SOAL RESPONSI UTS ELMAG

1.) Diketahui vektor $\vec{A} = 3a_r - 7a_\theta + 2a_\phi$ memiliki titik pangkal pada titik $(1, \frac{\pi}{2}, 0)$ Pada koordinat bola dan vektor $\vec{B} = -2a_r - 4a_\theta + 2a_\phi$ dengan titik pangkal Pada titik $(3, \frac{\pi}{2}, \frac{\pi}{2})$. Tentukan $\vec{A} - \vec{B}$.

Jawab:

* Untuk menjawab soal ini dapat menggunakan cara ⇝ Transformasi koordinat bola menjadi Kartesian.

Vektor A

$\vec{A}_{r,\theta,\phi} = 3a_r - 7a_\theta + 2a_\phi$ dengan titik pangkal $(1, \frac{\pi}{2}, 0)$

Maka $\theta = \frac{\pi}{2}$; $\phi = 0$, sehingga

$$\vec{A}_x = A_r \sin\theta \cos\phi + A_\theta \cos\theta \cos\phi - A_\phi \sin\phi$$

$$\vec{A}_x = 3 \sin\frac{\pi}{2} \cos 0 - 7 \cos\frac{\pi}{2} \cos 0 - 2 \sin 0$$

$$\vec{A}_x = 3.1.1 - 7.0.1 - 2.0$$

$$\vec{A}_x = 3$$

$$\vec{A}_y = A_r \sin\theta \sin\phi + A_\theta \cos\theta \sin\phi + A_\phi \cos\phi$$

$$\vec{A}_y = 3 \sin\frac{\pi}{2} \sin 0 - 7 \cos\frac{\pi}{2} \sin 0 + 2 \cos 0$$

$$\vec{A}_y = 3.1.0 - 7.0.0 + 2.1$$

$$\vec{A}_y = 2$$

$$\vec{A}_z = A_r \cos\theta - A_\theta \sin\theta$$

$$\vec{A}_z = 3 \cos\frac{\pi}{2} + 7 \sin\frac{\pi}{2}$$

$$\vec{A}_z = 3.0 + 7.1$$

$$\vec{A}_z = 7$$

Sehingga didapat $\vec{A}_{x,y,z} = 3a_x + 2a_y + 7a_z$

Vektor B

$\vec{B}_{r,\theta,\phi} = -2a_r - 4a_\theta + 2a_\phi$ dengan titik pangkal $(3, \frac{\pi}{2}, \frac{\pi}{2})$

Maka $\theta = \frac{\pi}{2}$; $\phi = \frac{\pi}{2}$, sehingga

$\vec{B_x} = A_r \sin\theta \cos\phi + A_\theta \cos\theta \cos\phi - A_\phi \sin\phi$

$\vec{B_x} = -2 \sin\frac{\pi}{2} \cos\frac{\pi}{2} - 4 \cos\frac{\pi}{2} \cos\frac{\pi}{2} - 2 \sin\frac{\pi}{2}$

$\vec{B_x} = -2.1.0 - 4.0.0 - 2.1$

$\vec{B_x} = -2$

$\vec{B_y} = A_r \sin\theta \sin\phi + A_\theta \cos\theta \sin\phi + A_\phi \cos\phi$

$\vec{B_y} = -2 \sin\frac{\pi}{2} \sin\frac{\pi}{2} - 4 \cos\frac{\pi}{2} \sin\frac{\pi}{2} + 2 \cos\frac{\pi}{2}$

$\vec{B_y} = -2.1.1 - 4.0.1 + 2.0$

$\vec{B_y} = -2$

$\vec{B_z} = A_r \cos\theta - A_\theta \sin\theta$

$\vec{B_z} = -2 \cos\frac{\pi}{2} + 4 \sin\frac{\pi}{2}$

$\vec{B_z} = -2.0 + 4.1$

$\vec{B_z} = 4$

Sehingga didapat $\vec{B}_{x,y,z} = -2a_x - 2a_y + 4a_z$

Sehingga $\vec{A} - \vec{B} = (3-(-2))a_x + (2-(-2))a_y + (7-4)a_z$

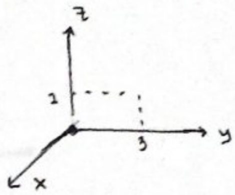
$\vec{A} - \vec{B} = 5a_x + 4a_y + 3a_z$

Dipindai dengan CamScanner

2. Pada sebuah ruang terdapat 2 buah muatan listrik. Muatan listrik pertama berupa muatan titik $Q_1 = 10$ nC terletak pada titik (2,2,2) dan muatan listrik kedua berupa muatan garis yang terdistribusi merata sebesar 5 nC/m, dengan panjang tak hingga yang terletak sepanjang sumbu y. Hitunglah besarnya intensitas medan listrik total pada titik P (0,3,2) !

Diketahui : • Q_1 : 10 nC pada (2,2,2)
• ρ_l : 5 nC/m sepanjang sumbu y

Ditanya : E pada (0, 3, 2)



Jawab

$$E_1 = \frac{k \cdot Q_1}{R^2}\vec{a}$$

$$R = \begin{pmatrix}0\\3\\2\end{pmatrix} - \begin{pmatrix}2\\2\\2\end{pmatrix} = \begin{pmatrix}-2\\1\\0\end{pmatrix}$$

$$= -2\vec{a_x} + \vec{a_y}$$

$$r = \sqrt{(-2)^2 + (1)^2}$$
$$= \sqrt{4+1}$$
$$= \sqrt{5}$$

$$E_1 = \frac{k \cdot Q_1}{R^2}\vec{a}$$

$$= \frac{9\times10^9 \cdot 10\times10^{-9}}{(\sqrt{5})^2} \cdot \left(\frac{-2\vec{a_x} + \vec{a_y}}{\sqrt{5}}\right)$$

$$= \frac{18}{\sqrt{5}}(-2\vec{a_x} + \vec{a_y})$$

$$= \frac{18\sqrt{5}}{5}(-2\vec{a_x} + \vec{a_y})$$

$$= 8.05(-2\vec{a_x} + \vec{a_y})$$

$$= -16,1\,\vec{a_x} + 8,05\,\vec{a_y} \quad \ldots (1)$$

$$E_{\rho l} = \frac{\rho l}{2\pi\varepsilon_0 l} \cdot \vec{a}$$

$$= \frac{5\times10^{-9}}{2\pi \cdot \frac{1}{36\pi} \cdot 10^{-9} \cdot 2} \cdot \frac{2\vec{a_z}}{2}$$

$$= \frac{5}{8/36}(2\vec{a_z})$$

$$= 22,5\,(2\vec{a_z})$$

$$= 45\,\vec{a_z} \quad \ldots (2)$$

→ Maka, E pada (0,3,2)

$$= E_1 + E_{\rho l}$$

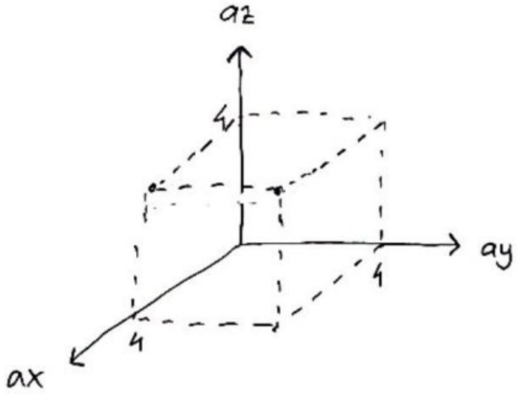
$$= -16,1\,\vec{a_x} + 8,05\,\vec{a_y} + 45\,\vec{a_z} \text{ V/m}$$

Type text here

③ Diketahui vektor B pada kartesian $B = (2x+4)\vec{a_x} + (4-y)\vec{a_y} + 5z\,\vec{a_z}$

Hitunglah jumlah vektor B yang menembus keluar permukaan dengan batas $0 \le x \le 4$, $0 \le y \le 4$, $0 \le z \le 4$

Jawab :



$$\oint \vec{B_0} \cdot \vec{ds} = \underset{\text{atas}}{\int \vec{B} \cdot \vec{ds}} + \underset{\text{depan}}{\int \vec{B} \cdot \vec{ds}} + \underset{\text{bawah}}{\int \vec{B} \cdot \vec{ds}} + \underset{\text{belakang}}{\int \vec{B} \cdot \vec{ds}} + \underset{\text{kanan}}{\int \vec{B} \cdot \vec{ds}} + \underset{\text{kiri}}{\int \vec{B} \cdot \vec{ds}}$$

$$\text{atas} = \int \vec{B} \cdot \vec{ds} = \int_0^4 \int_0^4 3z \, dx\, dy$$

$$= \int_0^4 3xz \Big|_0^4 dy$$

$$= \int_0^4 3z(4) - 3z(0) \, dy$$

$$= \int_0^4 12z \, dy$$

$$= 12yz \Big|_0^4$$

$$= 12z(4) - 12z(0)$$

$$= 48z$$

$$= 48(4)$$

$$= 192$$

$$\text{depan} = \int \vec{B} \cdot \vec{ds} = \int_0^4 \int_0^4 2x + 4 \, dy\, dz$$

$$= \int_0^4 2xy + 4y \Big|_0^4 dz$$

$$= \int_0^4 (2x(4) + 4(4)) - (2x(0) + 4(0)) \, dz$$

$$= \int_0^4 8x + 16 \, dz$$

$$= 8xz + 16z \Big|_0^4$$

$$= (8x(4) + 16(4)) - (8x(0) + 16(0))$$

$$= 32x + 64 = 32(4) + 64 = 192$$

$$\text{kanan} = \int \vec{B} \cdot \vec{ds} = \int_0^4 \int_0^4 4 - y \; dx\, dz$$

$$= \int_0^4 (4x - yx) \Big|_0^4 \; dz$$

$$= \int_0^4 4(4-0) - y(4-0) \; dz$$

$$= \int_0^4 16 - 4y \; dz$$

$$= 16z - 4yz \Big|_0^4$$

$$= 16(4-0) - 4y(4-0)$$

$$= 64 - 16y$$

$$= 64 - 16(4)$$

$$= 0$$

maka :

$$\oint \vec{B}_0 \; \vec{ds} = 192 + 192 + 0$$

$$= 384$$

jadi jumlah vektor B adalah 384

4. Kuat medan listrik statis dinyatakan dalam bentuk vektor $\bar{E} = 2a_r + 3a_\theta + 4a_\phi$ pada koordinat bola. Tentukan rapat muatan volume yang terkait dengan medan listrik tersebut pada titik $(1, \frac{\pi}{2}, \frac{\pi}{2})$

Jawab :

1) Persamaan maxwell 1 bentuk diverensial

$$\bar{\nabla} \cdot \varepsilon \bar{E} = \rho_v$$

$$\varepsilon (\bar{\nabla} \cdot \bar{E}) = \rho_v$$

1) Secara umum :

$$\bar{\nabla} \cdot \bar{E} = \left(\frac{1}{h_1 . h_2 . h_3} \left(\left(\frac{\partial (E_1 . h_2 . h_3)}{\partial u_1} \right) + \left(\frac{\partial (h_1 . E_2 . h_3)}{\partial u_2} \right) + \left(\frac{\partial (h_1 . h_2 . E_3)}{\partial u_3} \right) \right) \right)$$

Koordinat	u_1, u_2, u_3	h_1, h_2, h_3
Kartesian	x, y, z	$1, 1, 1$
Silinder	ρ, ϕ, z	$1, \rho, 1$
Bola	r, θ, ϕ	$1, r, r\sin\theta$

$$\bar{\nabla} \cdot \bar{E} = \rho_v = \left(\frac{1}{r . r\sin\theta} \left(\frac{\partial (2 . r . r\sin\theta)}{\partial r} + \frac{\partial (3r\sin\theta)}{\partial \theta} + \frac{\partial (4r)}{\partial \phi} \right) \right)$$

$$= \frac{1}{r . r\sin\theta} \cdot \frac{\partial (2 . r . r\sin\theta)}{\partial r} + \frac{1}{r . r\sin\theta} \frac{\partial (3 . r\sin\theta)}{\partial \theta} + \frac{1}{r . r\sin\theta} \cdot \frac{\partial (4r)}{\partial \phi}$$

$$= \frac{1}{r^2} \cdot \frac{\partial (2r^2)}{\partial r} + \frac{1}{r\sin\theta} \cdot \frac{\partial (3\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta} \cdot \frac{\partial (4)}{\partial \phi}$$

$$= \frac{1}{r^2} . 4r + \frac{1}{r\sin\theta} . 3\cos\theta + \frac{1}{r\sin\theta} . 0$$

$$= \frac{1}{(1)^2} . 4(1) + \frac{1}{1\sin\frac{\pi}{2}} . 3\cos\frac{\pi}{2} + \frac{1}{1 . \sin\frac{\pi}{2}} . 0$$

$$= 4 + \frac{0}{1} + 0$$

$$= 4$$

∴ Jadi, rapat muatan volume (ρ_v) adalah 4

Dipindai dengan CamScanner

Hukum Maxwell dalam bentuk Diferensial

- Gradien

Jika diketahui fungsi skalar f. maka gradien.

- Kartesian : $\nabla f = \frac{\partial f}{\partial x} a_x + \frac{\partial f}{\partial y} a_y + \frac{\partial f}{\partial z} a_z$
- Silinder : $\nabla f = \frac{\partial f}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \theta} a_\theta + \frac{\partial f}{\partial z} a_z$
- Bola : $\nabla f = \frac{\partial f}{\partial r} a_r + \frac{1}{r}\frac{\partial f}{\partial \theta} a_\theta + \frac{1}{r \sin\theta}\frac{\partial f}{\partial \varphi} a_\varphi$

- Divergensi

Jika diketahui medan vektor. $A = A_1 a_1 + A_2 a_2 + A_3 a_3$. Divergensi A

- Kartesian : $\nabla \cdot A = \frac{\partial}{\partial x} A_1 + \frac{\partial}{\partial y} A_2 + \frac{\partial}{\partial z} A_3$
- Silinder : $\nabla \cdot A = \nabla \cdot A = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{\partial}{\partial y} A_2 + \frac{\partial}{\partial z} A_3$
- Bola : $\nabla f = \frac{\partial f}{d r} a_r + \frac{1}{r}\frac{\partial f}{d \theta} a_\theta + \frac{1}{r \sin\theta}\frac{\partial f}{d \varphi} a_\varphi$

- Curl

Jika diketahui medan vektor $A = A_1 a_1 + A_2 a_2 + A_3 a_3$

- Kartesian : $\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$
- Silinder : $\nabla \times A = \begin{vmatrix} \frac{a_r}{r^2 \sin\theta} & \frac{a_\theta}{r \sin\theta} & \frac{a_\varphi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_1 & r \cdot A_2 & r \cdot \sin\theta \cdot A_3 \end{vmatrix}$
- Bola : $\nabla \times A = \begin{vmatrix} \frac{a_r}{r^2 \sin\theta} & \frac{a_\theta}{r \sin\theta} & \frac{a_\varphi}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_1 & r \cdot A_2 & r \cdot \sin\theta \cdot A_3 \end{vmatrix}$

- Hukum Maxwell dalam bentuk diferensial

- $\nabla \cdot (\varepsilon_0 E) = \rho_v$
- $\nabla \cdot B = 0$
- $\nabla \times \frac{B}{\mu_0} = J + \frac{\partial(\varepsilon_0 E)}{\partial t}$
- $\nabla \times E = 0$
- Dalam konteks medan statis, $\frac{\partial}{\partial t} = 0$