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1.) a. $dy - (y-1)^2 dx = 0$

$$dy = (y-1)^2 dx$$

$$\frac{1}{(y-1)^2} dy = dx$$

$$\int \frac{1}{(y-1)^2} dy = \int dx$$

$$-\frac{1}{y-1} + C_1 = x + C_2$$

$$-\frac{1}{y-1} = x + C$$

$$y-1 = -\frac{1}{x+C}$$

$$y = 1 - \frac{1}{x+C}$$

b. $\frac{dy}{dx} = x\sqrt{1-y^2}$

$$\frac{1}{\sqrt{1-y^2}} dy = x\,dx$$

$$\int \frac{1}{\sqrt{1-y^2}} dy = \int x\,dx$$

$$\sin^{-1} y + C_1 = \frac{1}{2}x^2 + C_2$$

$$\sin^{-1} y = \frac{1}{2}x^2 + C$$

$$y = \sin\left(\frac{1}{2}x^2 + C\right)$$

2.) a. $\dfrac{dy}{dx} = \dfrac{y^2-1}{x^2-1}$; $y(2) = 2$

$$\frac{1}{y^2-1}\,dy = \frac{1}{x^2-1}\,dx$$

$$\int \frac{1}{y^2-1}\,dy = \int \frac{1}{x^2-1}\,dx$$

$$\int \frac{-1}{2(y+1)} + \frac{1}{2(y-1)}\,dy = \int \frac{-1}{2(x+1)} + \frac{1}{2(x-1)}\,dx$$

$$\cancel{\tfrac{1}{2}} \int \frac{-1}{y+1} + \frac{1}{y-1}\,dy = \cancel{\tfrac{1}{2}} \int \frac{-1}{x+1} + \frac{1}{x-1}\,dx$$

$$-\ln|y+1| + \ln|y-1| + c = -\ln|x+1| + \ln|x-1| + c$$

$$\ln\left|\frac{y-1}{y+1}\right| + c = \ln\left|\frac{x-1}{x+1}\right| + c$$

$$\ln\left|\frac{y-1}{y+1}\right| - \ln\left|\frac{x-1}{x+1}\right| = c$$

$$\ln\left|\frac{2-1}{2+1}\right| - \ln\left|\frac{2-1}{2+1}\right| = c$$

$$\ln\left|\frac{1}{3}\right| - \ln\left|\frac{1}{3}\right| = c$$

$$c = 0$$

Penyelesaian : $\ln\left|\dfrac{y-1}{y+1}\right| - \ln\left|\dfrac{x-1}{x+1}\right| = 0$

$$\frac{y-1}{y+1} = \frac{x-1}{x+1}$$

$$\cancel{xy} + y - x \cancel{-1} = \cancel{xy} - y + x \cancel{-1}$$

$$2y = 2x$$

$$y = x$$

b. $x^2 \dfrac{dy}{dx} = y - xy \; ; \; y(-1) = 1$

$$x^2 dx = y(1-x)\, dx$$

$$\frac{1}{y} dy = \frac{1-x}{x^2} dx$$

$$\int \frac{1}{y} dy = \int \frac{1-x}{x^2} dx$$

$$\ln y + c = \int \frac{1}{x^2} - \frac{1}{x} dx$$

$$\ln y + c = -\frac{1}{x} - \ln x + c$$

$$\ln y = -\frac{1}{x} - \ln x + c$$

$$y = e^{-(\frac{1}{x} + \ln x) + c}$$

$$y = \frac{1}{e^{1/x}} \cdot \frac{1}{e^{\ln x}} \cdot e^{c}$$

$$y = e^{-1/x} \cdot \frac{1}{x} \cdot c$$

$$y = \frac{ce^{-1/x}}{x}$$

$$y(-1) = 1$$

$$\frac{ce^{-1/-1}}{-1} = 1$$

$$-ce = 1$$

$$c = -\frac{1}{e}$$

Penyelesaian : $y = -\frac{1}{e} \cdot \frac{e^{-1/x}}{x} = -\dfrac{1}{xe^{\frac{1+x}{x}}}$

3.) a. $(xy + \cos y)dx + \left(\frac{1}{2}x^2 - x\sin y - y\right)dy = 0$

$M = xy + \cos y$; $N = \frac{1}{2}x^2 - x\sin y - y$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial}{\partial y}(xy + \cos y) = \frac{\partial}{\partial x}\left(\frac{1}{2}x^2 - x\sin y - y\right)$$

$$x - \sin y = x - \sin y \rightarrow \text{Pers. eksak}$$

$$M = \frac{\partial f}{\partial x} \rightarrow f(x,y) = \int xy + \cos y \, dx + g(y)$$

$$f(x,y) = \frac{1}{2}x^2y + x\cos y + g(y)$$

$$N = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\frac{1}{2}x^2y + x\cos y + g(y)\right)$$

$$\frac{1}{2}x^2 - x\sin y - y = \frac{1}{2}x^2 - x\sin y + g'(y)$$

$$g'(y) = -y$$

$$g(y) = \int -y \, dy$$

$$g(y) = -\frac{1}{2}y^2$$

Solusi : $f(x,y) = c$

$$\frac{1}{2}x^2y + x\cos y + g(y) = c$$

$$\frac{1}{2}x^2y + x\cos y - \frac{1}{2}y^2 = c$$

3.) b. $(x - y^3 + y^2 \sin x)dx = (3xy^2 + 2y\cos x)dy$

$(x - y^3 + y^2 \sin x)dx - (3xy^2 + 2y\cos x)dy = 0$

$M = x - y^3 + y^2 \sin x$; $N = -(3xy^2 + 2y\cos x)$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow \frac{\partial}{\partial y}(x - y^3 + y^2 \sin x) = \frac{\partial}{\partial x} -(3xy^2 + 2y\cos x)$

$0 - 3y^2 + 2y\sin x = -3y^2 + 2y\sin x$ → Pers. eksak

$M = \frac{\partial f}{\partial x} \rightarrow f(x,y) = \int x - y^3 + y^2 \sin x \, dx + g(y)$

$f(x,y) = \frac{1}{2}x^2 - xy^3 - y^2\cos x + g(y)$

$N = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\frac{1}{2}x^2 - xy^3 - y^2\cos x + g(y)\right)$

$-(3xy^2 + 2y\cos x) = 0 - 3xy^2 - 2y\cos x + g'(y)$

$g'(y) = 0$

$g(x) = \int 0 \, dy$

$g(y) = c$

Solusi : $f(x,y) = c$

$\frac{1}{2}x^2 - xy^3 - y^2\cos x + g(y) = c$

$\frac{1}{2}x^2 - xy^3 - y^2\cos x + c = c$

$\frac{1}{2}x^2 - xy^3 - y^2\cos x = c$

4.) a. $x\,dx + (x^2y + 4y)\,dy = 0$; $y(4) = 0$

$$(x^2+4)y\,dy = -x\,dx$$

$$y\,dy = -\frac{x}{x^2+4}\,dx$$

$$\int y\,dy = -\int \frac{x}{x^2+4}\,dx \rightarrow$$

$$x^2 + 4 = u$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$\frac{1}{2}y^2 + c = -\int \frac{\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$$

$$\frac{1}{2}y^2 + c = -\frac{1}{2}\ln u + c$$

$$\frac{\cancel{1}}{\cancel{2}}y^2 + c = -\frac{\cancel{1}}{\cancel{2}}\ln(x^2+4) + c$$

$$y^2 = -\ln(x^2+4) + c$$

$$y = \sqrt{c - \ln(x^2+4)}$$

$$y(4) = 0$$

$$\sqrt{c - \ln(4^2+4)} = 0$$

$$c - \ln 20 = 0$$

$$c = \ln 20$$

Solusi : $y = \sqrt{\ln 20 - \ln(x^2+4)}$

$$y = \sqrt{\ln\left(\frac{20}{x^2+4}\right)}$$

4.) b. $(y^2\cos x - 3x^2y - 2x)\,dx + (2y\sin x - x^3 + \ln y)\,dy = 0$

$M = y^2\cos x - 3x^2y - 2x$; $N = 2y\sin x - x^3 + \ln y$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow \frac{\partial}{\partial y}(y^2\cos x - 3x^2y - 2x) = \frac{\partial}{\partial x}(2y\sin x - x^3 + \ln y)$$

$$2y\cos x - 3x^2 - 0 = 2y\cos x - 3x^2 + 0$$

(Pers. Eksak)

$$M = \frac{\partial f}{\partial x} \rightarrow f(x,y) = \int y^2\cos x - 3x^2y - 2x\,dx + g(y)$$

$$f(x,y) = y^2\sin x - x^3y - x^2 + g(y)$$

$$N = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(y^2\sin x - x^3y - x^2 + g(y))$$

$$2y\sin x - x^3 + \ln y = 2y\sin x - x^3 - 0 + g'(y)$$

$$g'(y) = \ln y$$

$$g(y) = \int \ln y\,dy$$

$$g(y) = \ln y \cdot y - \int y \cdot \frac{1}{y}\,dy$$

$$g(y) = y\ln y - y + C$$

$f(x,y) = c$

$y^2\sin x - x^3y - x^2 + g(y) = c$

$y^2\sin x - x^3y - x^2 + y\ln y - y = c$

$y(0) = e \rightarrow y = e$; $x = 0$

$e^2\sin 0 - 0^3e - 0^2 + e.\ln e - e = c$

$e - e = c$

$c = 0$

Solusi khusus : $\underline{\underline{y^2\sin x - x^3y - x^2 + y\ln y - y = 0}}$

5.) a. $\dfrac{dy}{dx} = \dfrac{1-2y-4x}{1+y+2x}$; $y+2x=0 \to y=-2x \to -2=\dfrac{y}{x}$

$(1+y+2x)dy = (1-2y-4x)dx$

$(1+(-2x)+2x)d{-2x} = (1-2(-2x)-4x)dx$

$1\,d(-2x) = 1\,dx$

$\int 1 d(-2x) = \int 1\,dx$

$-2x + C = x + C$

$\left(\dfrac{y}{x}\right)x + C = x + C$

$y + C = x + C$

$y = x + C$

5.) b. $(x^2+2y^2)\dfrac{dx}{dy} = xy$; $y = ux \to u = \dfrac{y}{x}$; $y(-1)=1$

$(x^2+2y^2)\,dx = xy\,dy$

$(x^2+2y^2)\,dx - xy\,dy = 0$

$(x^2+2(uy)^2)dx - x(ux)d(ux) = 0$

$\cancel{x^2}(1+2u^2)dx - u\cancel{x^2}(udx + xdu) = 0$

$(1+2u^2)dx - u^2dx - uxdu = 0$

$(1+2u^2-u^2)dx = (ux)du$

$(1+u^2)dx = (ux)\,du$

$\dfrac{1}{x}dx = \dfrac{u}{1+u^2}du$

$\int \dfrac{1}{x}dx = \int \dfrac{u}{1+u^2}du \longrightarrow$ $1+u^2 = t$

$\dfrac{dt}{du} = 2u \to du = \dfrac{dt}{2u}$

$$\ln|x| + c = \int \frac{\cancel{u}}{t} \cdot \frac{dt}{2\cancel{u}}$$

$$\ln|x| + c = \frac{1}{2}\ln|t| + c$$

$$\ln|x| + c = \frac{1}{2}\ln|1+u^2| + c$$

$$\ln|x| + c = \frac{1}{2}\ln\left|1 + \left(\frac{y}{x}\right)^2\right| + c$$

$$\ln\left|1 + \frac{y^2}{x^2}\right| = \ln|x| + c$$

$$\ln\left|\frac{x^2+y^2}{x^2}\right| = \ln|x| + c$$

$$\ln|x^2+y^2| - \ln|x^2| = \ln|x| + c$$

$$\ln|x^2+y^2| = \ln|x| + \ln|x^2| + c$$

$$\ln|x^2+y^2| = \ln|x^3| + c$$

$$e^{\ln|x^2+y^2|} = e^{\ln|x^3|+c}$$

$$x^2 + y^2 = x^3 \cdot e^c$$

$$y^2 = Cx^3 - x^2$$

$$y = \sqrt{Cx^3 - x^2}$$

$$y(-1) = 1$$

$$\sqrt{C(-1)^3 - (-1)^2} = 1$$

$$-C - 1 = 1$$

$$C = -2$$

Solusi umum : $y = \sqrt{Cx^3 - x^2}$ Solusi khusus : $y = \sqrt{-2x^3 - x^2}$