendicular bisectors of the sides of this triangle will intersect at the centre of the circle.



③ **Pythagorus:**

A right-angled triangle



$AB^2 = AC^2 + BC^2$

The hypotenuse is the DIAMETER of the circumcircle