INTEGRATION AND DIFFERENCIATION

THE POWER RULE:

the power rule is used to find the first derivative of any function.

If we have a function $f(x) = x^2$

We can apply the power rule as follows:

If $f(x) = x^2$ → $a = 1$, so not shown.

$f'(x) = 2 \times x^{2-1}$

another way to write this

STEPS: for any function $f(x) = ax^n$

$f'(x) = anx^{n-1}$

if $f(x)^n$ then $f'(x) = n \times f(x)^{n-1}$

① 1st we times the coefficient of x by the power (n)

② then we minus 1 from the power

in our example, $a = 1$, so it was not shown.

In short:

if $f(x) = x^2$

then $f'(x) = 2x$

another way of writing it:

$f(x) = x^2$

So:

$f'(x)$

$= 2 \times x^{2-1}$

REVERSING THE POWER RULE

The opposite of deriving is integrating.

So we do the exact opposite in order to get the integral of a function.

If $f(x) = 2x$

STEPS: ① first add 1 to the exponent 1
② then divide $f(x)$ by the a value 2

note that the steps are also reversed in ORDER. The order matters here.

$$\int 2x \, dx$$

integrate

our example $f(x)$

with respect to x.

Therefore becomes:

$$\frac{2x^{1+1}}{2}$$

So our formula for reverse power rule is:

$$\int f(x) \, dx$$

$$= \frac{f(x)^{n+1}}{n+1}$$

note that we are dividing by the new power.

THE CHAIN RULE (differenciation)

If we take the power rule above, but we have a question with brackets, we can use the chain rule so that we don't have to expand the brackets.

for examde if $f(x) = (3x-2)^4$
this would take long to expand.

The chain rule states that in this case, for any $f(x)$, $f'(x)$ is equal to the 1st derivative of outside the bracket times by the first derivative of the contents inside the bracket, like this:

If $g(x)$ → looks like this: $(f(x))^n$

Then: $g'(x) = n \times (f(x))^{n-1} \times f'(x)$

So using our example:

If $f(x) = (3x-2)^4$

then $f'(x) = 4 \times (3x-2)^3 \times 3$

note that the contents of the bracket are left the same here

this is the 1st derivative of the contents of the bracket.

to finish this: $4 \times 3 = 12$

$12(3x-2)^3$

REVERSING THE CHAIN RULE: (integration)

Just like the power rule, the chain rule is reversable.
This is used to INTEGRATE.

The reverse chain rule could be shown as follows:

In order to $\int f(x)^n \, dx$, where we have brackets:

Our integrated function would be found by:

$$\frac{f(x)^{n+1}}{(n+1) \times f'(x)}$$

original plus 1

divide n+1 instead of times to reverse it

divide by the first derivative of f(x) to reverse it.

NOTE: this is again the derivative, NOT the integral of f(x).

using our answer as an example:

$$\int 12(3x-2)^3 \, dx$$

$$= \frac{12(3x-2)^4}{4 \times 3}$$

$$= \frac{12(3x-2)^4}{12} = (3x-2)^4$$

QUICK VERSION:

POWER RULE:

if $f(x)^n$

then $f'(x) = n \times f(x)^{n-1}$

REVERSE POWER RULE:

if $\int f(x)\,dx$

then $\dfrac{f(x)^{n+1}}{n+1}$

CHAIN RULE:

If $g(x) \Rightarrow (f(x))^n$

then $g'(x) = n \times (f(x))^{n-1} \times f'(x)$

REVERSE CHAIN RULE:

If $\int f(x)^n\,dx$ => with brackets

then $\dfrac{(f(x))^{n+1}}{(n+1) \times (f'(x))} + C$

C represents the constant term. this is further explained on the next pg

WHERE DOES "C" go / come from?

When you differenciate $f(x)$ the constant term falls away.

When you integrate, you need to bring that constant term back.

Eg: If $f(x) = x^2+1$

$f'(x) = 2x$

but:

If $\int 2x \, dx$

we get x^2

So: we are missing the $+1$

to fix: we $+c$

If no point given, leave it like this:

$\therefore \; x^2+c$

If given:
sub point $(1;2)$

$2 = 1^2 + c$

$c = 1$

from here we would need further information to solve for c, such as subbing in a point on our graph.

This information will be given, or found, on the example, or the function can be left in this form.