FUNCTIONS

EXPONENTIAL AND LOG FUNCTIONS

general exponential: $f(x) = ab^{x+p} + q$

p: shift left/right ($+$ / $-$)

q: shift up/down ($+$ / $-$)

a: reflects over x-axis / the greater a is the STEEPER the curve is

$b > 0$ $\quad b \neq 1$

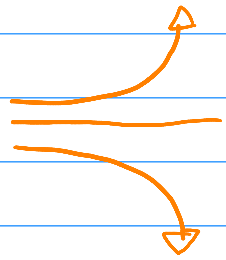
q: $y = q$ gives horizontal asymptote

$*$ ab^{x+p} can be expanded to $ab^{x} \times ab^{p}$

TRANSFORMATIONS:

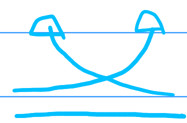
① reflection over the x-axis

$f(x)$ reflects to $-f(x)$



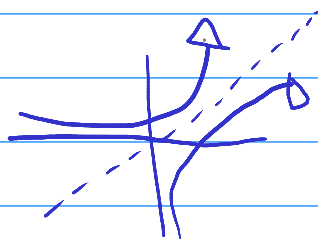
② reflect over the y-axis

$f(x)$ reflects to $f(-x)$



③ reflect over $y = x$ (inverse)

$f(x)$ inverts to $f^{-1}(x)$

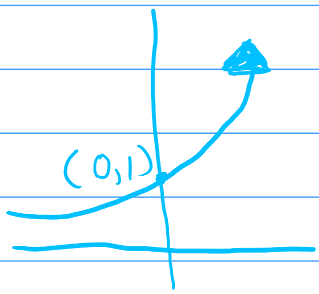


log functions are the inverse of exponential functions

method: swap x and y and make y the subject

EXPONENTIAL SHAPES

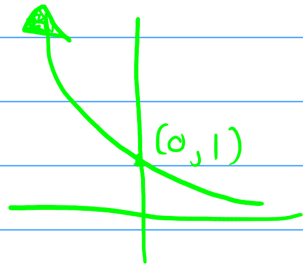
$y = 2^x$



$y = \frac{1}{2}^x$

y-axis

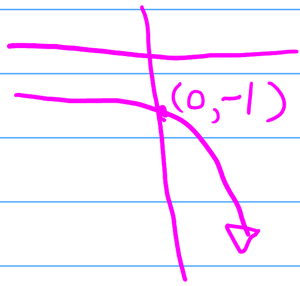
$y = 2^{-x}$



reflections

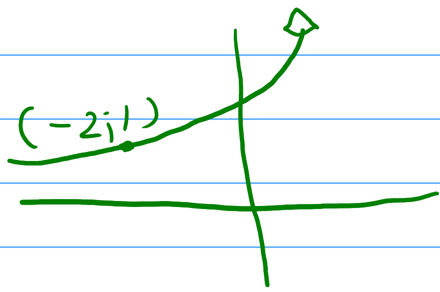
x-axis

$y = -2^x$



$y = 2^{x+2}$

shift two to the left (counter intuitive)



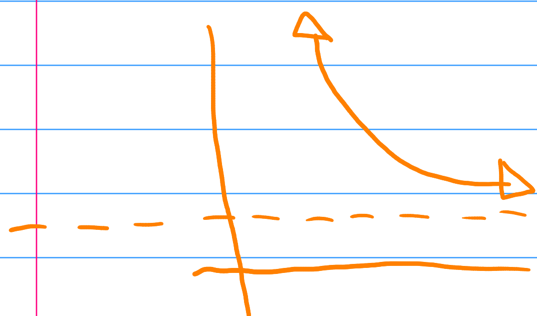
Horizontal

SHIFTS

Vertical

$y = 2\left(\frac{1}{3}\right)^{x-3} + 2$

shifted up two (intuitive)
shifted right three (counter intuitive)

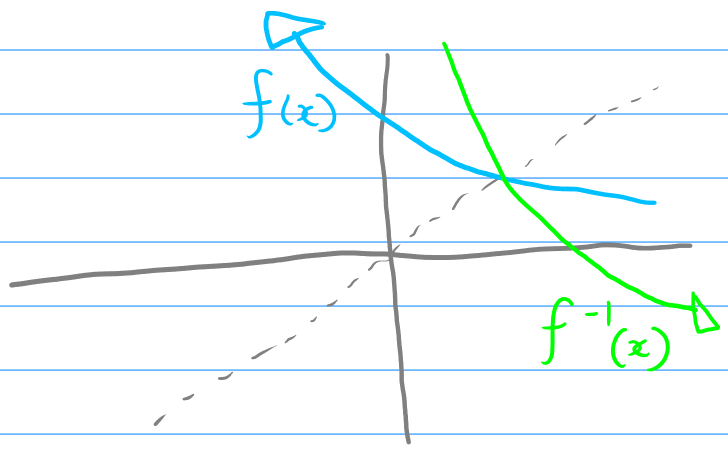


$y = 2$ horizontal asymptote

CONVERTING TO LOG FUNCTIONS

STEPS TO GRAPH THE INVERSE FUNCTION (LOG)

1. switch x/y
2. let $x=1$ to find a point on the graph
3. find y-intercept of original function (let $x=0$) then switch x/y to find x-intercept of inverse



If $f(x) = \frac{1}{2}^x$ then $f^{-1}(x) = \log_{\frac{1}{2}} x$

asymptote: $y=0$ asymptote: $x=0$

CONVERTING EXPONENTIAL TO LOG

STEPS:

1. Switch x/y
2. make y the subject using log law

If $a^b = c$

then $\log_a c = b$

EG: If $f(x) = \frac{1}{3}^x$

$y = \frac{1}{3}^x$

$x = \frac{1}{3}^y \quad \therefore \log_{\frac{1}{3}} x = y$

LOG LAWS FOR SIMPLIFYING / REARRANGING

1. if $a^b = c$
   then $b = \log_a c$

2. $\log_a c = \dfrac{\log_b c}{\log_b a}$

3. $\log_a a = 1 \qquad (a>0\ ;\ a\neq 1)$

4. $\log_a 1 = 0 \qquad (a>0\ ;\ a\neq 1)$

5. $\log_a x + \log_a y = \log_a xy$

6. $\log_a x - \log_a y = \log_a \dfrac{x}{y}$

7. $\log_a b^c = c \log_a b$

8. $\log_a b = \dfrac{1}{\log_b a}$

9. $\log_a b \times \log_a b \neq \log_a b^2$

   $\log_a b \times \log_a b = (\log_a b)^2$

   $= \left(\dfrac{\log_{10} b}{\log_{10} a}\right)^2$

this is often useful in log equations

$\boxed{\log}$ on the calculator assumes base 10.

eg: $\boxed{\log}\ \boxed{3}\ \boxed{=}$ is giving $\log_{10} 3$ answer

EXPONENT LAWS FOR SIMPLIFYING/REARRANGING

1. $b^x \cdot b^a = b^{x+a}$

2. $\dfrac{b^x}{b^a} = b^{x-a}$

All laws are reversable.

3. $(b^a)^x = b^{a \cdot x}$

4. $(a \cdot b)^x = a^x \cdot b^x$

5. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

6. $\sqrt[x]{b^y} = b^{\frac{y}{x}}$