Pre-Midterm Exam
Complex Variables

1. Find the real and imaginary parts, modulus, conjugate, and argument of z!

$$z = \frac{i}{-2-2i}$$

2. Find the solution of $z^2 - z + i(z-1) = 0$
3. Given complex function $f(z) = z^2 + z - 1$

   Find:

   a. $Re[f(z)]$

   b. $Im[f(z)]$

4. Is $f(z)$ analytic in region D?

   $f(z) = \frac{z^2+3}{z^2-7z+10}$; region D: $|z| < 3$

5. Is $f(z) = x^2 - 2xy + 5 + i(y^2 + x^2 + 1)$ differentiable at $z_0 = 2 + i$? Find $f'(z_0)$
6. Proof that $U(x, y)$ is harmonic function, then find its harmonic conjugate

$$U(x, y) = 2x(1 - y)$$

7. Find $\int_C f(z)\,dz$ with $f(z) = y - x - 3ix^2$ along a line segment from (0,0) to (1,1)
8. Find $\int_C f(z)dz$ with $f(z) = e^{2z}$ and C is a path from πi to 2πi
9. Find the integral of $g(z)$ over C if $g(z) = \frac{\sin z}{z+3i}$ and C: $|z + 3i| = 1$ (positive direction)