$$\int_{-\infty}^{\infty} \frac{\sin 2x}{x^2+2x+\frac{5}{2}}\,dx \rightarrow f(x) = \frac{1}{x^2+2x+\frac{5}{2}}$$

$$\int_{-\infty}^{\infty} \frac{1}{x^2+2x+\frac{5}{2}} \cdot e^{2ix}\,dx = \oint_C \frac{1}{z^2+2z+\frac{5}{2}} e^{2iz}\,dz$$

$$z^2+2z+\frac{5}{2} = 0$$

$$z_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-2 \pm \sqrt{4-4.1.\frac{5}{2}}}{2.1}$$

$$= \frac{-2 \pm \sqrt{-6}}{2}$$

$$= \frac{-2 \pm \sqrt{6}.\sqrt{-1}}{2}$$

$$= -1 \pm \frac{\sqrt{6}}{2} i$$

$$z_1 = -1 + \frac{\sqrt{6}}{2} i$$

$$z_2 = -1 - \frac{\sqrt{6}}{2} i$$

$$\oint_C \frac{e^{2iz}}{z^2+2z+\frac{5}{2}}\,dz = \oint_C \frac{e^{2iz}}{(z-z_1)(z-z_2)}\,dz = \oint_C \frac{e^{2iz}}{(z+1-\frac{\sqrt{6}}{2}i)(z+1+\frac{\sqrt{6}}{2}i)}\,dz$$

$$q(z) = \frac{e^{2iz}}{z+1+\frac{\sqrt{6}}{2}i}$$

$$q(z_1) = \frac{e^{2i(-1+\frac{\sqrt{6}}{2}i)}}{(-1+\frac{\sqrt{6}}{2}i)+1+\frac{\sqrt{6}}{2}i} = \frac{e^{-2i-\sqrt{6}}}{i\sqrt{6}}$$

$$\oint_C \frac{e^{2iz}}{z^2+2z+\frac{5}{2}}\,dz = 2\pi i\left(\operatorname{Res}_{z=z_1} = q(z_1)\right) = 2\pi i\left(\frac{e^{-2i-\sqrt{6}}}{i\sqrt{6}}\right)$$

$$= \frac{\sqrt{6}}{3}\pi e^{-2i-\sqrt{6}}$$

$$= \frac{\pi\sqrt{6}}{3e^{2i+\sqrt{6}}}$$

$$\therefore \int_{-\infty}^{\infty} \frac{\sin 2x}{x^2+2x+\frac{5}{2}}\,dx = \operatorname{Im}\left(\frac{\pi\sqrt{6}}{3e^{2i+\sqrt{6}}}\right) = 0$$

$$\int_{-\infty}^{\infty} \frac{x \sin x}{x^2+2x+2}\, dx = \int_{-\infty}^{\infty} \frac{x}{x^2+2x+2} e^{ix}\, dx = \oint_C \frac{z}{z^2+2z+2} e^{iz}\, dz$$

$$z^2+2z+2=0$$

$$z_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2-4.1.2}}{2.1}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm \sqrt{4}.\sqrt{-1}}{2}$$

$$= -1 \pm i \quad \begin{cases} z_1 = -1+i \\ z_2 = -1-i \end{cases}$$

$$\oint_C \frac{z}{z^2+2z+2} e^{iz}\, dz = \oint_C \frac{z e^{iz}}{(z+1-i)(z+1+i)}\, dz$$

$$g(z) = \frac{z e^{iz}}{z+1+i}$$

$$\text{Res}_{z=z_1} = \frac{z_1 e^{iz_1}}{z_1+1+i} = \frac{(-1+i)e^{i(-1+i)}}{(-1+i)+1+i} = \frac{(-1+i)e^{-1-i}}{2i} \times \frac{-i}{-i}$$

$$= \frac{(i+1)e^{-1-i}}{2}$$

$$= \frac{1}{2}e^{-1-i} + \frac{i}{2} e^{-1-i}$$

$$\oint_C \frac{z e^{iz}}{z^2+2z+2}\, dz = 2\pi i\left(\text{Res}_{z=z_1}\right) = \cancel{2}\pi i\left(\frac{1}{\cancel{2}}e^{-1-i} + \frac{i}{\cancel{2}} e^{-1-i}\right)$$

$$= i\pi e^{-1-i} - \pi e^{-1-i}$$

$$\int_{-\infty}^{\infty} \frac{x \sin x}{x^2+2x+2}\, dx = \text{Im}\left(i\pi e^{-1-i} - \pi e^{-1-i}\right) = \pi e^{-1-i}$$

$$\int_0^{2\pi} \frac{d\theta}{2-\cos\theta} = \oint_C \frac{1}{2-\left(\frac{z+\frac{1}{z}}{2}\right)} \cdot \frac{1}{iz}\, dz = \oint_C \frac{2}{4-\left(z+\frac{1}{z}\right)} \cdot \frac{1}{iz}\, dz$$

$$= \oint_C \frac{2}{-iz^2+4iz+i}\, dz$$

$$= \oint_C \frac{2}{(-z^2+4z+1)i}\, dz$$

$$-z^2+4z+1=0$$

$$z_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4.-1.1}}{2.-1}$$

$$= \frac{-4 \pm \sqrt{20}}{-2} = \frac{-4 \pm 2\sqrt{5}}{-2}$$

$$= 2 \mp \sqrt{5} \rightarrow z_1 = 2-\sqrt{5}, \quad z_2 = 2+\sqrt{5}$$

Yang di dalam $|z|=1 \rightarrow z_1 = 2-\sqrt{5}$

$$\oint_C \frac{2}{(z^2+4z+1)i}\, dz = \oint_C \frac{2}{(z-z_1)(z-z_2)i}\, dz$$

$$g(z) = \frac{2}{(z-z_2)i}$$

$$\text{Res}_{z=z_1} = \frac{2}{\left[(2-\sqrt{5})-(2+\sqrt{5})\right] i} = \frac{2}{-2i\sqrt{5}} = -\frac{1}{i\sqrt{5}}$$

$$\int_0^{2\pi} \frac{d\theta}{2-\cos\theta} = 2\pi i\left(\text{Res}_{z=z_1}\right) = 2\pi i\left(-\frac{1}{i\sqrt{5}}\right) = -\frac{2\pi}{\sqrt{5}} = -\frac{2\pi\sqrt{5}}{5}$$

$$\int_0^{2\pi} \frac{d\theta}{1+\frac{1}{4}\sin\theta} = \oint_C \frac{1}{1+\frac{1}{4}\left(\frac{z-\frac{1}{z}}{2i}\right)} \cdot \frac{1}{iz}\, dz = \oint_C \frac{8\cancel{i}}{8i+\left(z-\frac{1}{z}\right)} \cdot \frac{1}{\cancel{i}z}\, dz$$

$$= \oint_C \frac{8}{i8z+z^2-1}\, dz$$

$$= \oint \frac{8}{(z-z_1)(z-z_2)}\, dz$$

$$z^2 + i8z - 1 = 0$$

$$z_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-8i \pm \sqrt{(8i)^2-4.1.-1}}{2.1} = \frac{-8i \pm \sqrt{-60}}{2} = \frac{-8i \pm \sqrt{60}.\sqrt{-1}}{2}$$

$$= \frac{-8i \pm 2\sqrt{15}\, i}{2}$$

$$= -4i \pm i\sqrt{15}$$

$$z_1 = (-4+\sqrt{15})i = -0,---$$

$$z_2 = (-4-\sqrt{15})i = -7,---$$

Yang berada di dalam $|z|=1$ adalah z_1

$$g(z) = \frac{8}{z-z_2} = \frac{8}{z-(-4-\sqrt{15})i}$$

$$\text{Res}_{z=z_1} = \frac{8}{(-4+\sqrt{15})i-(-4-\sqrt{15})i} = \frac{8}{2i\sqrt{15}}$$

$$\int_0^{2\pi} \frac{d\theta}{1+\frac{1}{4}\sin\theta} = 2\pi i\left(\text{Res}_{z=z_1}\right) = 2\pi i\left(\frac{8}{2i\sqrt{15}}\right)$$

$$= \frac{8\pi}{\sqrt{15}} = \frac{8\pi\sqrt{15}}{15}$$

$$\int_{-\infty}^{\infty} \frac{\sin 3x}{x^2+2x+2}\,dx = \int_{-\infty}^{\infty} \frac{1}{x^2+2x+2} e^{3ix}\,dx = \oint_C \frac{e^{3iz}}{z^2+2z+2}\,dz$$

$$z^2+2z+2=0$$

$$z_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2-4.1.2}}{2.1}$$

$$\frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm \sqrt{4}.\sqrt{-1}}{}$$

$$= -1 \pm i \quad \begin{cases} z_1 = -1+i \\ z_2 = -1-i \end{cases}$$

$$\oint_C \frac{e^{3iz}}{z^2+2z+2}\,dz = \oint_C \frac{e^{3iz}}{(z+1-i)(z+1+i)}\,dz$$

$$q(z) = \frac{e^{3iz}}{z+1+i}$$

$$\operatorname{Res}_{z=z_1} = \frac{e^{3iz_1}}{z_1+1+i} = \frac{e^{3i(-1+i)}}{(-1+i)+1+i} = \frac{e^{-3-3i}}{2i}$$

$$\oint_C \frac{e^{3iz}}{z^2+2z+2}\,dz = 2\pi i\left(\operatorname{Res}_{z=z_1}\right) = 2\pi i\left(\frac{e^{-3-3i}}{2i}\right) = \pi e^{-3-3i}$$

$$\int_{-\infty}^{\infty} \frac{\sin 3x}{x^2+2x+2}\,dx = \operatorname{Im}\left(\pi e^{-3-3i}\right) = 0$$

$$\int_{-\infty}^{\infty} \frac{\cos 2x}{(x^2+1)(x^2+4)}\,dx = \int_{-\infty}^{\infty} \frac{e^{2ix}}{(x^2+1)(x^2+4)}\,dx = \oint_C \frac{e^{2iz}}{(z^2+1)(z^2+4)}\,dz$$

$$= \oint_C \frac{e^{2iz}}{(z+i)(z-i)(z+2i)(z-2i)}\,dz$$

$z_1 = -i$, $z_2 = i$, $z_3 = -2i$, $z_4 = 2i$

$$g(z_2) = \frac{e^{2iz}}{(z+i)(z+2i)(z-2i)} \qquad g(z_4) = \frac{e^{2iz}}{(z+i)(z-i)(z+2i)}$$

$$\operatorname{Res}_{z=z_2} = \frac{e^{2i\cdot i}}{(i+i)(i+2i)(i-2i)} \qquad \operatorname{Res}_{z=z_4} = \frac{e^{2i\cdot 2i}}{(2i+i)(2i-i)(2i+2i)}$$

$$= \frac{e^{-2}}{2i\cdot 3i\cdot -i} \qquad = \frac{e^{-4}}{3i\cdot i\cdot 4i}$$

$$= \frac{e^{-2}}{6i} \qquad = \frac{e^{-4}}{-12i}$$

$$\oint_C \frac{e^{2iz}}{(z^2+1)(z^2+4)}\,dz = 2\pi i\left(\operatorname{Res}_{z=z_2} + \operatorname{Res}_{z=z_4}\right) = 2\pi i\left(\frac{e^{-2}}{6i} - \frac{e^{-4}}{12i}\right)$$

$$= \pi\left(\frac{e^{-2}}{3} - \frac{e^{-4}}{6}\right)$$

$$\int_{-\infty}^{\infty} \frac{\cos 2x}{(x^2+1)(x^2+4)}\,dx = \operatorname{Re}\left(\pi\left(\frac{e^{-2}}{3} - \frac{e^{-4}}{6}\right)\right) = \pi\left(\frac{e^{-2}}{3} - \frac{e^{-4}}{6}\right)$$

$$\int_0^{2\pi} \frac{d\theta}{1+\cos\theta} = \oint_C \frac{1}{1+\left(\frac{z+\frac{1}{z}}{2}\right)} \cdot \frac{1}{iz}\, dz = \oint_C \frac{2}{2+\left(z+\frac{1}{z}\right)} \cdot \frac{1}{iz}\, dz$$

$$= \oint_C \frac{2}{(z^2+2z+1)i}\, dz$$

$$= \oint_C \frac{2}{(z+1)^2 i}\, dz$$

$z_1 = z_2 = -1$

$g(z) = \frac{2}{i}$

$$\mathrm{Res}_{z=z_1} = \frac{1}{(2-1)!} \cdot g^{2-1}(z) \Big|_{z=-1}$$

$$= \frac{1}{1!} g'(z) \Big|_{z=-1}$$

$$= 0$$

$$\int_0^{2\pi} \frac{d\theta}{1+\cos\theta} = 2\pi i \left(\mathrm{Res}_{z=z_1}\right) = 2\pi i (0) = 0$$