Polynomials

STEPS:

1. use calculator to find 1st term → this works with most CASIO fx calculators

   mode $\boxed{3}$ tables

   gives you $f(x) =$

   a. type in your polynomial
   b. press $\boxed{AC}$
   c. -7 to 7 should be enough
   d. make $g(x) = 0$
   e. check your table

x-values

x	$f(x)$	$g(x)$
3	0	

y-values

ignore $g(x)$ values for $f(x)$

look for where $f(x) = 0$

In this example

$f(x) = 0$

is when $x = 3$

$\therefore f(3) = 0$

If $f(3) = 0$

$x = 3$

$\therefore x - 3 = 0$

$(x-3)$ is therefore a factor of $f(x)$

Once you have your 1st factor there are three methods:

- Long division → (which is also called Factor theorium or remainder theorum)
- Two bracket quick method → the one most students like to use
- Using the calulator → easy but won't work for all examples

EXAMPLE ①

Long division

$f(x) = x^3 + 2x^2 - 17x + 6$

factor $(x-3)$ is given → note: this means $f(3) = 0$ and $x - 3 = 0$

$$
\begin{array}{r|l}
 & \quad x^2 \\
\hline
x-3 & x^3 + 2x^2 - 17x + 6 \\
 & -\ \underline{x^3 - 3x^2} \\
 & \qquad 5x^2 - 17x \\
 & \qquad \underline{5x^2 - 15x} \\
 & \qquad\qquad -2x + 6 \\
 & \qquad\qquad \underline{-2x + 6} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

This is one round.

Start here agian round 2

DMSB

tick as you go

* for the D, divide to term in x only

* for the M, multiply $x-3$ by x^2

* S is subtract

* B is bring down

If there is a remainder here, then $x-3$ is not a factor of $f(x)$

NOTE: For this method, if there is no x^2 term, write $0x^2$.
This will placehold the x^2 term for you.

Two bracket quick method

STEPS:

① use calculator to get the first factor if it is not given (as above)

Using the same example

$$f(x) = x^3+2x-17x+6$$

$f(3)=0$

$\therefore x-3$ is a factor of $f(x)$

② Open your two brackets

ⓐ

$$(x-3)(\qquad\qquad)$$

this is you first factor

this will be the answer or the quotient from the long division method.

ⓑ For the first term in the second bracket ask: what do I need to multiply x by in order to get x^3

$$f(x) = x^3+2x^2-17x+6$$

$$(x-3)(x^2\qquad\qquad)$$

$$x \times x^2 = x^3$$

(c) do the same for the third term, this time looking at the constant term

$(x-3)(x^2 \quad \underline{-2})$

What times -3 will equal $+6$

$-3 \times -2 = 6$

(d) last step is the middle term

$(x-3)(x^2 \underline{\qquad} -2)$

For this term we look only at the "x^2" term in $f(x)$.

First times -3 by x^2 like this:

$(x-3)(x^2 \quad -2)$

$\times$

$-3x^2$

then ask what you would need to add to $-3x^2$ in order to get your x^2 term.

In this case our x^2 term is $2x^2$ so:

$-3x^2 + 5x^2 = 2x^2$

$\therefore (x-3)(x^2+5x-2)$

note that your answer will be only x, not x^2, in this method.

(e) the final step is to find the remaining factors.

$(x-3)(x^2+5x-2)$

for this (x^2+5x-2) cannot be easily factored into two brackets so the quadratic formula can be used to finish

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

The last method is to simply use the calculator

First enter your $f(x)$ into tables on the calculator

from here you will be able to see all places where $f(x)=0$, and these are your factors.

EXAMPLE 2 → Example 1 wouldn't work this way as the quadratic formula was needed.

$2x^3+x^2-18x-9$

Calculator Steps:

1. mode $\boxed{3}$ tables
2. type in $f(x)=2x^3+x^2-18x-9$, press $\boxed{=}$
3. $g(x)=0$, press $\boxed{=}$
4. Start -7 ; end 7

5) press $\boxed{=}$

6) Steps $\boxed{1}$, press $\boxed{=}$

this will appear:

	x	$f(x)$	$g(x)$
1			
2			
3			

$\downarrow$

6) scroll down/up to see where $f(x) = 0$

in this example

$f(-3) = 0$ & $f(3) = 0$

$\therefore x + 3 = 0$ $\therefore x - 3 = 0$

Since only two factors are there, the third factor must be a fraction.

Therefore you would need to use the second method above to complete it.

EXAMPLE 3

$$f(x) = x^3 + 6x^2 + 5x - 12$$

Use the calculator method above.

read off the table:

$f(-4) = 0$; $f(-3) = 0$ & $f(1) = 0$

$\therefore (x+4) = 0$, $(x+3) = 0$ & $(x-1) = 0$

$\therefore (x+4)(x+3)(x-1) = 0$ $\rightarrow$ $f(x)$ has been fully factored using the calculator.

LAST NOTE:

If the question asks you to prove something is a factor:

$$f(x) = x^3 + 6x^2 + 5x - 12$$

for example, we proved that $x+4$ is a factor using the calculator method for $f(x)$ here.

We can further test our answer in the following way:

sub $x = -4$ into $f(x)$ — for this you need the calculator back in comp mode.

$$f(-4) = (-4)^3 + 6(-4)^2 + 5(-4) - 12$$

$f(-4)$ will equal zero if -4 is a factor of $f(x)$.

If $f(x)$ does not equal zero, we have a remainder.

For example, a quick check on the calculator of this example shows that

$$f(2) = 30$$

This information could be useful for solving unknown coefficients of the terms in x

NOTE: Using the calculator in this way is a good trial and error method to make guesses at factors if you get stuck on a tricky question.

OK this really is the last note.

What do you do if you get a question with a fraction factor?

for example:

$-\frac{1}{2}$ is a factor of $f(x)$

if $-\frac{1}{2}$ is a factor

then $f(-\frac{1}{2}) = 0$

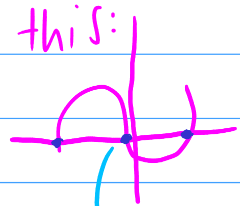
NOTE: this is an x-intercept of your curve with coordinates $(-\frac{1}{2}, 0)$

and $x = -\frac{1}{2}$

$\therefore 2x = -1$

$\therefore 2x + 1 = 0$

all factors of $f(x)$ are x-coordinates something like this:



$(-\frac{1}{2}, 0)$

Obviously this fraction method only really works for easy fractions.

With more tricky fractions it's best to go back to the drawing board and of course use the trusty quadratic formula as much as possible.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$