1. Tentukan Nilai Eigen dari SPL

$$X' = \underbrace{\begin{bmatrix} 4x & 2y \\ -3x & -y \end{bmatrix}}_{A} X$$

$$\det(A - \lambda I) = 0$$

$$\left| \begin{bmatrix} 4x & 2Y \\ -3x & -Y \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 4x - \lambda & 2Y \\ -3x & -Y - \lambda \end{vmatrix} = 0$$

$$(4x - \lambda)(-Y - \lambda) + 6xY = 0$$

$$(-4xY - 4x\lambda + Y\lambda + \lambda^2) + 6xY = 0$$

$$\lambda^2 + (-4x + Y)\lambda + 2xY = 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4x+Y) \pm \sqrt{(-4x+Y)^2 - 4.1.2xY}}{2.1}$$

$$= \frac{4x - Y \pm \sqrt{16x^2 - 8xY + Y^2 - 8xY}}{2}$$

$$= \frac{4x - Y \pm \sqrt{16x^2 - 16xY + Y^2}}{2}$$

$$\lambda_1 = \frac{4x - Y + \sqrt{16x^2 - 16xY + Y^2}}{2}$$

$$\lambda_2 = \frac{4x - Y - \sqrt{16x^2 - 16xY + Y^2}}{2}$$

2. Jika nilai eigen dari SPL

$$X' = \underbrace{\begin{bmatrix} 3x & -2y \\ 2x & -2y \end{bmatrix}}_{A} X$$ adalah $\lambda_1 = \lambda_2 = 1$, tentukan vector-vektor eigen-nya

$\lambda_1 = \lambda_2 = 1$

$(A - \lambda I)K = 0$

$$\left(\begin{bmatrix} 3x & -2y \\ 2x & -2y \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 3x-1 & -2y \\ 2x & -2y-1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0 \rightarrow$$

$$(3x-1)k_1 - 2yk_2 = 0$$

$$2yk_2 = (3x-1)k_1$$

$$k_2 = \frac{3x-1}{2} k_1$$

$$2xk_1 - (2y+1)k_2 = 0$$

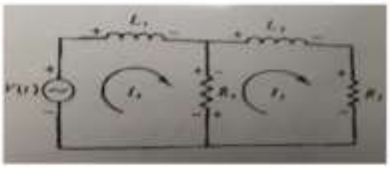
$$2xk_1 - (2y+1)\frac{3x-1}{2}k_1 = 0$$

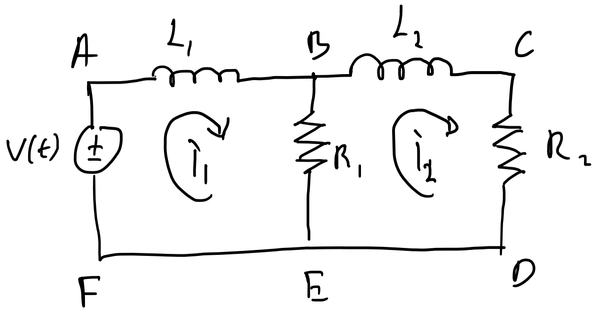
$$2xk_1 - \frac{6xy-2y+3x-1}{2}k_1 = 0$$

$$\frac{(6xy-2y+x-1)}{2}k_1 = 0$$

$i_1(0) = 0$; $i_2(0) = 0$

3. Dalam rangkaian listrik berikut,diketahui arus awalnya 0 dan $R_1 = 1{,}5\ ohm, R_2 = 4\ ohm, L_1 = L_2 = 1\ henry, V(t) = 2\ volt$. Tentukan arus $I_1(t)\ dan = I_2(t)$ pada setiap saat.





Loop 1 (ABEFA)

$$L_1 \frac{di_1}{dt} + R_1(i_1 - i_2) = V(t)$$

$$1 \cdot \frac{di_1}{dt} + 1{,}5(i_1 - i_2) = 2$$

$$\frac{di_1}{dt} = 2 - 1{,}5 i_1 + 1{,}5 i_2$$

$$\frac{di_1}{dt} = -1{,}5 i_1 + 1{,}5 i_2 + 2$$

Loop 2 (ABCDEFA)

$$L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + R_2 i_2 = V(t)$$

$$1 \frac{di_1}{dt} + 1 \cdot \frac{di_2}{dt} + 4 \cdot i_2 = 2$$

$$(2 - 1{,}5 i_1 + 1{,}5 i_2) + \frac{di_2}{dt} + 4 i_2 = 2$$

$$\frac{di_2}{dt} = 1{,}5 i_1 - 5{,}5 i_2 + 0$$

$$X' = \underbrace{\begin{bmatrix} -1{,}5 & 1{,}5 \\ 1{,}5 & -5{,}5 \end{bmatrix}}_{A} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} -1{,}5 - \lambda & 1{,}5 \\ 1{,}5 & -5{,}5 - \lambda \end{vmatrix} = 0$$

$$(-1{,}5 - \lambda)(-5{,}5 - \lambda) - 1{,}5^2 = 0$$

$$0{,}25 + 1{,}5\lambda + 5{,}5\lambda + \lambda^2 - 2{,}25 = 0$$

$$\lambda^2 + 7\lambda + 6 = 0$$

$$(\lambda + 6)(\lambda + 1) = 0$$

$$\lambda_1 = -6 \quad \lambda_2 = -1$$

$$\lambda_1 = -6$$

$$(A - \lambda_1 I)K_1 = 0$$

$$\left(\begin{bmatrix} -1{,}5 & 1{,}5 \\ 1{,}5 & -5{,}5 \end{bmatrix} - \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}\right)\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 4{,}5 & 1{,}5 \\ 1{,}5 & 0{,}5 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0 \longrightarrow$$

$$\begin{array}{l|l|} 4{,}5k_1 + 1{,}5k_2 = 0 & \times 1 \\ 1{,}5k_1 + 0{,}5k_2 = 0 & \times 3 \end{array}$$

$$4{,}5k_1 + 1{,}5k_2 = 0$$
$$4{,}5k_1 + 1{,}5k_2 = 0 \quad -$$
$$0 = 0$$

$$4{,}5k_1 + 1{,}5k_2 = 0$$

$$4{,}5k_1 = -1{,}5k_2$$

$$3k_1 = -k_2$$

$$K_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\lambda_2 = -1$$

$$(A - \lambda_2 I)K_2 = 0$$

$$\left(\begin{bmatrix} -1{,}5 & 1{,}5 \\ 1{,}5 & -5{,}5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\right)\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -0,5 & 1,5 \\ 1,5 & -4,5 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0 \rightarrow$$

$$-0,5\,k_1 + 1,5\,k_2 = 0$$
$$1,5\,k_2 = 0,5\,k_1$$
$$3k_2 = k_1$$

$$1,5\,k_1 - 4,5\,k_2 = 0$$
$$1,5\,k_1 = 4,5\,k_2$$
$$k_1 = 3k_2$$
$$K_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$X_H = c_1 K_1 e^{\lambda_1 t} + c_2 K_2 e^{\lambda_2 t}$$
$$= c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-6t} + c_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{-t}$$

$$F(t) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$
$$X_p = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$
$$X_p' = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$X_p' = A X_p + F(t)$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1,5 & 1,5 \\ 1,5 & -5,5 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1,5a_1 + 1,5b_1 \\ 1,5a_1 - 5,5b_1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$-1,5a_1 + 1,5b_1 + 2 = 0$$
$$1,5a_1 - 5,5b_1 = 0$$
$$1,5a_1 = 5,5\,b_1$$
$$3a_1 = 11\,b_1$$
$$a_1 = \frac{11}{3} b_1$$
$$-1,5\left(\frac{11}{3} b_1\right) - 5,5b_1 + 2 = 0$$

$$-5,5b_1 - 5,5b_1 = -2$$
$$11\,b_1 = 2$$
$$b_1 = \frac{2}{11} \longrightarrow a_1 = \frac{11}{3} \cdot \frac{2}{11} = \frac{2}{3}$$

$$X_p = \begin{bmatrix} 2/3 \\ 2/11 \end{bmatrix}$$

$$x = x_H + x_p$$

$$x = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-6t} + c_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{-t} + \begin{bmatrix} 2/3 \\ 2/11 \end{bmatrix}$$

$$i_1(t) = c_1 e^{-6t} + 3c_2 e^{-t} + 2/3$$

$$i_2(t) = -3c_1 e^{-6t} + c_2 e^{-t} + 2/11$$

$$i_1(0) = 0$$

$$c_1 e^0 + 3c_2 e^0 + 2/3 = 0$$

$$c_1 + 3c_2 + \frac{2}{3} = 0$$

$$c_1 + 3c_2 = -\frac{2}{3}$$

$$i_2(0) = 0$$

$$-3c_1 e^0 + c_2 e^0 + \frac{2}{11} = 0$$

$$-3c_1 + c_2 + \frac{2}{11} = 0$$

$$-3c_1 + c_2 = -\frac{2}{11}$$

$\times 3$

$$\begin{array}{r} 3c_1 + 9c_2 = -2 \\ \hline 10c_2 = -\frac{24}{11} \end{array} +$$

$$c_2 = -\frac{24}{110} = -\frac{12}{55}$$

$$c_1 + 3c_2 = -\frac{2}{3}$$

$$c_1 + 3\left(-\frac{24}{110}\right) = -\frac{2}{3}$$

$$c_1 = -\frac{2}{3} + \frac{72}{110}$$

$$c_1 = -\frac{2}{165}$$

$$i_1(t) = -\frac{2}{165} e^{-6t} + 3 \cdot -\frac{12}{55} e^{-t} + \frac{2}{3}$$

$$= -\frac{2}{165} e^{-6t} - \frac{36}{55} e^{-t} + \frac{2}{3}$$

$$i_2(t) = -3 \cdot \frac{-2}{165} e^{-6t} + -\frac{12}{55} e^{-t} + \frac{2}{11}$$

$$= \frac{2}{55} e^{-6t} - \frac{12}{55} e^{-t} + \frac{2}{11}$$

4. Tentukan

a. $L\{e^{-2t}\cos 3t\}$

b. $L^{-1}\left\{\frac{5}{(s-4)(s-1)}\right\}$

a. $L\{e^{-2t}\cos 3t\} = \dfrac{s-(-2)}{(s-(-2))^2+3^2} = \dfrac{s+2}{(s+2)^2+9}$

$= \dfrac{s+2}{s^2+4s+4+9}$

$= \dfrac{s+2}{s^2+4s+13}$

b. $L^{-1}\left\{\dfrac{5}{(s-4)(s-1)}\right\} = L^{-1}\left\{\dfrac{A}{s-4} + \dfrac{B}{s-1}\right\}$

$\dfrac{A}{s-4} + \dfrac{B}{s-1} = \dfrac{5}{(s-4)(s-1)}$

$\dfrac{A(s-1) + B(s-4)}{(s-4)(s-1)} = \dfrac{5}{(s-4)(s-1)}$

$As - A + Bs - 4B = 5$

$(A+B)s - A - 4B = 5$

$A + B = 0$

$-A - 4B = 5$ +

$-3B = 5$

$B = -\dfrac{5}{3}$

$A = \dfrac{5}{3}$

$= L^{-1}\left\{\dfrac{5/3}{s-4} + \dfrac{-5/3}{s-1}\right\}$

$= L^{-1}\left\{\dfrac{5/3}{s-4}\right\} + L^{-1}\left\{\dfrac{-5/3}{s-1}\right\}$

$= \dfrac{5}{3}L^{-1}\left\{\dfrac{1}{s-4}\right\} - \dfrac{5}{3}L^{-1}\left\{\dfrac{1}{s-1}\right\}$

$= \dfrac{5}{3}\cdot e^{4t} - \dfrac{5}{3}e^{t}$

$= \dfrac{5}{3}\left(e^{4t} - e^{t}\right)$

5. Selesaiakan Masalah Nilai Awal berikut dengan Transformasi Laplace

$y'' + 4y' = 4\cos 2t, y(0) = 0, y'(0) = 6$

$$L\{y'' + 4y'\} = L\{4\cos 2t\}$$

$$L\{y''\} + 4L\{y'\} = 4L\{\cos 2t\}$$

$$\left(s^2.Y(s) - s.Y(0) - Y'(0)\right) + 4\left(s.Y(s) - Y'(0)\right) = 4.\frac{s}{s^2+2^2}$$

$$s^2.Y(s) - s.0 - 6 + 4(s.Y(s) - 6) = \frac{4s}{s^2+4}$$

$$s^2Y(s) - 6 + 4sY(s) - 6 = \frac{4s}{s^2+4}$$

$$Y(s)(s^2+4s) - 12 = \frac{4s}{s^2+4}$$

$$Y(s)(s^2+4s) = \frac{4s}{s^2+4} + 12$$

$$Y(s) = \frac{4s}{(s^2+4)(s^2+4s)} + \frac{12}{s^2+4s}$$

$$Y(s) = \frac{4\cancel{s}}{\cancel{s}(s^2+4)(s+4)} + \frac{12}{s(s+4)}$$

$$Y(s) = \frac{4}{(s^2+4)(s+4)} + \frac{12}{s(s+4)}$$

$$Y(s) = \frac{-s+4}{5(s^2+4)} + \frac{1}{5(s+4)} + \frac{3}{s} - \frac{3}{s+4}$$

$$Y(s) = -\frac{s}{5(s^2+4)} + \frac{4}{5(s^2+4)} + \frac{1}{5(s+4)} + \frac{3}{s} - \frac{3}{s+4}$$

$$L^{-1}\{Y(s)\} = -\frac{1}{5}L^{-1}\left\{\frac{s}{s^2+2^2}\right\} + \frac{2}{5}L^{-1}\left\{\frac{2}{s^2+2^2}\right\} + \frac{1}{5}L^{-1}\left\{\frac{1}{s+4}\right\} + 3L^{-1}\left\{\frac{1}{s}\right\} - 3L^{-1}\left\{\frac{1}{s+4}\right\}$$

$$Y(t) = -\frac{1}{5}\cos 2t + \frac{2}{5}\sin 2t + \frac{1}{5}e^{-4t} + 3 - 3e^{-4t}$$

$$Y(t) = \frac{2}{5}\sin 2t - \frac{1}{5}\cos 2t - \frac{14}{5}e^{-4t} + 3$$