M. Hasyim Abdillah P. 1101191095 TT-43-11

2. $y[n] = x[n] + x[n-1] - x[n-2] - x[n-3]$

a. $y[n] = x[n] + x[n-1] - x[n-2] - x[n-3]$

$$Y(z) = X(z) + z^{-1}X(z) - z^{-2}X(z) - z^{-3}X(z)$$

$$\frac{Y(z)}{X(z)} = 1 + z^{-1} - z^{-2} - z^{-3}$$

$$H(z) = 1 + z^{-1} - z^{-2} - z^{-3}$$

$$= 1 + \frac{1}{z} - \frac{1}{z^2} - \frac{1}{z^3}$$

$$= \frac{z^3 + z^2 - z - 1}{z^3}$$

Zero : $z^3 + z^2 - z - 1 = 0$

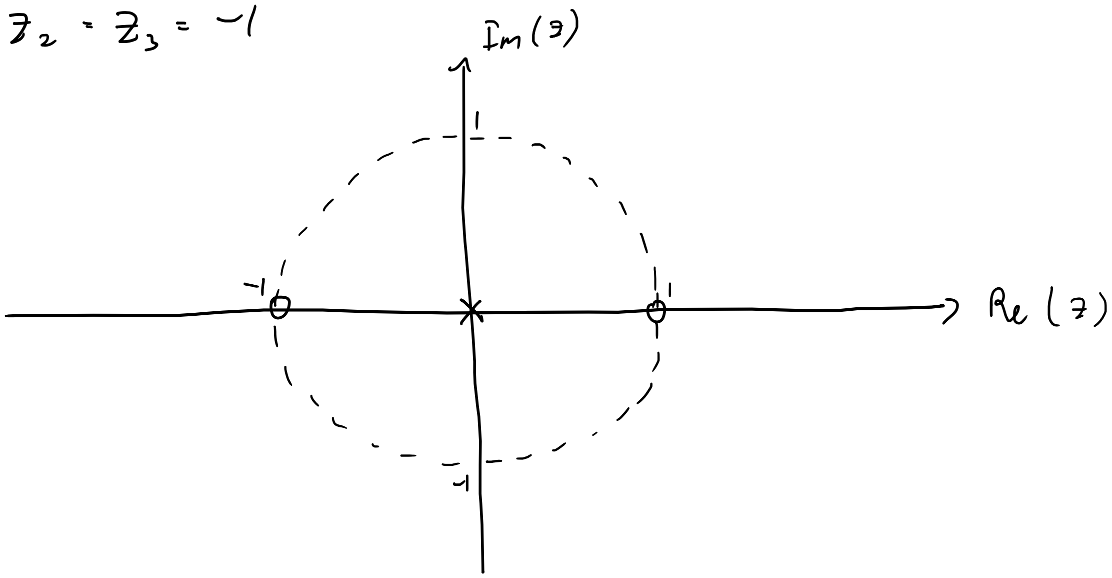
$(z+1)^2 \cdot (z-1) = 0$

Pole : $z^3 = 0$

$z = 0$

$z_1 = 1$

$z_2 = z_3 = -1$



b. $H(z) = 1 + z^{-1} - z^{-2} - z^{-3}$

$h[n] = \delta[n] + \delta[n-1] - \delta[n-2] - \delta[n-3]$

$H(e^{j\omega}) = 1 + e^{-j\omega.1} - e^{-j\omega.2} - e^{-j\omega.3}$

$= 1 + (\cos\omega - j\sin\omega) - (\cos 2\omega - j\sin 2\omega) - (\cos 3\omega - j\sin 3\omega)$

$H(e^{j\omega}) = 2\sin^2(\omega)(2\cos(\omega)+1) + j(-\sin\omega + \sin 2\omega + \sin 3\omega)$

c. $h[n] = \delta[n] + \delta[n-1] - \delta[n-2] - \delta[n-3]$

$= [1 \quad 1 \quad -1 \quad -1], \ 0 \leq n \leq 3$

$= [1 \quad 1 \quad -1 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0], \ 0 \leq n \leq 7$

$f[n] = h[2n] = [1 \quad -1 \quad 0 \quad 0]$

$g[n] = h[2n+1] = [1 \quad -1 \quad 0 \quad 0]$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

karenan $f[n] = g[n]$, maka $F_4 = G_4 = W_4 . g[n]$

$$F_4 = G_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1+j \\ 2 \\ 1-j \end{bmatrix}$$

$W_8^0 = 1 \qquad W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} \qquad W_8^2 = 0 \qquad W_8^3 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$H[0] = F[0] + W_8^0 G[0] = 0 + 1.0 = 0$

$H[1] = F[1] + W_8^1 G[1] = (1+j)\left(1 + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = 1 + \sqrt{2} + j$

$H[2] = F[2] + W_8^2 G[2] = 2 + 0.2 = 2$

$H[3] = F[3] + W_8^3 G[3] = (1-j)\left(1 - \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = 1 - \sqrt{2} - j$

$H[4] = F[0] - W_8^0 G[0] = 0 - 1.0 = 0$

$H[5] = F[1] - W_8^1 G[1] = (1+j)\left(1 - \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = 1 - \sqrt{2} + j$

$H[6] = F[2] - W_8^2 G[2] = 2 - 0.2 = 2$

$H[7] = F[3] - W_8^3 G[3] = (1-j)\left(1 + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = 1 + \sqrt{2} - j$

d. $|H[0]| = 0$

$|H[1]| = \sqrt{(1+\sqrt{2})^2 + (1)^2} = 2{,}61$

$|H[2]| = 2$

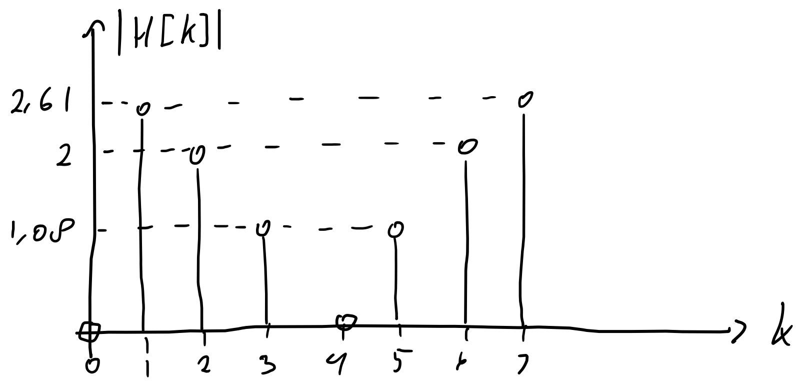
$|H[3]| = \sqrt{(1-\sqrt{2})^2 + 1^2} = 1{,}08$

$|H[4]| = 0$

$|H[5]| = \sqrt{(1-\sqrt{2})^2 + 1^2} = 1{,}08$

$|H[6]| = 2$

$|H[7]| = \sqrt{(1+\sqrt{2})^2 + 1^2} = 2{,}61$



e. $x[n] = \cos\left(\frac{\pi}{4} n\right) \qquad \omega = \frac{\pi}{4}$

$$H(e^{j\frac{\pi}{4}}) = 2\sin^2\left(\frac{\pi}{4}\right)\left(2\cos\frac{\pi}{4} + 1\right) + j\left(-\sin\frac{\pi}{4} + \sin\frac{2\pi}{4} + \sin\frac{3\pi}{4}\right)$$

$$= 1 + \sqrt{2} + j$$

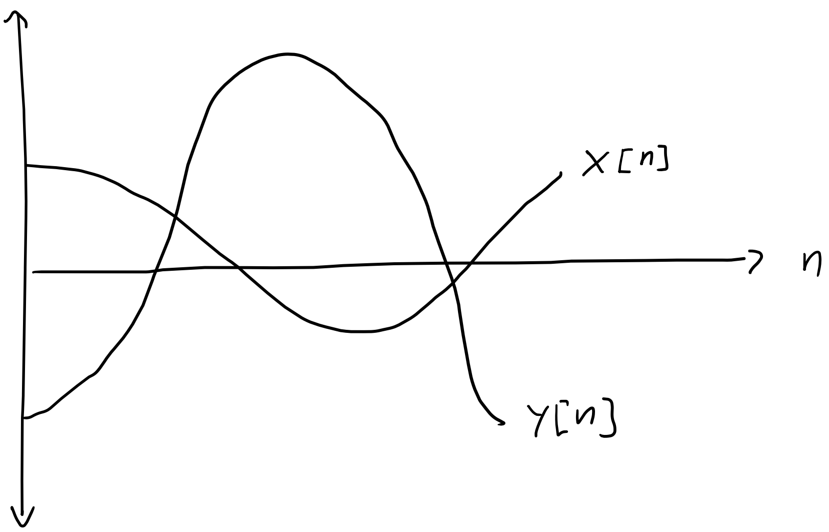
$$\left|H(e^{j\frac{\pi}{4}})\right| = 2{,}61$$

$$\arg\left\{H(e^{j\frac{\pi}{4}})\right\} = \tan^{-1}\left(\frac{1}{1+\sqrt{2}}\right) = 22{,}5^\circ$$

$$Y[n] = \left|H(e^{j\frac{\pi}{4}})\right| \cos\left(\frac{\pi}{4} n + \arg\right)$$

$$Y[n] = 2{,}61 \cos\left(\frac{\pi}{4} n + 22{,}5^\circ\right)$$

f.



g. Merupakan filter FIR Band-Stop Filter (BSF)