M. Hasyim Abdillah P.
1101191095 / TT-43-11

Tugas Pra Kuis

1. $A = \begin{bmatrix} 8 & -5 \\ 3 & -2 \end{bmatrix}$; $B = \begin{bmatrix} x & 2 \\ 3 & 2 \end{bmatrix}$; $C = \begin{bmatrix} 9 & 3y+5 \\ 3 & 4 \end{bmatrix}$

$$A \cdot B = A + C$$

$$\begin{bmatrix} 8 & -5 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 9 & 3y+5 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 8x-15 & 6 \\ 3x-6 & 2 \end{bmatrix} = \begin{bmatrix} 17 & 3y \\ 6 & 2 \end{bmatrix}$$

$8x - 15 = 17$ $\quad 3x - 6 = 6$ $\quad 6 = 3y$ $\quad x + y = 4 + 2$

$8x = 32$ $\quad 3x = 12$ $\quad y = 2$ $\quad = \underline{\underline{6}}$

$x = 4$ $\quad x = 4$

2. $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 5 \end{bmatrix}$; $B = A^{-1}$; ordo $A = 3 \times 3$

$|A| = (1.1.5) = 5$; $|B| = \frac{1}{|A|} = \frac{1}{5}$; $|A^T| = |A| = 5$

$$x = \frac{|2A^2| - |5B|}{|A^T B|}$$

$$= \frac{2^3 \cdot |A|^2 - 5^3 |B|}{|A^T| \cdot |B|}$$

$$= \frac{8 \cdot 5^2 - 5^{\cancel{3}2} \cdot \cancel{5}^{-1}}{\cancel{5} \cdot \cancel{5^{-1}}}$$

$$= (8-1)\,5^2$$

$$= 7 \cdot 25 = \underline{\underline{175}}$$

3.

$$R = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$$

a.

$$\det(R) = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{vmatrix} \quad \begin{matrix} -5b_1 + b_2 \\ b_1 + b_3 \\ -2b_1 + b_4 \\ \sim \end{matrix}$$

$$\det(R) = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 12 & 0 & -1 \end{vmatrix} \quad \begin{matrix} -12 b_2 + b_4 \\ \sim \end{matrix}$$

$$\det(R) = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 108 & 23 \end{vmatrix} \quad \begin{matrix} 36 b_3 + b_4 \\ \sim \end{matrix}$$

$$\det(R) = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & -13 \end{vmatrix}$$

$$\det(R) = 1 . 1 . -3 . -13 = \underline{\underline{39}}$$

b.

$$\det(R) = a_{11} C_{11} + a_{12} C_{12} + a_{13} C_{13} + a_{14} C_{14}$$

$$= 1. \begin{vmatrix} -9 & 6 & 3 \\ 2 & -6 & -2 \\ 8 & 6 & 1 \end{vmatrix} - (-2). \begin{vmatrix} 5 & 6 & 3 \\ -1 & -6 & -2 \\ 2 & 6 & 1 \end{vmatrix} + 3. \begin{vmatrix} 5 & -9 & 3 \\ -1 & 2 & -2 \\ 2 & 8 & 1 \end{vmatrix} -$$

$$1. \begin{vmatrix} 5 & -9 & 6 \\ -1 & 2 & -6 \\ 2 & 8 & 6 \end{vmatrix}$$

$\det(R) = 1.(18) + 2.(30) + 3.(81) - 1.(282)$

$\det(R) = 39$

4. $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 50 \rightarrow |A|$

$$|B| = 3 \begin{vmatrix} 2a & 2b & 2c \\ d-a & e-b & f-c \\ g+2a & h+2b & i+2c \end{vmatrix} \begin{matrix} -b_1 + b_3 \\ \sim \end{matrix}$$

$$|B| = 3 \begin{vmatrix} 2a & 2b & 2c \\ d-a & e-b & f-c \\ g & h & i \end{vmatrix} \begin{matrix} \frac{1}{2} b_1 \\ \sim \end{matrix}$$

$$\frac{1}{2}|B| = 3 \begin{vmatrix} a & b & c \\ d-a & e-b & f-c \\ g & h & i \end{vmatrix} -b_1 + b_2$$

$$|B| = 2.3. \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$|B| = 2.3.50$

$|B| = 300$

5. $A.X = B$

$$A = \begin{bmatrix} k & 5 & 5 \\ -1 & -1 & 0 \\ k & 2k & 3 \end{bmatrix} ; \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} ; \quad B = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$|A| = -1$

$(k.-1.3) + (5.0.k) + (5.-1.2k) - (5.-1.k) - (5.-1.3) - (k.0.2k) = -1$

$-3k + 0 - 10k + 5k + 15 - 0 = -1$

$-8k = -16$

$k = 2$

$$A = \begin{bmatrix} k & 5 & 5 \\ -1 & -1 & 0 \\ k & 2k & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 5 & 5 & 1 \\ -1 & -1 & 0 & 1 \\ 2 & 4 & 3 & -1 \end{array}\right] \underset{\sim}{b_1 \leftrightarrow b_2} \begin{bmatrix} -1 & -1 & 0 & 1 \\ 2 & 5 & 5 & 1 \\ 2 & 4 & 3 & -1 \end{bmatrix} \begin{matrix} 2b_1 + b_2 \\ 2b_1 + b_3 \\ \sim \end{matrix} \begin{bmatrix} -1 & -1 & 0 & 1 \\ 0 & 3 & 5 & 3 \\ 0 & 2 & 3 & 1 \end{bmatrix} \begin{matrix} -b_1 \\ -b_3 + b_2 \\ \sim \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 1 & 2 & 2 \\ 0 & 2 & 3 & 1 \end{bmatrix} \begin{matrix} -b_2 + b_1 \\ -2b_2 + b_3 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & -2 & -3 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & -1 & -3 \end{bmatrix} \begin{matrix} -2b_3 + b_1 \\ 2b_3 + b_2 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & -1 & -3 \end{bmatrix} \begin{matrix} -b_3 \\ \sim \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right] \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 3 \end{bmatrix} \to \begin{matrix} x_1 = 3 \\ x_2 = -4 \\ x_3 = 3 \end{matrix}$$

6. $2a - b = 1 - c$

$2b + c = a + d - 2$

$3a + 3c = d$

$\downarrow$

$$\begin{matrix} 2a - b + c = 1 \\ -a + 2b + c - d = -2 \\ 3a + 3c - d = 0 \end{matrix} \to \begin{bmatrix} 2 & -1 & 1 & 0 \\ -1 & 2 & 1 & -1 \\ 3 & 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & 0 & 1 \\ -1 & 2 & 1 & -1 & -2 \\ 3 & 0 & 3 & -1 & 0 \end{array}\right] \underset{\sim}{b_1 \leftrightarrow b_2} \begin{bmatrix} -1 & 2 & 1 & -1 & -2 \\ 2 & -1 & 1 & 0 & 1 \\ 3 & 0 & 3 & -1 & 0 \end{bmatrix} \begin{matrix} 2b_1 + b_2 \\ 3b_1 + b_3 \\ \sim \end{matrix}$$

$$\begin{bmatrix} -1 & 2 & 1 & -1 & -2 \\ 0 & 3 & 3 & -2 & -3 \\ 0 & 6 & 6 & -4 & -6 \end{bmatrix} \underset{\sim}{-2b_2 + b_3} \begin{bmatrix} -1 & 2 & 1 & -1 & -2 \\ 0 & 3 & 3 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} -b_1 \\ \sim \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & -1 & 1 & 2 \\ 0 & 3 & 3 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \underset{\sim}{\tfrac{1}{3} b_2} \begin{bmatrix} 1 & -2 & -1 & 1 & 2 \\ 0 & 1 & 1 & -2/3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \underset{\sim}{2b_2 + b_1}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1/3 & 0 \\ 0 & 1 & 1 & -2/3 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \to \begin{bmatrix} 1 & 0 & 1 & -1/3 \\ 0 & 1 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} a + c - \tfrac{1}{3} d &= 0 \\ b + c - \tfrac{2}{3} d &= -1 \end{aligned} \longrightarrow \begin{aligned} a &= -c + \tfrac{1}{3} d \\ b &= -c + \tfrac{2}{3} d - 1 \end{aligned} \qquad \begin{aligned} c &= t \\ d &= u \end{aligned}$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 1/3 \\ 2/3 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

7.

$$\begin{aligned} 2a - 2b + c + d &= 0 \\ a - b + 2d &= 0 \\ -a + b + c - d &= 0 \end{aligned} \to \begin{bmatrix} 2 & -2 & 1 & 1 \\ 1 & -1 & 0 & 2 \\ -1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 2 & -2 & 1 & 1 & 0 \\ 1 & -1 & 0 & 2 & 0 \\ -1 & 1 & 1 & -1 & 0 \end{array}\right] \underset{\sim}{\begin{matrix} b_3 + b_1 \\ b_3 + b_2 \end{matrix}} \left[\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ -1 & 1 & 1 & -1 & 0 \end{array}\right] \underset{\sim}{b_1 + b_3}$$

$$\begin{bmatrix} 1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & -1 & 0 \end{bmatrix} \underset{\sim}{b_2 + b_3} \begin{bmatrix} 1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 \end{bmatrix} \underset{\sim}{\tfrac{1}{4} b_3}$$

$$\begin{bmatrix} 1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \underset{\sim}{\begin{matrix} -2b_3 + b_1 \\ -b_3 + b_2 \end{matrix}} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \underset{\sim}{b_2 \leftrightarrow b_3}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \rightarrow \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$a - b = 0 \rightarrow a = b \rightarrow b = t$

$c = 0$

$d = 0$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} t$$

8. $a + 2b + c = 0$

$2a - kb + 2c = 0$

$-3ka - b + c = 0$

$$\rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & -k & 2 \\ -3k & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & -k & 2 & 0 \\ -3k & -1 & 1 & 0 \end{array}\right] \underset{\sim}{-2b_1 + b_2} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -k-4 & 0 & 0 \\ -3k & -1 & 1 & 0 \end{bmatrix} \underset{\sim}{b_2 \leftrightarrow b_3} \begin{bmatrix} 1 & 2 & 1 & 0 \\ -3k & -1 & 1 & 0 \\ 0 & -k-4 & 0 & 0 \end{bmatrix}$$

Agar memiliki solusi tak hingga banyak, maka :

$-k - 4 = 0$

$-k = 4$

$k = -4$

Substitusi kembali $k = -2$:

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -3(-4) & -1 & 1 & 0 \\ 0 & -(-4)-4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 12 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underset{\sim}{-12b_1 + b_2} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -25 & -11 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} -\frac{1}{25}b_2$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{11}{25} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underset{\sim}{-2b_2 + b_1} \left[\begin{array}{ccc|c} 1 & 0 & 3/25 & 0 \\ 0 & 1 & 11/25 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 3/25 \\ 0 & 1 & 11/25 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} a + \frac{3}{25}c &= 0 \\ b + \frac{11}{25}c &= 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} a &= -\frac{3}{25}c \\ b &= -\frac{11}{25}c \\ c &= t \end{aligned}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -3/25 \\ -11/25 \\ 1 \end{bmatrix} t$$