M. Hasyim Abdillah P. (1101191095) TT-43-11

1\. a. $\bar{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad \bar{v} = \begin{bmatrix} 6 \\ -8 \end{bmatrix}$

$|\bar{u}| = \sqrt{1^2 + 2^2}$ $\quad |\bar{v}| = \sqrt{6^2 + (-8)^2}$

$= \sqrt{1+4}$ $\quad |\bar{v}| = \sqrt{36+64}$

$= \sqrt{5}$ $\quad |\bar{v}| = \sqrt{100} = 10$

$\bar{u} \cdot \bar{v} = |\bar{u}||\bar{v}| \cos\theta$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ -8 \end{bmatrix} = \sqrt{5} \cdot 10 \cos\theta$

$6 - 16 = 10\sqrt{5} \cos\theta$

$-10 = 10\sqrt{5} \cos\theta$

$\cos\theta = -\frac{1}{\sqrt{5}} = -\frac{1}{5}\sqrt{5}$ //

b. $\bar{u} = \begin{bmatrix} 1 \\ -3 \\ 7 \end{bmatrix}$ $\quad \bar{v} = \begin{bmatrix} 8 \\ -2 \\ -2 \end{bmatrix}$

$|\bar{u}| = \sqrt{1^2 + (-3)^2 + 7^2}$ $\quad |\bar{v}| = \sqrt{8^2 + (-2)^2 + (-2)^2}$

$|\bar{u}| = \sqrt{1+9+49}$ $\quad |\bar{v}| = \sqrt{64+4+4}$

$|\bar{u}| = \sqrt{59}$ $\quad |\bar{v}| = \sqrt{72} = 6\sqrt{2}$

$\bar{u} \cdot \bar{v} = |\bar{u}| \cdot |\bar{v}| \cos\theta$

$\begin{bmatrix} 1 \\ -3 \\ 7 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ -2 \\ -2 \end{bmatrix} = \sqrt{59} \cdot 6\sqrt{2} \cos\theta$

$8 + 6 - 14 = 6\sqrt{118} \cos\theta$

$0 = 6\sqrt{118} \cos\theta$

$\cos\theta = 0$

$\theta = 90^\circ$ //

2\. a. $\bar{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\bar{b} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$$\text{Proy}_{\bar{b}}\, \bar{a} = \frac{\bar{a} \cdot \bar{b}}{|b|^2}\, \bar{b}$$

$$= \frac{-6+3}{(-3)^2 + 2^2} \cdot \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$$= \frac{-3}{13} \cdot \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$$\left|\text{Proy}_{\bar{b}}\, \bar{a}\right| = \left|\frac{-3}{13}\right| \sqrt{(-3)^2 + 2^2}$$

$$= \frac{3}{13} \sqrt{9+4}$$

$$= \frac{3}{13} \sqrt{13}$$

b. $\bar{a} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ $\bar{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$$\text{Proy}_{\bar{b}}\, \bar{a} = \frac{\bar{a} \cdot \bar{b}}{|b|^2}\, \bar{b}$$

$$= \frac{2-2+6}{1^2 + 2^2 + 2^2} \cdot \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= \frac{6}{9} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$= \frac{2}{3} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$$\left|\text{Proy}_{\bar{b}}\, \bar{a}\right| = \left|\frac{2}{3}\right| \sqrt{1^2 + 2^2 + 2^2}$$

$$= \frac{2}{3} \sqrt{1+4+4}$$

$$= \frac{2}{3} \sqrt{9}$$

$$= 2$$

3\. $\bar{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $\bar{v} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\bar{u} \perp \bar{v} \rightarrow \bar{u} \cdot \bar{v} = 0$$

$$3 \cdot x + (-2) \cdot y = 0$$

$$3x = 2y$$

$$y = \frac{3}{2} x$$

$$|\bar{v}| = 1$$

$$\sqrt{x^2 + y^2} = 1$$

$$\sqrt{x^2 + \left(\frac{3}{2} x\right)^2} = 1$$

$$\frac{13}{4} x^2 = 1$$

$$x^2 = \frac{4}{13}$$

$$x = \pm \frac{2}{\sqrt{13}}$$

$$\bar{v}_1 = \begin{bmatrix} 2/\sqrt{13} \\ 3/\sqrt{13} \end{bmatrix}$$

$$\bar{v}_2 = \begin{bmatrix} -2/\sqrt{13} \\ -3/\sqrt{13} \end{bmatrix}$$

4. $\bar{u} = \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix}$ $\quad \bar{v} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$ $\quad \bar{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\bar{u} \perp \bar{w} \rightarrow \bar{u} \cdot \bar{w} = 0$

$-7.x + 3.y + 1.z = 0$

$-7x + 3y + z = 0 \text{ --- } (1)$

$\bar{v} \perp \bar{w} \rightarrow \bar{v} \cdot \bar{w} = 0$

$2.x + 0.y + 4.z = 0$

$2x + 4z = 0$

$x = -2z \text{ --- } (2)$

$-7x + 3y + z = 0$

$-7(-2z) + 3y + z = 0$

$14z + 3y + z = 0$

$15z + 3y = 0$

$y = -5z \text{ --- } (3)$

$\bar{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2z \\ -5z \\ z \end{bmatrix}$

$\bar{w}_1 = \begin{bmatrix} -2 \\ -5 \\ 1 \end{bmatrix}$ //

5. $P(2, 0, -3),\ Q(1, 4, 5),\ R(7, 2, 9)$

$\bar{PQ} = Q - P = (1-2)\hat{i} + (4-0)\hat{j} + (5-(-3))\hat{k} = -\hat{i} + 4\hat{j} + 8\hat{k}$

$\bar{PR} = R - P = (7-2)\hat{i} + (2-0)\hat{j} + (9-(-3))\hat{k} = 5\hat{i} + 2\hat{j} + 12\hat{k}$

$\bar{PQ} \times \bar{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 8 \\ 5 & 2 & 12 \end{vmatrix} = 32\hat{i} - 52\hat{j} - 22\hat{k}$

$L_{\Delta} = \frac{1}{2} |\bar{PQ} \times \bar{PR}| = \frac{1}{2}\sqrt{32^2 + (-52)^2 + (-22)^2}$

$= \frac{1}{2}\sqrt{1024 + 2704 + 484}$

$= \frac{1}{2}\sqrt{4212} = \frac{1}{2}.18\sqrt{13} = 9\sqrt{13}$ //