





*11th Material Subject: Special Distribution of Continuous Random Variable

Undergraduate of Telecommunication Engineering

MUH1F3 - PROBABILITY AND STATISTICS

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TABLE OF CONTENTS:

- 1. Uniform
- 2. Exponential
- 3. Normal

LEARNING OBJECTIVES:

After careful study of this chapter, student should be able to do the following:

- 1. Understand the assumptions for some common continuous probability distributions
- 2. Calculate continuous probability distribution to calculate probabilities in specific applications
- Calculate probabilities and determine means and variances for some common continuous probability distributions





- Uniform random variables appear in situations where all values in a certain interval (a, b) have same probability.
 - If **X** is a random variable that is uniformly distributed at interval (**a**, **b**) then:

$$X \Rightarrow UNI(a,b) \tag{1}$$

• The **Probability Density Function** of Uniform Distribution:

$$\mathbf{f_X(x)} = \begin{cases} \frac{1}{b-1} & , a \le x \le b \\ 0 & , \text{otherwise} \end{cases}$$



UNIFORM



• The **Mean** of Uniform Distribution:

$$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x) \, dx = \int_a^b x \cdot \frac{1}{b-a} \, dx = \frac{x^2}{2(b-a)} \big|_a^b = \frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)}$$

$$E(X) = \frac{b+a}{2} \tag{3}$$

• The Variance of Uniform Distribution:

$$\begin{split} E(X^2) &= \int_{-\infty}^{\infty} x^2 \cdot f_X(x) \, dx = \int_a^b x^2 \cdot \frac{1}{b-a} \, dx = \frac{x^3}{3(b-a)} \Big|_a^b = \frac{b^3 - a^3}{2(b-a)} \\ E(X^2) &= \frac{(b-a)(b^2 + a^2 + ab)}{3(b-a)} = \frac{b^2 + a^2 + ab}{3} \end{split}$$

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UNIFORM



$$Var(X) = E(X^{2}) - (E(X))^{2} = \left(\frac{b^{2} + a^{2} + ab}{3}\right) - \left(\frac{b + a}{2}\right)^{2} = \frac{(b - a)^{2}}{12}$$
(4)

• The Moment Generation Function of Uniform Distribution:

$$\mathbf{f_X(x)} = \begin{cases} \frac{e^{bt} - e^{at}}{(b-a)t} &, \text{ for } t \neq 0\\ 1 &, \text{ for } t = 0 \end{cases}$$
 (5)

UNIFORM



Example:Let **X** be a random variable that states the amount of fading which occurs due to the number of obstacles in the propagation environment. Fading that occurs uniformly distributed in the range of **80 dB** up to **95 dB**. Determine:

- a. Probability Density Function (PDF) of X
- b. Mean / Expected Value of X
- c. Variance random variable of X
- d. Moment Generation Function of X
- e. P(X < 85), P(X < 85), P(X > 85) and P(X > 85)

Answer:

a. The PDF can be written:

$$\mathbf{f_X(x)} = \begin{cases} \frac{1}{95-80} & ,80 \le x \le 95\\ 0 & , \text{otherwise} \end{cases}$$







b. Mean / Expected Value of random variable X

$$\mathsf{E}(\mathsf{X}) = \frac{\mathsf{b} + \mathsf{a}}{\mathsf{2}} = \frac{\mathsf{175}}{\mathsf{2}}$$

c. Variance of random variable X

$$Var(X) = \frac{(b-a)^2}{12} = \frac{(95-80)^2}{12} = \frac{225}{12}$$

d. Moment Generation Function of random variable X

$$\mathbf{f_X(x)} = \begin{cases} \frac{e^{95t} - e^{80t}}{15t} & , \text{for } t \neq 0\\ 1 & , \text{for } t = 0 \end{cases}$$







d. $P(X \le 85)$, $P(X \le 85)$, P(X > 85) and $P(X \ge 85)$

$$P(X \le 85) = P(X < 85) = \int_{80}^{85} \frac{1}{15} dx = \frac{5}{15}$$

$$P(X \ge 85) = P(X > 85) = \int_{0.5}^{95} \frac{10}{15} dx = \frac{5}{15}$$



EXPONENTIAL



• The random variable **X** that equals the distance between successive events from a Poisson process with mean number of events $\lambda > \mathbf{0}$ per unit interval is an **Exponential** random variable with parameter λ .

$$X \Rightarrow EXP(\lambda)$$
 (6)

• The probability density function of **X** is:

$$\mathbf{f_X(x)} = \begin{cases} \lambda e^{-\lambda} & , 0 \le x < \lambda \\ 0 & , \text{otherwise} \end{cases}$$
 (7)

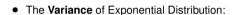
• The **Mean** of Exponential Distribution:

$$\mathsf{E}(\mathsf{X}) = \frac{\mathsf{1}}{\lambda}$$









$$Var(X) = \frac{1}{\lambda^2}$$
 (9)

• The Moment Generation Function of Binomial Distribution:

$$\mathbf{E}(\mathbf{e^{tx}}) = \frac{\lambda}{\lambda - \mathbf{t}}$$
 , for $\mathbf{t} < \lambda$ (10)





EXPONENTIAL



- **Example:** The average of telephone calls coming to the information center has a waiting time of 5 minutes / call. If **X** is random variable which states the average length of waiting time between call, specify:
 - a. Probability Density Function (PDF) of random variable X
 - b. Mean / Expected Value random variable **X**
 - c. Variance random variable X
 - d. Moment Generation Function random variable X
 - e. The probability of waiting no more than 1 minute
 - f. The probability of waiting between 1 minute 3 minutes
 - g. The probability of waiting more than 2 minute



EXPONENTIAL



Answer:

The PDF can be written:

$$\mathbf{f_X(x)} = \begin{cases} 5e^{-5} & , 0 \le x < \infty \\ 0 & , \text{otherwise} \end{cases} \tag{11}$$

b. Mean / Expected Value of random variable X

$$\mathsf{E}(\mathsf{X}) = \frac{\mathsf{1}}{\lambda} = \frac{\mathsf{1}}{\mathsf{5}}$$

c. Variance of random variable X

$$Var(X) = \frac{1}{\lambda^2} = \frac{1}{25}$$

d. Moment Generation Function of random variable X

$$\mathsf{E}(\mathsf{e^{tx}}) = \frac{\lambda}{\lambda \mathsf{t}} = \frac{\mathsf{5}}{\mathsf{5} - \mathsf{t}}, \text{ , for } \mathsf{t} < \lambda$$







e. The probability of waiting no more than 1 minute

$$P(t \le 1) = \int_0^1 5 \cdot e^{-5x} dx = -e^{-5x} \Big|_0^1 = 0.9933$$

f. The probability of waiting between 1 minute - 3 minutes

$$P(1 \le t \le 3) = \int_{1}^{2} 5 \cdot e^{-5x} dx = -e^{-5x} \Big|_{1}^{3} = 0.0067$$

g. The probability of waiting more than 2 minute

$$P(t>2) = \int_2^\infty 5 \cdot e^{-5x} dx = -e^{-5x} \Big|_2^\infty = 4.54 imes 10^{-5}$$

LECTURER CODE: NK





Undoubted the most widely used model for a continuous measurement is a Normal random variable. The Normal distribution often referred to as the Gauss distribution, taken from the German mathematician Carl Friedrich Gauss.

• A Normal random variable **X** is with an average value $\mathbf{E}(\mathbf{x}) = \mu$ and variance $\mathbf{Var}(\mathbf{x}) = \sigma^2$:

$$X \Rightarrow NOR(\mu, \sigma^2)$$
 (12)

• The **Probability Density Function** of Normal Distribution:

$$\mathbf{f}_{\mathbf{X}}(\mathbf{x}) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} &, -\infty < x < \infty\\ 0 &, \text{ otherwise} \end{cases}$$
(13)

(14)

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$$\mathbf{E}(\mathbf{X}) = \mu \tag{15}$$

• The Variance of Normal Distribution:

$$Var(X) = \sigma^2 \tag{16}$$

• The Moment Generation Function of Normal Distribution:

$$\mathbf{M}_{\mathbf{x}}(\mathbf{t}) = \mathbf{E}(\mathbf{e}^{\mathsf{tx}}) = \int_{-\infty}^{\infty} \mathbf{e}^{\mathsf{tx}} \frac{1}{\sigma_{\mathbf{x}} \sqrt{2\pi}} \mathbf{e}^{-\frac{1}{2} \left(\frac{\mathbf{x} - \mu}{\sigma}\right)^2} = \mathbf{e}^{\mu \mathbf{t} + \frac{\sigma^2 \mathbf{t}^2}{2}}$$
(17)





A normal random variable with $\mu = \mathbf{0}$ and $\sigma^2 = \mathbf{1}$ is called a Standard Normal random variable and is denoted as \mathbf{Z} . The cumulative distribution function of a standard normal random variable is denoted as:

$$\Phi(z) = P(Z \le z) \tag{18}$$

Probabilities that are not of the form $\Phi(z) = P(Z \le z)$ are found by using the basic rules of probability and the symmetry of the normal distribution along with Appendix Table. If **X** is a normal random variable with $\mathbf{E}(\mathbf{X}) = \mu$ and $\mathbf{Var}(\mathbf{X}) = \sigma^2$, the random variable:

$$z = \frac{x - \mu}{\sigma} \tag{19}$$

is a normal random variable with E(Z) = 0 and Var(Z) = 1. That is, Z is a standard normal random variable





Example: lamp life produced by PT. PIJAR JAYA normally distributed with an average of 1000 hours and standard deviations 100 hours. If X is a random variable state the lamp's lifespan as in the conditions above, determine:

- a. The Probability Density Function of X
- b. Mean, Standard Deviation and Variance of X
- c. $f_x(900)$, $P(X \le 850)$, $P(X \ge 1254)$, and P(975 < x < 1378)
- d. If the company advertises that 95% of the lights can ignite for at least 900 hours. Are these advertisements honest?





Answer:

a. The Probability Density Function of X

$$\mathbf{f_X(x)} = \begin{cases} \frac{1}{100\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-1000}{100}\right)^2} &, -\infty < x < \infty \\ 0 &, \text{otherwise} \end{cases}$$

b. Mean, Standard Deviation and Variance of X

$$\mu = 1000$$
 , $\sigma = 100$ and $\sigma^2 = 100^2 = 10000$



NORMAL



Answer

bc.
$$f_X(900)$$
, $P(X < 850)$, $P(X > 1254)$, and $P(975 < x < 1378)$

$$f_X(900) = \frac{1}{100\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{900-1000}{100}\right)^2} = 0.0024$$

$$P(X \le 850) = P\left(Z \le \frac{850 - 1000}{100}\right) = \Phi(-1.5) = 0.0681$$

$$P(X>1254)=1-P(X<1254)=1-P\left(Z\leq \frac{1254-1000}{100}\right)=1-\Phi(2.54)=0.00554$$

$$P(975 < x < 1254) = P(X < 1254) - P(X < 975)$$

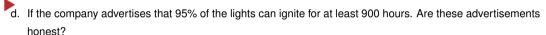
$$\mathsf{P}\left(\mathsf{Z} \leq \frac{1254-1000}{100}\right) - \mathsf{P}\left(\mathsf{Z} \leq \frac{975-1000}{100}\right) = \Phi(2.54) - \Phi(-0.25) = 0.59863$$

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NORMAL





Advertising is said to be honest if:

$$P(X > 900) = 0.95$$

Lets count:

$$P(X \geq 900) = 1 - P(X < 900) = 1 - P\left(Z < \frac{900 - 1000}{100}\right) = 1 - \Phi(1) = 1 - 0.15866 = 0.84134$$

It turns out that there are only about 84.134% of light that can light up for at least 900 hours, so the advertisement is dishonest.







Thank You

