NATURE OF ROOTS

for quadratics/parabolas

From the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

from form $ax^2+bx+c=0$

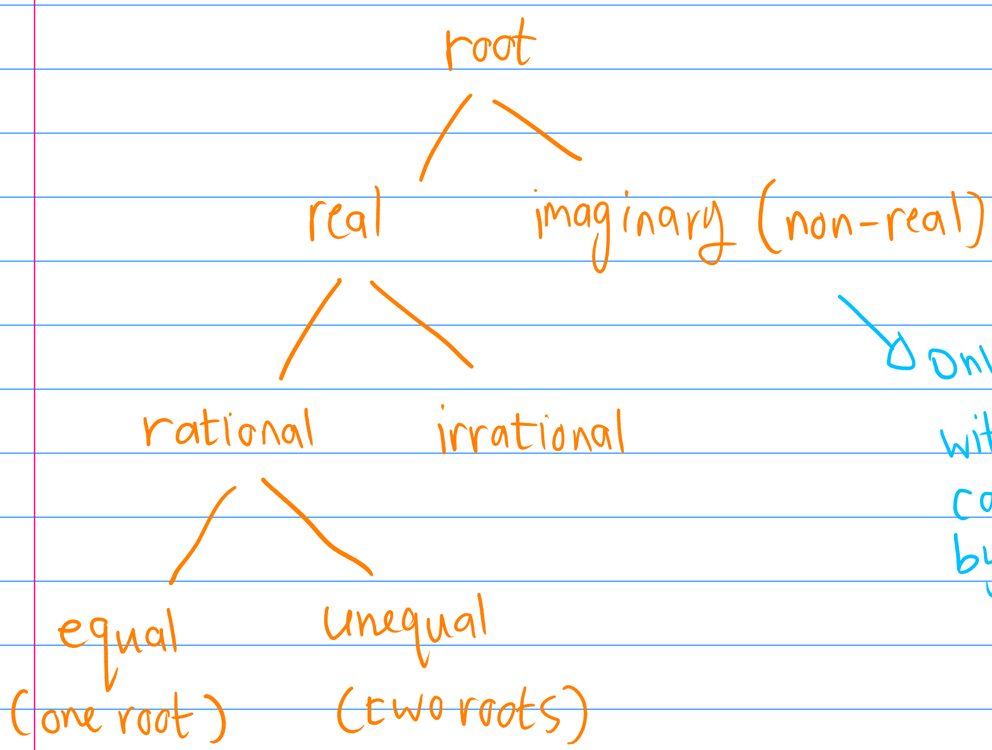
The content under the square root can be called the discriminant.

This is shown by the symbol δ or Δ → some textbooks will call this delta

$$\therefore \quad \Delta = b^2-4ac$$

The discriminant/delta tells us about the roots of a function in the following ways:

1. are the roots real or imaginary (non-real)
2. are the roots rational or irrational
3. are the roots equal (one root) or unequal (two roots)



Only equations with real roots can be solved by factorisation.

HOW TO USE THE DISCRIMINANT :

1. $b^2 - 4ac < 0$ → non-real / imaginary roots
   ↳ because $\sqrt{-ve}$ is imaginary
2. $b^2 - 4ac \geqslant 0$ → real roots
3. $b^2 - 4ac$ = perfect square → rational and unequal
   ↳ not including 0
4. $b^2 - 4ac = 0$ → rational and equal
   ↳ zero is a perfect square.
5. $b^2 - 4ac > 0$; but not a perfect square
   → irrational and unequal

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root x

- **real** — $\Delta \geqslant 0$
  - **rational** — Δ = perfect square, $\Delta > 0$, $\Delta = 0$
    - **equal** — $\Delta = 0$
    - **unequal** — $\Delta \neq$ perfect square, $\Delta > 0$, $\Delta \neq 0$
  - **irrational** — $\Delta \neq$ perfect square, $\Delta > 0$
- **imaginary** — $\Delta < 0$

EXAMPLES

1. Classify the roots of the following using the discriminant.

(a) $5x^2-2x+1=0$
(b) $7x^2-3x-4=0$
(c) $4x^2-12x+9=0$
(d) $3x^2+5x+1=0$

$\Delta = b^2-4ac$

when $ax^2+bx+c=0$

ANSWERS

(a) $5x^2-2x+1=0$

$\Delta=(-2)^2-4(5)(1)$
$=-16$ $\therefore \Delta<0$

$\therefore$ roots are imaginary / non-real

CHECK: CALCULATOR

$x=\dfrac{2\pm\sqrt{(-2)^2-4(5)(1)}}{2(5)}$

= error.

(b) $7x^2-3x-4=0$

$\Delta=(-3)^2-4(7)(-4)$
$=121$

$\therefore$ Δ is positive perfect square

$\therefore$ roots are real, rational and unequal

CHECK: $(7x+4)(x-1)$

$x=1$ or $x=-\frac{4}{7}$

(c) $4x^2-12x+9=0$

$\Delta=(-12)^2-4(4)(9)$
$=0$

$\therefore$ roots are real, rational & equal

CHECK: $(2x-3)(2x-3)$

$x=\frac{3}{2}$

(d) $3x^2+5x+1=0$

$\Delta=5^2-4(3)(1)$
$=13$

$\therefore$ Δ is positive but not a perfect square

$\therefore$ roots are real, irrational, unequal

CHECK: $\dfrac{-5\pm\sqrt{5^2-4(3)(1)}}{2(3)}$

$x=-0.23$ or $x=-1.43$