

# TRIGONOMETRY

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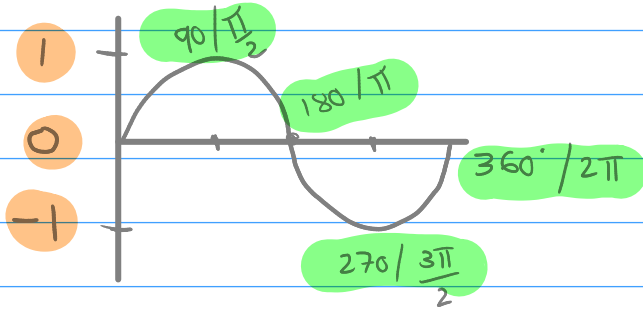
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# TRIG. GRAPHS

## sin θ



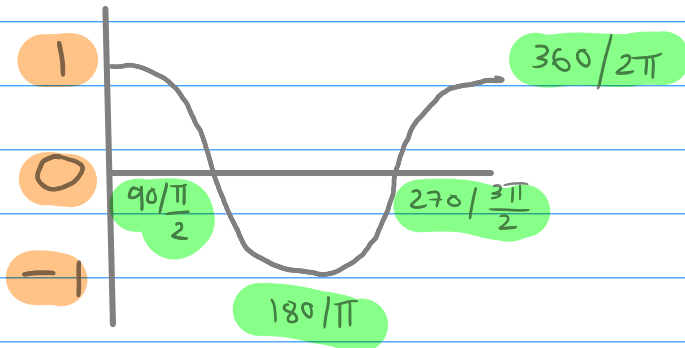
## degrees | radians

Convert:

radians to degrees

$$\theta \times \frac{180}{\pi}$$

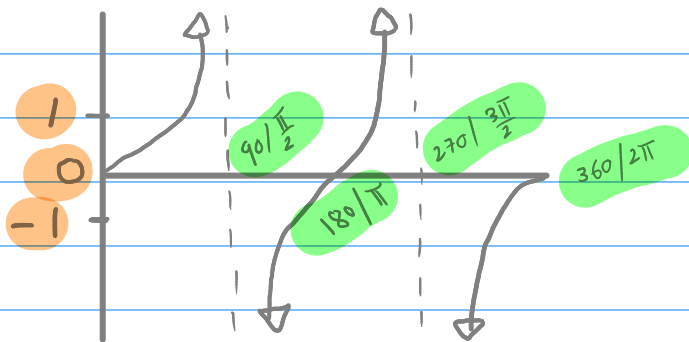
## cos θ



degrees to radians

$$\theta \times \frac{\pi}{180}$$

## tan θ



shift  $\uparrow +$   
 $\downarrow -$

General form:  $a \cdot \sin bx + q$

reflect over x-axis

amplitude

max to min

horizontal stretch/squish

intuitive

$a > 1$  longer

$-1 < a < 1$  shorter

vertical squish/stretch  
Counter intuitive

$b > 1$

shorter

$-1 < b < 1$

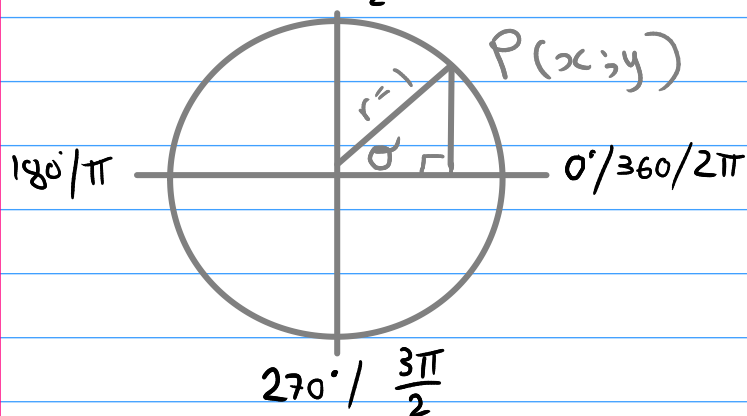
$b \neq 0$

longer

## UNIT CIRCLE

A circle with radius = 1

$$x^2 + y^2 = 1$$



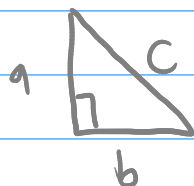
$$\begin{aligned}\cos \theta &= x \\ \sin \theta &= y \\ \tan \theta &= \frac{y}{x}\end{aligned}$$

## OTHER USEFUL NOTES:

S<sub>H</sub> C<sub>H</sub> T<sub>A</sub>

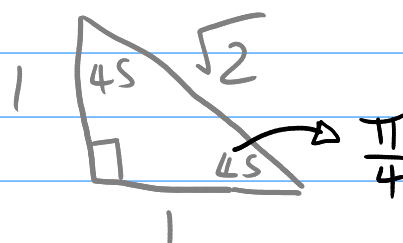
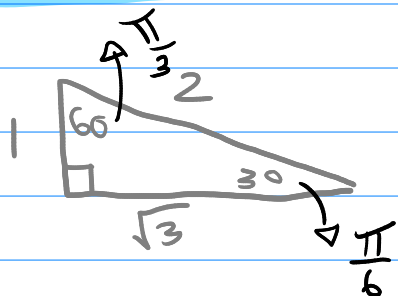
S<sub>r</sub><sup>y</sup> C<sub>r</sub><sup>x</sup> T<sub>x</sub><sup>y</sup>

Pythagoras



$$a^2 + b^2 = c^2$$

## SPECIAL TRIANGLES

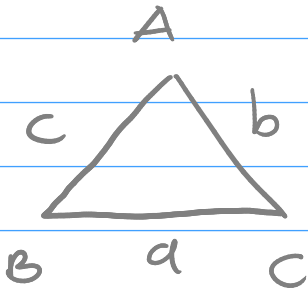


## ANGLES OF INCLINATION

$$\tan \theta = m$$

## AREA RULE

In  $\triangle ABC$



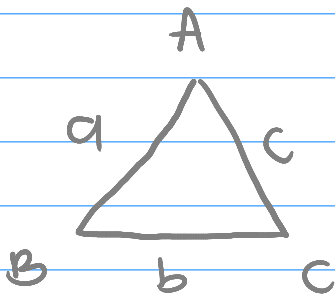
$$\text{Area} = \frac{1}{2} \cdot a \cdot b \cdot \sin \hat{C}$$

$$= \frac{1}{2} \cdot a \cdot c \cdot \sin \hat{B}$$

$$= \frac{1}{2} \cdot b \cdot c \cdot \sin \hat{A}$$

## Cos rule

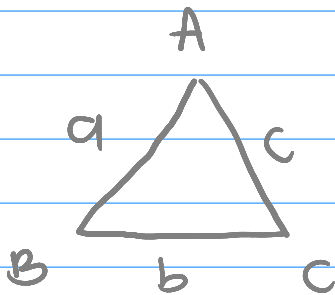
In triangle ABC



$$a^2 = b^2 + c^2 - 2(a)(b) \cos \hat{C}$$

## Sin rule

In triangle ABC



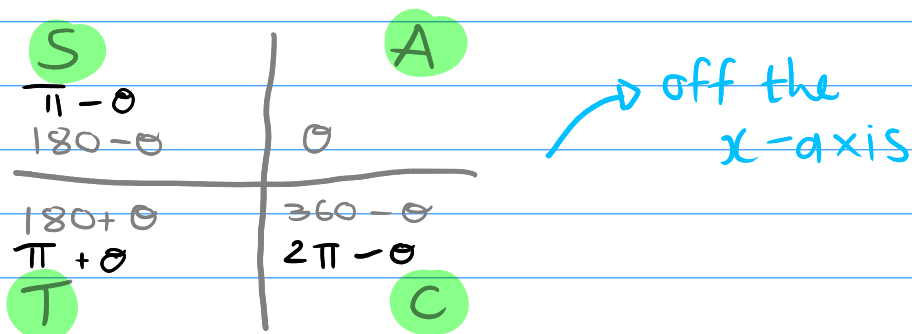
$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

# REDUCTION FORMULAE (180° / 360°)

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$$\begin{matrix} \sin \\ \cos \\ \tan \end{matrix} \begin{pmatrix} \text{obtuse or} \\ \text{reflex} \\ \text{angle} \end{pmatrix} \Leftrightarrow \begin{matrix} \pm \sin \\ \pm \cos \\ \pm \tan \end{matrix} \begin{pmatrix} \text{acute} \\ < \end{pmatrix}$$

the CAST diagram shows where each will be positive: degrees or radians



## CO-RATIOS (for 90°)

$$\sin(90^\circ + \theta) \Leftrightarrow \cos \theta$$

$$\sin(90^\circ - \theta) \Leftrightarrow \cos \theta$$

$$\cos(90^\circ + \theta) \Leftrightarrow -\sin \theta$$

$$\cos(90^\circ - \theta) \Leftrightarrow \sin \theta$$

if you remember this one, the others just switch with cos/sin

in radians:

$$\sin\left(\frac{\pi}{2} + \theta\right) \Leftrightarrow \cos \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) \Leftrightarrow \cos \theta$$

$$\cos\left(\frac{\pi}{2} + \theta\right) \Leftrightarrow -\sin \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) \Leftrightarrow \sin \theta$$

# TRIG IDENTITIES

## Quotient identity:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

## Pyth. Identity

$$\cos^2 \theta + \sin^2 \theta = 1$$

## DOUBLE ANGLE FORMULAE

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\begin{aligned} \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta \end{aligned}$$

## COMPOUND ANGLE FORMULAE

$$\begin{aligned} \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ \sin(A-B) &= \sin A \cos B - \cos A \sin B \end{aligned}$$

$$\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \cos(A-B) &= \cos A \cos B + \sin A \sin B \end{aligned}$$

## GENERAL SOLUTION

Gives every possible answer considering that trig graphs are repeating.

- STEPS:**
- ① Determine the positive reference angle
  - ② Apply the reference angle to the relevant quadrants
  - ③ Simplify and don't forget  $n\pi$

### Example 1

given  $\tan x \sin x + \cos x \tan x = 0$

Determine the general solution.

**Answer:**

$$\textcircled{1} \quad \tan x \sin x + \cos x \tan x = 0$$

$$\tan x (\sin x + \cos x) = 0$$

$$\therefore \tan x = 0$$

OR

$$\sin x + \cos x = 0$$

$$x = \tan^{-1}(0)$$

$$\sin x = -\cos x$$

$$= 0$$

$$\frac{\sin x}{\cos x} = -1$$

$$\tan x = -1$$

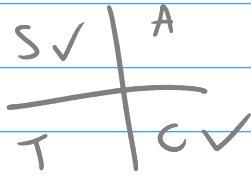
$$x = \tan^{-1}(-1)$$

$$= \left(-\frac{\pi}{4}\right) / (-)45$$

$\therefore$  reference angles :

$$x = 0 \text{ or } x = 45 \mid \frac{\pi}{4}$$

(2) tan is **negative** in quadrant 2 and 4



tan period of 180

(3)  $\therefore x = 180 - 0 + n180$   
 $= 180 + n180 \quad (n \in \mathbb{Z})$   
 or  $x = 360 - 0 + n180$   
 $= 360 + n180 \quad (n \in \mathbb{Z})$

or

$$x = 180 - 45 + n180$$

$$= 135 + n180 \quad (n \in \mathbb{Z})$$

or

$$x = 360 - 45 + n180$$

$$= 315 + n180 \quad (n \in \mathbb{Z})$$