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1. Misalkan $\vec{u}, \vec{v}, \vec{w} \in V$ dan $k, l \in$ Riil, V dinamakan ruang vektor jika terpenuhi aksioma :

1.) V tertutup terhadap operasi penjumlahan untuk setiap $\vec{u}, \vec{v} \in V$ maka $\vec{u} + \vec{v} \in V$

2.) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

3.) $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$

4.) Terdapat $\vec{0} \in V$ sehingga untuk setiap $\vec{u} \in V$ berlaku $\vec{u} + \vec{0} = \vec{0} + \vec{u} = \vec{u}$

5.) Untuk setiap $\vec{u} \in V$ terdapat $(-\vec{u})$ sehingga $\vec{u} + (-\vec{u}) = (-\vec{u}) + \vec{u} = \vec{0}$

6.) V tertutup terhadap operasi perkalian dengan skalar. Untuk setiap $\vec{u} \in V$ dan $k \in$ Riil maka $k\vec{u} \in V$

7.) $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$

8.) $(k + l)\vec{u} = k\vec{u} + l\vec{u}$

9.) $k(l\vec{u}) = l(k\vec{u}) = (kl)\vec{u}$

10.) $1 . \vec{u} = \vec{u}$

2. Misalkan W merupakan subhimpunan dari sebuah ruang vektor V. W dinamakan subruang V jika W juga merupakan ruang vektor yang tertutup terhadap operasi penjumlahan dan perkalian dengan skalar Syarat W disebut subruang dari V adalah :

1.) $W \neq \{\}$

2.) $W \subseteq V$

3.) $\vec{u}, \vec{v} \in W$ maka $\vec{u} + \vec{v} \in W$

4.) $\vec{u} \in W$ dan $k \in$ Riil maka $k\vec{u} \in W$

3. $\langle \vec{u}, \vec{v} \rangle$ dinamakan ruang hasil kali dalam jika memenuhi :

1.) $\langle \vec{u}, \vec{v} \rangle = \langle \vec{v}, \vec{u} \rangle$ (Simetris)

2.) $\langle \vec{u} + \vec{v}, \vec{w} \rangle = \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$ (Aditivitas)

3.) Untuk $k \in \mathbb{R}$, $\langle k\vec{u}, \vec{v} \rangle = \langle \vec{u}, k\vec{v} \rangle = k \langle \vec{u}, \vec{v} \rangle$ (Homogenitas)

4.) $\langle \vec{u}, \vec{u} \rangle \geq 0$, untuk setiap $\vec{u}$ dan $\langle \vec{u}, \vec{u} \rangle = 0 \leftrightarrow \vec{u} = \vec{0}$ (Positifitas)

4. Misalkan V dan W adalah ruang vektor, $T : V \to W$ dinamakan transformasi linear, jika untuk setiap $\vec{a}, \vec{b} \in V$ dan $\alpha \in \mathbb{R}$ berlaku :

1.) $T(\vec{a} + \vec{b}) = T(\vec{a}) + T(\vec{b})$

2.) $T(\alpha \vec{a}) = \alpha . T(\vec{a})$

Jika $V = W$ maka T dinamakan operator linear

5. $S = \{1, 2 - x^2, (2 - x)^2\}$

$S = \{1, 2 - x^2, 4 - 4x + x^2\}$

a. $P_2 = a + bx + cx^2$

$S_1 = 1 = 1 + 0.x + 0x^2$

$S_2 = 2 - x^2 = 2 + 0.x - x^2$

$S_3 = 4 - 4x + x^2$

$k_1 S_1 + k_2 S_2 + k_3 S_3 = \vec{0}$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & -4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & k_1 \\ 0 & 0 & -4 & k_2 \\ 0 & -1 & 1 & k_3 \end{array}\right] \begin{array}{c} b_2 \leftrightarrow b_3 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 4 & k_1 \\ 0 & -1 & 1 & k_3 \\ 0 & 0 & -4 & k_2 \end{array}\right] \begin{array}{c} 2b_2 + b_1 \\ -\frac{1}{4} b_3 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 6 & k_1 + 2k_3 \\ 0 & -1 & 1 & k_3 \\ 0 & 0 & 1 & -\frac{1}{4} k_2 \end{array}\right] \begin{array}{c} -b_3 + b_2 \\ -6b_3 + b_1 \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & k_1+2k_3+\frac{3}{2}k_2 \\ 0 & -1 & 0 & k_3+\frac{1}{4}k_2 \\ 0 & 0 & 1 & -\frac{1}{4}k_2 \end{array}\right] \overset{-b_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & k_1+\frac{3}{2}k_2+2k_3 \\ 0 & 1 & 0 & -\frac{1}{4}k_2-k_3 \\ 0 & 0 & 1 & -\frac{1}{4}k_2 \end{array}\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$k_2 = 0$, $-\frac{1}{4}.0 - k_3 = 0$, $k_1 + \frac{3}{2}.0 + 2.0 = 0$

$k_3 = 0$ $\qquad$ $k_1 = 0$

∴ Karena SPL homogen bersolusi tunggal, maka S bebas linear

b. $k_1 S_1 + k_2 S_2 + k_3 S_3 = \vec{u}$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & -4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 + \frac{3}{2}k_2 + 2k_3 \\ -\frac{1}{4}k_2 - k_3 \\ -\frac{1}{4}k_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$-\frac{1}{4}k_2 = u_3$ $\qquad$ $-\frac{1}{4}(-4u_3) - k_3 = u_2$ $\qquad$ $k_1 + \cancel{\frac{3}{2}}(-\cancel{4}^{2}u_3) + 2(-u_2+u_3) = u_1$

$k_2 = -4u_3$ $\qquad$ $k_3 = -u_2 + u_3$ $\qquad$ $k_1 = u_1 + 2u_2 + 4u_3$

∴ Karena SPL bersolusi tunggal (konsisten), maka membangun P_2

c. Karena S bebas linear dan membangun P_2, maka S merupakan basis bagi Polinom orde 2 (P_2)

6. a. $\langle \vec{u}, \vec{v} \rangle = u_1v_1 + u_2v_2 + 2u_3v_3$

$\vec{u} = (1, 1, -1)$

$\vec{v} = (2, 2, 1)$

$\langle (1,1,-1), (2,2,1) \rangle = 1.2 + 1.2 + 2.-1.1$

$= 2 + 2 - 2$

$= 2$

6. $S = \{ \vec{w}_1 = (1,0,1),\ \vec{w}_2 = (0,1,1) \}$

$$\vec{u}_1 = \frac{\vec{w}_1}{\|\vec{w}_1\|} = \frac{(1,0,1)}{\sqrt{1^2+0^2+1^2}} = \frac{(1,0,1)}{\sqrt{2}} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\vec{P}_1 = \vec{w}_2 - \text{proy}_{\vec{u}_1} \vec{w}_2 = \vec{w}_2 - \langle \vec{w}_2, \vec{u}_1 \rangle \vec{u}_1$$

$$\vec{P}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \left\langle \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \right\rangle \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\vec{P}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \left(0 \cdot \frac{1}{\sqrt{2}} + 1 \cdot 0 + 2 \cdot 1 \cdot \frac{1}{\sqrt{2}} \right) \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\vec{P}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{2}{\sqrt{2}} \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{P}_1}{\|\vec{P}_1\|} = \frac{(-1,1,0)}{\sqrt{(-1)^2+1^2+0^2}} = \frac{(-1,1,0)}{\sqrt{2}}$$

$$= \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$u = \{ \vec{u}_1, \vec{u}_2 \}$

$$u = \left\{ \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \right\}$$ → Himpunan ortonormal dari S

7. $T : R^3 \to R^2$

$\vec{v}_1 = (1, 0, -1)$ $\quad T(\vec{v}_1) = (0, 3)$

$\vec{v}_2 = (1, 1, 1)$ $\quad T(\vec{v}_2) = (3, 1)$

$\vec{v}_3 = (1, -1, 0)$ $\quad T(\vec{v}_3) = (0, -1)$

a. $T(\vec{v}_1, \vec{v}_2, \vec{v}_3) = A(\vec{v}_1, \vec{v}_2, \vec{v}_3)$

$$\begin{bmatrix} 0 & 3 & 0 \\ 3 & 1 & -1 \end{bmatrix} = A \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1/3 & 1/3 & -2/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & -2/3 & 1/3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}$$

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y+z \\ x+2y-2z \end{bmatrix}$$

b.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$a + 4c = 0$ $\quad$ $b - 3c = 0$ $\quad$ $c = p \rightarrow$ parameter

$a = -4c$ $\quad$ $b = 3c$

$$\ker(T) = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix} p \right\} \qquad R(T) = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

Dimensi $\ker(T) = 1$ $\qquad$ Dimensi $R(T) = 2$

8. $T: R^3 \rightarrow R^2$ $\quad T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} x+y-z \\ y-z \end{bmatrix}$

a. $T\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{bmatrix} 1+1-(-1) \\ 1-(-1) \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

b. $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} x + y - z \\ y - z \end{bmatrix}$

$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y - z \\ y - z \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$

$a = 0$, $b - c = 0$, $c = p \rightarrow$ parameter

$b = c$

$\ker(T) = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} p \right\}$

Nulitas $T = 1$

c. $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

Basis ruang kolom = 2

$A^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ -1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

Basis ruang baris = 2

Rank $(T) = 2$

9. $A = \begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix}$

a. $\det(A - \lambda I) = 0$

$$\left| \begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 5-\lambda & -3 & 2 \\ 15 & -9-\lambda & 6 \\ 10 & -6 & 4-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(-9-\lambda)(4-\lambda) + -3.6.10 + 2.15.-6 - 2.(-9-\lambda).10 - (-3)15(4-\lambda) - (5-\lambda)6.-6 = 0$$

$$-\lambda^3 + 61\lambda \cancel{-180} \cancel{-180} \cancel{-180} \cancel{+180} + 20\lambda \cancel{+180} - 45\lambda \cancel{+180} - 36\lambda = 0$$

$$-\lambda^3 = 0$$

$$\lambda^3 = 0$$

$$\lambda = 0$$

b. $\lambda = 0$

$$(A - \lambda I)X = 0$$

$$\left(\begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix} - 0I \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 5 & -3 & 2 & 0 \\ 15 & -9 & 6 & 0 \\ 10 & -6 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 5 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$5x - 3y + 2z = 0 \qquad y = s \qquad z = t$$

$$5x = 3y - 2z$$

$$x = 0{,}6y - 0{,}4z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0{,}6s - 0{,}4t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0{,}6 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -0{,}4 \\ 0 \\ 1 \end{bmatrix} t \rightarrow \left\{ \begin{bmatrix} 0{,}6 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -0{,}4 \\ 0 \\ 1 \end{bmatrix} \right\}$$

c. Matriks A tidak dapat didiagonalkan karena matriks P yang terbentuk dari basis ruang eigen berukuran 3x2 sehingga tidak dapat diinverskan

10. $Y_1' = 2Y_1 - 2Y_2$

$Y_2' = -Y_1 + 2Y_2$

a. $Y' = AY$

$$\begin{bmatrix} Y_1' \\ Y_2' \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$$

b. $\det(A - \lambda I) = 0$

$$\left| \begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & -2 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)^2 - 2 = 0$$

$$4 - 4\lambda + \lambda^2 - 2 = 0$$

$$\lambda^2 - 4\lambda + 2 = 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4.1.2}}{2.1}$$

$$= \frac{4 \pm \sqrt{8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2}$$

$$= 2 \pm \sqrt{2}$$

$\lambda_1 = 2 + \sqrt{2}$ $\qquad$ $\lambda_2 = 2 - \sqrt{2}$

$\lambda_1 = 2 + \sqrt{2}$

$(A - \lambda I) P_1 = 0$

$$\left(\begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 2+\sqrt{2} & 0 \\ 0 & 2+\sqrt{2} \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\sqrt{2} & -2 \\ -1 & -\sqrt{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x - y\sqrt{2} = 0$ $\qquad$ $-\sqrt{2}x - 2y = 0$ $\qquad$ $y = t \rightarrow$ parameter

$-x = y\sqrt{2}$ $\qquad$ $\sqrt{2}x = -2y$

$x = -y\sqrt{2}$ $\qquad$ $x = -y\sqrt{2}$

$$P_1 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\sqrt{2} \\ 1 \end{bmatrix} t \right\} = \left\{ \begin{bmatrix} -\sqrt{2} \\ 1 \end{bmatrix} \right\}$$

$\lambda_2 = 2 - \sqrt{2}$

$(A - \lambda_2 I) P_2 = 0$

$$\left(\begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 2-\sqrt{2} & 0 \\ 0 & 2-\sqrt{2} \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{2} & -2 \\ -1 & \sqrt{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x + y\sqrt{2} = 0$ $\qquad$ $y = t \rightarrow$ parameter

$x = y\sqrt{2}$

$$P_2 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ 1 \end{bmatrix} t \right\} = \left\{ \begin{bmatrix} \sqrt{2} \\ 1 \end{bmatrix} \right\}$$

c. $P = \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ 1 & 1 \end{bmatrix}$

$$P^{-1} = \begin{bmatrix} -\frac{\sqrt{2}}{4} & \frac{1}{2} \\ \frac{\sqrt{2}}{4} & \frac{1}{2} \end{bmatrix}$$

$$D = P^{-1} A P = \begin{bmatrix} -\frac{\sqrt{2}}{4} & \frac{1}{2} \\ \frac{\sqrt{2}}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{2}+1}{2} & \frac{\sqrt{2}+2}{2} \\ \frac{\sqrt{2}-1}{2} & -\frac{\sqrt{2}-2}{2} \end{bmatrix} \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ 1 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 2+\sqrt{2} & 0 \\ 0 & 2-\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 2+\sqrt{2} & 0 \\ 0 & 2-\sqrt{2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$u_1 = C_1 e^{(2+\sqrt{2})t} \qquad u_2 = C_2 e^{(2-\sqrt{2})t}$$

$$Y = PU \rightarrow \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 e^{(2+\sqrt{2})t} \\ C_2 e^{(2-\sqrt{2})t} \end{bmatrix}$$

$$Y_1 = -C_1\sqrt{2}\, e^{(2+\sqrt{2})t} + C_2\sqrt{2}\, e^{(2-\sqrt{2})t}$$

$$Y_2 = C_1 e^{(2+\sqrt{2})t} + C_2 e^{(2-\sqrt{2})t}$$