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1. b. $\langle \vec{u}, \vec{v} \rangle = 5u_1v_1 - u_1v_2 - u_2v_1 + 10u_2v_2$ di R^2

a) $\langle \vec{v}, \vec{u} \rangle = 5v_1u_1 - v_1u_2 - v_2u_1 + 10u_2v_2$

$\qquad = 5u_1v_1 - u_2v_1 - u_1v_2 + 10u_2v_2$

$\qquad = 5u_1v_1 - u_1v_2 - u_2v_1 + 10u_2v_2$

$\qquad = \langle \vec{u}, \vec{v} \rangle$

b) $\langle \vec{u} + \vec{v}, \vec{w} \rangle = \langle (u_1 + v_1, u_2 + v_2), (w_1, w_2) \rangle$

$\qquad = 5(u_1 + v_1)w_1 - (u_1 + v_1)w_2 - (u_2 + v_2)w_1 + 10(u_2 + v_2)w_2$

$\qquad = 5u_1w_1 + 5v_1w_1 - u_1w_2 - v_1w_2 - u_2w_1 - v_2w_1 + 10u_2w_2 + 10v_2w_2$

$\qquad = (5u_1w_1 - u_1w_2 - u_2w_1 + 10u_2w_2) + (5v_1w_1 - v_1w_2 - v_2w_1 + 10v_2w_2)$

$\qquad = \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$

c) $\langle k\vec{u}, \vec{v} \rangle = 5 \cdot ku_1v_1 - ku_1v_2 - ku_2v_1 + 10 \cdot ku_2v_2$

$\qquad = 5u_1 \cdot kv_1 - u_1 \cdot kv_2 - u_2 \cdot kv_1 + 10u_2 \cdot kv_2 = \langle \vec{u}, k\vec{v} \rangle$

$\qquad = k(5u_1v_1 - u_1v_2 - u_2v_1 + 10u_2v_2) = k\langle \vec{u}, \vec{v} \rangle$

d) $\langle \vec{u}, \vec{u} \rangle = 5u_1^2 - u_1u_2 - u_2u_1 + 10u_2^2$

$\qquad = 5u_1^2 - 2u_1u_2 + 10u_2^2$

Saat $2u_1u_2 > 5u_1^2 + 10u_2^2$ maka $\langle \vec{u}, \vec{u} \rangle < 0$

Tidak memenuhi positivitas

$\therefore \langle \vec{u}, \vec{v} \rangle = 5u_1v_1 - u_1v_2 - u_2v_1 + 10u_2v_2$ bukan merupakan RHD

1. c. $\langle \vec{u}, \vec{v} \rangle = u_1v_1^2 - u_2^2v_2$ di R^2

a) $\langle \vec{v}, \vec{u} \rangle = v_1u_1^2 - v_2^2u_2$

$\therefore$ Tidak simetris karena $\langle \vec{u}, \vec{v} \rangle \neq \langle \vec{v}, \vec{u} \rangle$

4. a. $\vec{a} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right)$ $\vec{b} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right)$ $\vec{c} = \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right)$

$S = \{ \vec{a}, \vec{b}, \vec{c} \}$

Syarat vektor orthogonal $\langle \vec{u}, \vec{v} \rangle = 0$

a) $\langle \vec{a}, \vec{b} \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$

$$0 = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{3}} + 0 \cdot \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}} \cdot \frac{-1}{\sqrt{3}}$$

$$0 = \frac{1}{\sqrt{6}} + 0 - \frac{1}{\sqrt{6}} \quad \checkmark$$

b) $\langle \vec{a}, \vec{c} \rangle = a_1 c_1 + a_2 c_2 + a_3 c_3$

$$0 = \frac{1}{\sqrt{2}} \cdot \frac{-1}{\sqrt{2}} + 0 \cdot 0 + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$$

$$0 = -\frac{1}{2} + 0 + \frac{1}{2} \quad \checkmark$$

c) $\langle \vec{b}, \vec{c} \rangle = b_1 c_1 + b_2 c_2 + b_3 c_3$

$$0 = \frac{1}{\sqrt{3}} \cdot \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \cdot 0 + \frac{-1}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}}$$

$$0 = \frac{-1}{\sqrt{6}} + 0 - \frac{1}{\sqrt{6}} \quad \times$$

$\therefore S = \{\vec{a}, \vec{b}, \vec{c}\}$ bukan himpunan vektor orthogonal

b. $S = \{\vec{a}, \vec{b}, \vec{c}\}$

$$\vec{a} = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right) \quad \vec{b} = \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right) \quad \vec{c} = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$$

Syarat orthogonal : $\langle \vec{u}, \vec{v} \rangle = 0$

a) $\langle \vec{a}, \vec{b} \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$

$$0 = \frac{2}{3} \cdot \frac{2}{3} + \frac{-2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{-2}{3}$$

$$0 = \frac{4}{9} - \frac{2}{9} - \frac{2}{9} \quad \checkmark$$

b) $\langle \vec{a}, \vec{c} \rangle = a_1 c_1 + a_2 c_2 + a_3 c_3$

$$0 = \frac{2}{3} \cdot \frac{1}{3} + \frac{-2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{3}$$

$$0 = \frac{2}{9} - \frac{4}{9} + \frac{2}{9} \quad \checkmark$$

d) $\langle \vec{b}, \vec{c} \rangle = b_1 c_1 + b_2 c_2 + b_3 c_3$

$$0 = \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} + \frac{-2}{3} \cdot \frac{2}{3}$$

$$0 = \frac{2}{9} + \frac{2}{9} - \frac{4}{9} \checkmark$$

$\therefore S = \{\vec{a}, \vec{b}, \vec{c}\}$ merupakan himpunan vektor orthogonal

c. $S = \{\vec{a}, \vec{b}, \vec{c}\}$

$\vec{a} = (1,0,0) \quad \vec{b} = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \quad \vec{c} = (0,0,1)$

Syarat vektor orthogonal $\langle \vec{u}, \vec{v} \rangle = 0$

$\Rightarrow \langle \vec{a}, \vec{b} \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$

$$0 = 1 \cdot 0 + 0 \cdot \frac{1}{\sqrt{2}} + 0 \cdot 1$$

$$0 = 0 + 0 + 0 \checkmark$$

$\Rightarrow \langle \vec{a}, \vec{c} \rangle = a_1 c_1 + a_2 c_2 + a_3 c_2$

$$0 = 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1$$

$$0 = 0 + 0 + 0 \checkmark$$

$\Rightarrow \langle \vec{b}, \vec{c} \rangle = b_1 c_1 + b_2 c_2 + b_3 c_3$

$$0 = 0 \cdot 0 + \frac{1}{\sqrt{2}} \cdot 0 + \frac{1}{\sqrt{2}} \cdot 1$$

$$0 = 0 + 0 + \frac{1}{\sqrt{2}} \textcolor{red}{\times}$$

$\therefore S = \{\vec{a}, \vec{b}, \vec{c}\}$ bukan merupakan himpunan vektor orthogonal

5. b. $\quad \vec{u} = (k, -6, -7) \quad \vec{v} = (k, -k, 1)$

Syarat agar vektor saling orthogonal $(\vec{u} \perp \vec{v})$:

$\langle \vec{u}, \vec{v} \rangle = 0$

$u_1 v_1 + u_2 v_2 + u_3 v_3 = 0$

$k \cdot k + (-6) \cdot (-k) + (-7) \cdot 1 = 0$

$k^2 + 6k - 7 = 0$

$k^2 + 6k - 7 = 0$

$(k - 1)(k + 7) = 0$

$k = 1 \quad \vee \quad k = -7$

6. $\vec{u_1} = (0, 1, 2)$ $\vec{u_2} = (-1, 0, 1)$ $\vec{u_3} = (-1, 1, 4)$

$U = \{\vec{u_1}, \vec{u_2}, \vec{u_3}\}$ basis bagi R^3

$$\vec{v_1} = \frac{\vec{u_1}}{\|\vec{u_1}\|} = \frac{(0,1,2)}{\sqrt{0^2 + 1^2 + 2^2}} = \frac{(0,1,2)}{\sqrt{0+1+4}} = \left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$\vec{u_2} - proy_{\vec{v_1}} \vec{u_2} = \vec{u_2} - \langle \vec{u_2}, \vec{v_1} \rangle \vec{v_1}$$

$$= (-1,0,1) - \left(-1 \cdot 0 + 1 \cdot \frac{1}{\sqrt{5}} + 1 \cdot \frac{2}{\sqrt{5}}\right) \cdot \left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$= (-1,0,1) - \frac{3}{\sqrt{5}} \left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$= (-1,0,1) - \left(0, \frac{3}{5}, \frac{6}{5}\right)$$

$$= \left(-1, -\frac{3}{5}, -\frac{1}{5}\right)$$

$$\vec{v_2} = \frac{\vec{u_2} - proy_{\vec{v_1}} \vec{u_2}}{\|\vec{u_2} - proy_{\vec{v_1}} \vec{u_2}\|} = \frac{\left(-1, -\frac{3}{5}, -\frac{1}{5}\right)}{\sqrt{(-1)^2 + \left(-\frac{3}{5}\right)^2 + \left(-\frac{1}{5}\right)^2}}$$

$$= \frac{\left(-1, -\frac{3}{5}, -\frac{1}{5}\right)}{\sqrt{1 + \frac{9}{25} + \frac{1}{25}}} = \frac{\left(-1, -\frac{3}{5}, -\frac{1}{5}\right)}{\sqrt{\frac{35}{25}}}$$

$$= \frac{5\left(-1, -\frac{3}{5}, -\frac{1}{5}\right)}{\sqrt{35}}$$

$$= \left(-\frac{5}{\sqrt{35}}, -\frac{3}{\sqrt{35}}, -\frac{1}{\sqrt{35}}\right)$$

$$\vec{u_3} - proy_W \vec{u_3} = \vec{u_3} - \langle \vec{u_3}, \vec{v_1} \rangle v_1 - \langle \vec{u_3}, \vec{v_2} \rangle \vec{v_2}$$

$$= (-1,1,4) - \left(-1 \cdot 0 + 1 \cdot \frac{1}{\sqrt{5}} + 4 \cdot \frac{2}{\sqrt{5}}\right)\left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) - \left(-1 \cdot \frac{-5}{\sqrt{35}} + 1 \cdot \frac{-3}{\sqrt{35}} + \right.$$

$$\left. 4 \cdot \frac{-1}{\sqrt{35}}\right)\left(-\frac{5}{\sqrt{35}}, \frac{-3}{\sqrt{35}}, \frac{-1}{\sqrt{35}}\right)$$

$$= (-1,1,4) - \frac{9}{\sqrt{5}}\left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) - \frac{-12}{\sqrt{35}}\left(-\frac{5}{\sqrt{35}}, -\frac{3}{\sqrt{35}}, \frac{-1}{\sqrt{35}}\right)$$

$$= (-1,1,4) - \left(0, \frac{9}{5}, \frac{18}{5}\right) - \left(\frac{60}{35}, \frac{36}{35}, \frac{12}{35}\right)$$

$$= \left(-\frac{95}{35}, -\frac{64}{35}, \frac{2}{35}\right)$$

$$\left\| \vec{u}_3 - proy_w \vec{u}_3 \right\| = \sqrt{\left(\frac{-95}{35}\right)^2 + \left(\frac{-64}{35}\right)^2 + \left(\frac{2}{35}\right)^2}$$

$$= \frac{5\sqrt{3}}{\sqrt{7}} = \frac{5\sqrt{21}}{7}$$

$$\vec{v}_3 = \frac{\vec{u}_3 - proy_w \vec{u}_2}{\left\| \vec{u}_3 - proy_w \vec{u}_3 \right\|} = \frac{\left(\frac{-95}{35}, \frac{-64}{35}, \frac{2}{35}\right)}{\frac{5\sqrt{21}}{7}}$$

$$= \left(-\frac{95}{25\sqrt{21}}, -\frac{64}{25\sqrt{21}}, \frac{2}{25\sqrt{21}}\right)$$

$$= \left(-\frac{19}{5\sqrt{21}}, -\frac{64}{25\sqrt{21}}, \frac{2}{25\sqrt{21}}\right)$$

V merupakan basis orthonormal dari u dengan:

$$V = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$$

$$V = \left\{ \left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right), \left(-\frac{5}{\sqrt{35}}, -\frac{3}{\sqrt{35}}, -\frac{1}{\sqrt{35}}\right), \left(-\frac{19}{5\sqrt{21}}, -\frac{64}{25\sqrt{21}}, \frac{2}{25\sqrt{21}}\right) \right\}$$