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1.) a.

$$K = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

$$|K| = \begin{vmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{vmatrix} \begin{matrix} -6b_1 + b_2 \\ 3b_1 + b_3 \\ \sim \end{matrix} = \begin{vmatrix} 1 & -2 & 3 \\ 0 & 19 & -19 \\ 0 & -5 & 13 \end{vmatrix} \begin{matrix} \frac{1}{19} b_2 \\ \sim \end{matrix}$$

$$\frac{1}{19} |K| = \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 0 & -5 & 13 \end{vmatrix} \begin{matrix} 5b_2 + b_3 \\ \sim \end{matrix}$$

$$|K| = 19 \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 8 \end{vmatrix}$$

$$|K| = 19(1)(1)(8) = 152$$

1.) b.

$$L = \begin{bmatrix} a-3 & 5 \\ -3 & a-2 \end{bmatrix}$$

$$|L| = \begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix} \begin{matrix} \frac{1}{a-3} b_1 \\ \sim \end{matrix} \quad \frac{1}{a-3} |L| = \begin{vmatrix} 1 & \frac{5}{a-3} \\ -3 & a-2 \end{vmatrix} \begin{matrix} 3b_1 + b_2 \\ \sim \end{matrix}$$

$$\frac{1}{a-3} |L| = \begin{vmatrix} 1 & \frac{5}{a-3} \\ 0 & \frac{15}{a-3} + (a-2) \end{vmatrix}$$

$$|L| = (a-3)(1)\left(\frac{15}{a-3} + (a-2)\right)$$

$$|L| = 15 + (a-3)(a-2)$$

$$|L| = a^2 - 5a + 21$$

2.) a. $A = \begin{bmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{bmatrix}$

$|A| = 0$ jika $\begin{vmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{vmatrix} \overset{OBE}{\sim} \begin{vmatrix} \lambda-2 & 1 \\ 0 & 0 \end{vmatrix}$ atau $\begin{vmatrix} 0 & 0 \\ -5 & \lambda+4 \end{vmatrix}$

$b_1 = b_2 \rightarrow \lambda - 2 = -5 \qquad \lambda + 4 = 1$

$\lambda = -3 \qquad \lambda = -3$

$\therefore \lambda = -3$

2.) b. $B = \begin{bmatrix} 1 & \lambda & \lambda^2 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda & \lambda^2 \end{bmatrix} \rightarrow b_1 = b_2 = b_3$

Karena $b_1 = b_2 = b_3$ maka apabila dilakukan OBE akan menghasilkan baris nol untuk berapa pun nilai λ

$\therefore \lambda \in R$

3.) $A = \begin{bmatrix} 2 & 3 & -1 & 1 \\ -3 & 2 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 3 & -2 & 1 & 4 \end{bmatrix}$

a. $M_{32} = \begin{vmatrix} 2 & -1 & 1 \\ -3 & 0 & 3 \\ 3 & 1 & 4 \end{vmatrix}$

$M_{32} = (2)(0)(4) + (-1)(3)(3) + (1)(-3)(1) - (1)(0)(3) - (-1)(-3)(4) - (2)(3)(1)$

$= 0 - 9 - 3 - 0 - 12 - 6$

$= -30$

$C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & -1 & 1 \\ -3 & 0 & 3 \\ 3 & 1 & 4 \end{vmatrix} = -(-30) = 30$

b. $M_{44} = \begin{vmatrix} 2 & 3 & -1 \\ -3 & 2 & 0 \\ 3 & -2 & 1 \end{vmatrix}$

$= (2)(2)(1) + (3)(0)(3) + (-1)(-3)(-2) - (-1)(2)(3) - (3)(-3)(1) - (2)(0)(-2)$

$= 4 + 0 - 6 + 6 + 9 - 0$

$= 13$

$C_{44} = (-1)^{4+4} \begin{vmatrix} 2 & 3 & -1 \\ -3 & 2 & 0 \\ 3 & -2 & 1 \end{vmatrix} = 1 . 13 = 13$

c.

$M_{41} = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ -2 & 1 & 0 \end{vmatrix}$

$= (3)(0)(0) + (-1)(3)(-2) + (1)(2)(1) - (1)(0)(-2) - (-1)(2)(0) - (3)(3)(1)$

$= 0 + 6 + 2 + 2 + 2 - 9$

$= 3$

$C_{41} = (-1)^{4+1} \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ -2 & 1 & 0 \end{vmatrix} = (-1)(3) = -3$

d.

$M_{24} = \begin{vmatrix} 2 & 3 & -1 \\ 3 & -2 & 1 \\ 3 & -2 & 1 \end{vmatrix}$

$= (2)(-2)(1) + (3)(1)(3) + (-1)(3)(-2) - (-1)(-2)(3) - (3)(3)(1) - (2)(1)(-2)$

$= -4 + 9 + 6 - 6 - 9 + 4$

$= 0$

$C_{24} = (-1)^{2+4} \begin{vmatrix} 2 & 3 & -1 \\ 3 & -2 & 1 \\ 3 & -2 & 1 \end{vmatrix} = (-1)\, 0 = 0$

4.) $X = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$

a. $|X| = x_{11}C_{11} + x_{12}C_{12} + X_{13}C_{13} + X_{14}C_{14}$

$= 3.(-1)^{1+1}\begin{vmatrix} 2 & 0 & -2 \\ 1 & -3 & 0 \\ 10 & 3 & 2 \end{vmatrix} + 3(-1)^{1+2}\begin{vmatrix} 2 & 0 & -2 \\ 4 & -3 & 0 \\ 2 & 3 & 2 \end{vmatrix} + 0(-1)^{1+3}\begin{vmatrix} 2 & 2 & -2 \\ 4 & 1 & 0 \\ 2 & 10 & 2 \end{vmatrix} +$

$5.(-1)^{1+4}\begin{vmatrix} 2 & 2 & 0 \\ 4 & 1 & -3 \\ 2 & 10 & 3 \end{vmatrix}$

$= 3.(-78) - 3.(-48) + 0 - 5(30) = -240$

b. $|X| = x_{21}C_{21} + x_{22}C_{22} + x_{23}C_{23} + x_{24}C_{24}$

$= 2(-1)^{2+1}\begin{vmatrix} 3 & 0 & 5 \\ 1 & -3 & 0 \\ 10 & 3 & 2 \end{vmatrix} + 2.(-1)^{2+2}\begin{vmatrix} 3 & 0 & 5 \\ 4 & -3 & 0 \\ 2 & 3 & 2 \end{vmatrix} + 0.(-1)^{2+3}\begin{vmatrix} 3 & 3 & 5 \\ 4 & 1 & 0 \\ 2 & 10 & 2 \end{vmatrix} +$

$-2.(-1)^{2+4}\begin{vmatrix} 3 & 3 & 0 \\ 4 & 1 & -3 \\ 2 & 10 & 3 \end{vmatrix}$

$= -2(147) + 2(72) + 0 - 2(45) = -240$

c. $|X| = x_{13}C_{13} + x_{23}C_{23} + x_{33}C_{33} + x_{43}C_{43}$

$= 0.(-1)^{1+3}\begin{vmatrix} 2 & 2 & -2 \\ 4 & 1 & 0 \\ 2 & 10 & 2 \end{vmatrix} + 0(-1)^{2+3}\begin{vmatrix} 3 & 3 & 5 \\ 4 & 1 & 0 \\ 2 & 10 & 2 \end{vmatrix} + -3(-1)^{3+3}\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} +$

$3.(-1)^{4+3}\begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix}$

$= 0 + 0 - 3(120) - 3(-48) = -240$

d. $|X| = x_{14}C_{14} + x_{24}C_{24} + x_{34}C_{34} + x_{44}C_{44}$

$$= 5.(-1)^{1+4}\begin{vmatrix} 2 & 2 & 0 \\ 4 & 1 & -3 \\ 2 & 10 & 3 \end{vmatrix} + -2(-1)^{2+4}\begin{vmatrix} 3 & 3 & 0 \\ 4 & 1 & -3 \\ 2 & 10 & 3 \end{vmatrix} + 0(-1)^{3+4}\begin{vmatrix} 3 & 3 & 0 \\ 2 & 2 & 0 \\ 2 & 10 & 3 \end{vmatrix} +$$

$$2(-1)^{4+4}\begin{vmatrix} 3 & 3 & 0 \\ 2 & 2 & 0 \\ 4 & 1 & -3 \end{vmatrix}$$

$$= -5(30) - 2(45) + 0 + 2(0) = -240$$

5.)

$$B = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta - \cos\theta & \sin\theta + \cos\theta & 1 \end{bmatrix}$$

$$|B| = \sin^2\theta + 0 + 0 - 0 - (-\cos^2\theta) - 0$$

$$|B| = \sin^2\theta + \cos^2\theta = 1$$