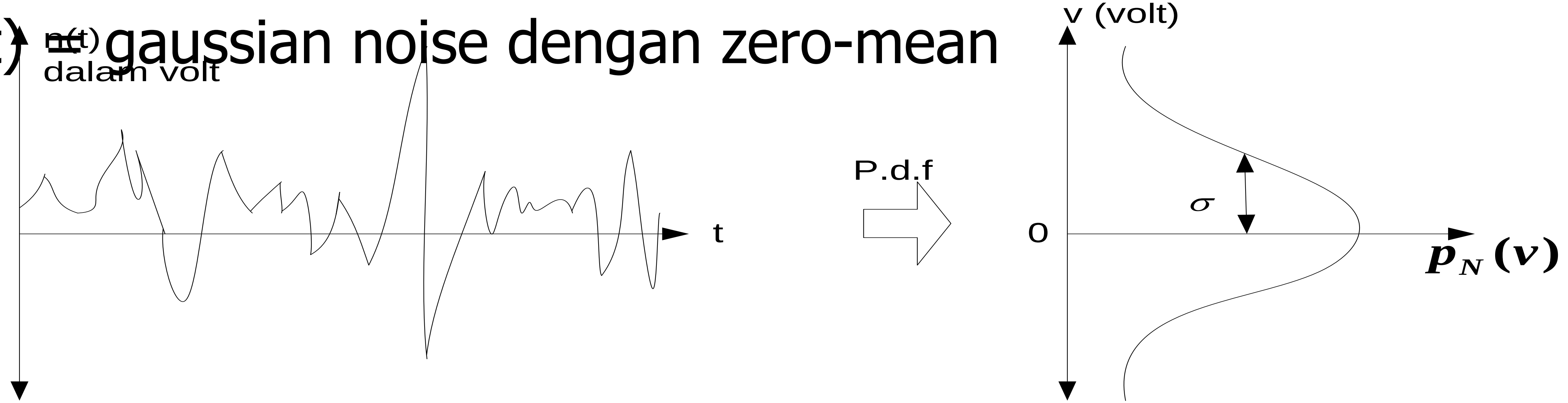
AWGN

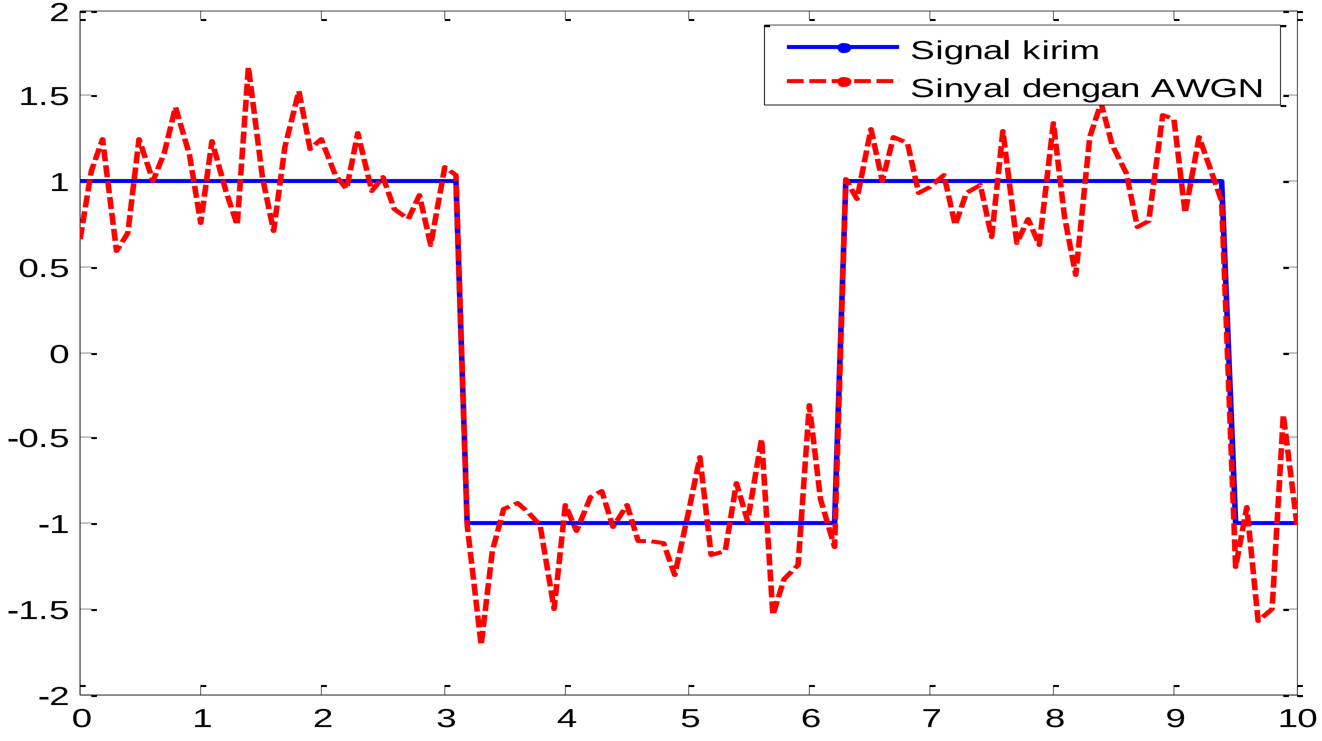
- n(t) = gaussian noise dengan zero-mean

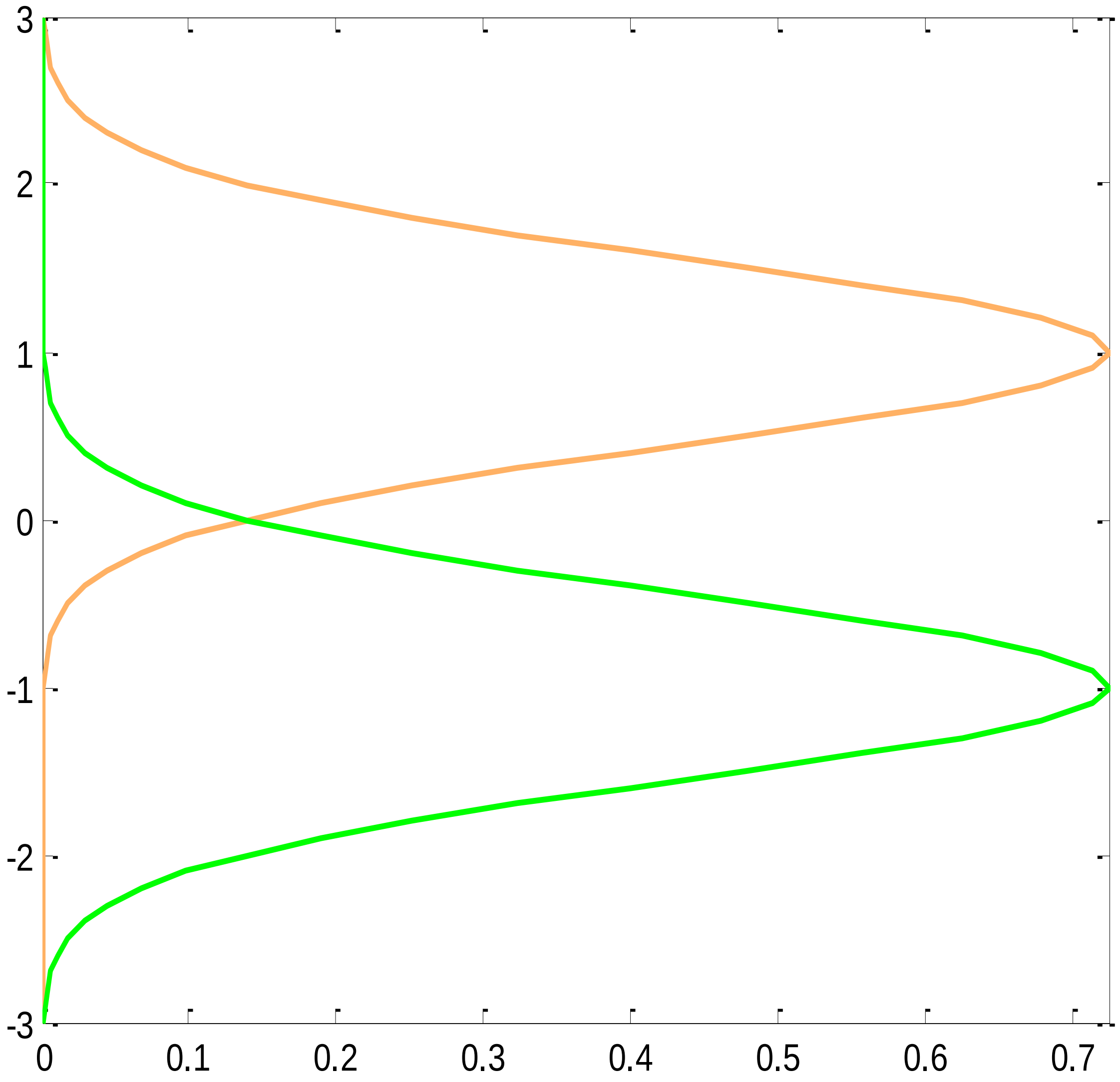


$$p_N(v) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{v^2}{2\sigma^2}\right]$$

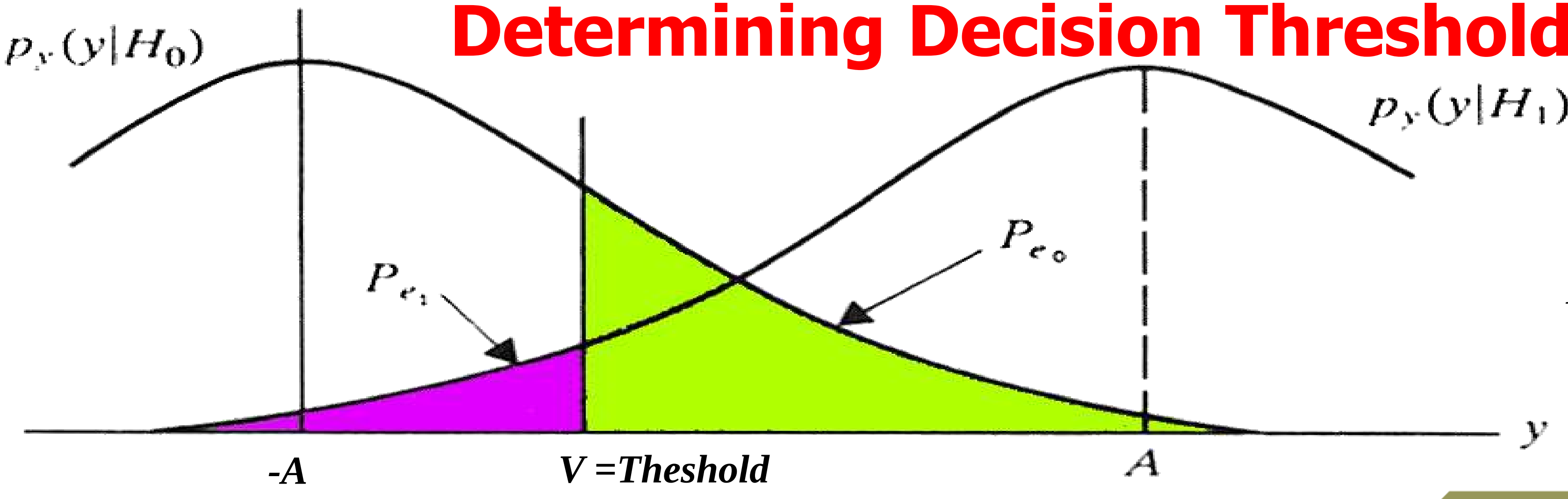
- σ = standar deviasi = tegangan effektive noise

Gangguan Noise Terhadap Sinyal   Digital





Determining Decision Threshold



$$H_0 : a_k = 0, Y = -A + n$$

$$p_Y(y/H_0) = p_N(y + A)$$

$$H_1 : a_k = 1, Y = A + n$$

$$p_Y(y \mid H_1) = p_N(y - A)$$

The comparator implements decision rule:

Choose Ho *(a_k=0)* if *Y<V*
Choose H1 *(a_k=1)* if *Y>V*

$$p_{e1} \equiv P(Y < V \mid H_1) = \int_{-\infty}^{V} p_Y(y \mid H_1)dy$$

$$p_{eo} \equiv P(Y > V \mid H_0) = \int_{V}^{\infty} p_Y(y \mid H_0)dy$$

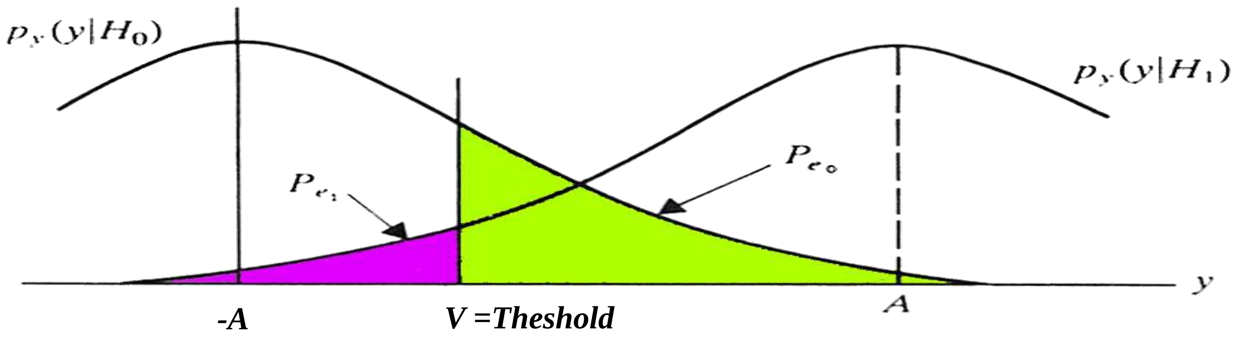
Transmitted ‘0’
but detected as ‘1’

Average error error probability: $P_e = P_0 P_{e0} + P_1 P_{e1}$

$$P_0 = P_1 = 1/2 \Rightarrow P_e = \tfrac{1}{2}(P_{e0} + P_{e1})$$

Channel noise is Gaussian with the pdf: $p_N(y) = \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left[-\dfrac{y^2}{2\sigma^2}\right]$

Error rate and Q-function



$$P_0 = P_1 = 1/2 \Rightarrow P_e = \tfrac{1}{2}(P_{e0} + P_{e1})$$

V threshold = 0

$$P_{e0} = \int_V^\infty p_N(y)dy$$

$$p_e = p_{e0} = \frac{1}{\sigma\sqrt{2\pi}} \int_{V=0}^{\infty} \exp\left[-\frac{(y+A)^2}{2\sigma^2}\right] dy$$

This can be expressed by using the Q-function

$$Q(z) \triangleq p(x > z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy.$$

by

$$p_{e0} = \int_V^\infty p_N(y)dy = p_e = Q\left(\frac{A}{\sigma}\right) = Q\left(\sqrt{\frac{A^2}{\sigma^2}}\right)$$

Baseband Binary Error Rate in Terms of Pulse Shape and S/N

setting *V=0* yields then

$$p_e = \tfrac{1}{2}(p_{e0} + p_{e1}) = p_{e0} = p_{e1} \Rightarrow p_e = Q\left(\frac{A}{\sigma}\right) = Q\left(\sqrt{\frac{A^2}{\sigma^2}}\right) = Q\left(\sqrt{\frac{S}{N}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

for polar, **rectangular** NRZ [-A,A] bits

Probability of occurrence

Signal power:

$$S = \frac{1}{2}A^2 + \frac{1}{2}(-A)^2 = A^2$$

Noise power:

$$N = \sigma^2 = \eta . BW_N = N_0 \cdot \frac{R_b}{2} = N_0 \cdot \frac{1}{2T_b}$$

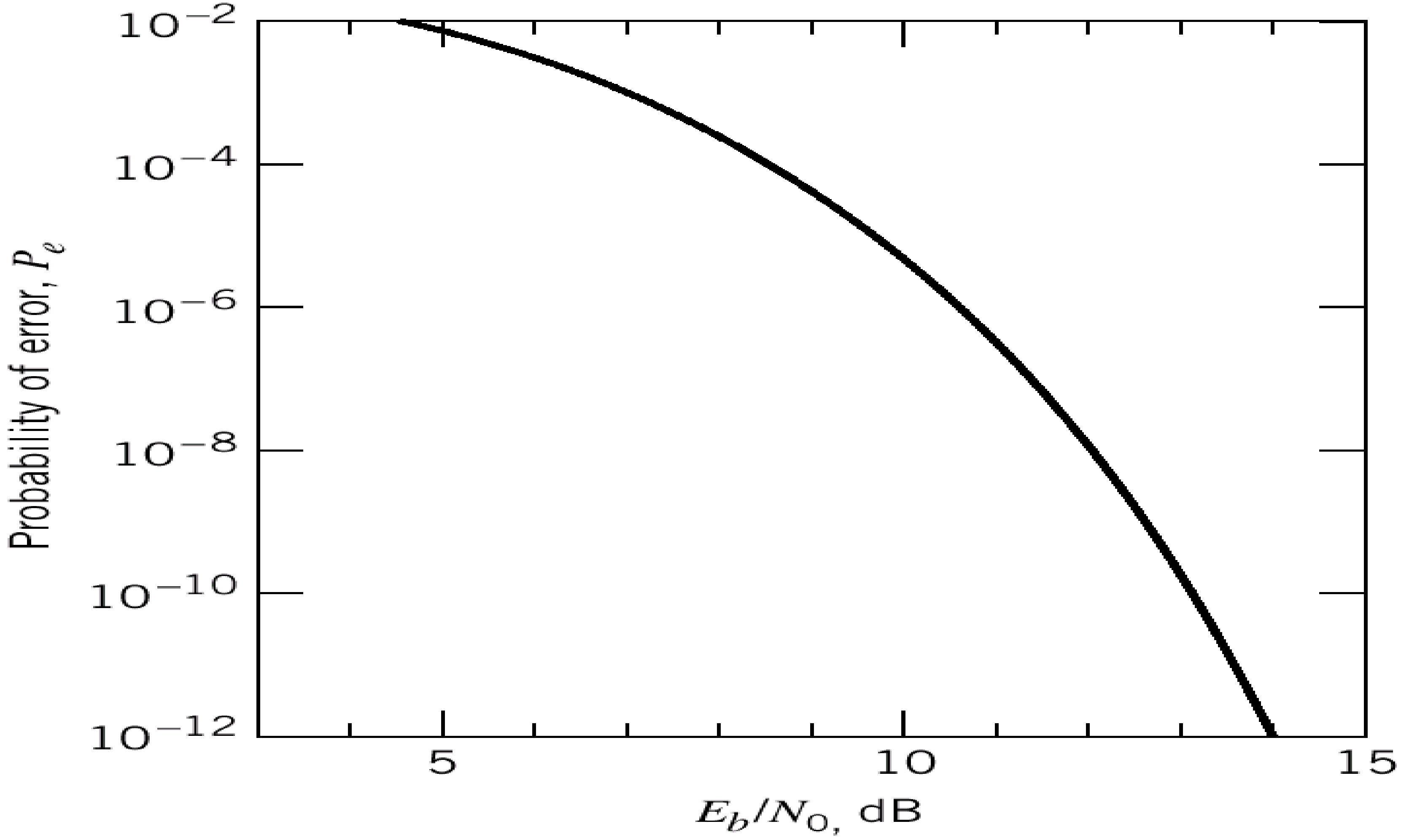
Energy Bit to Noise Spectral Density Ratio

$$\frac{E_b}{N_0} = \frac{S.T_b}{N/BW_N} = \frac{S.T_b}{N/\left(Rb/2\right)} = \frac{S.T_b}{N} \cdot \frac{R_b}{2} = \frac{S.T_b}{N} \cdot \frac{1}{2.T_b} = \frac{1}{2} \cdot \frac{S}{N}$$

Note that $BW_N = \dfrac{R_b}{2}$ (BW pulse shapping filter)

When $p_0 = p_1 = 1/2$, the value of V that minimizes the probability of error is V = 0.

$$p_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{S}{N}}\right)$$

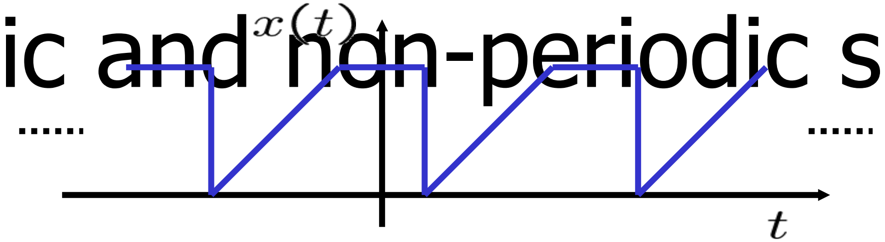


Classification of signals

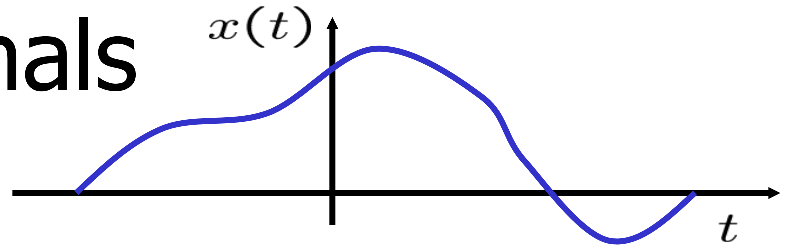
- Deterministic and random signals
  - Deterministic signal: No uncertainty with respect to the signal value at any time.
  - Random signal: Some degree of uncertainty in signal values before it actually occurs.
    - Thermal noise in electronic circuits due to the random movement of electrons
    - Reflection of radio waves from different layers of ionosphere

Classification of signals …

- Periodic and non-periodic signals

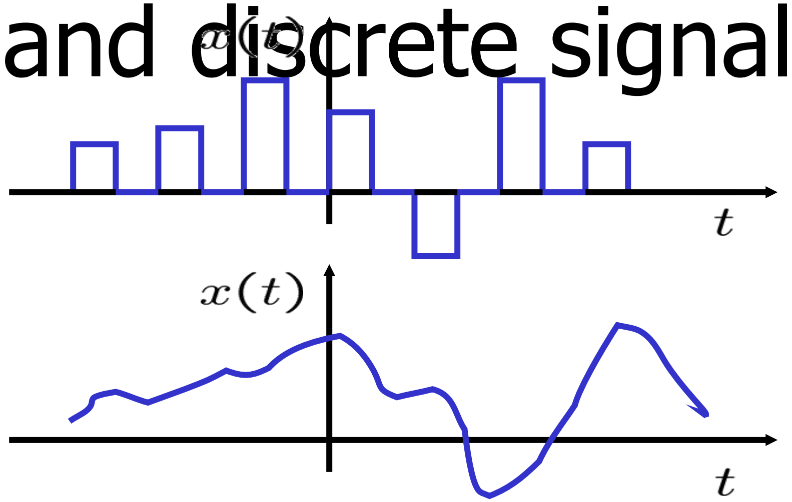


A periodic signal

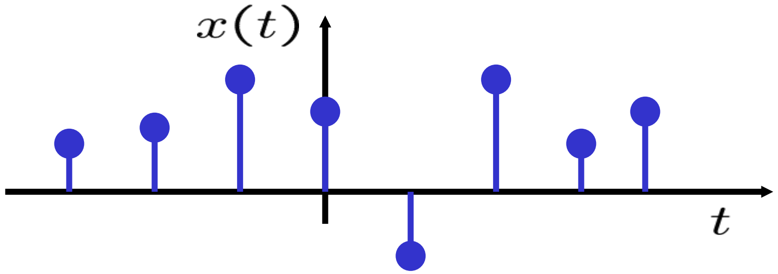


A non-periodic signal

- Analog and discrete signals



Analog signals



A discrete signal

Classification of signals ..

- Energy and power signals
  - A signal is an <u>energy signal</u> if, and only if, it has nonzero but finite energy for all time:

$$(0 < E_x < \infty)$$

$$E_x = \lim_{T \to \infty} \int_{-T/2}^{T/2} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

  - A signal is a <u>power signal</u> if, and only if, it has finite but nonzero power for all time:

$$(0 < P_x < \infty)$$

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

  - General rule: Periodic and random signals are power signals. Signals that are both deterministic and non-periodic are energy signals.

Random process

- A random process is a collection of time functions, or signals, corresponding to various outcomes of a random experiment. For each outcome, there exists a deterministic function, which is called a sample function or a realization.