

$$z = x + iy$$

$$r = |z| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$z = |z| \angle \theta \rightarrow z = r e^{i\theta}$$

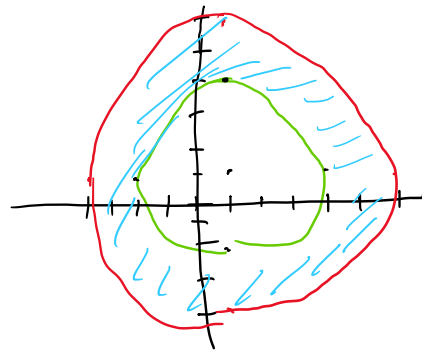
$$z_1 = r_1 e^{i\theta_1} \quad z_2 = r_2 e^{i\theta_2}$$

$$z_1 + z_2 = r_1 e^{i\theta_1} + r_2 e^{i\theta_2}$$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$$

$$r_1 \leq |z - 1 - i| < r_2$$

$$|z - (1+i)| \rightarrow \text{TP}$$

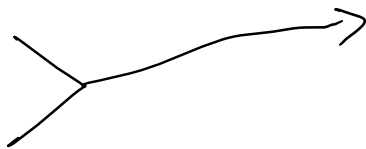


$f(z) \rightarrow \mathbb{C} \rightarrow \text{harmonic} \rightarrow \text{differentiable}$

$$f(z) = u + iv$$

$$u_x = v_y$$

$$u_y = -v_x$$



Pers. Laplace

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

$$f(x+iy) = (x^2 - y^2 + 5) + i(2xy)$$

$$\begin{aligned} * \text{ Cek PCR : } u_x = 2x &= v_y = 2y \\ u_y = -2y &= v_x = 2x \end{aligned}$$

$$* \text{ Turunan : } f'(x+iy) = 2x + i2y$$

$$f'(x+iy) = 2(x+iy)$$

$$f'(z) = 2z$$

$$f(z) = \int 2z \, dz$$

$$f(z) = z^2 + C$$

$$* f(x+iy) = (x+iy)^2 + C$$

$$= \cancel{x^2} - \cancel{y^2} + \cancel{2ixy} + C = (\cancel{x^2} - \cancel{y^2} + 5) + \cancel{i(2xy)}$$

$$C = 5$$

$$* f(z) = z^2 + C = z^2 + 5$$

$$u(x, y) = xy \quad v(x, y)$$

$$* \text{ PCR : } u_x = y = v_y = \frac{\partial v}{\partial y}$$

$$u_y = x = -v_x = \frac{\partial v}{\partial x}$$

$$u_x = v_y$$

$$y = \frac{\partial v}{\partial y} \rightarrow v = \int y \, dy$$

$$v = \frac{1}{2}y^2 + \overset{C}{g(x)}$$

$$u_y = -v_x$$

$$x = \frac{\partial}{\partial x} \cdot \left(\frac{1}{2} y^2 + g(x) \right)$$

$$x = -g'(x)$$

$$g(x) = \int -x dx$$

$$g(x) = -\frac{1}{2}x^2 + C$$

$$v = \frac{1}{2}y^2 + g(x)$$

$$v = \frac{1}{2}y^2 - \frac{1}{2}x^2 + C$$

$$z = x + iy \quad ; \quad \bar{z} = x - iy \quad z = u + iv$$

$$z + \bar{z} = 2x + i \cdot 0 \quad u_x = v_y$$

$$z - \bar{z} = i(2y) + 0 \quad u_x = v_y$$

$$\int (2z + 5) dz \quad \text{dari } (2, 0) \text{ ke } (4, 2)$$

$$\frac{x-0}{4-0} = \frac{y-0}{2-0}$$

$$\frac{x}{4} = \frac{y}{2}$$

$$2y = x$$

$$y = t$$

$$x = 2t$$

$$f(z) = (2z + 5)$$

$$f(x + iy) = 2(x + iy) + 5$$

$$f(x + iy) = 2x + 5 + i2y$$

$$f(t) = 2(2t) + 5 + i(2t)$$

$$f(t) = 4t + 5 + 2it$$

$$\int_0^2 (4t + 5 + 2it) dt = 2t^2 + 5t + it^2 \Big|_0^2$$

$$= 8 + 10 + 4i = 18 + 4i$$