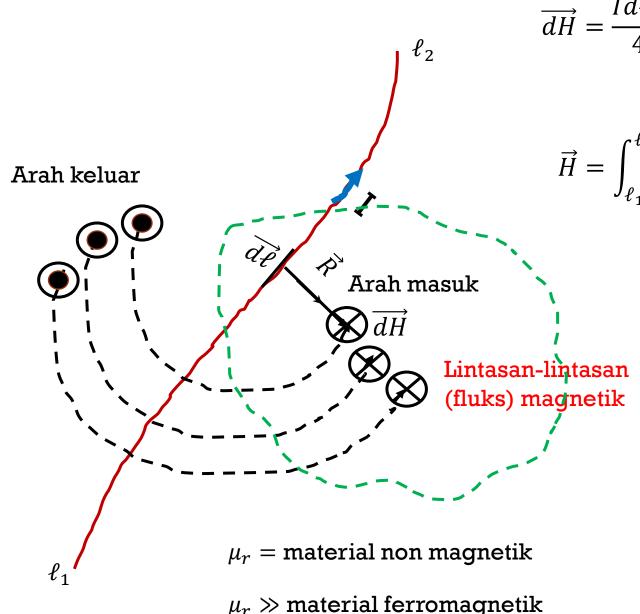


DIFERENSIAL VEKTOR



Tentang Fluks Magnetik



$$\overrightarrow{dH} = \frac{I\overrightarrow{d\ell} \times \overrightarrow{a}_R}{4\pi R^2}$$

 \overrightarrow{dH} adalah medan magnet yang dirasakan di titik observator akibat adanya arus I yang melewati potongan panjang $\overrightarrow{d\ell}$

$$\vec{H} = \int_{\ell_1}^{\ell_2} \vec{dH} = \int_{\ell_1}^{\ell_2} \frac{I \vec{d\ell} \times \vec{a}_R}{4\pi R^2}$$

$$\vec{B} = \mu \vec{H}$$

 $\mu = \text{permeabilitas medium (material)}$

$$\mu = \mu_r \mu_0$$
; $\mu_0 = \text{permeabilitas ruang}$
hampa
= $4\pi \times 10^{-7}$

 μ_r = permeabilitas relatif medium terhadap permeabilitas ruang hampa



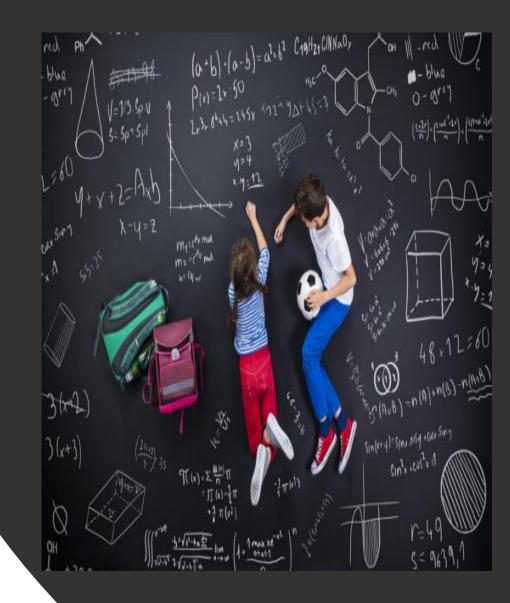
DIFERENSIAL VEKTOR

- Merupakan operasi matematika berupa diferensiasi terhadap vektor.
- Tools ini digunakan untuk mencari sumber sumber terkecil dari medan listrik dan medan magnet.
- Terdapat 3 tools (operasi diferensial vector) :
 - Gradien
 - Divergensi
 - Curl



GRADIEN

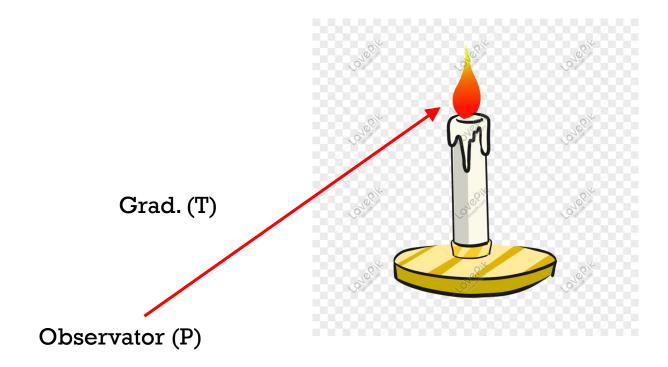
- Sebuah tools matematik yaitu diferensiasi vektor untuk menentukan posisi maksimum suatu besaran listrik skalar/suatu besaran fisik skalar tertentu dan magnitudo maksimum besaran tersebut;
- Hasil akhir operasi gradien adalah besaran vektor;
- Besaran vektor memiliki arah dan magnitudo;
- Arah menunjukan ke posisi maksimum;
- Magnitudo menunjukan magnitudo maksimum suatu besaran skalar yang diuji



CONTOH GRADIEN

Gradien Suhu

Gradien Suhu : ∇T (symbol)



∇V (simbol untuk Gradien Potensial Listrik)



DEL FACTOR V

Merupakan simbol diferensial vector dan bergantung pada sistem koordinat

$$\nabla = \frac{\partial()}{\partial x} \vec{a}_x + \frac{\partial()}{\partial y} \vec{a}_y + \frac{\partial()}{\partial z} \vec{a}_z$$
 (kartesian)

$$\nabla = \frac{\partial()}{\partial\rho} \vec{a}_{\rho} + \frac{1}{\rho} \frac{\partial()}{\partial\phi} \vec{a}_{\phi} + \frac{\partial()}{\partial z} \vec{a}_{z}$$
 (silinder)

$$\nabla = \frac{\partial()}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial()}{\partial \theta}\vec{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial()}{\partial \theta}\vec{a}_\theta$$
 (bola)



CONTOH GRADIEN GRADIEN POTENSIAL LISTRIK

$$\nabla V = \frac{\partial V}{\partial x} \vec{a}_x + \frac{\partial V}{\partial y} \vec{a}_y + \frac{\partial V}{\partial z} \vec{a}_z$$

(sistem koordinat kartesian)

$$\nabla V = \frac{\partial V}{\partial \rho} \vec{a}_{\rho} + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \vec{a}_{\phi} + \frac{\partial V}{\partial z} \vec{a}_{z}$$

(sistem koordinat silinder)

$$\nabla V = \frac{\partial V}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\vec{a}_\phi \qquad \text{(sistem koordinat bola)}$$



Contoh:

Diketahui : potensial listrik memenuhi persamaan : $V = 6x^2y^3 + e^z$

Tentukan: Gradien potensial listrik di titik P(2,1,0)

Solusi:

$$\nabla V = \frac{\partial (6x^2y^3 + e^z)}{\partial x} \vec{a}_x + \frac{\partial (6x^2y^3 + e^z)}{\partial y} \vec{a}_y + \frac{\partial (6x^2y^3 + e^z)}{\partial z} \vec{a}_z$$
$$= 12xy^3 \vec{a}_x + 18x^2y^2 \vec{a}_y + e^z \vec{a}_z$$

Gradien *V* di titik P(2,1,0) =
$$24\vec{a}_x + 72\vec{a}_y + \vec{a}_z$$

$$|\nabla V| = \sqrt{24^2 + 72^2 + 1^2} = 75,9$$
 (volt/meter)

$$\vec{a}_{\nabla V} = \frac{\nabla V}{|\nabla V|} = \frac{24\vec{a}_x + 72\vec{a}_y + \vec{a}_z}{75,9} = 0,316\vec{a}_x + 0,95\vec{a}_y + 0,013\vec{a}_z$$



Contoh:

Potensial listrik memenuhi persamaan : $V = 2\rho z^2 \sin \phi$ (volt)

Tentukan : Gradien potensial listrik di titik $P(\rho = 1m, \phi = 45^{\circ}, z = 2m)$

Solusi:

$$\nabla V = \frac{\partial V}{\partial \rho} \vec{a}_{\rho} + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \vec{a}_{\phi} + \frac{\partial V}{\partial z} \vec{a}_{z}$$

$$\nabla V = \frac{\partial (2\rho z^2 \sin \phi)}{\partial \rho} \vec{a}_{\rho} + \frac{1}{\rho} \frac{\partial (2\rho z^2 \sin \phi)}{\partial \phi} \vec{a}_{\phi} + \frac{\partial (2\rho z^2 \sin \phi)}{\partial z} \vec{a}_{z}$$

DIVERGENSI

Divergensi Kerapatan Fluks Listrik:

$$\nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

(Sistem Koordinat

Kartesian)

$$\nabla \cdot \overrightarrow{D} = \frac{1}{\rho} \frac{\partial (\rho D_{\rho})}{\partial \rho} + \frac{1}{\rho} \frac{\partial D_{\phi}}{\partial \phi} + \frac{\partial D_{z}}{\partial z}$$

(Sistem Koordinat

Silinder)

$$\nabla \cdot \vec{D} = \frac{1}{r^2} \frac{\partial (r^2 D_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (\sin \theta D_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$$

(Sistem Koordinat Bola)

CURL

Curl dari Medan Magnet

$$\nabla \times \vec{H} = \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] \vec{a}_x + \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right] \vec{a}_y + \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right] \vec{a}_z \qquad \text{Kartesian}$$

$$\nabla \times \vec{H} = \left[\frac{1}{\rho} \frac{\partial H_Z}{\partial \phi} - \frac{\partial H_Y}{\partial z}\right] \vec{a}_\rho + \left[\frac{\partial H_\rho}{\partial z} - \frac{\partial H_Z}{\partial \rho}\right] \vec{a}_\phi + \left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho H_\phi\right) - \frac{1}{\rho} \frac{\partial H_\rho}{\partial \phi}\right] \vec{a}_Z \qquad \text{Silinder}$$

$$\nabla \times \vec{H} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} \left(\sin \theta \, H_{\phi} \right) - \frac{\partial H_{\theta}}{\partial \phi} \right] \vec{a}_r + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial H_r}{\partial \phi} - \frac{\partial}{\partial r} \left(r H_{\phi} \right) \right] \vec{a}_{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} \left(r H_{\theta} \right) - \frac{\partial H_r}{\partial \theta} \right] \vec{a}_{\phi}$$



