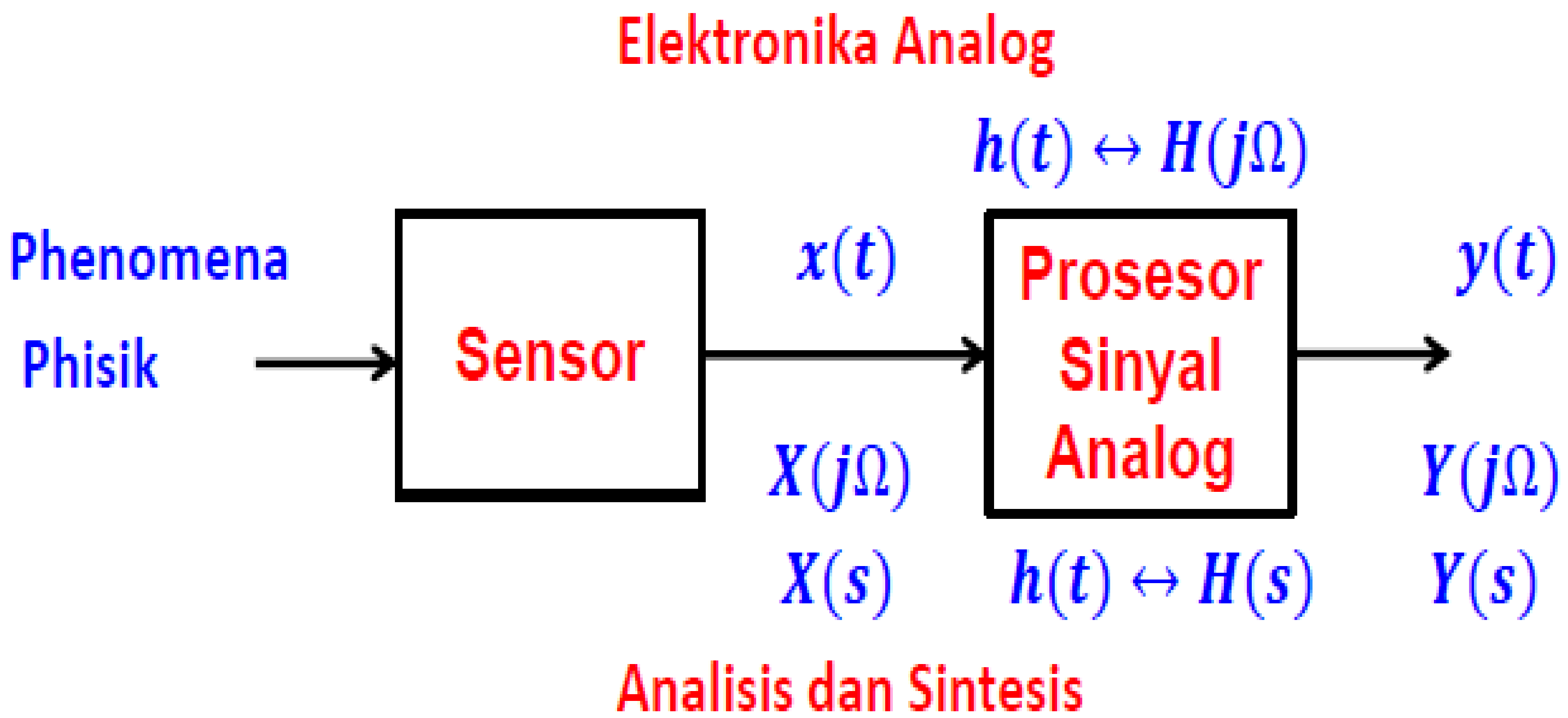
Pengolahan Sinyal Dalam Waktu Kontinyu

Bab 6. Pengantar Filter Analog



Dosen:

Suhartono Tjondronegoro

Isi Kuliah

- Bab 0. Pendahuluan.
- Bab 1. Sinyal.
- Bab 2. Sistem.
- Bab 3. Deret Fourier.
- Bab 4. Transformasi Fourier.
- Bab 5. Transformasi Laplace.
- **Bab 6. Pengantar Filter Analog.**
- Bab 7. Pengantar Sistem Umpan Balik Linier.

Pengantar Filter Analog

- Pendahuluan.
- Filter Butterworth.
- Transformasi Filter Analog ke Filter Analog.
- Perancangan filter Analog Butterworth Low-Pass.
- Perancangan filter Analog Butterworth Band-Pass.
- Filter Chebyshev.

Pendahuluan (1)

- Perkalian yang terjadi direpresentasi kawasan frekuensi memberikan pengertian tentang ”filtering”.

$$y(t) = x(t) * h(t) \overset{TF}{\Leftrightarrow} Y(j\Omega) = X(j\Omega)H(j\Omega)$$

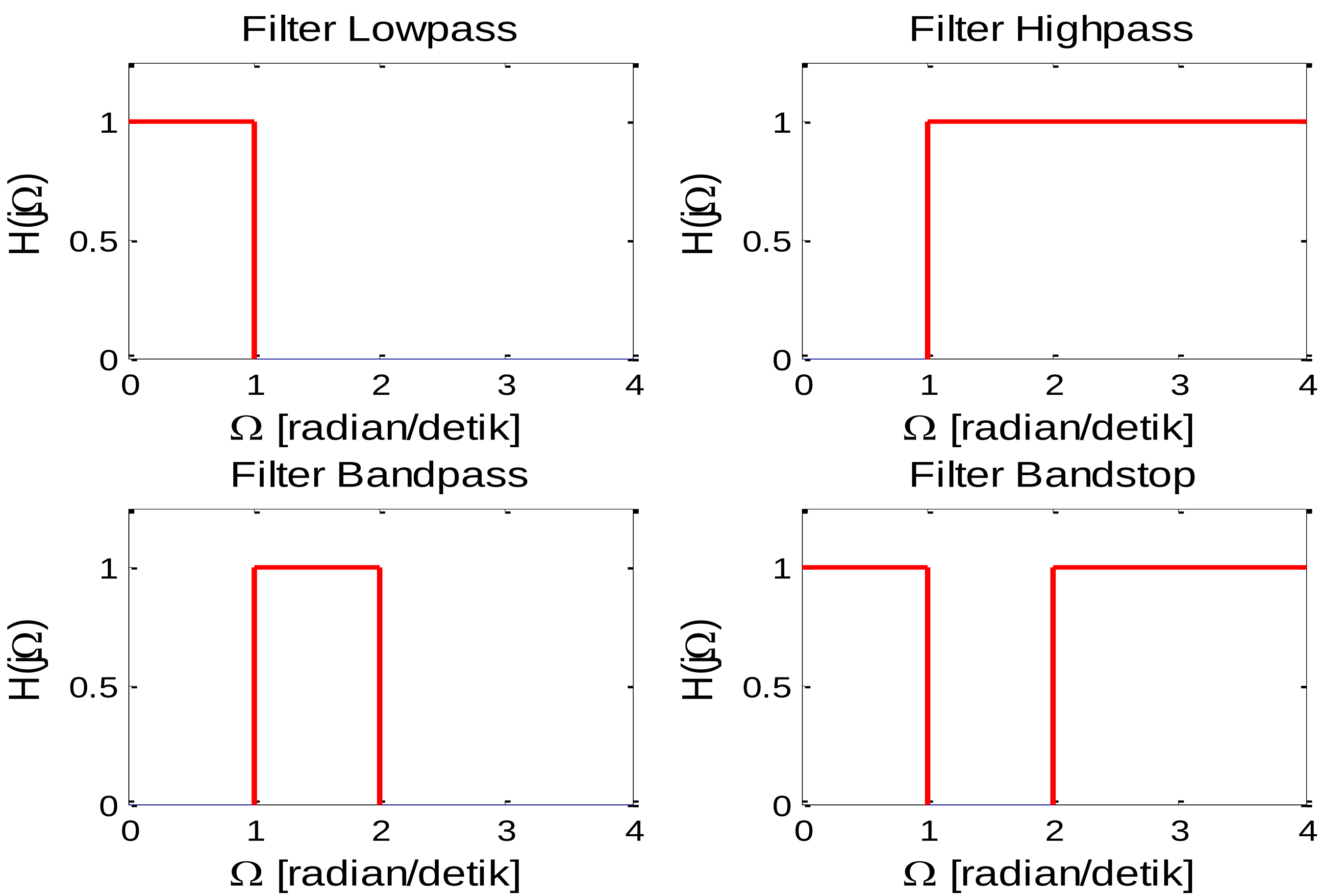
- Sebuah sistem melakukan filtering terhadap sinyal input dengan memberikan respons yang berbeda terhadap komponen input dengan frekuensi berbeda. Istilah ”filtering” mempunyai arti bahwa ada komponen frekuensi input yang dihilangkan, sedangkan yang lain dilewatkan oleh sistem tanpa perubahan.
- Diperlukan adanya pengetahuan filter analog yang dapat bertindak sebagai prototype untuk mendapatkan fungsi transfer filter $H(s)$ yang dikehendaki.
- Fungsi transfer filter $H(s)$ diperoleh dari respons frekuensi filter $H(j\Omega)$ yang dirancang.

Pendahuluan (2)

- **Jenis filter:**
- **Filter Low-pass:**
- Meredam komponen frekuensi tinggi input dan melewatkan komponen frekuensi rendah.
- **Filter High-pass:**
- Meredam komponen frekuensi rendah input dan melewatkan komponen frekuensi tinggi.
- **Band-pass filter:**
- Melewatkan sinyal-sinyal didalam band frekuensi tertentu dan meredam sinyal-sinyal diluar band tersebut.
- **Band-stop filter:**
- Meredam sinyal-sinyal didalam band frekuensi tertentu dan melewatkan sinyal-sinyal diluar band tersebut.

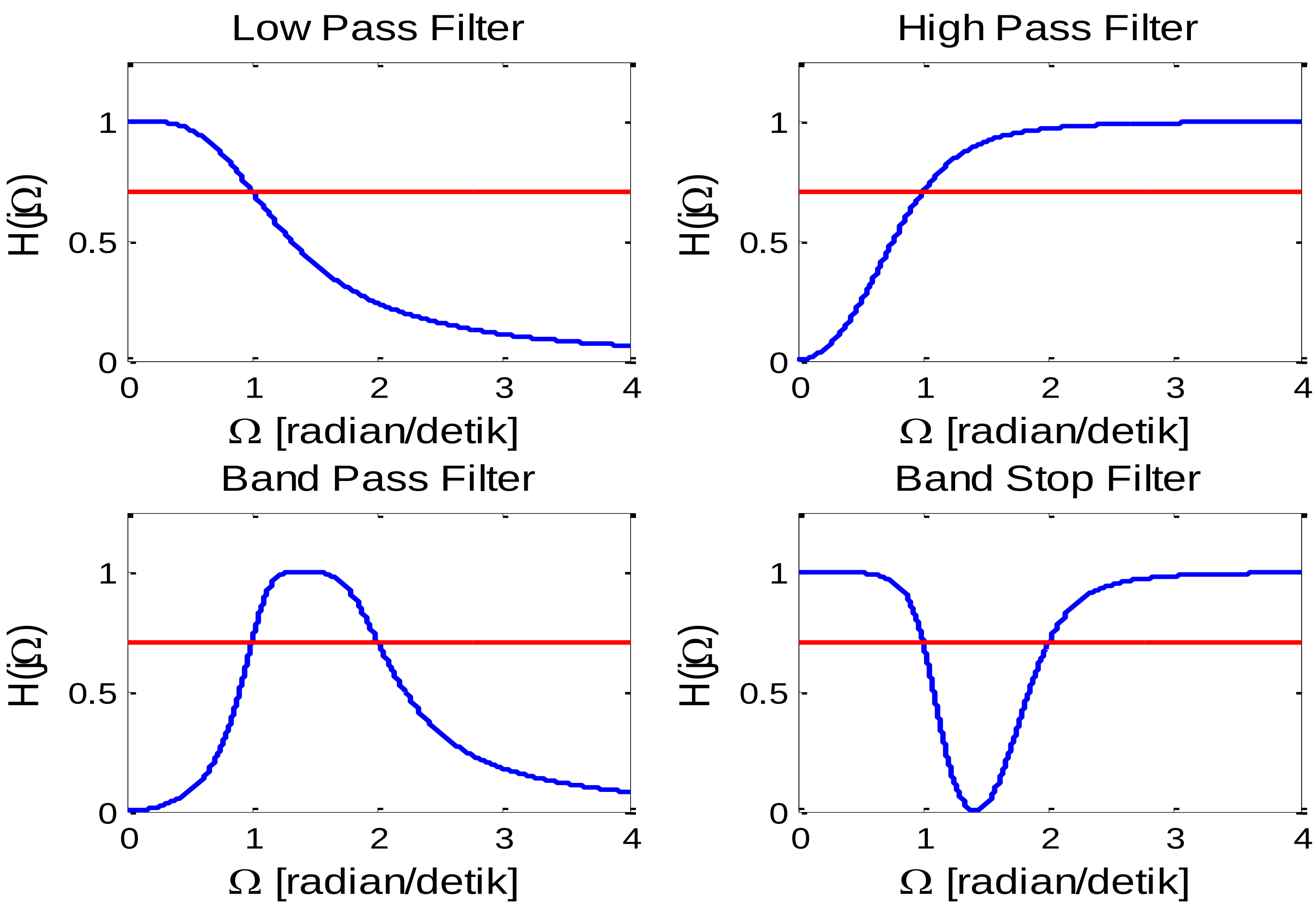
Pendahuluan (3)

- **Respons frekuensi ideal (respons frekuensi satu sisi):**



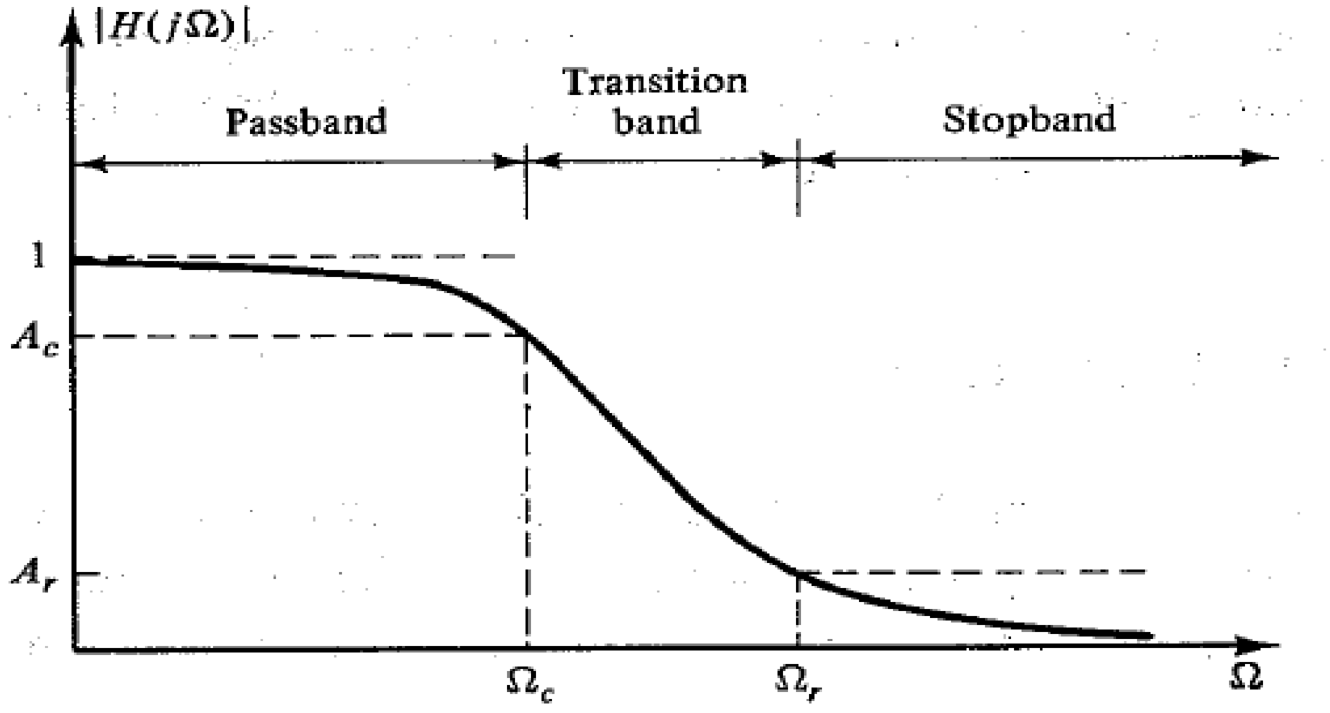
Pendahuluan (4)

- **Respons frekuensi riil (respons frekuensi satu sisi):**



Pendahuluan (5)

- **Desain filter low-pass**.
- **Respons frekuensi yang dibutuhkan**:

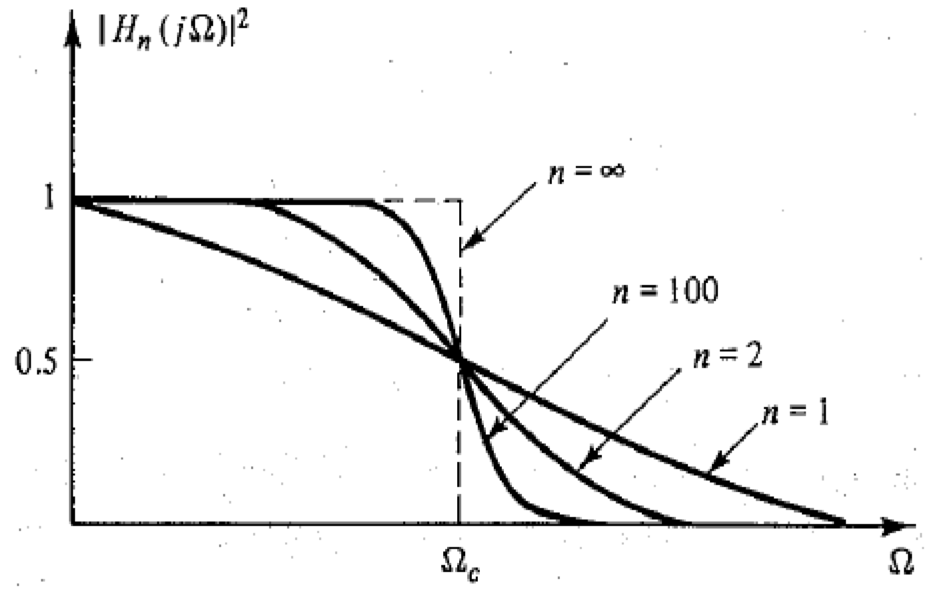


Filter Butterworth (1)

- **Magnitude squared respons frekuensi**:

$$|H_n(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}}$$

- $|H_n(j\Omega)|^2|_{\Omega=0} = 1.$ untuk semua n.
- $|H_n(j\Omega)|^2|_{\Omega=\Omega_c} = 0{,}5.$ untuk semua n terbatas.
- $|H_n(j\Omega)||_{\Omega=\Omega_c} = 0{,}707.$
- $20\log|H_n(j\Omega)||_{\Omega=\Omega_c} = -3{,}01$ dB.
- $|H_n(j\Omega)|^2$ fungsi monoton menurun variabel Ω.
- Bila $n \to \infty$, $|H_n(j\Omega)|^2$ mendekati respons frekuensi LP ideal.
- $|H_n(j\Omega)|^2$ disebut “maximally flat” di Ω = 0.

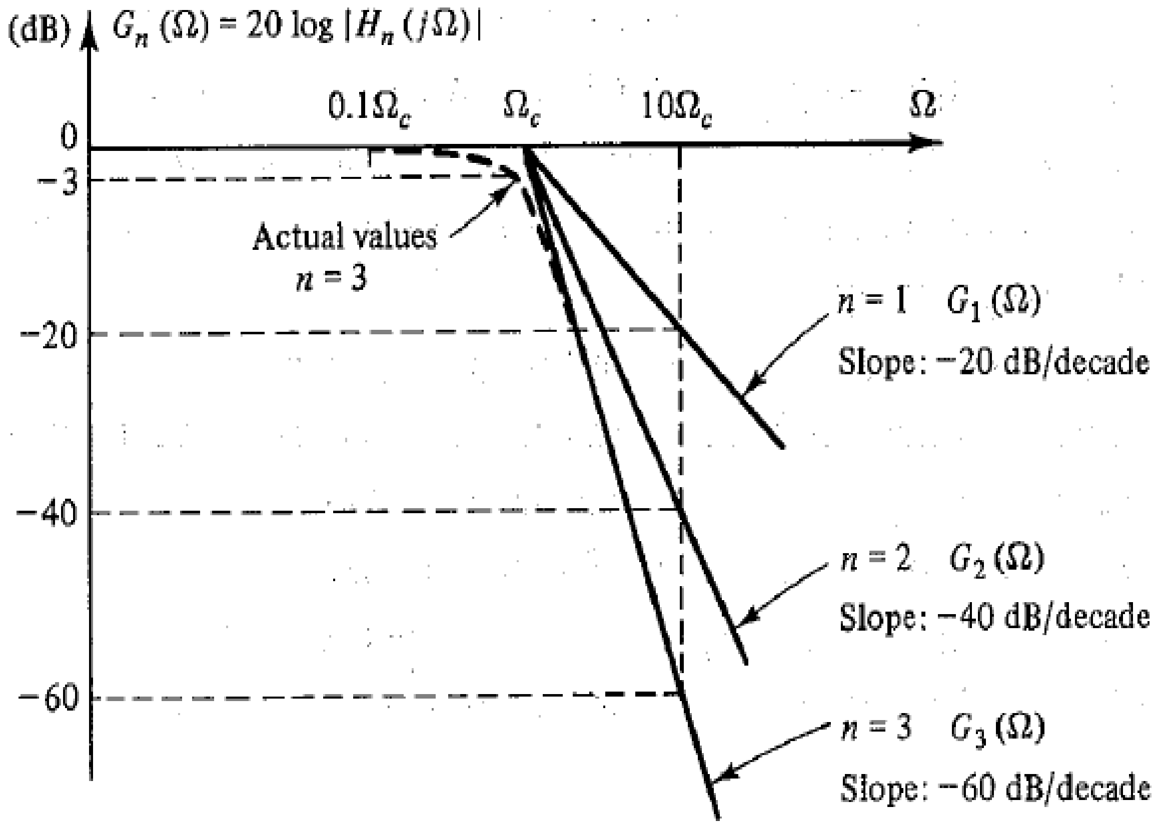


Filter Butterworth (2)

- **Gain**: $G_n(\Omega) = 20\log|H_n(j\Omega)| = 10\log|H_n(j\Omega)|^2$
- $G_n(\Omega) = 10\log\left[\frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}}\right] = -10\log 1 + \left(\frac{\Omega}{\Omega_c}\right)^{2n}$
- Fungsi n.
- Untuk $\Omega \ll \Omega_c$

  $G_n(\Omega) \approx 0$ dB.
- Untuk $\Omega \gg \Omega_c$

  $G_n(\Omega) \approx -20n\log\left|\frac{\Omega}{\Omega_c}\right|$



Filter Butterworth (3)

- **Filter Butterworth Low-pass ternormalisasi**: $\Omega_c = 1$ rad/detik.
- **Magnitude squared respons frekuensi**:

$$|H_n(j\Omega)|^2 = \frac{1}{1 + (\Omega)^{2n}}$$

- Fungsi transfer Filter Butterworth Low-pass ternormalisasi: $H(s)$
- Umumnya $s = \sigma + j\Omega$, bila $s = j\Omega \rightarrow \Omega = \frac{s}{j}$.

$$|H_n(j\Omega)|^2 = H_n(j\Omega)H_n(-j\Omega) = \frac{1}{1 + (\Omega)^{2n}}$$

$$H_n(s)H_n(-s) = \frac{1}{1 + \left(\frac{s}{j}\right)^{2n}}$$

- Pole-pole dari $H_n(s)H_n(-s)$ dihitung dari akar penyebut:

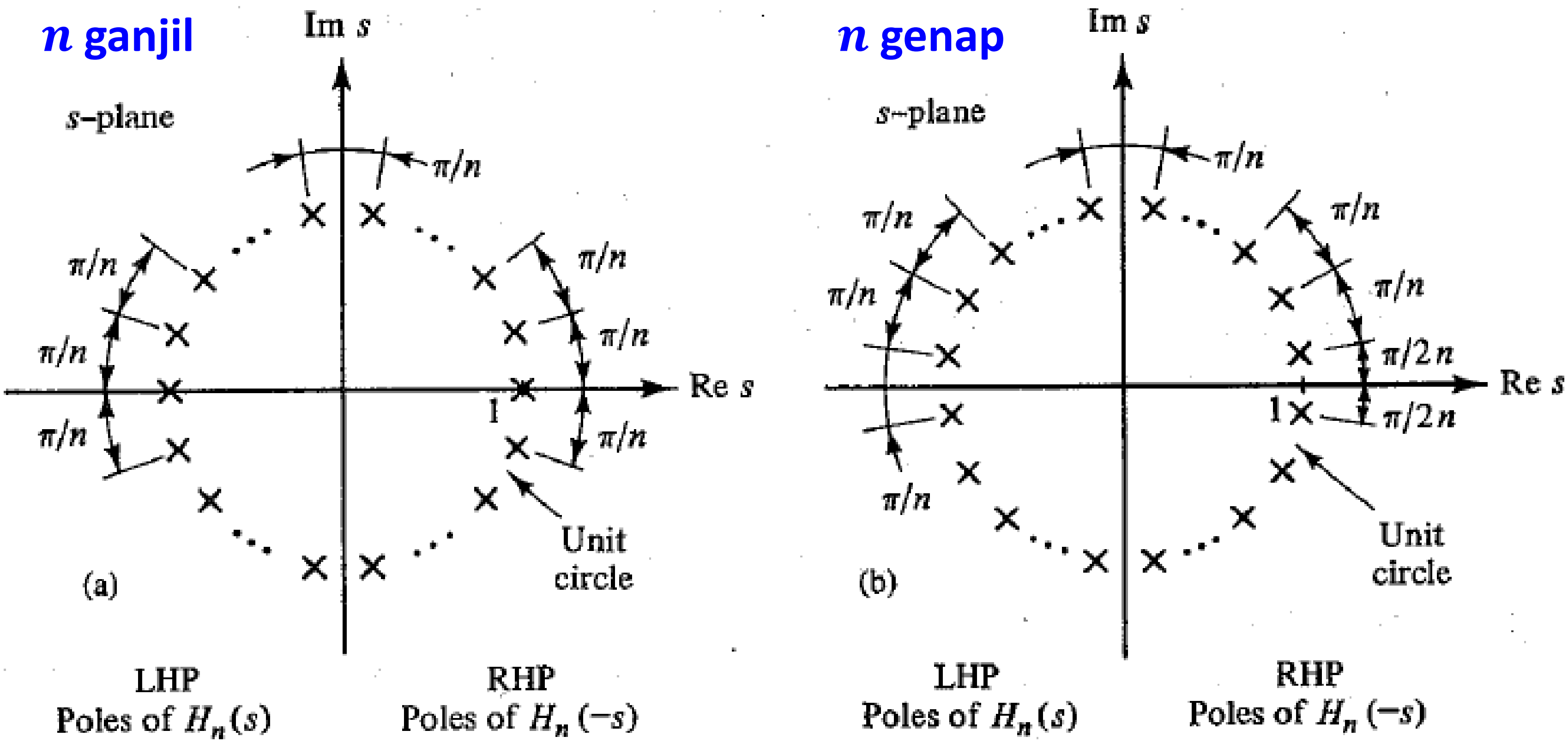
$$1 + \left(\frac{s}{j}\right)^{2n} \quad atau \;\; s^{2n} = -1(j)^{2n} = (-1)^{n+1}$$

Filter Butterworth (4)

- **Bila n ganjil**:
- $s^{2n} = 1 \rightarrow s_k = 1\angle k\pi/n, \quad k = 0, 1, 2, \dots, 2n-1.$
- **Bila n genap**:
- $s^{2n} = -1 \rightarrow s_k = 1\angle \pi/2n + k\pi/n, \quad k = 0, 1, 2, \dots, 2n-1.$

***n* ganjil**

***n* genap**



Filter Butterworth (5)

- Fungsi transfer filter $H(s)$ harus stabil dan kausal, maka pole-pole $H_n(s)$ harus berada disebelah kiri sumbu $j\Omega$ pada bidang s.

$$H_n(s) = \frac{1}{\prod_{pole\ seb\ kiri}(s - s_k)} = \frac{1}{B_n(s)}$$

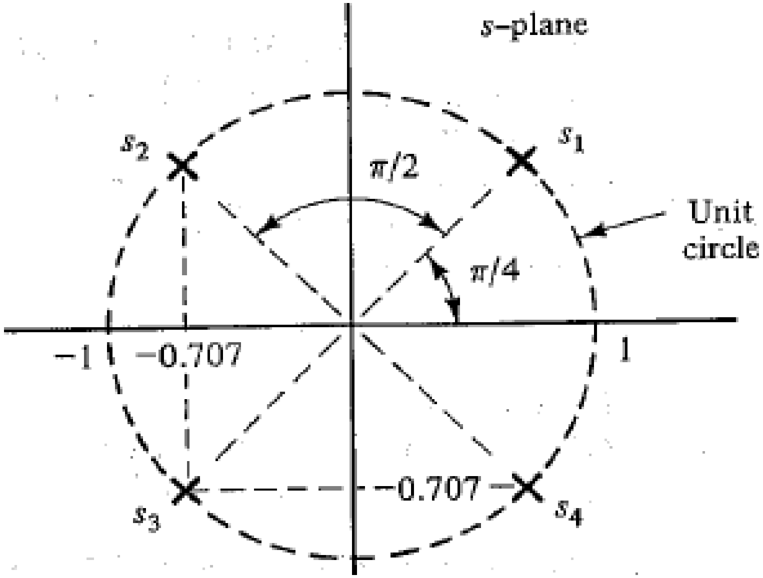
- Dimana s_k adalah pole-pole $H_n(s)H_n(-s)$ yang berada disebelah kiri sumbu $j\Omega$ pada bidang s.
- Polinomial $B_n(s)$ adalah polinomial Butterworth orde n.
- **Fungsi transfer filter Butterworth ternormalisasi orde 1**:
- Pole: $s_1 = 1\angle 0 = 1$, $s_2 = 1\angle\pi = -1$.

$$H_1(s) = \frac{1}{s - (-1)} = \frac{1}{s + 1}$$

- **Fungsi transfer filter Butterworth ternormalisasi orde 2**:
- Pole: $s_k = 1\angle\,\pi/2n + k\pi/n, \quad k = 0, 1, 2, 3.$
-

Filter Butterworth (6)

- **Fungsi transfer filter Butterworth ternormalisasi orde 2**:
- Pole: $s_k = 1\angle\pi/2n + k\pi/n, \quad k = 0, 1, 2, 3.$
- $s_1 = 1\angle\pi/4 = 0{,}707 + j0{,}707$
- $s_2 = 1\angle 3\pi/4 = -0{,}707 + j0{,}707$
- $s_3 = 1\angle 5\pi/4 = -0{,}707 - j0{,}707$
- $s_4 = 1\angle 7\pi/4 = 0{,}707 - j0{,}707$



- Fungsi transfer:

$$H_2(s) = \frac{1}{(s - s_2)(s - s_3)} = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Filter Butterworth (7)

- Fungsi transfer Filter Butterworth ternormalisasi, $\Omega_c = 1$ rad/det

Orde Filter	$H_n(s) = \dfrac{1}{a_n s^n + a_{n-1} s^{n-1} \ldots + a_1 s + 1} = \dfrac{1}{B_n(s)}$
n	Polynomial $B_n(s)$
1	$s + 1$
2	$s^2 + \sqrt{2}s + 1$
3	$s^3 + 2s^2 + 2s + 1$
4	$s^4 + 2{,}613s^3 + 3{,}414s^2 + 2{,}613s + 1$
5	$s^5 + 3{,}236s^4 + 5{,}236s^3 + 5{,}236s^2 + 3{,}236s + 1$
6	$s^6 + 3{,}863s^5 + 7{,}464s^4 + 9{,}141s^3 + 7{,}464s^2 + 3{,}863s + 1$
7	$s^7 + 4{,}494s^6 + 10{,}103s^5 + 14{,}606s^4 + 14{,}606s^3 + 10{,}103s^2 + 4{,}494s + 1$

Transformasi Analog ke Analog (1)

- Dalam praktek umumnya diinginkan membuat filter low-pass dengan $\Omega_c = \Omega_u \neq 1$ rad/detik.
- Harus ditentukan $H(s)$ filter berdasarkan $H_n(s)$ LPF yang dipakai.
- Bila s di $H_n(s)$ diganti dengan $\frac{s}{\Omega_u}$, maka akan diperoleh

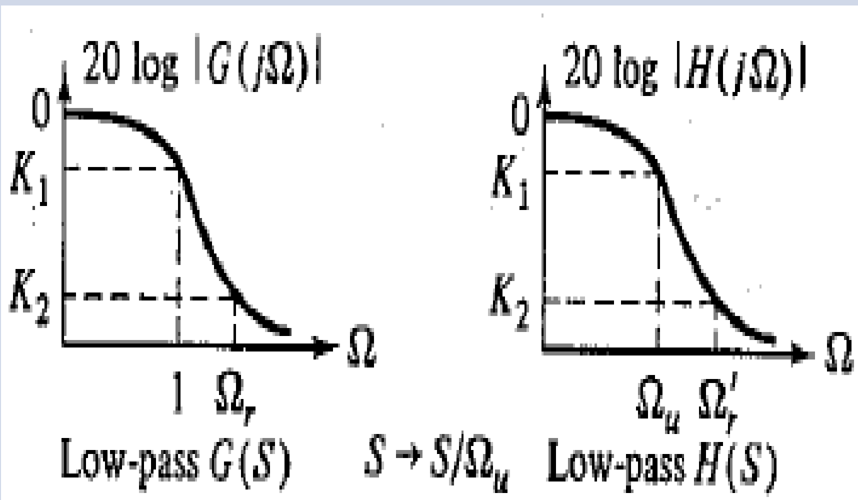
$$H(s) = H_n(s)\Big|_{s \to \frac{s}{\Omega_u}} = H_n\left(\frac{s}{\Omega_u}\right)$$

- Magnituda di $s = j\Omega$: $|H(j\Omega)| = \left|H\left(j\frac{\Omega}{\Omega_u}\right)\right|$
- Magnituda di $s = j\Omega_u$: $|H(j\Omega_u)| = |H(j1)|$
- Artinya frekuensi cut-off $\Omega_c = 1$ rad/det pindah $\Omega_c = \Omega_u$.

- Pada slide berikutnya ditampilkan persamaan transformasi LPF → LPF, LPF → HPF, LPF → BPF, dan LPF → BSF.

Transformasi Analog ke Analog (2)

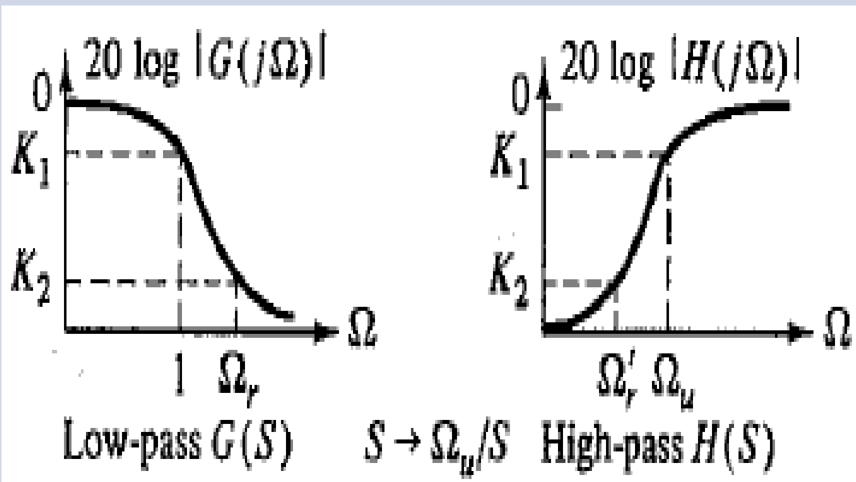
- **LPF ke LPF**:

Respons prototype	Respons filter tertransformasi	Persamaan Desain
		Forward: $\Omega_r' = \Omega_r \Omega_u$ Backward: $\Omega_r = \Omega_r'/\Omega_u$

$$H_1(s) = \frac{1}{s+1} \rightarrow H(s) = \frac{1}{\frac{s}{\Omega_u}+1} = \frac{\Omega_u}{s+\Omega_u}$$

Transformasi Analog ke Analog (3)

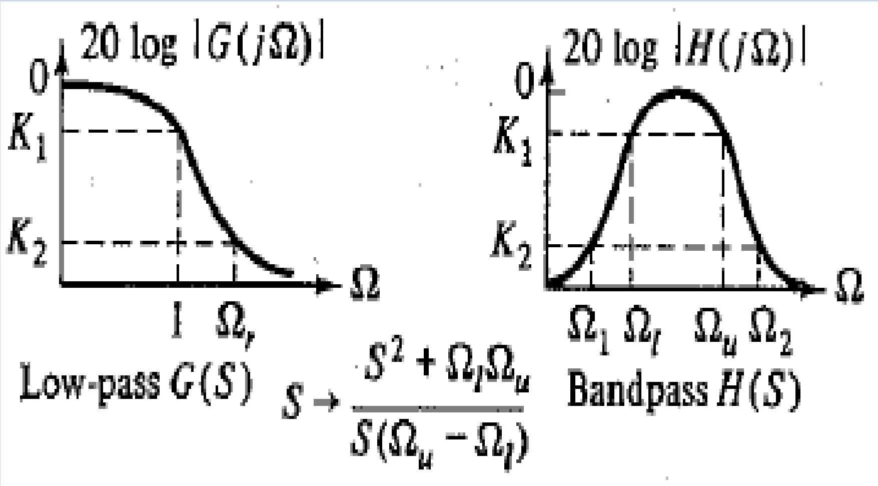
- **LPF ke HPF**:

Respons prototype	Respons filter tertransformasi	Persamaan Desain
		Forward: $\Omega_r' = \Omega_u/\Omega_r$ Backward: $\Omega_r = \Omega_u/\Omega_r'$

$$H_1(s) = \frac{1}{s+1} \rightarrow H(s) = \frac{1}{\frac{\Omega_u}{s}+1} = \frac{s}{s+\Omega_u}$$

Transformasi Analog ke Analog (4)

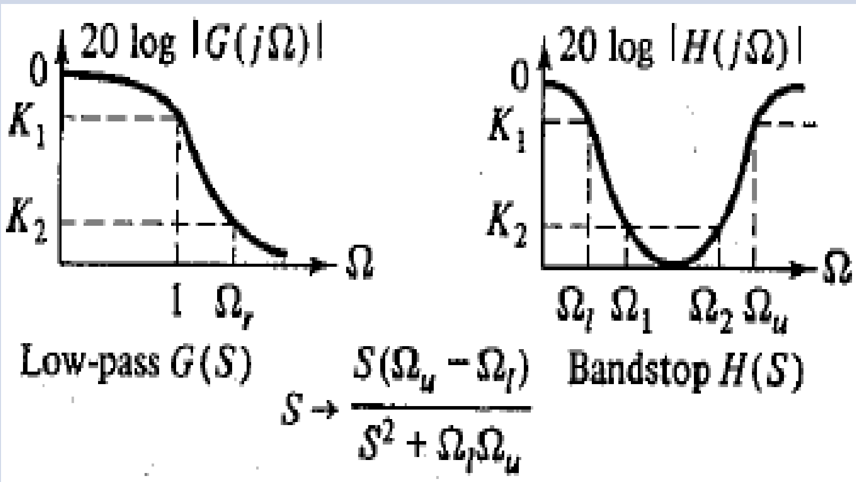
- **LPF ke BPF**:

Respons prototype	Respons filter tertransformasi	Persamaan Desain
		Forward: $\Omega_{av} = (\Omega_u - \Omega_l)/2$ $\Omega_1 = [\Omega_{av}/\Omega_r)^2 + \Omega_l\Omega_u]^{1/2} - \Omega_{av}/\Omega_r$ $\Omega_2 = [(\Omega_{av}/\Omega_r)^2 + \Omega_l\Omega_u]^{1/2} + \Omega_{av}/\Omega_r$ Backward: $\Omega_r = \min\{\lvert A\rvert, \lvert B\rvert\}$ $A = \Omega_1(\Omega_u - \Omega_l)/[-\Omega_1^2 + \Omega_l\Omega_u]$ $B = \Omega_2(\Omega_u - \Omega_l)/[-\Omega_2^2 + \Omega_l\Omega_u]$

$$H_1(s) = \frac{1}{s+1} \rightarrow H(s) = \frac{1}{\dfrac{s^2 + \Omega_l\Omega_u}{s(\Omega_u - \Omega_l)} + 1} = \frac{s(\Omega_u - \Omega_l)}{s^2 + s(\Omega_u - \Omega_l) + \Omega_l\,\Omega_u}$$

Transformasi Analog ke Analog (5)

- **LPF ke BSF**:

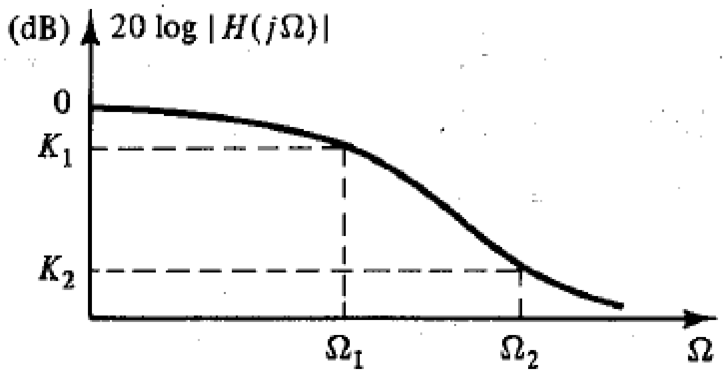
<table>
<tr><th>Respons prototype</th><th>Respons filter tertransformasi</th><th>Persamaan Desain</th></tr>
<tr><td colspan="2">
</td><td>Forward: $\Omega_{av} = (\Omega_u - \Omega_l)/2$
$\Omega_1 = [(\Omega_{av}/\Omega_r)^2 + \Omega_l\Omega_u]^{1/2} - \Omega_{av}/\Omega_r$
$\Omega_2 = [(\Omega_{av}/\Omega_r)^2 + \Omega_l\Omega_u]^{1/2} + \Omega_{av}/\Omega_r$

Backward: $\Omega_r = \min\{|A|, |B|\}$
$A = \Omega_1(\Omega_u - \Omega_l)/[-\Omega_1^2 + \Omega_l\Omega_u]$
$B = \Omega_2(\Omega_u - \Omega_l)/[-\Omega_2^2 + \Omega_l\Omega_u]$</td></tr>
</table>

$$H_1(s) = \frac{1}{s+1} \rightarrow H(s) = \frac{1}{\frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_l\Omega_u} + 1} = \frac{s^2 + \Omega_l\Omega_u}{s^2 + s(\Omega_u - \Omega_l) + \Omega_l\,\Omega_u}$$

Desain Filter LP-Butterworth (1)

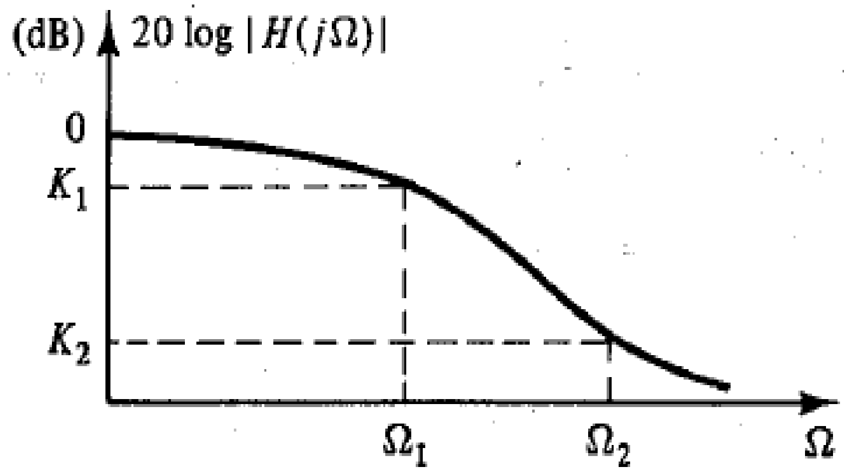
- **Respons frekuensi yang diinginkan:**



- **Persyaratan**:
- $0 \geq 20\log|H(j\Omega)| \geq K_1$, untuk semua $\Omega \leq \Omega_1$
- $20\log|H(j\Omega)| \leq K_2$, untuk semua $\Omega \geq \Omega_2$.
- Respons frekuensi filter LP Butterworth hanya ditentukan oleh n dan Ω_c.
- Dari persamaan: $|H_n(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}}$
- Maka: $10\log\left[\frac{1}{1+\left(\frac{\Omega_1}{\Omega_c}\right)^{2n}}\right] = K_1$, dan $10\log\left[\frac{1}{1+\left(\frac{\Omega_2}{\Omega_c}\right)^{2n}}\right] = K_2$

Desain Filter LP-Butterworth (2)

- **Respons frekuensi LPF yang diinginkan:**



- **Diperoleh**:

$$\left(\frac{\Omega_1}{\Omega_c}\right)^{2n} = 10^{-\frac{K_1}{10}} - 1, \mathrm{dan} \left(\frac{\Omega_2}{\Omega_c}\right)^{2n} = 10^{-\frac{K_2}{10}} - 1$$

$$\left(\frac{\Omega_1}{\Omega_2}\right)^{2n} = \frac{10^{-\frac{K_1}{10}} - 1}{10^{-\frac{K_2}{10}} - 1} \rightarrow\rightarrow n = \left\lceil \frac{\log_{10}\left[\frac{10^{-\frac{K_1}{10}} - 1}{10^{-\frac{K_2}{10}} - 1}\right]}{2\log_{10}\left(\frac{\Omega_1}{\Omega_c}\right)} \right\rceil$$

Desain Filter LP-Butterworth (3)

- Nilai n yang diperoleh akan memberikan 2 nilai Ω_c yang berbeda:

$$\left(\frac{\Omega_1}{\Omega_c}\right)^{2n} = 10^{-\frac{K_1}{10}} - 1, \text{dan } \left(\frac{\Omega_2}{\Omega_c}\right)^{2n} = 10^{-\frac{K_2}{10}} - 1$$

- Bila diinginkan memenuhi persyaratan dengan tepat untuk nilai Ω_1, dan mendapat hasil lebih baik untuk Ω_2, dipakai:

$$\Omega_{c1} = \frac{\Omega_1}{\left(10^{-\frac{K_1}{10}} - 1\right)^{\frac{1}{2n}}}$$

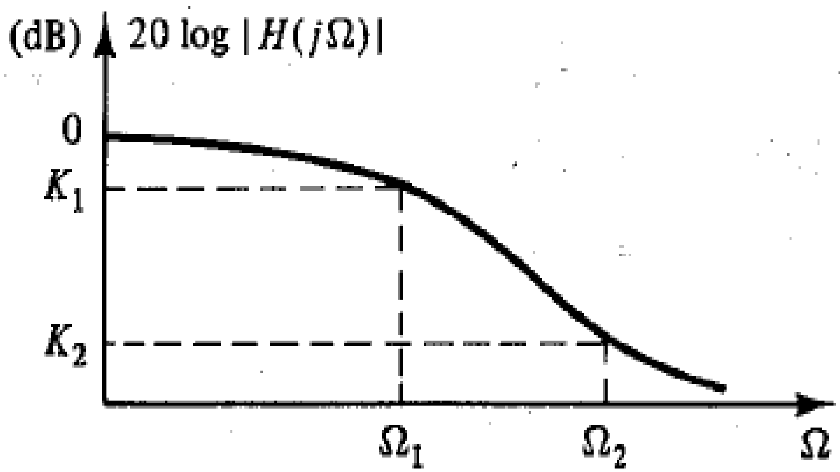
- Bila diinginkan memenuhi persyaratan dengan tepat untuk nilai Ω_2, dan mendapat hasil lebih baik untuk Ω_1, dipakai:

$$\Omega_{c2} = \frac{\Omega_2}{\left(10^{-\frac{K_2}{10}} - 1\right)^{\frac{1}{2n}}}$$

- Bisa dipilih: $\Omega_{c1} < \Omega_c < \Omega_{c2}$

Desain Filter LP-Butterworth (4)

- **Aplikasi numerik**:
- Rancang filter LP analog Butterworth mempunyai gain −2 dB atau lebih baik pada frekuensi 20 rad/det, dan mempunyai redaman paling sedikit 10 dB pada frekuensi 30 rad/det.
- $\Omega_1 = 20, K_1 = -2$
- $\Omega_2 = 30, K_2 = -10$



- $n = \left\lceil \dfrac{\log_{10}\left[\dfrac{10^{-\frac{K_1}{10}}-1}{10^{-\frac{K_2}{10}}-1}\right]}{2\log_{10}\left(\frac{\Omega_1}{\Omega_c}\right)} \right\rceil = 3{,}37 = 4.$

$$\Omega_c = \frac{\Omega_1}{\left(10^{-\frac{K_1}{10}}-1\right)^{\frac{1}{2n}}} = \frac{20}{(10^{0{,}2}-1)^{\frac{1}{8}}} = 21{,}3868$$

Desain Filter LP-Butterworth (5)

- Filter low-pass Butterworth ternormalisasi dengan $\Omega_c = 1$ rad/det dan $n = 4$.

$$H_4(s) = \frac{1}{s^4 + 2{,}613s^3 + 3{,}414s^2 + 2{,}613s + 1}$$

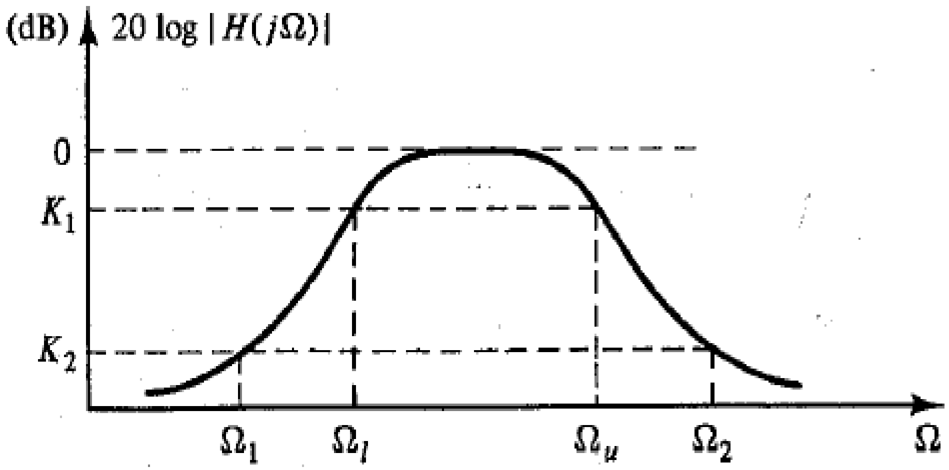
- Dengan transformasi LP ke LP, $s \to \frac{s}{\Omega_c}$, dengan $\Omega_c = 21{,}3868$ rad/det.

$$H(s) = H_4(s)\Big|_{s \to \frac{s}{21{,}3868}}$$

$$H(s) = \frac{1}{\left(\frac{s}{21{,}3868}\right)^4 + 2{,}613\left(\frac{s}{21{,}3868}\right)^3 + 3{,}414\left(\frac{s}{21{,}3868}\right)^2 + 2{,}613\left(\frac{s}{21{,}3868}\right) + 1}$$

Desain Filter BP-Butterworth (1)

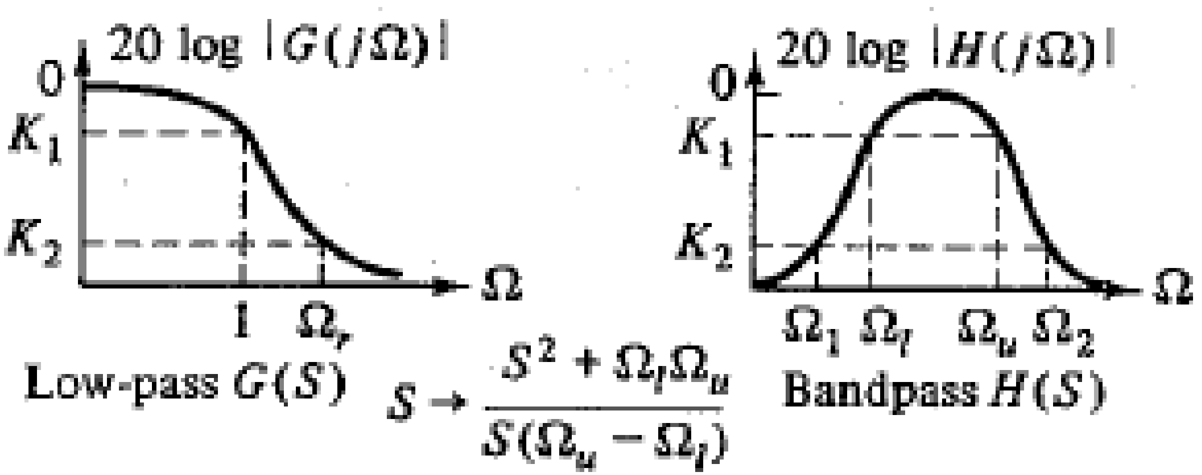
- **Respons frekuensi yang diinginkan:**



- **Persyaratan**:
- $20\log|H(j\Omega)| \le K_2$, untuk semua $\Omega \le \Omega_1$.
- $0 \ge 20\log|H(j\Omega)| \ge K_1$, untuk semua $\Omega_l \le \Omega \le \Omega_u$
- $20\log|H(j\Omega)| \le K_2$, untuk semua $\Omega \ge \Omega_2$.

Desain Filter BP-Butterworth (2)

- Bila $H_{LP}(s)$ adalah LPF dengan $\Omega_c = 1$ rad.det, dan frekuensi stop-band adalah Ω_r



$$H_{BP}(s) = H_{LP}(s)\Big|_{s \to \frac{s^2+\Omega_l\Omega_u}{s(\Omega_u-\Omega_l)}}$$

$$\Omega_r = \min\{|A|, |B|\}$$

$$A = \frac{-{\Omega_1}^2 + \Omega_l\Omega_u}{\Omega_1(\Omega_u - \Omega_l)} \text{ dan } B = \frac{{\Omega_2}^2 - \Omega_l\Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$$

Desain Filter BP-Butterworth (3)

- **Aplikasi numerik**:
- Rancang filter BP analog dengan spesifikasi:
  – Penguatan (Gain) = - 3,0103 dB pada $\Omega_l = 2\pi(50)$ rad/det dan $\Omega_u = 2\pi(20000)$ rad/det.
  – Redaman di stop-band minimum 20 dB pada $\Omega_1 = 2\pi(20)$ rad/det dan $\Omega_2 = 2\pi(50000)$ rad/det.
  – Respons frekuensi monotonic.
- **Solusi**:
- Persyaratan monotonic dipenuhi oleh filter Butterworth.
- Frekuensi kritis:
- $\Omega_1 = 2\pi(20) = 125{,}663$ rad/det.
- $\Omega_l = 2\pi(50) = 314{,}159$ rad/det.
- $\Omega_u = 2\pi(20000) = 1{,}2566.10^5$ rad/det.
- $\Omega_2 = 2\pi(50000) = 2{,}8274.10^5$ rad/det

Desain Filter BP-Butterworth (4)

- Filter LP prototype harus memenuhi:
- $0 \geq 20\log|H_{lP}(j1)| \geq -3{,}0103$ dB.
- $20\log|H_{lP}(j\Omega_r)| \leq -20$ dB.
- $$\left.\begin{aligned} A &= \frac{-\Omega_1{}^2+\Omega_l\Omega_u}{\Omega_1(\Omega_u-\Omega_l)} = 2{,}5053. \\ B &= \frac{\Omega_2{}^2-\Omega_l\Omega_u}{\Omega_2(\Omega_u-\Omega_l)} = 2{,}2545. \end{aligned}\right\} \rightarrow \Omega_r = 2{,}2545.$$
- $$n = \left\lceil \frac{\log_{10}\left[\frac{10^{0,301}-1}{10^2-1}\right]}{2\log_{10}(2{,}2545)} \right\rceil = \lceil 2{,}829 \rceil = 3$$

$$H_{LP}(s) = \frac{1}{s^3+2s^2+2s+1}$$

- Transformasi LP→BP:

$$s \rightarrow \frac{s^2+\Omega_l\Omega_u}{s(\Omega_u-\Omega_l)} = \frac{s^2+3{,}94784.10^7}{s(1{,}25349.10^5)}$$

Desain Filter BP-Butterworth (5)

$$H_{BP}(s) = \frac{1}{\left(\frac{s^2 + 3{,}94784.10^7}{s(1{,}25349.10^5)}\right)^3 + 2\left(\frac{s^2 + 3{,}94784.10^7}{s(1{,}25349.10^5)}\right)^2 + 2\left(\frac{s^2 + 3{,}94784.10^7}{s(1{,}25349.10^5)}\right) + 1}$$

$$H_{BP}(s) = \frac{gs^3}{s^6 + a\,s^5 + bs^4 + cs^3 + ds^2 + es + f}$$

- $a = 2{,}50699.10^5$
- $b = 3{,}15434.10^{10}$
- $c = 1{,}9893.10^{15}$
- $d = 1{,}24528.10^{18}$
- $e = 3{,}90726.10^{20}$
- $f = 6{,}15289.10^{22}$
- $g = 1{,}96953.10^{15}$

Filter Chebyshev (1)

- Ada 2 type filter Chebyshev:
- Type 1: mempunyai ripple di pass-band.
- Type 2: mempunyai ripple di stop-band.
- Filter low-pass Chebyshev type 1 ternormalisasi mempunyai persamaan respons frekuensi magnitude squared:

$$|H_n(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 {T_n}^2(\Omega)}, \qquad n = 1,2,3, \dots \dots$$

- Dimana $T_n(\Omega)$ adalah polinomial Chebyshev orde n.
- Polinomial Chebyshev dapat dihasilkan melalui rumus rekursif:

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x), \qquad n > 2$$

- Dimana $T_0(x) = 1$ dan $T_1(x) = x$

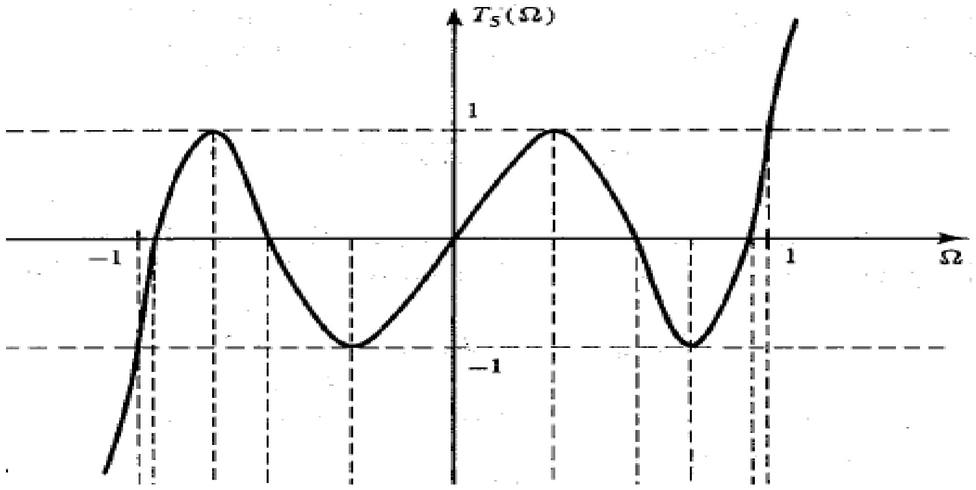
Filter Chebyshev (2)

- Polinomial Chebyshev: $T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x), \qquad n > 2$

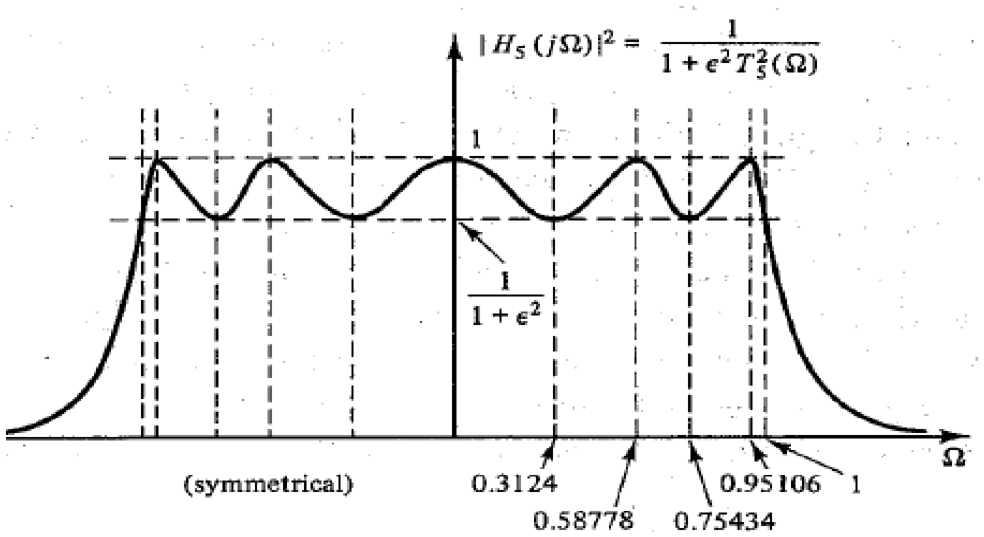
n	$T_n(x)$
0	1
1	x
2	$2x^2 - 1$
3	$4x^3 - 3x$
4	$8x^4 - 8x^2 + 1$
5	$16x^5 - 20x^3 + 5x$
6	$32x^6 - 48x^4 + 18x^2 - 1$
7	$64x^7 - 112x^5 + 56x^3 - 7x$
8	$128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$
9	$256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x$
10	$512x^{10} - 1280x^8 + 1120x^6 - 400x^4 + 50x^2$-1

Filter Chebyshev (3)

- Polinomial $T_5(x)$

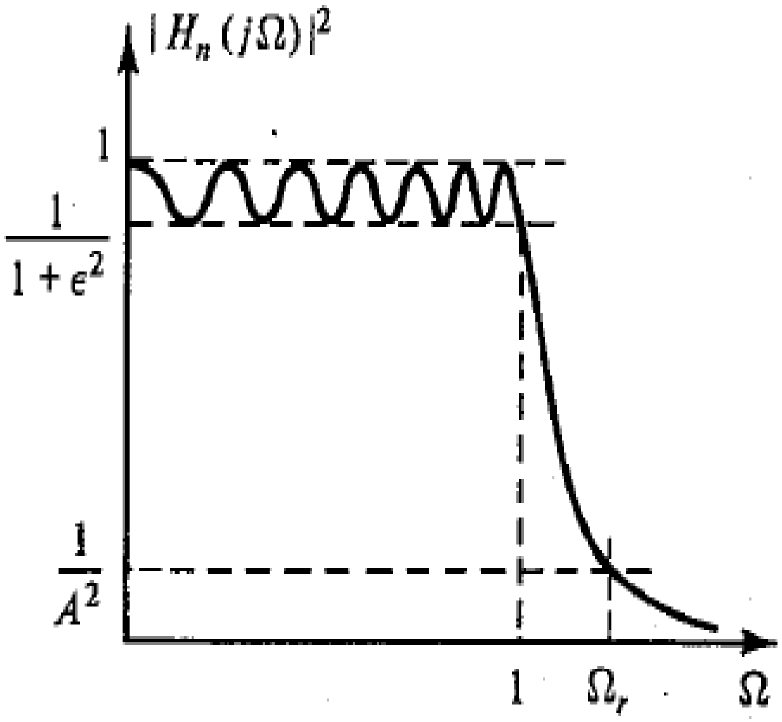


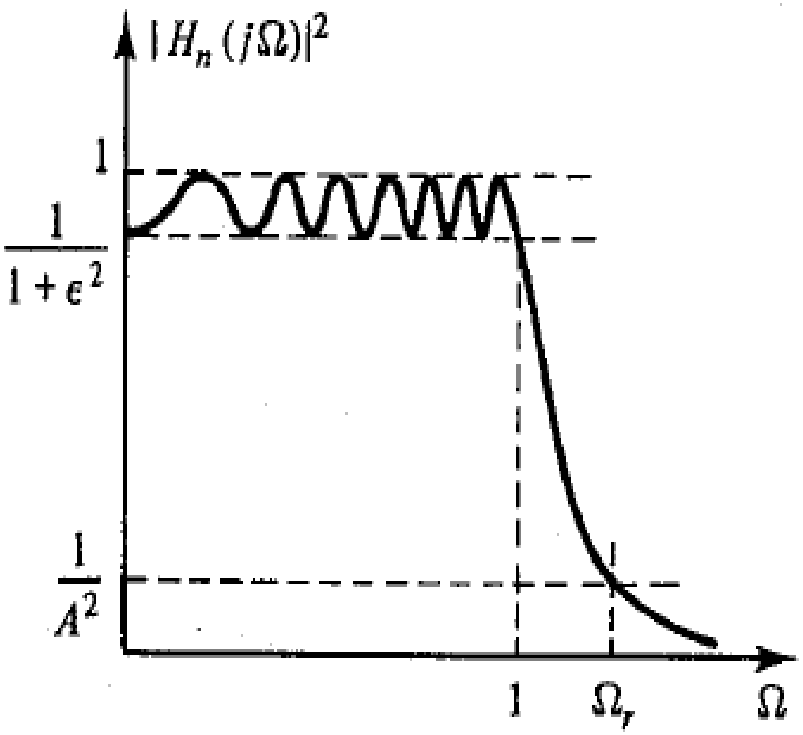
- $|H_5(j\Omega)|^2$ type 1



Filter Chebyshev (4)

- n ganjil $\qquad$ n genap





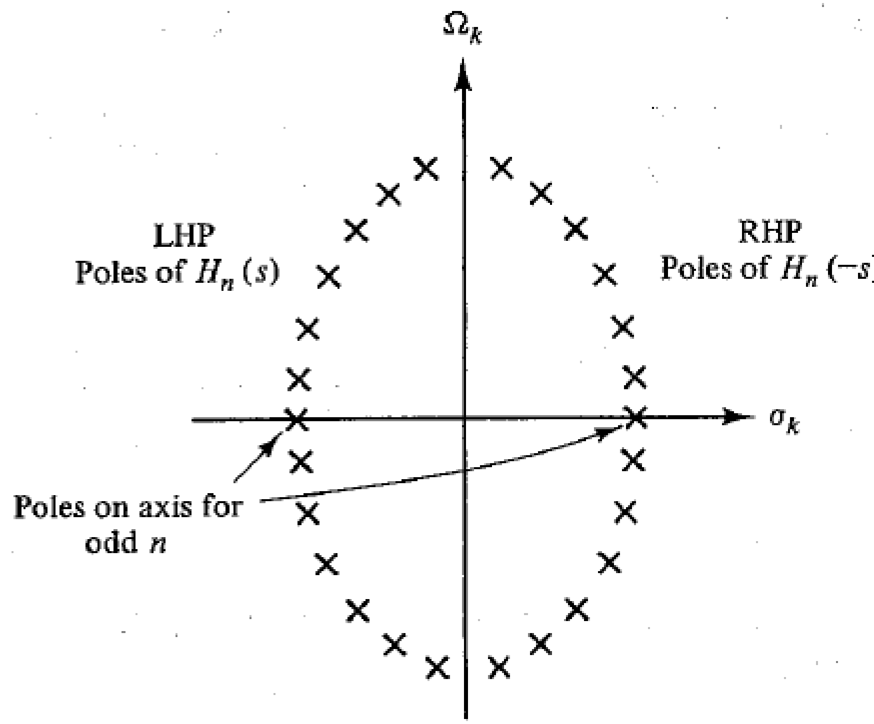
Filter Chebyshev (5)

- **Sifat filter Chebyshev type 1**:
- Didaerah pass-band: $|H_n(j\Omega)|^2$ berosilasi antara 1 dan $\frac{1}{(1+\varepsilon^2)}$, disebut equiripple, dan pada frekuensi cutoff Ω = 1 nilainya $\frac{1}{(1+\varepsilon^2)}$.
- Didaerah band transisi dan stop-band: $|H_n(j\Omega)|^2$ nilainya monoton turun. Stop-band dimulai di Ω_r, dimana $|H_n(j\Omega_r)|^2 = \frac{1}{A^2}$.
- **Fungsi transfer filter**:
- $H(s)$ harus stabil dan kausal, maka pole-pole $H_n(s)$ harus berada disebelah kiri sumbu $j\Omega$ pada bidang s.
- Pole-pole dari $H_n(s)H_n(-s)$ dihitung dari akar penyebut:

$$1 + \varepsilon^2 {T_n}^2\left(\frac{s}{j}\right) = 0$$

Filter Chebyshev (6)

- Tempat kedudukan pole-pole:
- $H_n(s)H_n(-s)$
- Bila pole: $s_k = \sigma_k + j\Omega_k$



- Memenuhi persamaan:
- $\frac{{\sigma_k}^2}{a^2} + \frac{{\Omega_k}^2}{b^2} = 1$
- dimana

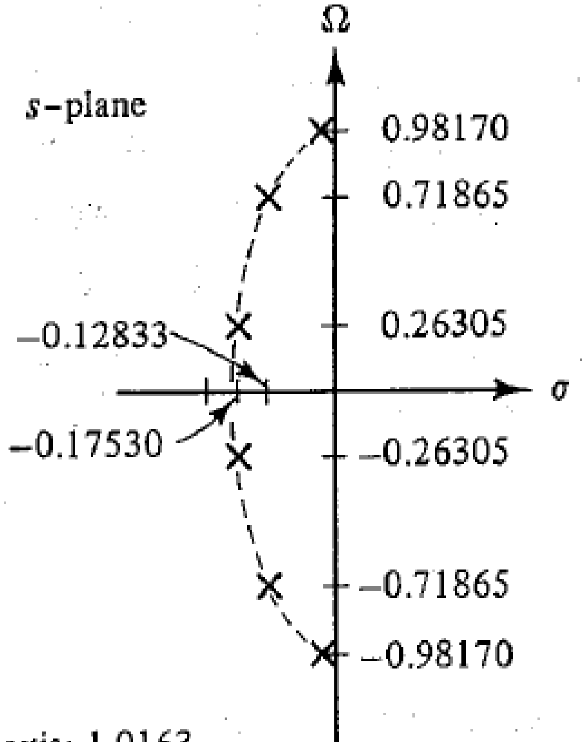
$$a = \frac{1}{2}\left\{\frac{1 + \sqrt{1 + \varepsilon^2}}{\varepsilon}\right\}^{\frac{1}{n}} - \frac{1}{2}\left\{\frac{1 + \sqrt{1 + \varepsilon^2}}{\varepsilon}\right\}^{-\frac{1}{n}}$$

$$b = \frac{1}{2}\left\{\frac{1 + \sqrt{1 + \varepsilon^2}}{\varepsilon}\right\}^{\frac{1}{n}} + \frac{1}{2}\left\{\frac{1 + \sqrt{1 + \varepsilon^2}}{\varepsilon}\right\}^{-\frac{1}{n}}$$

Filter Chebyshev (7)

- $\sigma_k = -a \sin\left[\frac{(2k-1)\pi}{2K}\right], \quad k = 1,.,2n$
- $\Omega_k = b \cos\left[\frac{(2k-1)\pi}{2K}\right], \quad k = 1,.,2n$

- Pole $H(s)$
- $n = 6$
- $\varepsilon = 0{,}7647831$



Major axis: 1.0163
Minor axis: −0.181637

$$\sigma_k = -a \sin\left[\frac{(2k-1)}{2n}\pi\right]$$

$$\Omega_k = b \cos\left[\frac{(2k-1)}{2n}\pi\right]$$

k	σ_k	Ω_k
1	−0.0469732	0.9817052
2	−0.1283332	0.7186581
3	−0.1753064	0.2630471
4	−0.1753064	−0.2630471
5	−0.1283332	−0.7186581
6	−0.0469732	−0.9817052

Filter Chebyshev (8)

- Fungsi transfer filter $H(s)$ harus stabil dan kausal, maka pole-pole $H_n(s)$ harus berada disebelah kiri sumbu $j\Omega$ pada bidang s.

$$H_n(s) = \frac{K}{\prod_{pole\ seb\ kiri}(s - s_k)} = \frac{K}{V_n(s)}$$

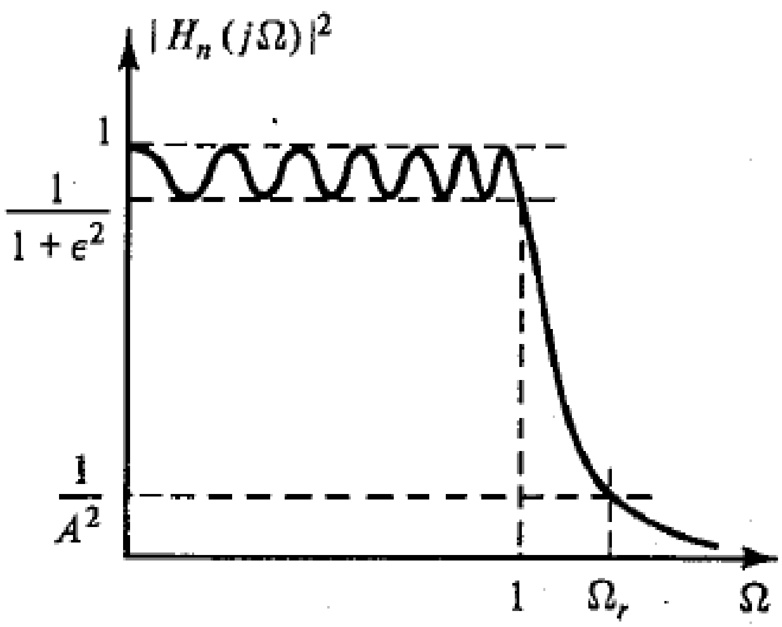
- Dimana s_k adalah pole-pole $H_n(s)H_n(-s)$ yang berada disebelah kiri sumbu $j\Omega$ pada bidang s.
- K adalah faktor normalisasi, yang membuat nilai:

$$H(0) = \begin{cases} 1 & n\ ganjil \\ \dfrac{1}{\sqrt{1+\varepsilon^2}} & n\ genap \end{cases} \rightarrow K = \begin{cases} V_n(0) = b_0 & n\ ganjil \\ \dfrac{V_n(0)}{\sqrt{1+\varepsilon^2}} & n\ genap \end{cases}$$

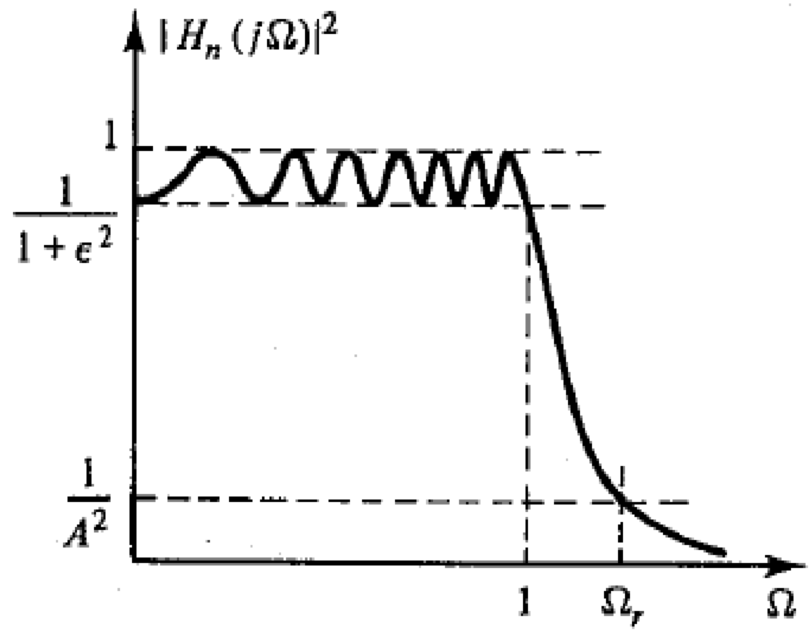
- $V_n(s) = s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0$

Filter Chebyshev (9)

- Derajat filter: $n = \left\lceil \dfrac{\log_{10}\left[g+\sqrt{g^2-1}\right]}{\log_{10}\left[\Omega_r+\sqrt{\Omega_r{}^2-1}\right]} \right\rceil$
- Dimana $A = \dfrac{1}{|H_n(j\Omega_r)|}$ dan $g = \sqrt{\dfrac{A^2-1}{\varepsilon^2}}$
- n ganjil



n genap



Chebyshev filters

$$|H_n(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 T_n^2(\Omega)}, \quad H_n(s) = \frac{K_n}{V_n(s)}, \quad K_n = \begin{cases} \dfrac{b_0}{\sqrt{1+\varepsilon^2}} & \text{untuk n genap} \\ b_0 & \text{untuk n ganjil} \end{cases}$$

$$V_n(s) = s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0$$

0.5 dB ripple, $\varepsilon = 0.3493114$, $\varepsilon^2 = 0.1220184$						
n	b_0	b_1	b_2	b_3	b_4	b_5
1	2.8627752					
2	1.5162026	1.4256245				
3	0.7156938	1.5348954	1.2529130			
4	0.3790506	1.0254553	1.7168662	1.1973856		
5	0.1789234	0.7525181	1.3095747	1.9373675	1.1724909	
6	0.0947626	0.4323669	1.1718613	1.5897635	2.1718446	1.1591761

$$H_1(s) = \frac{2.8627752}{s + 2.8627752}$$

$$H_2(s) = \frac{1.5162026}{\sqrt{1.1220184}\left(s^2 + 1.4256245s + 1.5162026\right)}$$

Chebyshev filters

$$\left|H_n(j\Omega)\right|^2 = \frac{1}{1+\varepsilon^2 T_n^2(\Omega)},\quad H_n(s) = \frac{K_n}{V_n(s)},\quad K_n = \begin{cases} \dfrac{b_0}{\sqrt{1+\varepsilon^2}} & \text{untuk n genap} \\ b_0 & \text{untuk n ganjil} \end{cases}$$

$$V_n(s) = s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0$$

1 dB ripple, $\varepsilon = 0.5088471,\ \varepsilon^2 = 0.2589254$						
n	b_0	b_1	b_2	b_3	b_4	b_5
1	1.9652267					
2	1.1025103	1.0977343				
3	0.4913067	1.2384092	0.9883412			
4	0.2756276	0.7426194	1.4539248	0.9528114		
5	0.1228267	0.5805342	0.9743961	1.6888160	0.9368201	
6	0.0689069	0.3070808	0.9393461	1.2021409	1.9308256	0.9282510

$$H_1(s) = \frac{1.9652267}{s + 1.9652267}$$

$$H_2(s) = \frac{1.1025103}{\sqrt{1.2589254}\left(s^2 + 1.0977343s + 1.1025103\right)}$$

Chebyshev filters

$$|H_n(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 T_n^2(\Omega)}, \; H_n(s) = \frac{K_n}{V_n(s)}, \; K_n = \begin{cases} \dfrac{b_0}{\sqrt{1+\varepsilon^2}} & \text{untuk n genap} \\ b_0 & \text{untuk n ganjil} \end{cases}$$

$$V_n(s) = s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0$$

2 dB ripple, $\varepsilon = 0.7647831$, $\varepsilon^2 = 0.5848932$						
n	b_0	b_1	b_2	b_3	b_4	b_5
1	1.3075603					
2	0.6367681	0.8038164				
3	0.3268901	1.0221903	0.7378216			
4	0.2057651	0.5167981	1.2564819	0.7162150		
5	0.0817225	0.4593491	0.6934770	1.4995433	0.7064606	
6	0.0514413	0.2102706	0.7714618	0.8670149	1.7458587	0.7012257

$$H_1(s) = \frac{1.3075603}{s + 1.3075603}$$

$$H_2(s) = \frac{0.6367681}{\sqrt{1.5848932}\left(s^2 + 0.8038164s + 0.6367681\right)}$$

Filter Chebyshev

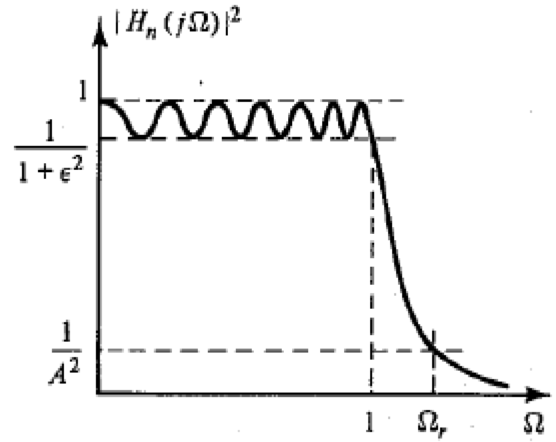
$$H_n(s) = \frac{K_n}{V_n(s)}\text{, Zero polinomial } V_n(s) = s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0$$

0.5 dB ripple, $\varepsilon = 0.3493114$, $\varepsilon^2 = 0.1220184$					
$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
$-2{,}8627752$	$-0{,}7128122 \pm j1{,}0040425$	$-0{,}6264565$	$-0{,}1753531 \pm j1{,}0162529$	$-0{,}3623196$	$-0{,}0776501 \pm j1{,}0084608$
		$-0{,}3132282 \pm j1{,}0219275$	$-0{,}4233398 \pm j0{,}4209457$	$-0{,}1119629 \pm j1{,}0115574$	$-0{,}2121440 \pm 0{,}7382446$
				$-0{,}2931227 \pm j0{,}6251768$	$-0{,}2897940 \pm j0{,}2702162$

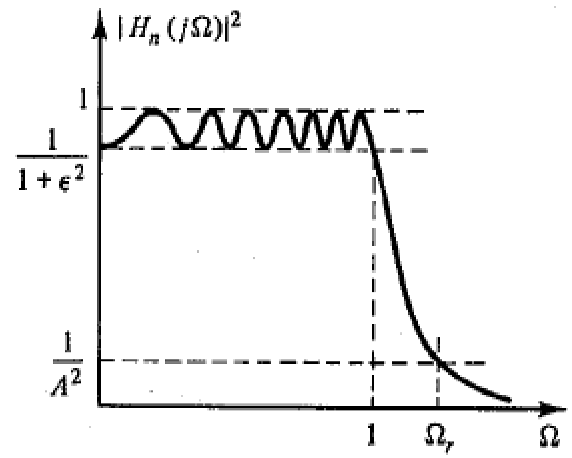
2 dB ripple, $\varepsilon = 0.7647831$, $\varepsilon^2 = 0.5848932$					
$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
$-1{,}3075603$	$-0{,}4019082 \pm j0{,}6893750$	$-0{,}3689108$	$-0{,}1048872 \pm j0{,}9579530$	$-0{,}2183083$	$-0{,}0469732 \pm j0{,}9817052$
		$-0{,}1844554 \pm j0{,}9230771$	$-0{,}2532202 \pm j0{,}3967971$	$-0{,}0674610 \pm j0{,}9734557$	$-0{,}1283332 \pm 0{,}7186581$
				$-0{,}1766151 \pm j0{,}6016287$	$-0{,}1753064 \pm j0{,}2630471$

Contoh no 1 (1)

- **Rancang filter low-pass Chebyshev, bandwidth 1 rad/det dengan spesifikasi sebagai berikut**:



- Ripple di passband adalah 2 dB.
- Frekuensi cutoff 1 rad/det.
- Redaman pada stop-band (diluar 1,3 rad/det) ≥ 20 dB.
- **Solusi**:



- $20\log|H_n(j1)| = 20\log\sqrt{\frac{1}{1+\varepsilon^2}}$
- $20\log|H_n(j1)| = -2$ dB.
- $20\log|H_n(j1{,}3)| = 20\log\sqrt{\frac{1}{A^2}}$
- $20\log|H_n(j1{,}3)| = -20$ dB.

Contoh no 1 (2)

- Diperoleh: $\varepsilon = 0{,}76478$, dan $A = 10$
- $g = \sqrt{\frac{A^2-1}{\varepsilon^2}} = \sqrt{\frac{100-1}{(0{,}76478)^2}} = 13{,}01$
- $n = \left\lceil \frac{\log_{10}\left[g+\sqrt{g^2-1}\right]}{\log_{10}\left[\Omega_r+\sqrt{\Omega_r{}^2-1}\right]} \right\rceil = \left\lceil \frac{\log_{10}\left[13.01+\sqrt{(13{,}01)^2-1}\right]}{\log_{10}\left[1{,}3+\sqrt{(1{,}3)^2-1}\right]} \right\rceil = \lceil 4{,}3 \rceil = 5$

$$H_5(s) = \frac{K}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$

$$H_5(s) = \frac{0{,}08172}{s^5 + 0{,}70646 s^4 + 1{,}4995 s^3 + 0{,}6934 s^2 + 0{,}45935 s + 0{,}08172}$$

Contoh no 1 (3)

- Dengan memakai nilai-nilai pole orde 5, dan $\varepsilon = 0{,}76478$

2 dB ripple, $\varepsilon = 0.7647831$, $\varepsilon^2 = 0.5848932$					
$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
$-1{,}3075603$	$-0{,}4019082 \pm j0{,}6893750$	$-0{,}3689108$	$-0{,}1048872 \pm j0{,}9579530$	$-0{,}2183083$	$-0{,}0469732 \pm j0{,}9817052$
		$-0{,}1844554 \pm j0{,}9230771$	$-0{,}2532202 \pm j0{,}3967971$	$-0{,}0674610 \pm j0{,}9734557$	$-0{,}1283332 \pm 0{,}7186581$
				$-0{,}1766151 \pm j0{,}6016287$	$-0{,}1753064 \pm j0{,}2630471$

- Diperoleh bentuk kuadratik:

$$H_5(s) = \frac{0{,}08172}{(s+0{,}218303)(s^2+0{,}134922s+0{,}95215)(s^2+0{,}35323s+0{,}393115)}$$

Referensi:

1. Fundamentals of Digital Signal Processing; Lonnie C. Ludeman; John Wiley & Sons, Inc. 1987. Chapter 3.
2. Passive and Active Filters, Theory and Implementations; Wai-Kai Chen; John Wiley & Sons, Inc. 1986. Chapter 2.
3. Active and Passive Analog Filter Design; Lawrence P Huelsman; McGraw-Hill International Editions, 1992. Chapter 2.

- **Bab 6. Pengantar Filter Analog.**
- **Selesai.**