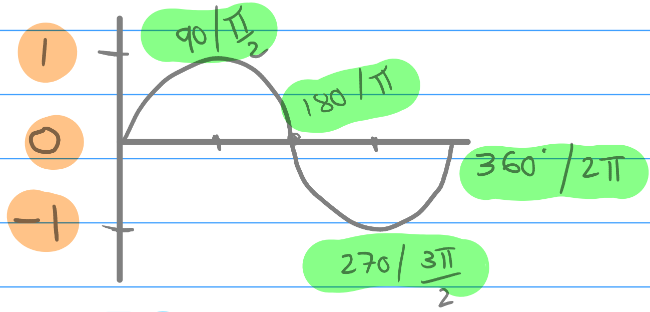
TRIGONOMETRY

Including

TRIG GRAPHS

$\sin\theta$



degrees | radians

Convert:

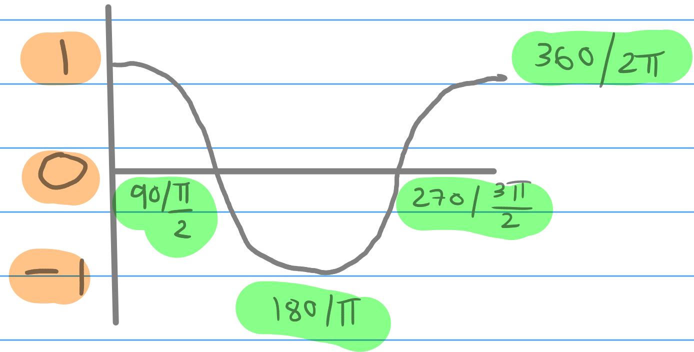
radians to degrees

$\theta \times \frac{180}{\pi}$

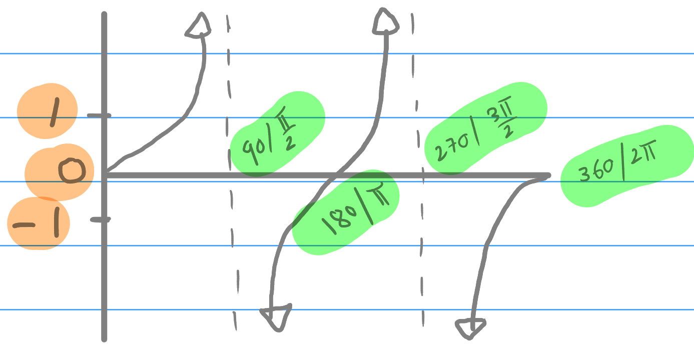
degrees to radians

$\theta \times \frac{\pi}{180}$

$\cos\theta$



$\tan\theta$



shift + / −

General form: $a \cdot \sin bx + q$

reflect over x-axis

amplitude
max to min

vertical
squish/stretch
Counter intuitive

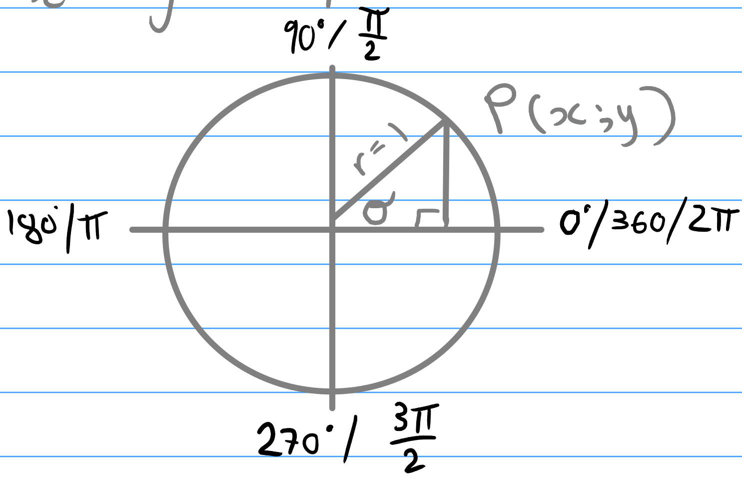
horizontal stretch/squish
intuitive
$a > 1$ longer
$-1 < a < 1$ $a \neq 0$ shorter

$b > 1$
shorter
$-1 < b < 1$
$b \neq 0$
longer

UNIT CIRCLE

A circle with radius $= 1$

$x^2 + y^2 = 1$



$\cos\theta = x$

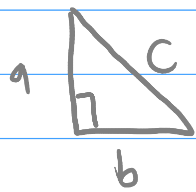
$\sin\theta = y$

$\tan\theta = \frac{y}{x}$

OTHER USEFUL NOTES:

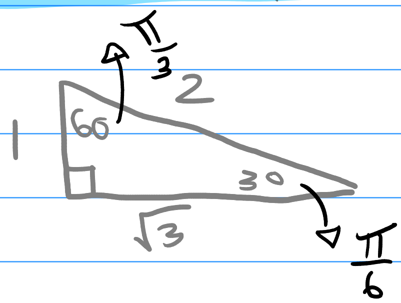
$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$ $\qquad$ $S\frac{y}{r}\ C\frac{x}{r}\ T\frac{y}{x}$

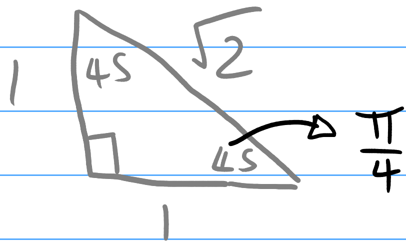
Pythagorus



$a^2 + b^2 = c^2$

SPECIAL TRIANGLES



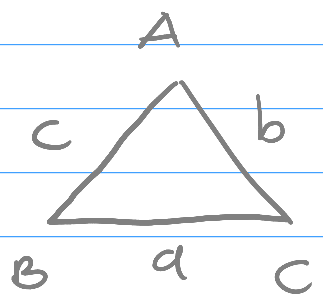


ANGLES OF INCLINATION

$\tan\theta = m$

AREA RULE

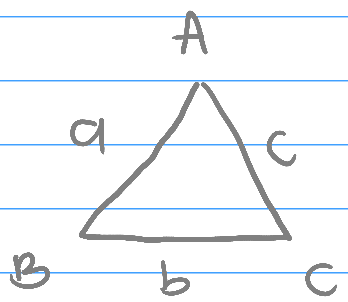
In $\triangle ABC$



$$\text{Area} = \frac{1}{2}\cdot a \cdot b \cdot \sin \hat{C}$$
$$= \frac{1}{2}\cdot a \cdot c \cdot \sin \hat{B}$$
$$= \frac{1}{2}\cdot b \cdot c \cdot \sin \hat{A}$$

Cos rule

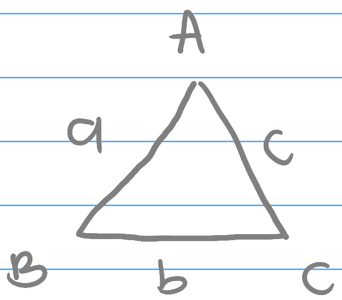
In triangle ABC



$$a^2 = b^2 + c^2 - 2(a)(b)\cos \hat{C}$$

Sin rule

In triangle ABC



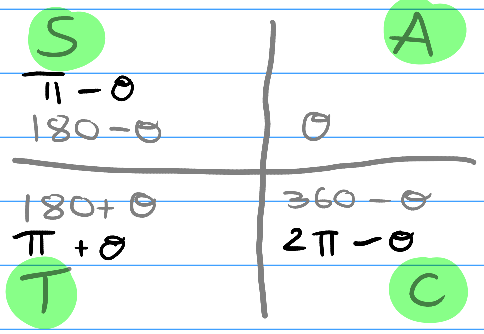
$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

REDUCTION FORMULAE $(180° / 360°)$

sin, cos, tan (obtuse or reflex angle) <=> $\pm$ sin, $\pm$ cos, $\pm$ tan (acute <)

the CAST diagram shows where each will be positive:

degrees or radians



off the x-axis

CO-RATIOS (for 90°)

$\sin(90° + \theta) \iff \cos\theta$

$\sin(90° - \theta) \iff \cos\theta$

$\cos(90° + \theta) \iff -\sin\theta$

$\cos(90° - \theta) \iff \sin\theta$

if you remember this one, the others just swith cos/sin

in radians:

$\sin\left(\frac{\pi}{2} + \theta\right) \iff \cos\theta$

$\sin\left(\frac{\pi}{2} - \theta\right) \iff \cos\theta$

$\cos\left(\frac{\pi}{2} + \theta\right) \iff -\sin\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) \iff \sin\theta$

TRIG IDENTITIES

Quotient identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Pyth. Identity

$$\cos^2\theta + \sin^2\theta = 1$$

DOUBLE ANGLE FORMULAE

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$

COMPOUND ANGLE FORMULAE

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

GENERAL SOLUTION

Gives every possible answer considering that trig graphs are repeating.

STEPS:
1. Determine the positive reference angle
2. Apply the reference angle to the relevent quadrants
3. Simplify and don't forget $n \in \mathbb{Z}$

Example 1

given $\tan x \sin x + \cos x \tan x = 0$

Determine the general solution.

Answer:

1 $\tan x \sin x + \cos x \tan x = 0$

$\tan x(\sin x + \cos x) = 0$

$\therefore \tan x = 0$ OR $\sin x + \cos x = 0$

$x = \tan^{-1}(0)$

$= 0$

$\sin x = -\cos x$

$\frac{\sin x}{\cos x} = -1$

$\tan x = -1$

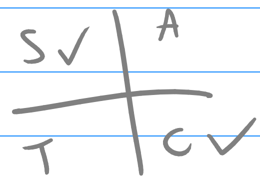
$x = \tan^{-1}(-1)$

$= (-)\frac{\pi}{4} \,/\, (-)45$

∴ reference angles :

$x = 0$ or $x = 45 \;/\; \frac{\pi}{4}$

(2) tan is negative in quadrant 2 and 4



tan period of 180

(3) $\therefore x = 180 - 0 + n180$

$= 180 + n180 \quad (n \in \mathbb{Z})$

or $x = 360 - 0 + n180$

$= 360 + n180 \quad (n \in \mathbb{Z})$

or

$x = 180 - 45 + n180$

$= 135 + n180 \; (n \in \mathbb{Z})$

or

$x = 360 - 45 + n180$

$= 315 + n180 \; (n \in \mathbb{Z})$