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1101191095 / TT-43-11

1.) $(x+y)\,dx - x\,dy = 0 \quad ; \; y = ux$

$(x + ux)dx - x(u\,dx + x\,du) = 0$

$x(1+u)dx - x(u\,dx + x\,du) = 0$

$(1+u)\,dx - u\,dx - x\,du = 0$

$(1+u-u)\,dx = x\,du$

$dx = x\,du$

$\frac{1}{x}\,dx = du$

$\int \frac{1}{x}\,dx = \int du$

$\ln|x| + C = u + C$

$\ln|x| = \frac{y}{x} + C$

$y = x(\ln|x| + C)$

2.) $(x^2 + 3y^2)dx - 2xy\,dy = 0 \quad ; \; y = ux$

$(x^2 + 3(ux)^2)dx - 2x(ux)(u\,dx + x\,du) = 0$

$x^2(1+3u^2)dx - 2ux^2(u\,dx + x\,du) = 0$

$(1+3u^2)dx - 2u^2dx - 2ux\,du = 0$

$(1+u^2)dx = 2ux\,du$

$\frac{1}{x}dx = \frac{2u}{u^2+1}\,du$

$$\int \frac{1}{x}dx = \int \frac{2u}{u^2+1}\,du$$

$$\ln|x| + c = \int \frac{\cancel{2u}}{t}\frac{dt}{\cancel{2u}}$$

$$\ln|x| + c = \ln|t| + c$$

$$\ln|x| + c = \ln|u^2+1| + c$$

$$\ln\left|\left(\frac{y}{x}\right)^2 + 1\right| = \ln|x| + c$$

$$\frac{y^2}{x^2} + 1 = x + e^c$$

$$y^2 + x^2 = x^3 + Cx^2$$

$$y^2 = x^3 + x^2(c-1)$$

$$y = x\sqrt{x - 1 + C}$$

7.) $(x^2+y^2)dx - 2xy\,dy = 0$; $y(1) = 1$; $y = ux$

$$(x^2 + (ux)^2)dx - 2x(ux)(u\,dx + x\,d(u)) = 0$$

$$x^2(1+u^2)dx - 2ux^2(u\,dx + x\,du) = 0$$

$$(1+u^2)dx - 2u^2dx - 2ux\,du = 0$$

$$(1-u^2)dx = 2ux\,du$$

$$\frac{1}{x}dx = \frac{u}{1-u^2}du$$

$$\int \frac{1}{x}dx = \int \frac{u}{1-u^2}du$$

$$\ln|x| + c = \int \frac{\cancel{u}}{t}\frac{dt}{2u}$$

$$\ln|x| + c = \frac{1}{2}\ln|t| + c$$

$$2\ln|x| + c = \ln|1-u^2| + c$$

$$\ln|1-u^2| = 2\ln|x| + C$$

$$1-\left(\frac{y}{x}\right)^2 = x^2 + e^c$$

$$1-\frac{y^2}{x^2} = x^2 + C$$

$$x^2 - y^2 = x^4 + Cx^2$$

$$y^2 = x^2(1-C) - x^4$$

$$y^2 = x^2(1-x^2-C)$$

$$y = x\sqrt{1-x^2-C}$$

$$y(1) = 1$$

$$1\sqrt{1-1-C} = 1$$

$$\sqrt{-C} = 1$$

$$-C = 1$$

$$C = -1$$

$\therefore$ Penyelesaian khusus : $y = x\sqrt{1-x^2-(-1)}$

$$y = x\sqrt{2-x^2}$$

5.) $(2x+y)dx + (x-y)dy = 0$; $y(2) = 2$; $y = ux$

$$(2x+(ux))dx + (x-(ux))(udx + xdu) = 0$$

$$x(2+u)dx + x(1-u)(udx + xdu) = 0$$

$$(2+u)dx + (u-u^2)dx + x(1-u)du = 0$$

$$(2+2u-u^2)dx = -x(1-u)du$$

$$\frac{1}{x}dx = -\frac{1-u}{2+2u-u^2}du$$

$$\int \frac{1}{x}dx = -\int \frac{1-u}{2+2u-u^2}du$$

$$\ln|x| + c = -\int \frac{1-u}{t} \cdot \frac{dt}{2-2u}$$

$$\ln|x| + c = -\int \frac{1-u}{t} \cdot \frac{dt}{2(1-u)}$$

$$\ln|x| + c = -\frac{1}{2} \ln|t| + c$$

$$2\ln|x| + c = -\ln|2+2u-u^2|$$

$$\ln|2+2u-u^2| = -\ln|x^2| + c$$

$$2+2u-u^2 = -x^2 + e^c$$

$$2 + 2\frac{y}{x} - \left(\frac{y}{x}\right)^2 = -x^2 + C$$

$$2x^2 + 2xy - y^2 = -x^4 + Cx^2$$

$$y^2 - 2xy - x^4 + x^2(c-2) = 0$$

$$y(2) = 2$$

$$2^2 - 2.2.2 - 2^4 + 2^2(c-2) = 0$$

$$4 - 8 - 16 + 4(c-2) = 0$$

$$4(c-2) = 20$$

$$c - 2 = 5$$

$$c = 7$$

$\therefore$ Penyelesaian khusus : $y^2 - 2xy - x^4 + x^2(7-2) = 0$

$$y^2 - 2xy - x^4 + 5x^2 = 0$$