Assignment 1: Report

Team:

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2.1 Task 1: Linear Regression

The linear regression model takes given pairs of x and y values from the training data as arguments and fits them along a straight line/curve depending on the order of the x value projection.

LinearRegression().fit() does this in the following steps:

- → Arbitrarily initializing a set of coefficients for these models.
- → Calculating mean squared error for a predicted model
- → The regressor now penalizes the existing model and consequently provides us a new set of coefficients.
- → These two steps are repeated until the penalizing has little to no effect on the existing model

Parameters:

fit_intercept : bool, default=True

Whether to calculate the intercept for this model. If set to False, no intercept will be used in calculations (i.e. data is expected to be centered).

normalize: bool, default=False

This parameter is ignored when fit_intercept is set to False. If True, the regressors X will be normalized before regression by subtracting the mean and dividing by the I2-norm. If you wish to standardize, please use StandardScaler before calling fit on an estimator with normalize=False.

copy_X : bool, default=True

If True, X will be copied; else, it may be overwritten.

n_jobs : int, default=None

The number of jobs to use for the computation. This will only provide speedup for n_targets > 1 and sufficient large problems. None means 1 unless in a joblib.parallel_backend context. -1 means using all processors. See Glossary for more details.

positive : bool, default=False

When set to True, forces the coefficients to be positive. This option is only supported for dense arrays.

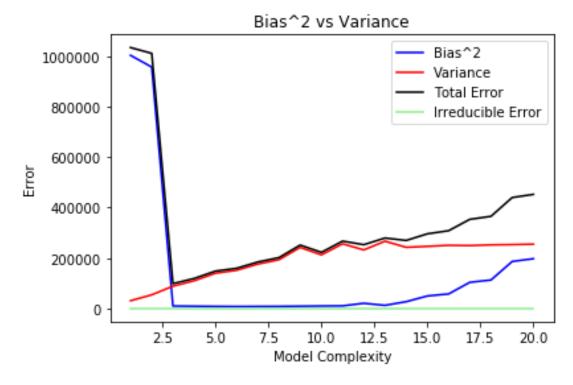
2.2.2 Task

Bias and variance change on varying function classes

Here is a table of **Degree-Bias-Variance-TotalError**

| Degree | Bias | Variance | Total Error |
|--------|------------|---------------|--------------|
| 1 | 231.357297 | 30861.144128 | 1.033871e+06 |
| 2 | 227.326495 | 54270.382667 | 1.010896e+06 |
| 3 | 15.970239 | 88571.607355 | 9.860308e+04 |
| 4 | 10.159046 | 109998.124526 | 1.190728e+05 |
| 5 | 6.952178 | 139727.351411 | 1.481205e+05 |
| 6 | 8.811795 | 151462.424236 | 1.593715e+05 |
| 7 | 2.629138 | 176036.484613 | 1.842606e+05 |
| 8 | 7.689096 | 193223.168482 | 2.016299e+05 |
| 9 | 6.241115 | 242173.902770 | 2.513633e+05 |
| 10 | 11.898679 | 212535.367371 | 2.226044e+05 |
| 11 | 9.204463 | 256419.447705 | 2.670020e+05 |
| 12 | 5.094152 | 232219.386098 | 2.528695e+05 |
| 13 | 12.316387 | 266680.328945 | 2.792715e+05 |
| 14 | 21.939739 | 242568.968659 | 2.696649e+05 |
| 15 | 30.129640 | 246238.836329 | 2.959643e+05 |
| 16 | 31.325810 | 250506.007469 | 3.081951e+05 |
| 17 | 43.970706 | 249467.889831 | 3.532988e+05 |
| 18 | 46.828925 | 252249.732550 | 3.652708e+05 |
| 19 | 61.438390 | 253243.693744 | 4.397213e+05 |
| 20 | 65.461974 | 254879.277230 | 4.522052e+05 |

And here is the **Bias^2 vs Variance** graph for reference (Task 4)



- → The best fit possible (Minimum Total Error) is observed around a polynomial of degree 3.
- → Bias stays low until around degree 10-11, but the variance increases steadily.
- → Hence, the total error increases as well.

Polynomials of degree 1 & 2:

- → They are oversimplified and do not generalize the data well, leading to a high bias
- → They are consistent with not very scattered predicted values (lesser features), which is why they have low variance.
- → These models are overall inaccurate, evident from the total error value.

Polynomial of degree 3:

 \rightarrow The lowest total error is demonstrated by this regression.

→ We can see that it generalizes best to data not seen by the model before.

Polynomials of degree 4+:

- → Variance increases gradually with an increase in complexity.
- → The is expected since the number of features is higher, leading to overfitting of models over the trained data, and can be explained by regression models trying to fit the noise from the training data.
- → The flexibility provided by having more features leads to noise being captured which causes incorrect prediction for unseen data.
- → There is relatively low bias as the more complex model is able to model the training set well.

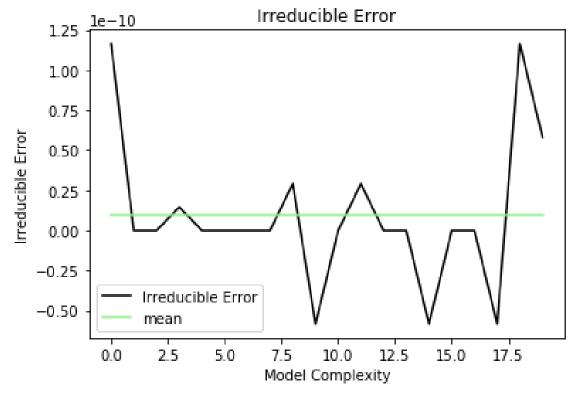
2.3 Task 3 : Calculating Irreducible Error

Here is the irreducible error by degree table

| Degree | Irreducible Error | |
|--------|-------------------|--|
| 1 | 1.164153e-10 | |
| 2 | 0.000000e+00 | |
| 3 | 0.000000e+00 | |
| 4 | 1.455192e-11 | |
| 5 | 0.000000e+00 | |
| 6 | 0.000000e+00 | |
| 7 | 0.000000e+00 | |
| 8 | 0.000000e+00 | |
| 9 | 2.910383e-11 | |
| 10 | -5.820766e-11 | |
| 11 | 0.000000e+00 | |
| 12 | 2.910383e-11 | |

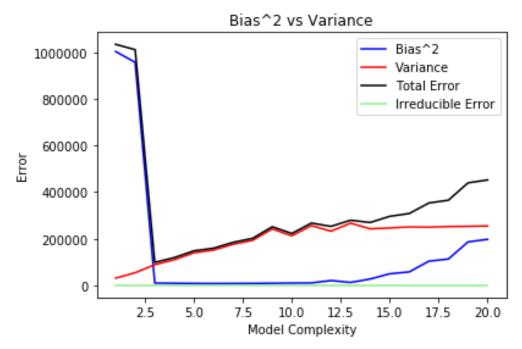
| 13 | 0.000000e+00 | |
|----|---------------|--|
| 14 | 0.000000e+00 | |
| 15 | -5.820766e-11 | |
| 16 | 0.000000e+00 | |
| 17 | 0.000000e+00 | |
| 18 | -5.820766e-11 | |
| 19 | 1.164153e-10 | |
| 20 | 5.820766e-11 | |

Here is the irreducible error vs degree graph



- ightharpoonup We can see that the irreducible error shows no consistent patterns or variations.
- → They cannot be reduced, regardless of the algorithm applied
- → It is a measure of noise in the data, which is independent of the model or regression analysis used to make predictions.

2.4 Task 4 : Plotting bias ^2 - variance graph



Observations above, in 2.2.2.