4. $\frac{dx}{dt} = x + y - z$ $\qquad x' = A.x$

$\frac{dy}{dt} = 2y$ $\qquad x' = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix} x$

$\frac{dz}{dt} = y - z$

$$\det(A - \lambda I) = \left| \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 1 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = \begin{vmatrix} 1-\lambda & 1 & -1 \\ 0 & 2-\lambda & 0 \\ 0 & 1 & -1-\lambda \end{vmatrix}$$

$$0 = (1-\lambda)(2-\lambda)(-1-\lambda) + 0 + 0 - 0 - 0 - 0$$

$$0 = (1-\lambda)(2-\lambda)(-1-\lambda)$$

$$\lambda_1 = 1 \qquad \lambda_2 = 2 \qquad \lambda_3 = -1$$

Untuk $\lambda_1 = 1$

$(A - \lambda_1 I) K_1 = 0$

$$\begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = 0 \rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix} \begin{array}{l} -b_2 + b_1 \\ -b_2 + b_3 \\ \sim \end{array} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{array}{l} -b_1 \\ \sim \end{array}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{array}{l} 2b_1 + b_3 \\ \sim \end{array} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = 0 \rightarrow \begin{array}{l} k_1 = 0 \\ k_2 = 0 \\ k_3 = 0 \end{array} \rightarrow K_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Untuk $\lambda_2 = 2$

$(A - \lambda_2 I) K_2 = 0$

$$\begin{bmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow$$

$-k_1 + 2k_3 = 0$

$2k_3 = k_1$

$k_2 - 3k_3 = 0$

$k_2 = 3k_3$

$k_3 = 1$

$$K_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

Untuk $\lambda_3 = -1$

$(A - \lambda_3 I) K_3 = 0$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow$$

$k_2 = 0$

$2k_1 - k_3 = 0$

$2k_1 = k_3$

$$K_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$X = C_1 K_1 e^{\lambda_1 t} + C_2 K_2 e^{\lambda_2 t} + C_3 K_3 e^{\lambda_3 t}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C_1 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} e^{t} + C_2 \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} e^{2t} + C_3 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} e^{-t}$$

$$X(t) = 2C_2 e^{2t} + C_3 e^{-t}$$
$$Y(t) = 3C_2 e^{2t}$$
$$Z(t) = C_2 e^{2t} + 2C_3 e^{-t}$$

5.
$$X' = \begin{bmatrix} 10 & -5 \\ 8 & -12 \end{bmatrix} X$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 10-\lambda & -5 \\ 8 & -12-\lambda \end{vmatrix} = 0$$

$$(10-\lambda)(-12-\lambda) - (-5 \cdot 8) = 0$$

$$-120 - 10\lambda + 12\lambda + \lambda^2 + 40 = 0$$
$$\lambda^2 + 2\lambda - 80 = 0$$
$$(\lambda + 10)(\lambda - 8) = 0$$
$$\lambda_1 = -10 \qquad \lambda_2 = 8$$

Untuk $\lambda_1 = -10$

$$(A - \lambda_1 I)K_1 = 0$$

$$\begin{bmatrix} 20 & -5 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$$20k_1 - 5k_2 = 0 \qquad\qquad 8k_1 - 2k_2 = 0$$
$$4k_1 = k_2 \qquad\qquad 4k_1 = k_2 \to k_1 = 1$$

$$K_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Untuk $\lambda_2 = 8$

$(A - \lambda_2 I) K_2 = 0$

$$\begin{bmatrix} 2 & -5 \\ 8 & -16 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -5 \\ 8 & -16 \end{bmatrix} \underset{\sim}{\tfrac{1}{8} b_2} \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix} \underset{\sim}{-2b_2 + b_1} \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} \underset{\sim}{-b_1 + b_2}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} \underset{\sim}{-b_2 + b_1} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \underset{\sim}{-b_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{array}{l} k_1 = 0 \\ k_2 = 0 \end{array} \rightarrow K_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$X = C_1 K_1 e^{\lambda_1 t} + C_2 K_2 e^{\lambda_2 t}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} e^{-10t} + C_2 \begin{bmatrix} 0 \\ 0 \end{bmatrix} e^{8t}$$

$$x(t) = C_1 e^{-10t}$$

$$Y(t) = 4C_1 e^{-10t}$$