Random process …

- Strictly stationary: If none of the statistics of the random process are affected by a shift in the time origin.
- Wide sense stationary (WSS): If the mean and autocorrelation function do not change with a shift in the origin time.
- Cyclostationary: If the mean and autocorrelation function are periodic in time.
- Ergodic process: A random process is ergodic in mean and autocorrelation, if

$$m_X = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t)dt$$

and

$$R_X(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t)X^*(t-\tau)dt$$

, respectively.

Autocorrelation

- Autocorrelation of an energy signal

$$R_x(\tau) = x(\tau) \star x^*(-\tau) = \int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt$$

- Autocorrelation of a power signal

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t-\tau)dt$$

  - For a periodic signal:

$$R_x(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)x^*(t-\tau)dt$$

- Autocorrelation of a random signal

$$R_X(t_i, t_j) = \mathsf{E}[X(t_i)X^*(t_j)]$$

  - For a WSS process:

$$R_X(\tau) = \mathsf{E}[X(t)X^*(t-\tau)]$$

Spectral density

- Energy signals:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \qquad X(f) = \mathcal{F}[x(t)]$$

  - Energy spectral density (ESD):

$$\boxed{\Psi_x(f) = |X(f)|^2}$$

- Power signals:

$$P_x = \frac{1}{T_0} \int_{T_0/2}^{T_0/2} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2 \qquad \{c_n\} = \mathcal{F}[x(t)]$$

  - Power spectral density (PSD):

$$\boxed{G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0) \qquad f_0 = 1/T_0}$$

- Random process:
  - Power spectral density (PSD):

$$\boxed{G_X(f) = \mathcal{F}[R_X(\tau)]}$$

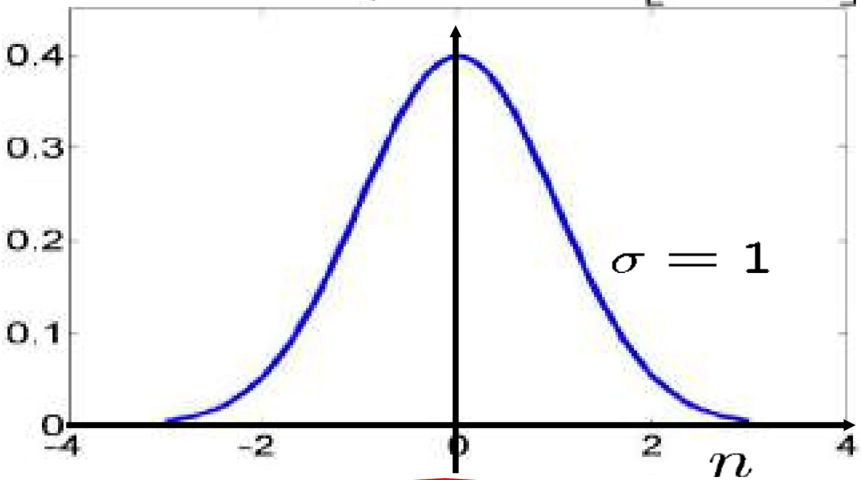
Properties of an autocorrelation function

- For real-valued (and WSS in case of random signals):
  1. Autocorrelation and spectral density form a Fourier transform pair.
  2. Autocorrelation is symmetric around zero.
  3. Its maximum value occurs at the origin.
  4. Its value at the origin is equal to the average power or energy.

Noise in communication systems

- Thermal noise is described by a zero-mean Gaussian random process, n(t).
- Its PSD is flat, hence, it is called white noise.

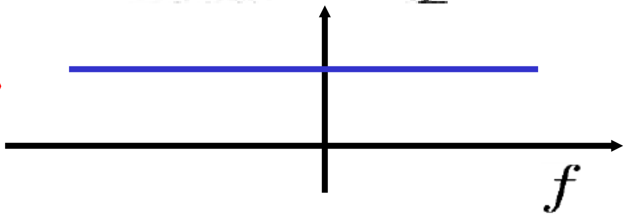
$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{n^2}{2\sigma^2}\right]$$



Probability density function

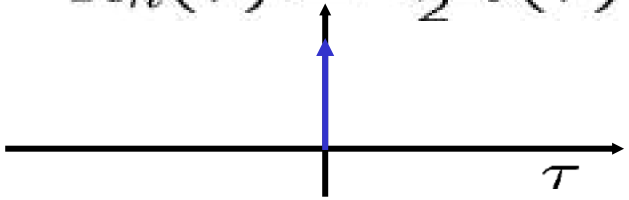
$$G_n(f) = \frac{N_0}{2} \text{ [w/Hz]}$$

Power spectral density



$$R_n(\tau) = \frac{N_0}{2}\delta(\tau)$$

Autocorrelation function



Signal transmission through linear systems

Input $\begin{matrix} x(t) \\ X(f) \end{matrix} \longrightarrow \boxed{\begin{matrix} h(t) \\ H(f) \end{matrix}} \longrightarrow \begin{matrix} y(t) \\ Y(f) \end{matrix}$ Output

Linear system

- Deterministic signals: $\boxed{Y(f) = X(f)H(f)}$
- Random signals: $\boxed{G_Y(f) = G_X(f)|H(f)|^2}$

- Ideal distortionless transmission:

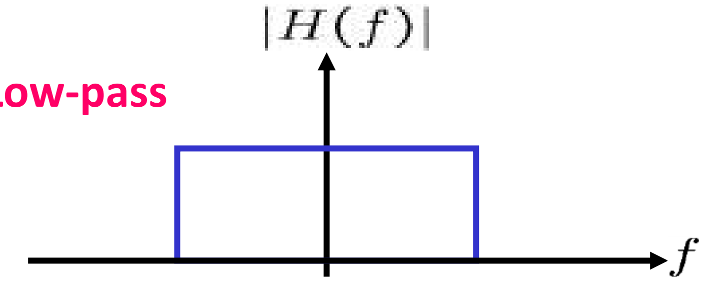
  All the frequency components of the signal not only arrive with an identical time delay, but also are amplified or attenuated equally.

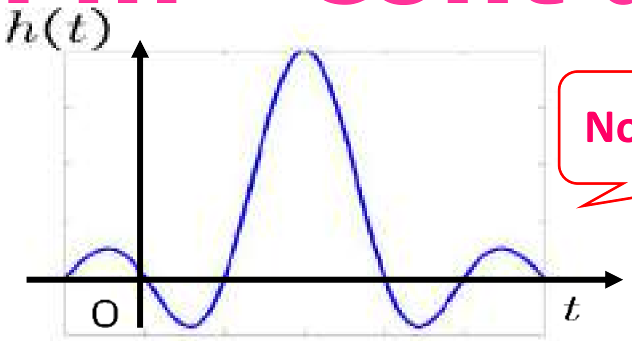
$$\boxed{y(t) = Kx(t - t_0) \text{ or } H(f) = Ke^{-j2\pi f t_0}}$$

Signal transmission … - cont'd

- Ideal filters:

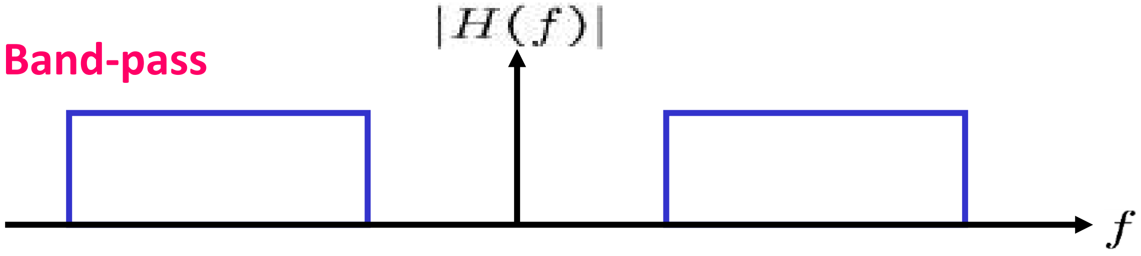
Low-pass



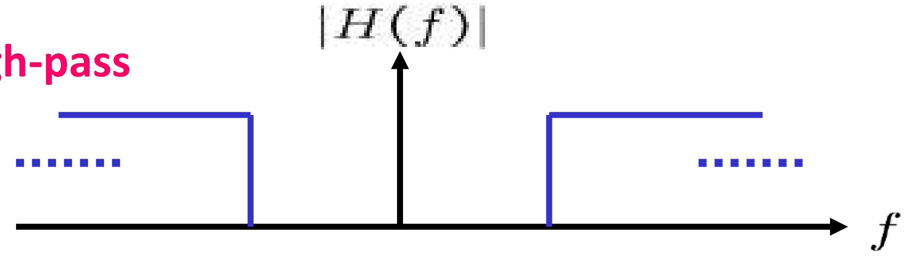


Non-causal!

Band-pass



High-pass



- Realizable filters:

RC filters

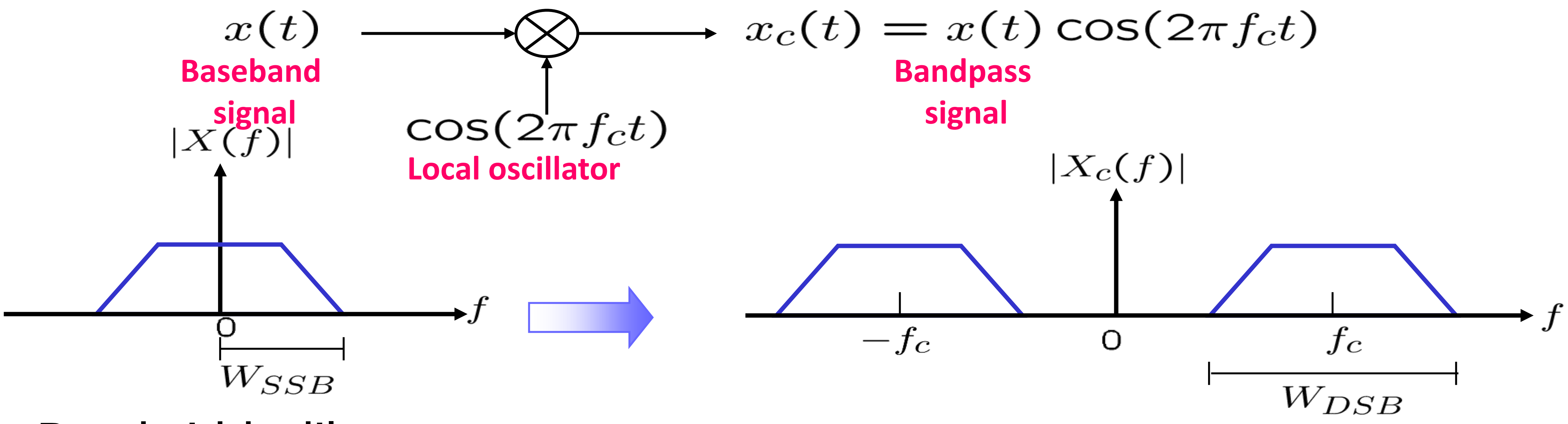
$$H(f) = \frac{1}{1 + j2\pi fRC}$$

Butterworth filter

$$|H_n(f)| = \frac{1}{\sqrt{1 + (f/f_u)^{2n}}}$$

Bandwidth of signal

- Baseband versus bandpass:

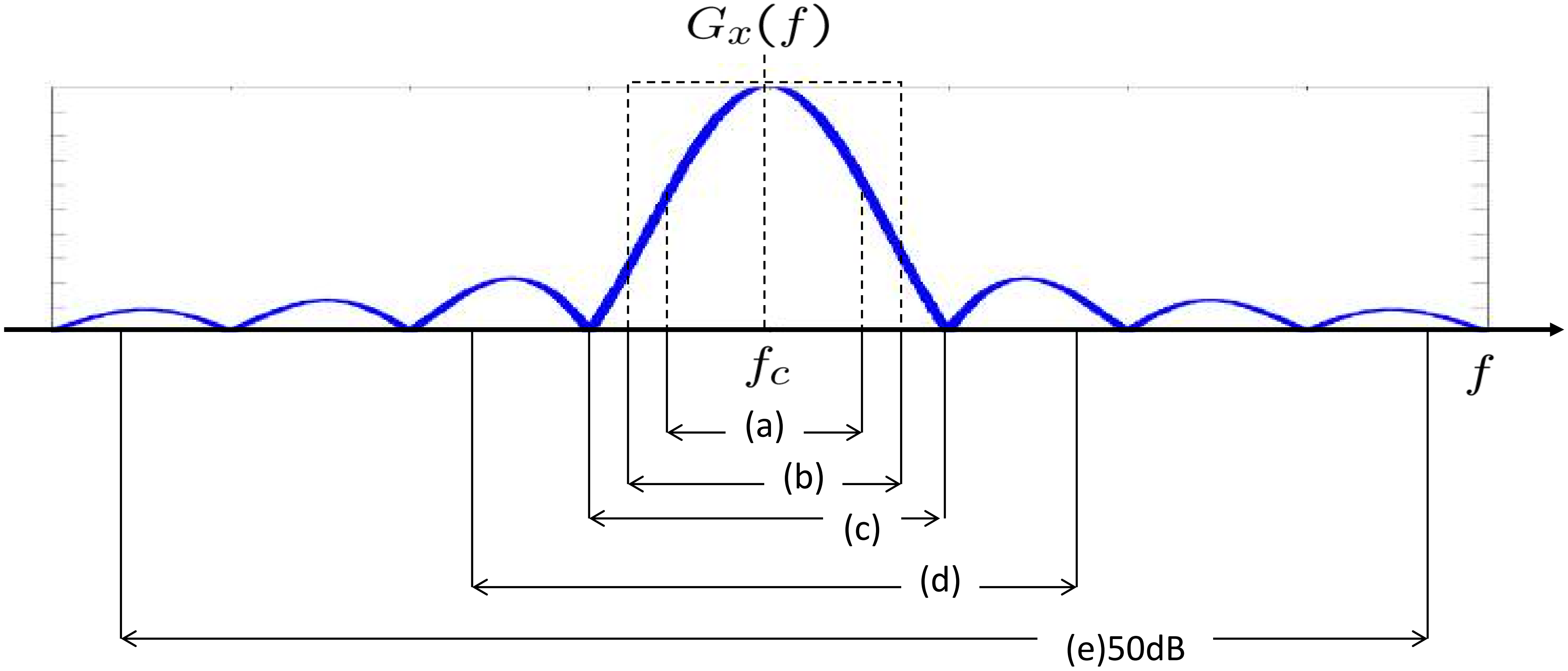


- Bandwidth dilemma:
  - Bandlimited signals are not realizable!
  - Realizable signals have infinite bandwidth!

Bandwidth of signal – cont'd

- Different definition of bandwidth:

a) Half-power bandwidth
b) Noise equivalent bandwidth
c) Null-to-null bandwidth
d) Fractional power containment bandwidth
e) Bounded power spectral density
f) Absolute bandwidth



End of Module 10