1. Diketahui $A=\begin{pmatrix}1 & 2 & 3\\ 0 & -1 & 1\\ -1 & 2 & -2\end{pmatrix}$, $B=\begin{pmatrix}x & x+y & y+z\\ z-a & b & b+2c\\ x+d & y-e & e+f\end{pmatrix}$, dan $C=\begin{pmatrix}-a & 2b & x\\ 2c-1 & y & -d\\ e-y & x+c & b+y\end{pmatrix}$. Jika $A=B$, maka tentukan $A^{-1}(BC^T)$!

$A = B$

$$\begin{pmatrix}1 & 2 & 3\\ 0 & -1 & 1\\ -1 & 2 & -2\end{pmatrix} = \begin{pmatrix}x & x+y & y+z\\ z-a & b & b+2c\\ x+d & y-e & e+f\end{pmatrix}$$

$x = 1$

$x + y = 2$

$1 + y = 2$

$y = 1$

$y + z = 3$

$z = 2$

$z - a = 0$

$z = a$

$a = 2$

$b = -1$

$b + 2c = 1$

$c = 1$

$x + d = -1$

$d = -2$

$y - e = 2$

$e = -1$

$e + f = -2$

$f = -1$

$$C = \begin{bmatrix}-a & 2b & x\\ 2c-1 & y & -d\\ e-y & x+c & b+y\end{bmatrix} = \begin{bmatrix}-2 & -2 & 1\\ 1 & 1 & 2\\ -2 & 2 & 0\end{bmatrix} \rightarrow C^T = \begin{bmatrix}-2 & 1 & -2\\ -2 & 1 & 2\\ 1 & 2 & 0\end{bmatrix}$$

$$A^{-1}(BC^T) = A^{-1} \cdot A \cdot C^T = \begin{bmatrix}-2 & 1 & -2\\ -2 & 1 & 2\\ 1 & 2 & 0\end{bmatrix}$$

3. Tentukan solusi dari SPL berikut dengan menggunakan salah satu metode yang Anda ketahui.

$$\begin{aligned} 2x - 4y + z &= 0 \\ -4x + 3z &= 0 \\ y - z &= 0 \end{aligned}$$

$$\begin{aligned} 2x - 4y + z &= 0 \\ -4x + 3z &= 0 \\ y - z &= 0 \end{aligned} \rightarrow \begin{bmatrix} 2 & -4 & 1 \\ -4 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Metode cramer

$$D = \begin{vmatrix} 2 & -4 & 1 \\ -4 & 0 & 3 \\ 0 & 1 & -2 \end{vmatrix} = (2.0.-2) + (-4.3.0) + (1.-4.1) - (1.0.0) - (-4.-4.-2) - (2.3.1)$$

$D = 22 \rightarrow D \neq 0$

$$D_x = \begin{vmatrix} 0 & -4 & 1 \\ 0 & 0 & 3 \\ 0 & 1 & -2 \end{vmatrix} = 0 + 0 + 0 - 0 - 0 - 0 = 0$$

$$D_y = \begin{vmatrix} 2 & 0 & 1 \\ -4 & 0 & 3 \\ 0 & 0 & 2 \end{vmatrix} = 0 + 0 + 0 - 0 - 0 - 0 = 0$$

$$D_z = \begin{vmatrix} 2 & -4 & 0 \\ -4 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0 + 0 + 0 - 0 - 0 - 0 = 0$$

$$x = \frac{D_x}{D} = \frac{0}{22} = 0 \qquad y = \frac{D_y}{D} = \frac{0}{22} = 0 \qquad z = \frac{D_z}{D} = \frac{0}{22} = 0$$

Metode OBE

$$\left[\begin{array}{cc|c} 2 & -4 & 0 \\ -4 & 3 & 0 \\ 0 & -1 & 0 \end{array}\right] \overset{A \quad B}{} \quad b_2 \leftrightarrow b_3 \sim \left[\begin{array}{cc|c} 2 & -4 & 0 \\ 0 & -1 & 0 \\ -4 & 3 & 0 \end{array}\right] \begin{matrix} \\ -b_2 \\ 2b_1 + b_3 \end{matrix} \sim \left[\begin{array}{cc|c} 2 & -4 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 0 \end{array}\right] \begin{matrix} 4b_2 + b_1 \\ 5b_2 + b_3 \end{matrix} \sim$$

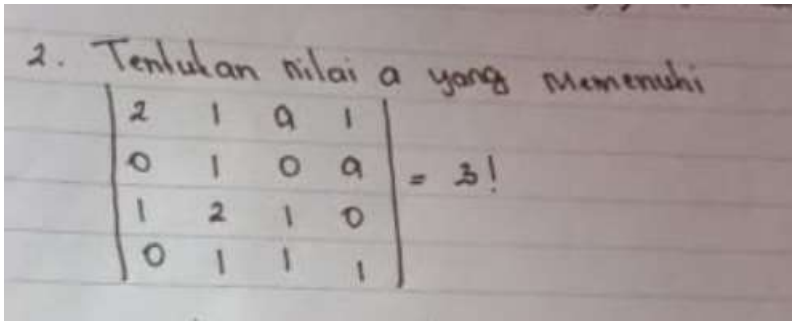
$$\left[\begin{array}{ccc|c} 2 & 0 & -7 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -11 & 0 \end{array}\right] -\frac{1}{11} b_3 \sim \left[\begin{array}{ccc|c} 2 & 0 & -7 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{matrix} 2b_3 + b_2 \\ 7b_3 + b_1 \end{matrix} \sim \left[\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \frac{1}{2} b_1$$

$$\left[\begin{array}{ccc|c} 1x & 0 & 0 & 0 \\ 0 & 1y & 0 & 0 \\ 0 & 0 & 1z & 0 \end{array}\right] \qquad \begin{aligned} x &= 0 \\ y &= 0 \\ z &= 0 \end{aligned}$$

Metode invers matriks → det, adj (A)

↓

$C_A^T \rightarrow C_A$



$$\begin{vmatrix} 2 & 1 & a & 1 \\ 0 & 1 & 0 & a \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{vmatrix} = 3$$

$\det = 3$

$a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23} + a_{24}C_{24} = 3$

$$0 \cdot C_{21} + 1 \cdot (-1)^{2+2} \begin{vmatrix} 2 & a & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} + 0 \cdot C_{23} + a \cdot (-1)^{2+4} \begin{vmatrix} 2 & 1 & a \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 3$$

$$0 + (2 + 0 + 1 - 0 - a0) + 0 + a(4 + 0 + a - 0 - 1 - 2) = 3$$

$$3 - a + a(a + 1) = 3$$

$$3 - a + a^2 + a = 3$$

$$a^2 + 3 = 3$$

$$a^2 = 0$$

$$a = 0$$