FEH3A3– PENGOLAHAN SINYAL WAKTU DISKRIT

PERANCANGAN FILTER DIGITAL RESPON IMPULS TERBATAS (FIR)

FAKULTAS TEKNIK ELEKTRO

2 Methods FIR Filter Design

Windowing Method

$$H(\omega)$$

Inverse Discrete Time Fourier Transform / ITFWD

$$h_i(n)$$

Windowing

$$h(n) = h_i(n)w(n)$$

Frequency Sampling Method

$$H(\omega)$$

Sampling

$$H(k)$$

Inverse Discrete Fourier Transform/IDFT/IFFT

$$h(n)$$

Langkah-langkah Merancang Filter Digital FIR Menggunakan Metode Windowing

1. **Sketch Magnitude Response of Digital Filter as the specification needed**
2. **Determine the ideal impulse response h_i(n) from Magnitude Response 1st step by Inverse DTFT (look up the table)**
3. **Determine the delay /symmetrical axis (α), filter order (N), Filter length (M)**
4. **Determine and calculate the delayed impulse response in which the delay was determined from 3rd step, from 0 to N (N-filter order with N+1 filter length)**
5. **Calculate the coefficient of the window used from 0 to N (N-filter order with N+1 filter length) (given)**
6. **Multiply the result of 4th and 5th step to determine the overall filter coefficient**

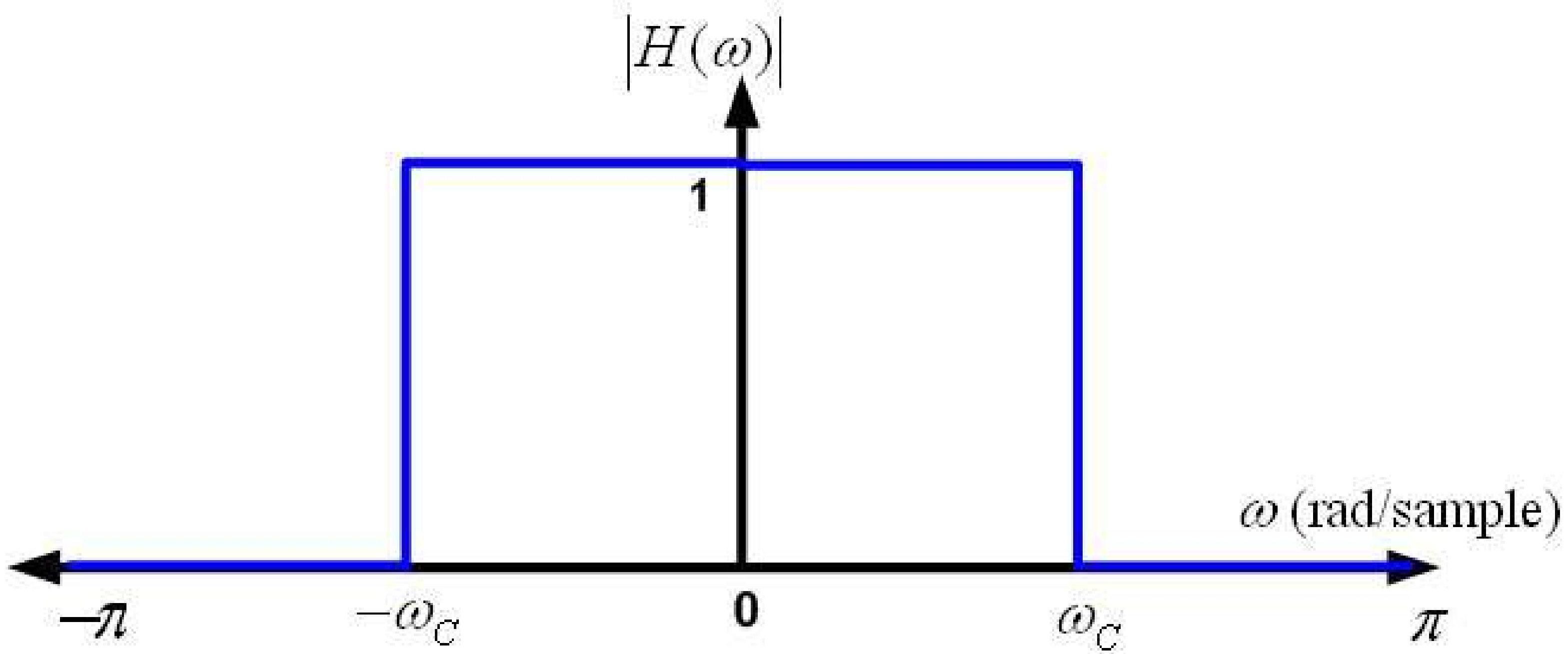
N-order Windowing Methods FIR Filter Design

$$
\begin{tikzcd}
H(\omega) \arrow[r] \arrow[d] & \left|H(\omega)\right| \arrow[r, "\text{Inverse Discrete Time Fourier Transform / ITFWD}"'] & h_i(n) \arrow[d] \\
\alpha \arrow[rr] & & h_i(n-\alpha) \arrow[d, "\text{Windowing}"] \\
\text{window used : } w(n) \arrow[rr] & & h(n) = h_i(n-\alpha)w(n)
\end{tikzcd}
$$

Filter length : N+1

Steps 1-2 (Several Ideal Magnitude Response

LPF

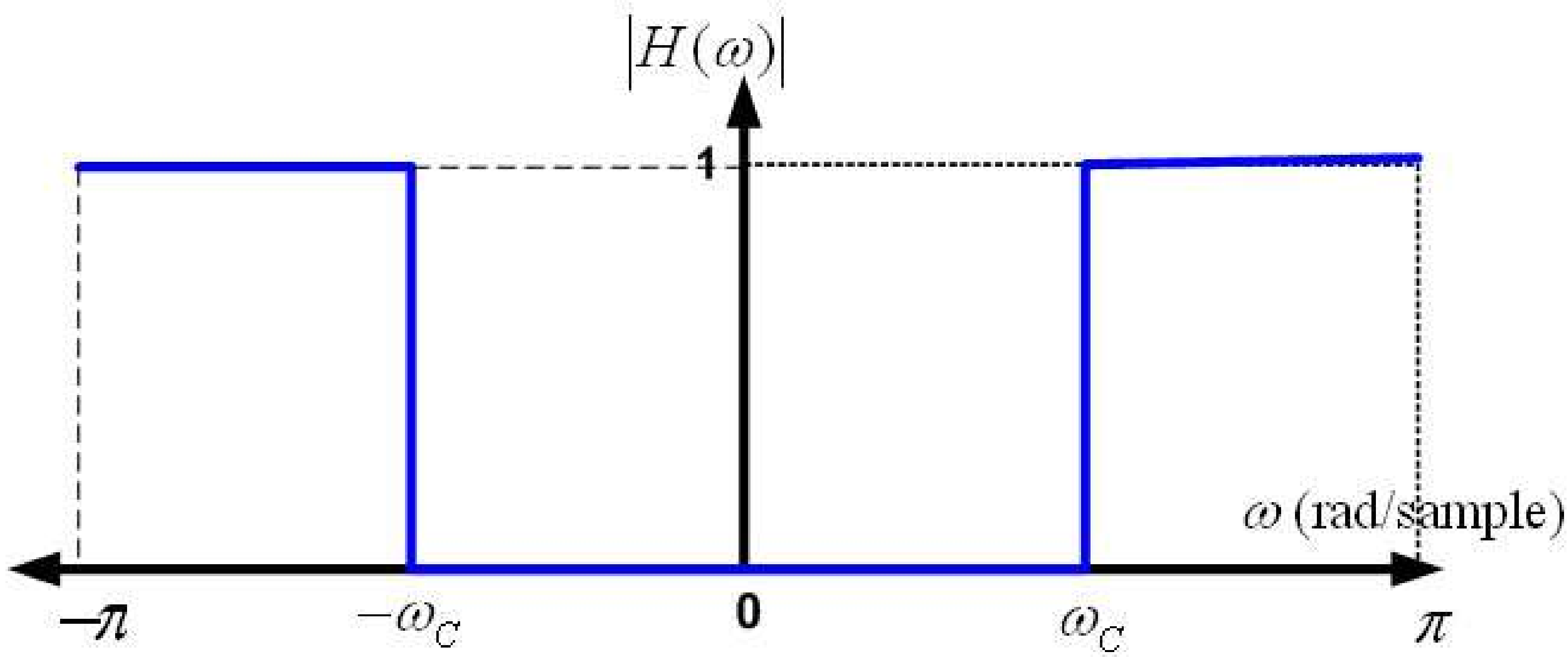


$$H_i\left(e^{j\omega}\right)=\begin{cases}1.e^{-j\alpha\omega}, & |\omega|\le\omega_c\\ 0, & \omega_c<|\omega|\le\pi\end{cases}$$

$$h_i(n)=\frac{\sin\left[\omega_c(n-\alpha)\right]}{\pi(n-\alpha)}$$

Steps 1-2 (Several Ideal Magnitude Response

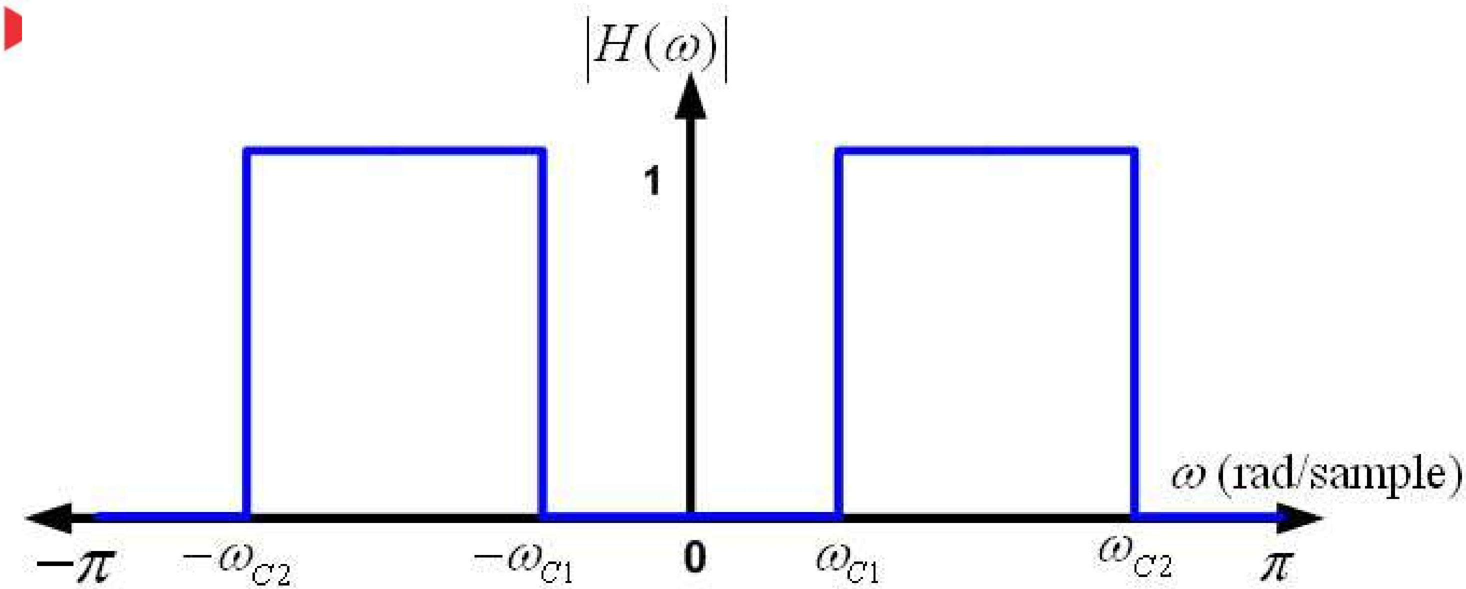
HPF



$$H_i\left(e^{j\omega}\right)=\begin{cases}1.e^{-j\alpha\omega}, & \omega_c<|\omega|\le\pi\\ 0, & |\omega|\le\omega_c\end{cases}\quad h_i(n)=\frac{\sin\left[\pi(n-\alpha)\right]-\sin\left[\omega_c(n-\alpha)\right]}{\pi(n-\alpha)}$$

Steps 1-2 (Several Ideal Magnitude Response)

BPF

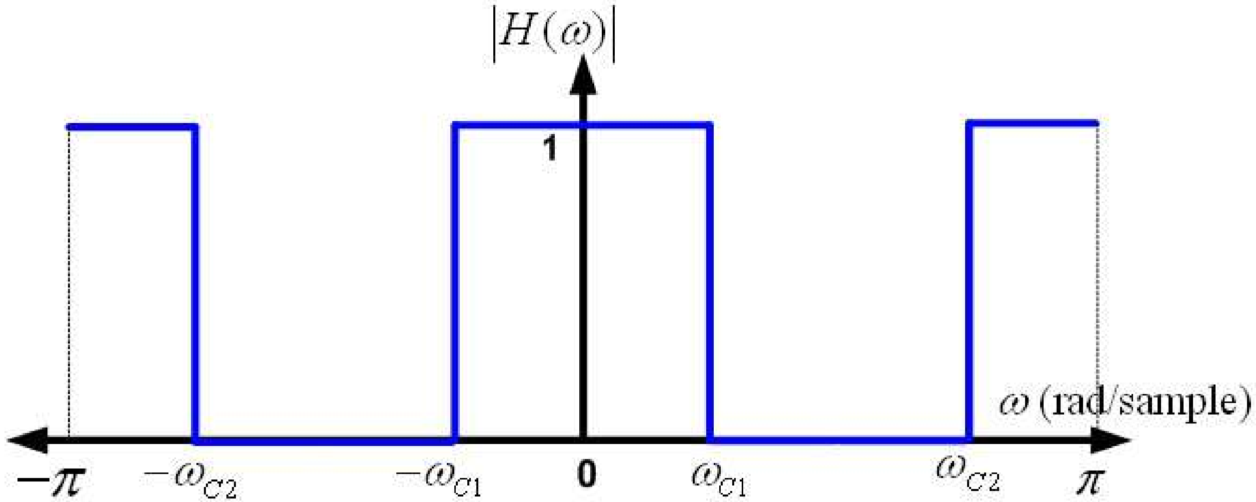


$$H_i\left(e^{j\omega}\right)=\begin{cases}0, & 0\le|\omega|<\omega_{c1}\\ 1.e^{-j\alpha\omega}, & \omega_{c1}\le|\omega|\le\omega_{c2}\\ 0, & \omega_{c2}<|\omega|\le\pi\end{cases}$$

$$h_i(n)=\frac{\sin\left[\omega_{c2}(n-\alpha)\right]-\sin\left[\omega_{c1}(n-\alpha)\right]}{\pi(n-\alpha)}$$

Steps 1-2 (Several Ideal Magnitude Response)

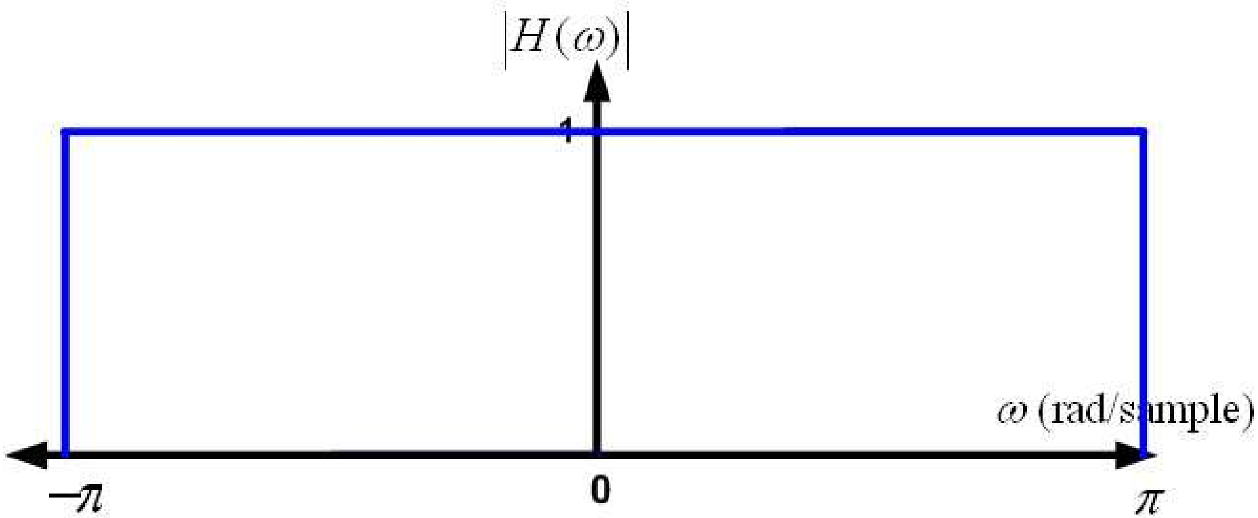
BSF



$$H_i\left(e^{j\omega}\right)=\begin{cases}1.e^{-j\alpha\omega}, & 0\le|\omega|<\omega_{c1}\\ 0, & \omega_{c1}\le|\omega|\le\omega_{c2}\\ 1.e^{-j\alpha\omega}, & \omega_{c2}<|\omega|\le\pi\end{cases}$$

$$h_i(n)=\frac{\sin\left[\omega_{c1}(n-\alpha)\right]-\sin\left[\omega_{c2}(n-\alpha)\right]+\sin\left[\pi(n-\alpha)\right]}{\pi(n-\alpha)}$$

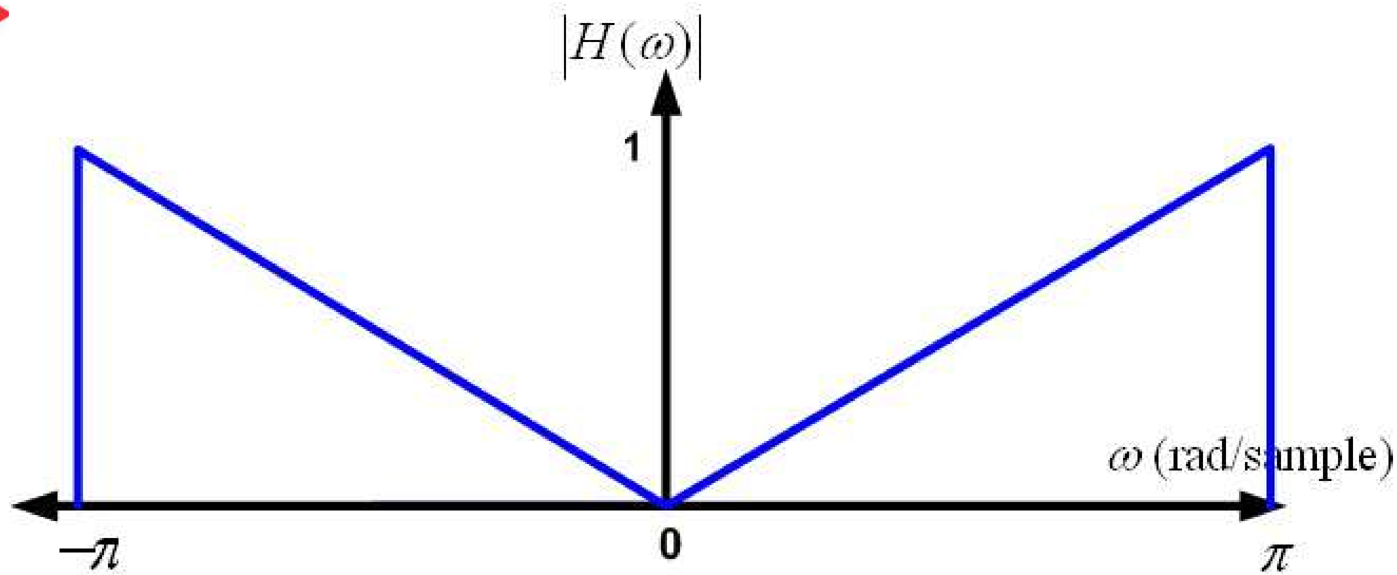
Steps 1-2 (Several Ideal Magnitude Response Filter/Hilbert



$$H_i\left(e^{j\omega}\right)=\begin{cases}-j.e^{-j\alpha\omega}, & 0<\omega<\pi\\ j.e^{-j\alpha\omega}, & -\pi<\omega<0\end{cases}\qquad h_i(n)=\begin{cases}\dfrac{2}{\pi}\dfrac{\sin^2\left[\dfrac{\pi(n-\alpha)}{2}\right]}{n-\alpha}, & n\neq\alpha\\ 0, & n=\alpha\end{cases}$$

1-2 (Several Ideal Magnitude Response)

Differensiator



$$H_i\left(e^{j\omega}\right)=\begin{cases} j\omega.e^{-j\alpha\omega}, & 0<\omega\leq\pi \\ -j\omega.e^{-j\alpha\omega}, & -\pi<\omega<0 \end{cases}$$

$$h_i\left(n\right)=\begin{cases} \dfrac{\cos\left[\pi\left(n-\alpha\right)\right]}{\pi\left(n-\alpha\right)}, & n\neq\alpha \\ 0, & n=\alpha \end{cases}$$

Respon Filter Ideal Untuk Perancangan FIR Windowing

Jenis Filter Ideal	Respon Filter Ideal dlm Respon Frekuensi (Frekuensi domain)	Respon Impuls Ideal (Time domain)
LPF	$H_d(e^{j\omega}) = \begin{cases} 1.e^{-j\alpha\omega}, & \lvert\omega\rvert \le \omega_c \\ 0, & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$	$h_d(n) = \dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}$
HPF	$H_d(e^{j\omega}) = \begin{cases} 1.e^{-j\alpha\omega}, & \omega_c < \lvert\omega\rvert \le \pi \\ 0, & \lvert\omega\rvert \le \omega_c \end{cases}$	$h_d(n) = \dfrac{\sin[\pi(n-\alpha)] - \sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}$
BPF	$H_d(e^{j\omega}) = \begin{cases} 0, & 0 \le \lvert\omega\rvert < \omega_{c1} \\ 1.e^{-j\alpha\omega}, & \omega_{c1} \le \lvert\omega\rvert \le \omega_{c2} \\ 0, & \omega_{c2} < \lvert\omega\rvert \le \pi \end{cases}$	$h_d(n) = \dfrac{\sin[\omega_{c2}(n-\alpha)] - \sin[\omega_{c1}(n-\alpha)]}{\pi(n-\alpha)}$
BSF	$H_d(e^{j\omega}) = \begin{cases} 1.e^{-j\alpha\omega}, & 0 \le \lvert\omega\rvert < \omega_{c1} \\ 0, & \omega_{c1} \le \lvert\omega\rvert \le \omega_{c2} \\ 1.e^{-j\alpha\omega}, & \omega_{c2} < \lvert\omega\rvert \le \pi \end{cases}$	$h_d(n) = \dfrac{\sin[\omega_{c1}(n-\alpha)] - \sin[\omega_{c2}(n-\alpha)] + \sin[\pi(n-\alpha)]}{\pi(n-\alpha)}$
Differensiator	$H_d(e^{j\omega}) = \begin{cases} j\omega e^{-j\alpha\omega}, & 0 < \omega \le \pi \\ -j\omega e^{-j\alpha\omega}, & -\pi < \omega < 0 \end{cases}$	$h_d(n) = \begin{cases} \dfrac{\cos[\pi(n-\alpha)]}{\pi(n-\alpha)}, & n \ne \alpha \\ 0, & n = \alpha \end{cases}$
Hilbert Transform	$H_d(e^{j\omega}) = \begin{cases} -j.e^{-j\alpha\omega}, & 0 < \omega < \pi \\ j.e^{-j\alpha\omega}, & -\pi < \omega < 0 \end{cases}$	$h_d(n) = \begin{cases} \dfrac{2}{\pi} \dfrac{\sin^2\left[\dfrac{\pi(n-\alpha)}{2}\right]}{n-\alpha}, & n \ne \alpha \\ 0, & n = \alpha \end{cases}$

Steps 3
Determining α, N (Filter Order), M (Filter length)

$$\alpha = \left| \frac{\angle H_i(\omega)}{\omega} \right|$$

$$M = 2\alpha + 1 \qquad N = M - 1$$

$$\text{Example} \;:\; \text{If } H_i\left(e^{j\omega}\right) = \begin{cases} 1.e^{-j2\omega}, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$

$$\text{Then } \alpha = 2$$

Steps 4

Calculating $h_i(n-\alpha)$

Calculate $h_i(n-\alpha)$ from $n=0$ to $n=N$

Then

$$h_i(n-\alpha)$$

$$=$$

$$\begin{bmatrix} h_i(-\alpha) & h_i(-\alpha+1) & h_i(-\alpha+2) & \ldots & h_i(-\alpha+N-1) & h_i(-\alpha+N) \end{bmatrix}$$

Steps 5
Calculating w(n)

Example : If window used is rectangular

Then : $w(n) = \begin{cases} 1, 0 \le n \le N \\ 0, otherwise \end{cases}$

$$w(n) = \begin{bmatrix} w(0) & w(1) & \ldots\ldots & w(N) \end{bmatrix}$$

Karakteristik Jendela/Window Pada Perancangan FIR

Nama Jendela	Lebar Transisi $\Delta\omega$		Redaman Stopband Minimal (dB)	Rumus
	Pendekatan	Nilai Exact		
Rectangular	$\frac{4\pi}{M}$	$\frac{1{,}8\pi}{M}$	21	$w(n) = \begin{cases} 1, & 0 \le n \le M-1 \\ 0, & otherwise \end{cases}$
Bartlett	$\frac{8\pi}{M}$	$\frac{6{,}1\pi}{M}$	25	$w(n) = \begin{cases} \frac{2n}{M-1}, & 0 \le n \le \frac{M-1}{2} \\ 2 - \frac{2n}{M-1}, & \frac{M-1}{2} \le n \le M-1 \\ 0, & otherwise \end{cases}$
Hanning	$\frac{8\pi}{M}$	$\frac{6{,}2\pi}{M}$	44	$w(n) = \begin{cases} 0{,}5\left[1 - \cos\left(\frac{2\pi n}{M-1}\right)\right], & 0 \le n \le M-1 \\ 0, & otherwise \end{cases}$
Hamming	$\frac{8\pi}{M}$	$\frac{6{,}6\pi}{M}$	53	$w(n) = \begin{cases} 0{,}54 - 0{,}46\cos\left(\frac{2\pi n}{M-1}\right), & 0 \le n \le M-1 \\ 0, & otherwise \end{cases}$
Blackman	$\frac{12\pi}{M}$	$\frac{11\pi}{M}$	74	$w(n) = \begin{cases} 0{,}42 - 0{,}5\cos\left(\frac{2\pi n}{M-1}\right) + 0{,}08\cos\left(\frac{4\pi n}{M-1}\right), & 0 \le n \le M-1 \\ 0, & otherwise \end{cases}$

Steps 6
Calculating $h(n)=h_i(n)w(n)$

Example :

$$h(n) = \left[h_i(-\alpha)\, w(0) \quad h_i(-\alpha+1) w(1) \quad \ldots\ldots \quad h_i(-\alpha+N) w(N) \right]$$

Rancanglah suatu filter FIR dengan respon frekuensi diinginkan sbb :

$$H(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & -0{,}5\pi \le \omega \le -\pi \\ 0 & ,-0{,}5\pi \le \omega \le 0{,}5\pi \\ e^{-j3\omega}, & 0{,}5\pi \le \omega \le \pi \end{cases}$$

Akan dirancang dengan metoda *windowing* menggunakan *window* hamming.

$$w(n) = 0{,}54 - 0{,}46\cos\left(\frac{2\pi n}{M-1}\right), \ 0 \le n \le M\text{-}1$$

Frekuensi pencuplikan yang dipakai 20 kHz.

Tentukan :

a. Hitunglah koefisien filter digital tersebut !

b. Apakah filter stabil dan kausal? Jelaskan!

c. Menurut anda sistem ini berfungsi sbg apa ? (LPF, HPF, BPF, BSF, Differensiator, atau Hilbert transform)

d. Realisasikan filter !

a. $\omega_c = 0{,}5\Pi$

$\alpha=3$

Panjang filter M= 2α+1= 7

Orde filter N= M-1= 6

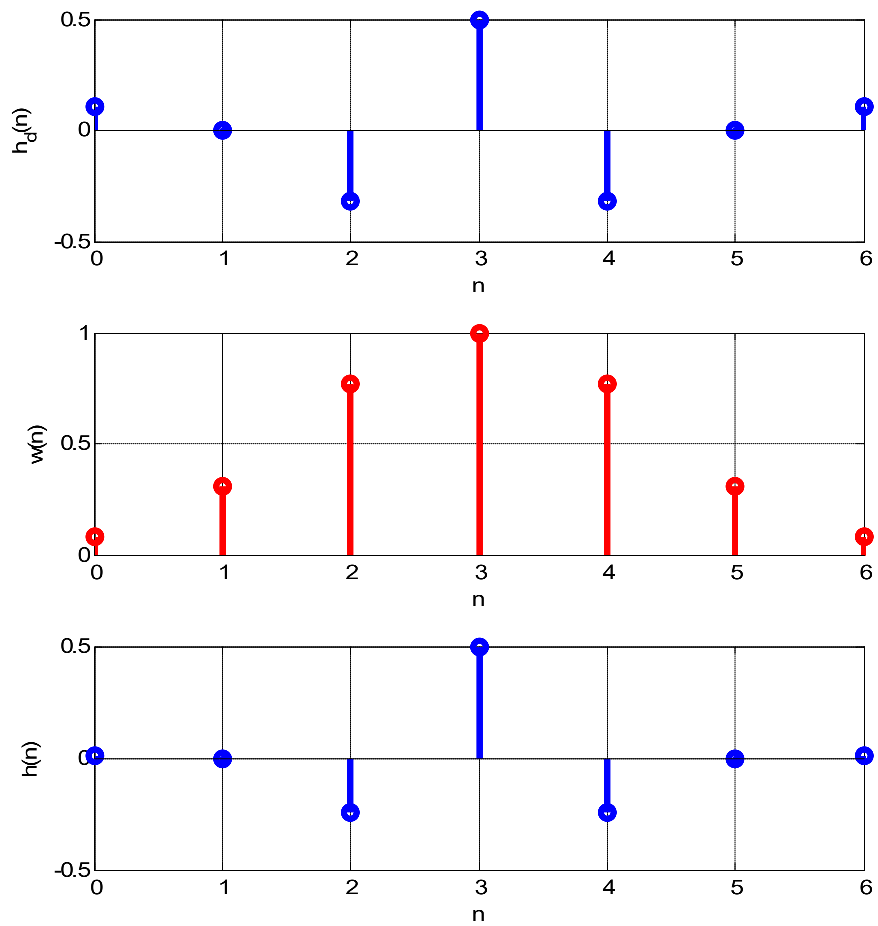
$$h(n) = h_d(n).w(n) = \frac{\sin\pi(n-3) - \sin 0{,}5\pi(n-3)}{\pi(n-3)}\left[0{,}54 - 0{,}46\cos\left(\frac{2\pi n}{M-1}\right)\right]$$

$$,0 \le n \le M-1$$

$$h_d(n) = [0.1061 \;\; 0 \;\; -0.3183 \;\; 0.5000 \;\; -0.3183 \;\; 0 \;\; 0.1061]$$

$$w(n) = [\,0.0800 \;\; 0.3100 \;\; 0.7700 \;\; 1.0000 \;\; 0.7700 \;\; 0.3100 \;\; 0.0800]$$

$$h(n) = [0.0085 \;\; 0 \;\; -0.2451 \;\; 0.5000 \;\; -0.2451 \;\; 0 \;\; 0.0085]$$

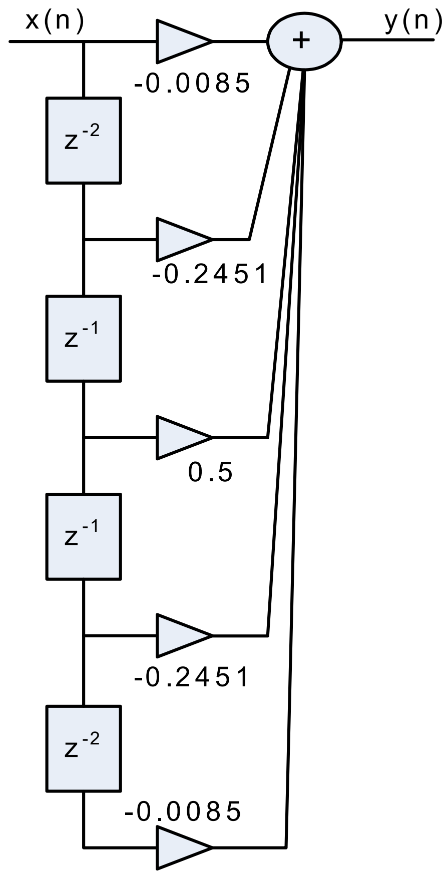


0.5
0
-0.5
$h_d(n)$
0
1
2
3
4
5
6
n
1
0.5
0
w(n)
0
1
2
3
4
5
6
n
0.5
0
-0.5
h(n)
0
1
2
3
4
5
6
n

b. Filter stabil dan kausal karena jenis fiter adalah FIR yang non rekursif dan BIBO

c. Jika dilihat dari respon magnitudenya maka Filter tersebut berfungsi sebagai HPF

d. Realisasi filter:



1. Sampling
2. Hitung dan Gambarkan Respon Magnituda Diskrit Hasil Sampling |H(k)|
3. Menghitung koefisien filter digital h(n) sesuai jumlah Koefisien N (Genap atau Ganjil)
4. Gambar Struktur Realisasi Sistem

Suatu filter akan dirancang dengan metode sampling frekuensi dengan banyak 4 sampel. Filter dapat meloloskan frekuensi diantara 3 kHz dan 6 kHz . Akan dilakukan perancangan filter tersebut dengan frekuensi sampling sebesar 20 kHz.

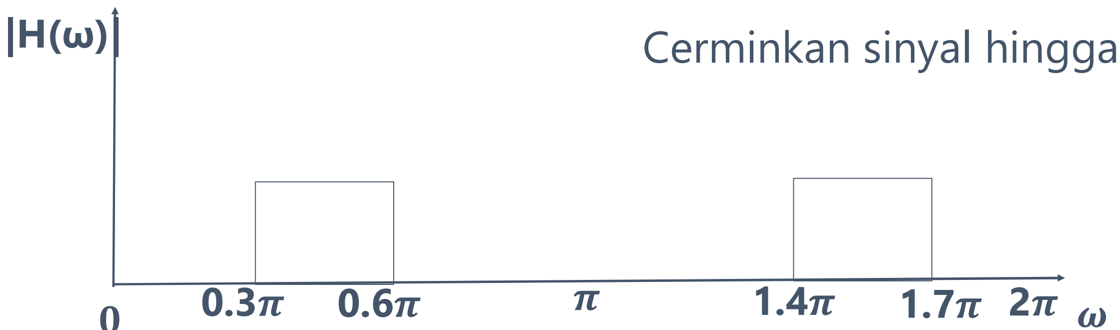
a. Gambarkan respon magnitudo filter yang diinginkan (dari 0 rad/sampel s.d 2π rad/sampel
b. Hitung dan Gambarkan Respon Magnituda Diskrit Hasil sampling
c. Hitung Orde Filter
d. Hitunglah Koefisien Filter Digital

Tahap 1 : Sampling

1. Tentukan dan Gambar Respon Frekuensi Sistem
2. Menentukan jumlah sampling (sesuai kebutuhan). Sampel = N
3. Sinyal terdefinisi dari 0 sampai π, dan sinyal harus dicerminkan hingga 2π.
4. Jarak antar sampel $\Delta\omega = \frac{2\pi}{N}$ .Sampel pertama dimulai dari nol

$$\omega_1 = \frac{2\pi f_1}{Fs} = \frac{2\pi .3kHz}{20kHz} = 0,3\ \pi$$

$$\omega_2 = \frac{2\pi f_1}{Fs} = \frac{2\pi .6kHz}{20kHz} = 0,6\pi$$



Cerminkan sinyal hingga 2π

$$\omega = \frac{2\pi k}{N} = \frac{2\pi k}{4}$$