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1. Misalkan $\vec{u}, \vec{v}, \vec{w} \in V$ dan $k, l \in$ Riil, V dinamakan ruang vektor jika terpenuhi aksioma :

1.) V tertutup terhadap operasi penjumlahan untuk setiap $\vec{u}, \vec{v} \in V$ maka $\vec{u} + \vec{v} \in V$

2.) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

3.) $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$

4.) Terdapat $\vec{0} \in V$ sehingga untuk setiap $\vec{u} \in V$ berlaku $\vec{u} + \vec{0} = \vec{0} + \vec{u} = \vec{u}$

5.) Untuk setiap $\vec{u} \in V$ terdapat $(-\vec{u})$ sehingga $\vec{u} + (-\vec{u}) = (-\vec{u}) + \vec{u} = \vec{0}$

6.) V tertutup terhadap operasi perkalian dengan skalar. Untuk setiap $\vec{u} \in V$ dan $k \in$ Riil maka $k\vec{u} \in V$

7.) $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$

8.) $(k + l)\vec{u} = k\vec{u} + l\vec{u}$

9.) $k(l\vec{u}) = l(k\vec{u}) = (kl)\vec{u}$

10.) $1.\vec{u} = \vec{u}$

2. Misalkan W merupakan subhimpunan dari sebuah ruang vektor V. W dinamakan subruang V jika W juga merupakan ruang vektor yang tertutup terhadap operasi penjumlahan dan perkalian dengan skalar. Syarat W disebut subruang dari V adalah :

1.) $W \neq \{\}$

2.) $W \subseteq V$

3.) $\vec{u}, \vec{v} \in W$ maka $\vec{u} + \vec{v} \in W$

4.) $\vec{u} \in W$ dan $k \in$ Riil maka $k\vec{u} \in W$

3. $\langle \vec{u}, \vec{v} \rangle$ dinamakan ruang hasil kali dalam jika memenuhi :

1.) $\langle \vec{u}, \vec{v} \rangle = \langle \vec{v}, \vec{u} \rangle$ (Simetris)

2.) $\langle \vec{u} + \vec{v}, \vec{w} \rangle = \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$ (Aditivitas)

3.) Untuk $k \in \mathbb{R}$, $\langle k\vec{u}, \vec{v} \rangle = \langle \vec{u}, k\vec{v} \rangle = k \langle \vec{u}, \vec{v} \rangle$ (Homogenitas)

4.) $\langle \vec{u}, \vec{u} \rangle \geq 0$, untuk setiap $\vec{u}$ dan $\langle \vec{u}, \vec{u} \rangle = 0 \leftrightarrow \vec{u} = \vec{0}$
(Positivitas)

4. Misalkan V dan W adalah ruang vektor, $T : V \to W$ dinamakan transformasi linear, jika untuk setiap $\vec{a}, \vec{b} \in V$ dan $\alpha \in \mathbb{R}$ berlaku :

1.) $T(\vec{a} + \vec{b}) = T(\vec{a}) + T(\vec{b})$

2.) $T(\alpha \vec{a}) = \alpha . T(\vec{a})$

Jika $V = W$ maka T dinamakan operator linear

5. $S = \{ 1, 2 - x^2, (2 - x)^2 \}$

$S = \{ 1, 2 - x^2, 4 - 4x + x^2 \}$

a. $P_2 = a + bx + cx^2$

$S_1 = 1 = 1 + 0.x + 0x^2$

$S_2 = 2 - x^2 = 2 + 0.x - x^2$

$S_3 = 4 - 4x + x^2$

$k_1 S_1 + k_2 S_2 + k_3 S_3 = \vec{0}$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & -4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & k_1 \\ 0 & 0 & -4 & k_2 \\ 0 & -1 & 1 & k_3 \end{array}\right] \begin{array}{c} b_2 \leftrightarrow b_3 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 4 & k_1 \\ 0 & -1 & 1 & k_3 \\ 0 & 0 & -4 & k_2 \end{array}\right] \begin{array}{c} 2b_2 + b_1 \\ -\frac{1}{4} b_3 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 6 & k_1 + 2k_3 \\ 0 & -1 & 1 & k_3 \\ 0 & 0 & 1 & -\frac{1}{4} k_2 \end{array}\right] \begin{array}{c} -b_3 + b_2 \\ -6b_3 + b_1 \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & k_1+2k_3+\frac{3}{2}k_2 \\ 0 & -1 & 0 & k_3+\frac{1}{4}k_2 \\ 0 & 0 & 1 & -\frac{1}{4}k_2 \end{array}\right] \overset{-b_2}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & k_1+\frac{3}{2}k_2+2k_3 \\ 0 & 1 & 0 & -\frac{1}{4}k_2-k_3 \\ 0 & 0 & 1 & -\frac{1}{4}k_2 \end{array}\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$k_2 = 0$, $-\frac{1}{4}.0 - k_3 = 0$, $k_1 + \frac{3}{2}.0 + 2.0 = 0$

$k_3 = 0$ $\quad$ $k_1 = 0$

∴ Karena SPL homogen bersolusi tunggal, maka S bebas linear

b. $k_1 S_1 + k_2 S_2 + k_3 S_3 = \vec{u}$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & -4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_1+\frac{3}{2}k_2+2k_3 \\ -\frac{1}{4}k_2-k_3 \\ -\frac{1}{4}k_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$-\frac{1}{4}k_2 = u_3$ $\quad$ $-\frac{1}{4}(-4u_3) - k_3 = u_2$ $\quad$ $k_1 + \frac{3}{\cancel{2}}(-\cancel{4}^{2}u_3) + 2(-u_2+u_3) = u_1$

$k_2 = -4u_3$ $\quad$ $k_3 = -u_2 + u_3$ $\quad$ $k_1 = u_1 + 2u_2 + 4u_3$

∴ Karena SPL bersolusi tunggal (konsisten), maka membangun P_2

c. Karena S bebas linear dan membangun P_2, maka S merupakan basis bagi Polinom orde 2 (P_2)

6. a. $\langle \vec{u}, \vec{v} \rangle = u_1v_1 + u_2v_2 + 2u_3v_3$

$\vec{u} = (1,1,-1)$

$\vec{v} = (2,2,1)$

$\langle (1,1,-1),(2,2,1) \rangle = 1.2 + 1.2 + 2.-1.1$

$= 2 + 2 - 2$

$= 2$

6. $S = \{ \vec{w_1} = (1,0,1), \vec{w_2} = (0,1,1) \}$

$$\vec{u_1} = \frac{\vec{w_1}}{\|\vec{w_1}\|} = \frac{(1,0,1)}{\sqrt{1^2+0^2+1^2}} = \frac{(1,0,1)}{\sqrt{2}} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\vec{P_1} = \vec{w_2} - \text{proy}_{\vec{u_1}} \vec{w_2} = \vec{w_2} - \langle \vec{w_2}, \vec{u_1} \rangle \vec{u_1}$$

$$\vec{P_1} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \left\langle \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \right\rangle \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\vec{P_1} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \left(0 . \frac{1}{\sqrt{2}} + 1 . 0 + 2 . 1 . \frac{1}{\sqrt{2}} \right) \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$\vec{P_1} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{2}{\sqrt{2}} \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\vec{u_2} = \frac{\vec{P_1}}{\|\vec{P_1}\|} = \frac{(-1,1,0)}{\sqrt{(-1)^2+1^2+0^2}} = \frac{(-1,1,0)}{\sqrt{2}}$$

$$= \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$u = \{ \vec{u_1}, \vec{u_2} \}$$

$$u = \left\{ \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \right\}$$ → Himpunan ortonormal dari S

7. $T : R^3 \to R^2$

$\vec{v_1} = (1, 0 -1)$ $\quad T(\vec{v_1}) = (0,3)$

$\vec{v_2} = (1,1,1)$ $\quad T(\vec{v_2}) = (3,1)$

$\vec{v_3} = (1,-1,0)$ $\quad T(\vec{v_3}) = (0,-1)$

a. $T(\vec{v_1}, \vec{v_2}, \vec{v_3}) = A(\vec{v_1}, \vec{v_2}, \vec{v_3})$

$$\begin{bmatrix} 0 & 3 & 0 \\ 3 & 1 & -1 \end{bmatrix} = A \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 0 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1/3 & 1/3 & -2/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & -2/3 & 1/3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}$$

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y + z \\ x + 2y - 2z \end{bmatrix}$$

b.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$a + 4c = 0$ $\qquad$ $b - 3c = 0$ $\qquad$ $c = p \rightarrow$ parameter

$a = -4c$ $\qquad$ $b = 3c$

$$\ker(T) = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix} p \right\} \qquad R(T) = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

Dimensi $\ker(T) = 1$ $\qquad$ Dimensi $R(T) = 2$