1. Diketahui Matriks A sebagai berikut:

$$A = \begin{pmatrix} 2 & 2 & 2 & 2 \\ 3 & 1 & 1 & 0 \\ 4 & 2 & 0 & 0 \\ 2 & 0 & 4 & 2 \end{pmatrix}$$

Cari determinan matriks tersebut dengan menggunakan:

a. OBE

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 3 & 1 & 1 & 0 \\ 4 & 2 & 0 & 0 \\ 2 & 0 & 4 & 2 \end{bmatrix} \longrightarrow |A| = \begin{vmatrix} 2 & 2 & 2 & 2 \\ 3 & 1 & 1 & 0 \\ 4 & 2 & 0 & 0 \\ 2 & 0 & 4 & 2 \end{vmatrix} \begin{matrix} \frac{1}{2} b_1 \\ \sim \end{matrix}$$

$$\frac{1}{2}|A| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 \\ 4 & 2 & 0 & 0 \\ 2 & 0 & 4 & 2 \end{vmatrix} \begin{matrix} -3b_1 + b_2 \\ -4b_1 + b_3 \\ -2b_1 + b_4 \\ \sim \end{matrix}$$

OBE :

1. $b_1 \leftrightarrow b_2 \Rightarrow (-)|A|$
2. $k b_1 \Rightarrow k|A|$
3. $b_1 + b_2 \Rightarrow |A|$

$$\boxed{\frac{1}{2}|A|} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & -3 \\ 0 & -2 & -4 & -4 \\ 0 & -2 & 2 & 0 \end{vmatrix} \begin{matrix} -b_2 + b_3 \\ -b_2 + b_4 \\ \sim \end{matrix}$$

$$\frac{1}{2}|A| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & -3 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 4 & 3 \end{vmatrix} \begin{matrix} 2b_3 + b_4 \\ \sim \end{matrix}$$

$$\frac{1}{2}|A| = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & -3 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\frac{1}{2}|A| = 1 \cdot -2 \cdot -2 \cdot 1$$

$$|A| = 2 \cdot (4) = 8$$

b. kofaktor

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 3 & 1 & 1 & 0 \\ 4 & 2 & 0 & 0 \\ 2 & 0 & 4 & 2 \end{bmatrix}$$

kofaktor baris 3 atau kolom 4

$$C_{mn} = (-1)^{m+n} M_{mn}$$

$$C_{32} = (-1)^{3+2} M_{32}$$
$$= (-1)^5 M_{32}$$

baris 3

$$|A| = a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33} + a_{34}C_{34}$$

$$= 4 \cdot \begin{vmatrix} 2 & 2 & 2 \\ 1 & 1 & 0 \\ 0 & 4 & 2 \end{vmatrix} + 2 \cdot - \begin{vmatrix} 2 & 2 & 2 \\ 3 & 1 & 0 \\ 2 & 4 & 2 \end{vmatrix} + 0 \cdot C_{33} + 0 \cdot C_{34}$$

$$= 4 \cdot (\cancel{4} + 0 + 8 - 0 - \cancel{4} - 0) - 2(\cancel{4} + 0 + 24 - \cancel{4} - 12 - 0)$$

$$= 32 - 24 = 8$$

kolom 4

$$|A| = a_{14}C_{14} + a_{24}C_{24} + a_{34}C_{34} + a_{44}C_{44}$$

$$= 2 \cdot - \begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 0 \\ 2 & 0 & 4 \end{vmatrix} + 0 \cdot C_{24} + 0 \cdot C_{34} + 2 \cdot \begin{vmatrix} 2 & 2 & 2 \\ 3 & 1 & 1 \\ 4 & 2 & 0 \end{vmatrix}$$

$$= -2(24 + 0 + 0 - 4 - 16 - 0) + 2(0 + \cancel{8} + 12 - \cancel{8} - 0 - 4)$$

$$= -2 \cdot 4 + 2 \cdot 8$$

$$= -8 + 16 = 8$$

2. **[Nilai: 20]** Dengan **Operasi Baris Elementer**, tentukan invers dari matrik berikut:

$$B = \begin{pmatrix} 1 & -2 & 2 & 2 \\ 3 & -1 & 1 & 0 \\ -2 & 2 & 0 & 0 \\ 2 & 0 & 4 & 2 \end{pmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 2 & 2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ -2 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & 4 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} -3b_1+b_2 \\ 2b_1+b_3 \\ -2b_1+b_4 \\ \sim \end{array} \left[\begin{array}{cccc|cccc} 1 & -2 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 5 & -5 & -6 & -3 & 1 & 0 & 0 \\ 0 & -2 & 4 & 4 & 2 & 0 & 1 & 0 \\ 0 & 4 & 0 & -2 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \frac{1}{5}b_2 \\ \sim \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -6/5 & -3/5 & 1/5 & 0 & 0 \\ 0 & -2 & 4 & 4 & 2 & 0 & 1 & 0 \\ 0 & 4 & 0 & -2 & -2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} 2b_2+b_3 \\ -4b_2+b_4 \\ \sim \end{array} \left[\begin{array}{cccc|cccc} 1 & -2 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -6/5 & -3/5 & 1/5 & 0 & 0 \\ 0 & 0 & 2 & 8/5 & 4/5 & 2/5 & 1 & 0 \\ 0 & 0 & 4 & 15/5 & 4/5 & -4/5 & 0 & 1 \end{array}\right] \begin{array}{l} -2b_3+b_4 \\ \sim \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -6/5 & -3/5 & 1/5 & 0 & 0 \\ 0 & 0 & 2 & 8/5 & 4/5 & 2/5 & 1 & 0 \\ 0 & 0 & 0 & -1/5 & -4/5 & -8/5 & -2 & 1 \end{array}\right] \begin{array}{l} \frac{1}{2}b_3 \\ -5b_4 \\ \sim \end{array} \left[\begin{array}{cccc|cccc} 1 & -2 & 2 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -6/5 & -3/5 & 1/5 & 0 & 0 \\ 0 & 0 & 1 & 4/5 & 2/5 & 1/5 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 4 & 8 & 10 & -5 \end{array}\right] \begin{array}{l} 2b_2+b_1 \\ \sim \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2/5 & -1/5 & 2/5 & 0 & 0 \\ 0 & 1 & -1 & -6/5 & -3/5 & 1/5 & 0 & 0 \\ 0 & 0 & 1 & 4/5 & 2/5 & 1/5 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 4 & 8 & 10 & -5 \end{array}\right] \begin{array}{l} b_3+b_2 \\ \sim \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -2/5 & -1/5 & 2/5 & 0 & 0 \\ 0 & 1 & 0 & -2/5 & -1/5 & 2/5 & 1/2 & 0 \\ 0 & 0 & 1 & 4/5 & 2/5 & 1/5 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 4 & 8 & 10 & -5 \end{array}\right] \begin{array}{l} \frac{2}{5}b_4+b_1 \\ \frac{2}{5}b_4+b_2 \\ -\frac{4}{5}b_4+b_3 \\ \sim \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 7/5 & 18/5 & 4 & 2 \\ 0 & 1 & 0 & 0 & 7/5 & 18/5 & 9/2 & 2 \\ 0 & 0 & 1 & 0 & -6/5 & -31/5 & -7/2 & 4 \\ 0 & 0 & 0 & 1 & 4 & 8 & 10 & -5 \end{array}\right] \qquad B^{-1} = \begin{bmatrix} 7/5 & 18/5 & 4 & 2 \\ 7/5 & 18/5 & 9/2 & 2 \\ -6/5 & -31/5 & -7/2 & 4 \\ 4 & 8 & 10 & -5 \end{bmatrix}$$

3. Diketahui Persamaan Linear sebagai berikut:

$$\underbrace{\boxed{\begin{array}{l} 2x + y - z \\ -x + 2y + 2z \\ 3x - 2y - z \end{array}}}_{A} \; \underbrace{\boxed{\begin{array}{l} = 4 \\ = 3 \\ = 1 \end{array}}}_{B}$$

Tentukan solusi dari persamaan linear tersebut dengan menggunakan:

$$A = \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 2 & 1 & -1 \\ -1 & 2 & 2 \\ 3 & -2 & -1 \end{bmatrix} \end{array} \qquad B = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

$$\left.\begin{array}{ccc} 2 & 1 & -1 \\ -1 & 2 & 2 \\ 3 & -2 & -1 \end{array}\right| \begin{array}{cc} 2 & 1 \\ -1 & 2 \\ 3 & -2 \end{array}$$

$$|A| = D = \begin{vmatrix} 2 & 1 & -1 \\ -1 & 2 & 2 \\ 3 & -2 & -1 \end{vmatrix} = -4 + 6 + -2 - -6 - 1 - -8 = 13$$

$$D_x = \begin{vmatrix} 4 & 1 & -1 \\ 3 & 2 & 2 \\ 1 & -2 & -1 \end{vmatrix} = -8 + 2 + 6 - (-2) - (-3) - (-16) = 21$$

$$D_y = \begin{vmatrix} 2 & 4 & -1 \\ -1 & 3 & 2 \\ 3 & 1 & -1 \end{vmatrix} = -6 + 24 + 1 - (-9) - 4 - 4 = 20$$

$$D_z = \begin{vmatrix} 2 & 1 & 4 \\ -1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = 4 + 9 + 8 - 24 - (-1) - (-12) = 10$$

$$x = \frac{D_x}{D} = \frac{21}{13} \qquad y = \frac{D_x}{D} = \frac{20}{13} \qquad z = \frac{D_z}{D} = \frac{10}{13}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A)$$

4. **[Nilai: 20]** Tentukan solusi SPL homogen berikut:

SPL → OBE, ~~invers~~, crammer > $|A| \neq 0$

$$\begin{array}{c} A \quad\quad B \\ \begin{aligned} a - b + 2c + d &= 0 \\ -3a + 5b - 4c + d &= 0 \\ 2a - 2b + 4c + 2d &= 0 \\ 3a - 3b + 6c + 3d &= 0 \end{aligned} \end{array}$$

$$A = \begin{bmatrix} 1 & -1 & 2 & 1 \\ -3 & 5 & -4 & 1 \\ 2 & -2 & 4 & 2 \\ 3 & -3 & 6 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 0 \\ -3 & 5 & -4 & 1 & 0 \\ 2 & -2 & 4 & 2 & 0 \\ 3 & -3 & 6 & 3 & 0 \end{array}\right] \begin{array}{l} -2b_1 + b_3 \\ -3b_1 + b_4 \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 0 \\ \boxed{-3} & 5 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} 3b_1 + b_2 \\ \sim \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \frac{1}{2} b_2 \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & \boxed{-1} & 2 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} b_2 + b_1 \\ \sim \end{array}$$

$$\begin{array}{ccccc} a & b & c & d & = B \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 3 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} a + 3c + 3d &= 0 \quad \rightarrow \quad a + 3u + 3t = 0 \quad \rightarrow \quad a = -3u - 3t \\ b + c + 2d &= 0 \quad \rightarrow \quad b + u + 2t = 0 \quad \rightarrow \quad b = -u - 2t \\ c &= u \quad\quad\quad\quad c = u \quad\quad\quad\quad c = u + 0.t \\ d &= t \quad\quad\quad\quad d = t \quad\quad\quad\quad d = 0.u + t \end{aligned}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -3u \\ -u \\ u \\ 0.u \end{bmatrix} + \begin{bmatrix} -3t \\ -2t \\ 0.t \\ t \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \end{bmatrix} t$$

5. **[Nilai: 15]** Diketahui matriks E sebagai berikut

$$E = \begin{pmatrix} 1 & 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Jika F adalah matriks E invers dan G adalah matriks E transpose, tentukan:

$$X = \frac{Det(2E^4F) - Det(3E)}{Det(F^TG^2)}$$

$$|E| = \begin{vmatrix} 1 & 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{vmatrix} \quad b_1 \leftrightarrow b_2$$

$$-|E| = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ \boxed{1} & 2 & 1 & 1 & 0 \\ \boxed{1} & 1 & 2 & 0 & 1 \\ \boxed{1} & 0 & 2 & 2 & 1 \\ \boxed{1} & 1 & 1 & 0 & 0 \end{vmatrix} \quad \begin{matrix} -b_1 + b_2 \\ -b_1 + b_3 \\ -b_1 + b_4 \\ -b_1 + b_5 \end{matrix}$$

$$-|E| = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 2 & 2 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{vmatrix} \quad b_2 \leftrightarrow b_5$$

$$|E| = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & \boxed{1} & 2 & 0 & 1 \\ 0 & \boxed{0} & 2 & 2 & 1 \\ 0 & \boxed{2} & 1 & 1 & 0 \end{vmatrix} \quad \begin{matrix} -b_2 + b_3 \\ -2b_2 + b_5 \end{matrix}$$

$$|E| = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & \boxed{2} & 2 & 1 \\ 0 & 0 & \boxed{-1} & 1 & 0 \end{vmatrix} \quad \begin{matrix} -2b_3 + b_4 \\ b_3 + b_5 \end{matrix}$$

$$|E| = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & \boxed{1} & 1 \end{vmatrix} \quad -\frac{1}{2} b_4 + b_5$$

$$|E| = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 3/2 \end{vmatrix} = 1 \cdot 1 \cdot 1 \cdot \cancel{2} \cdot \frac{3}{\cancel{2}} = 3$$

$$|F| = |E^{-1}| = \frac{1}{|E|} \quad , \quad |G| = |E^T| = |E|$$

$$x = \frac{|2E^4F| - |3E|}{|F^T G^2|} = \frac{|2E^4E^{-1}| - |3E|}{|E^{-1} \cdot E^2|}$$

$$= \frac{|2E^3| - |3E|}{|E|}$$

$|kA| = k^n |A|$

$|A^n| = |A|^n$

$$= \frac{2^5 \cdot |E|^{\cancel{3}\,2} - 3^5 \cancel{|E|}}{\cancel{|E|}}$$

$$= 2^5 \cdot |E|^2 - 3^5$$

$$= 32 \cdot 3^2 - 243$$

$$= 288 - 243 = 45$$