5. $\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + 49y = 5x + 3 + e^{3x}$; $y(0) = 1$, $y'(0) = 3$

$r^2 + 14r + 49 = 0$

$(r+7)^2 = 0$

$r_1 = r_2 = -7$

$y_h = (c_1 + c_2 x)e^{-7x}$

karena $f(x) = 5x + 3 + e^{3x}$, maka :

$y_p = Ax + B + Ce^{3x}$

$\frac{d^2y_p}{dx^2} = 9Ce^{3x}$

$\frac{dy_p}{dx} = A + 3Ce^{3x}$

$\frac{d^2y_p}{dx^2} + 14\frac{dy_p}{dx} + 49y = 5x + 3 + e^{3x}$

$9Ce^{3x} + 14(A + 3Ce^{3x}) + 49(Ax + B + Ce^{3x}) = 5x + 3 + e^{3x}$

$49Ax + 14A + 49B + 100Ce^{3x} = 5x + 3 + e^{3x}$

$49A = 5$ $\quad 14A + 49B = 3$ $\quad 100C = 1$

$A = \frac{5}{49}$ $\quad \frac{70}{49} + 49B = 3$ $\quad C = \frac{1}{100}$

$B = \frac{11}{343}$

$y_p = \frac{5}{49}x + \frac{11}{343} + \frac{1}{100}e^{3x}$

$y_T = y_h + y_p$

$y_T = (c_1 + c_2 x)e^{-7x} + \frac{5}{49}x + \frac{11}{343} + \frac{1}{100}e^{3x}$

$y_T' = c_2 e^{-7x} + -7(c_1 + c_2 x)e^{-7x} + \frac{5}{49} + \frac{3}{100}e^{3x}$

$y(0) = 1$

$(c_1 + c_2 \cdot 0)e^{-7\cdot 0} + \frac{5}{49}\cdot 0 + \frac{11}{343} + \frac{1}{100}e^{3\cdot 0} = 1$

$c_1 + \frac{1443}{34300} = 1 \rightarrow c_1 = 0,96$

$Y'(0) = 3$

$C_2 e^{-7.0} + -7(C_1 + C_2 \cdot 0) e^{-7.0} + \frac{5}{49} + \frac{3}{100} e^{3.0} = 3$

$C_2 - 7(0,96) + \frac{5}{49} + \frac{3}{100} = 3$

$C_2 - 6,72 + 0,13 = 3$

$C_2 = 9,59$

$\therefore$ Solusi khusus : $Y = (C_1 + C_2 x) e^{-7x} + \frac{5}{49} x + \frac{11}{343} + \frac{1}{100} e^{3x}$

$Y = (0,96 + 9,59x) e^{-7x} + \frac{5}{49} x + \frac{11}{343} + \frac{1}{100} e^{3x}$

6. $\frac{d^2Y}{dx^2} + 6\frac{dY}{dx} + 10Y = \cos 4x \; ; \; Y(0) = 2 \; , \; Y'(0) = 5$

$r^2 + 6r + 10 = 0$

$r_{1,2} = \frac{-6 \pm \sqrt{6^2 - 4.1.10}}{2.1}$

$= \frac{-6 \pm \sqrt{36 - 40}}{2}$

$= \frac{-6 \pm \sqrt{-4}}{2}$

$= \frac{-6 \pm 2i}{2}$

$= -3 \pm i$ / $r_1 = -3 + i$, $r_2 = -3 - i$

$Y_u = e^{-3x}(C_1 \cos x + C_2 \sin x)$

Karena $f(x) = \cos 4x$, maka :

$Y_p = A \sin 4x + B \cos 4x$

$\frac{dY_p}{dx} = 4A \cos 4x - 4B \sin 4x$

$\frac{d^2Y_p}{dx^2} = -16A \sin 4x - 16B \cos 4x$

$\frac{d^2Y_p}{dx^2} + 6\frac{dY_p}{dx} + 10Y_p = \cos 4x$

$-16A \sin 4x - 16B \cos 4x + 6(4A \cos 4x - 4B \sin 4x) + 10(A \sin 4x + B \cos 4x) = \cos 4x$

$(-6A - 24B) \sin 4x + (24A - 6B) \cos 4x = \cos 4x$

$-6A - 24B = 0$ $\qquad 24A - 6B = 1$

$-6A - 24B$ $\qquad 24(-4B) - 6B = 1$

$A = -4B$ $\qquad -96B - 6B = 1$

$-102B = 1$

$B = -\frac{1}{102}$

$A = \frac{2}{51}$

$y_p = A\sin 4x + B\cos 4x$

$y_p = \frac{2}{51}\sin 4x - \frac{1}{102}\cos 4x$

$y_T = y_h + y_p$

$y_T = e^{-3x}(C_1\cos x + C_2\sin x) + \frac{2}{51}\sin 4x - \frac{1}{102}\cos 4x$

$y_T' = -3e^{-3x}(C_1\cos x + C_2\sin x) + e^{-3x}(-C_1\sin x + C_2\cos x) + \frac{8}{51}\cos 4x + \frac{2}{51}\sin 4x$

$y(0) = 2$

$e^{-3.0}(C_1\cos 0 + C_2\sin 0) + \frac{2}{51}\sin 4.0 - \frac{1}{102}\cos 4.0 = 2$

$C_1 - \frac{1}{102} = 2$

$C_1 = \frac{205}{102}$

$y'(0) = 5$

$-3e^{-3.0}(C_1\cos 0 + C_2\sin 0) + e^{-3.0}(-C_1\sin 0 + C_2\cos 0) + \frac{8}{51}\cos 4.0 + \frac{2}{51}\sin 4.0 = 5$

$-3.\frac{205}{201} + C_2 + \frac{8}{51} = 5$

$C_2 - \frac{599}{102} = 5 \rightarrow C_2 = \frac{1109}{102}$

$\therefore$ Solusi khusus : $y = e^{-3}(C_1\cos x + C_2\sin x) + \frac{2}{51}\sin 4x - \frac{1}{102}\cos 4x$

$y = e^{-3}\left(\frac{205}{102}\cos x + \frac{1109}{102}\sin x\right) + \frac{2}{51}\sin 4x - \frac{1}{102}\cos 4x$