M. Hasyim Abdillah P. 1101191095 TT-43-11

Diketahui $< \vec{u}, \vec{v} >= u_1v_1 + 8u_2v_2$ adalah hasil kali dalam Euclides di R^2 jika $\vec{a} = (1,1),\ \vec{b} = (2,3), \vec{c} = (0,1)$, dan $k = 3$. Maka tentukan

1. $< \vec{a}, \vec{b} >$
2. $< k\vec{a}, \vec{b} >$
3. $< \vec{a} + \vec{b}, \vec{c} >$
4. $|\vec{a}|$

1. $\langle \vec{a}, \vec{b} \rangle = a_1 b_1 + 8 a_2 b_2$

$= 1.2 + 8.1.3$

$= 2 + 24$

$= 26$

2. $\langle k\vec{a}, \vec{b} \rangle = ka_1 . b_1 + 8 ka_2 . b_2$

$= 3.1.2 + 8.3.1.3$

$= 6 + 72$

$= 78$

3. $\langle \vec{a} + \vec{b}, \vec{c} \rangle = \langle (a_1 + b_1, a_2 + b_2), (c_1, c_2) \rangle$

$= (a_1 + b_1) c_1 + 8 (a_2 + b_2) c_2$

$= (1 + 2) 0 + 8 (1 + 3) . 1$

$= 0 + 32$

$= 32$

4. $\|\vec{a}\| = \langle \vec{a}, \vec{a} \rangle^{1/2}$

$= (a_1^2 + 8a_2^2)^{1/2}$

$= (1^2 - 8.1^2)^{1/2}$

$= (1 + 8)^{1/2}$

$= 9^{1/2}$

$= 3$

Diketahui $<\vec{u},\vec{v}>$ adalah hasil kali dalam Euclides R^3. Tentukan nilai k agar himpunan vektor dibawah ini saling orthogonal

1. $\vec{u}=(3,5,-8),\ \vec{v}=(5,k,5),$
2. $\vec{u}=(k,-3,0),\ \vec{v}=(k,3,13),$

Syarat agar vektor dikatakan saling orthogonal adalah $<\vec{u},\vec{v}> = 0$

1. $<\vec{u},\vec{v}> = u_1v_1 + u_2v_2 + u_3v_3$

$$0 = 3.5 + 5.k + (-8).5$$

$$0 = 15 + 5k - 40$$

$$0 = 5k - 25$$

$$5k = 25$$

$$k = 5$$

2. $<\vec{u},\vec{v}> = u_1v_1 + u_2v_2 + u_3v_3$

$$0 = k.k + (-3).3 + 0.13$$

$$0 = k^2 - 9 + 0$$

$$k^2 - 9 = 0$$

$$(k+3)(k-3) = 0$$

$$k_1 = -3 \qquad k_2 = 3$$

Diketahui

$B = \{(0, -4, 0), (5, 12, 0), (1, 0, -2)\}$

Menggunakan proses Gramm Schmidt, transformasikan basis B menjadi basis orthonormal

$$B = \left\{ \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 12 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \right\} \qquad \vec{u}_1 = \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} 5 \\ 12 \\ 0 \end{bmatrix} \qquad \vec{u}_3 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\vec{v}_1 = \frac{\vec{u}_1}{\|\vec{u}_1\|} = \frac{(0,-4,0)}{(0^2+(-4)^2+0^2)^{1/2}} = \frac{(0,-4,0)}{4} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$\vec{u}_2 - \text{proy}_{\vec{v}_1} \vec{u}_2 = \vec{u}_2 - \langle \vec{u}_2, \vec{v}_1 \rangle \vec{v}_1$$

$$= \begin{bmatrix} 5 \\ 12 \\ 0 \end{bmatrix} - (5.0 + 12.(-1) + 0.0) \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 12 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 12 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

$$\|\vec{u}_2 - \text{proy}_{\vec{v}_1} \vec{u}_2\| = \sqrt{5^2+0^2+0^2} = 5$$

$$\vec{v}_2 = \frac{\vec{u}_2 - \text{proy}_{\vec{v}_1} \vec{u}_2}{\|\vec{u}_2 - \text{proy}_{\vec{v}_1} \vec{u}_2\|} = \frac{(5,0,0)}{5} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{u}_3 - \text{proy}_w \vec{u}_3 = \vec{u}_3 - \langle \vec{u}_3 . \vec{v}_1 \rangle \vec{v}_1 - \langle \vec{u}_3, \vec{v}_2 \rangle \vec{v}_2$$

$$= \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} - (1.0 + 0.(-1) + (-2).0) . \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} - (1.1 + 0.0 + (-2).0) . \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$$

$$\vec{v}_3 = \frac{\vec{u}_3 - \text{proy}_w \vec{u}_3}{\|\vec{u}_3 - \text{proy}_w \vec{u}_3\|} = \frac{(0,0,-2)}{2}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \left\{ \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \right\}$$ merupakan basis orthonormal dari basis B untuk ruang vektor R^3 dengan RHD Euklides

M. Hasyim Abdillah P. 1101101095 TT-43-11

1.b $\langle \vec{u}, \vec{v} \rangle = 5u_1v_1 - u_1v_2 - u_2v_1 + 10u_2v_2$ di R^2

$\circ$ $\langle \vec{v}, \vec{u} \rangle = 5v_1u_1 - v_1u_2 - v_2u_1 + 10u_2v_2$

$= 5u_1v_1 - u_2v_1 - u_1v_2 + 10u_2v_2$

$= 5u_1v_1 - u_1v_2 - u_2v_1 + 10u_2v_2$

$= \langle \vec{u}, \vec{v} \rangle$

$\circ$ $\langle \vec{u}+\vec{v}, \vec{w} \rangle = \langle (u_1+v_1, u_2+v_2), (w_1, w_2) \rangle$

$= 5(u_1+v_1)w_1 - (u_1+v_1)w_2 - (v_2+u_2)w_1 + 10(u_2+v_2)w_2$

$= 5u_1w_1 + 5v_1w_1 - u_1w_2 - v_1w_2 - u_2w_1 - v_2w_1 + 10u_2w_2 + 10v_2w_2$

$= (5u_1w_1 - u_1w_2 - u_2w_1 + 10u_2w_2) + (5v_1w_1 - v_1w_2 - v_2w_1 + 10u_2w_2)$

$= \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$

$\circ$ $\langle k\vec{u}, \vec{v} \rangle = 5.ku_1v_1 - ku_1v_2 - ku_2v_1 + 10.ku_2v_2$

$= 5u_1.kv_1 - u_1.kv_2 - u_2.kv_1 + 10u_2.kv_2 = \langle \vec{u}, k\vec{v} \rangle$

$= k(5u_1v_1 - u_1v_2 - u_2v_1 + 10u_2v_2) = k\langle \vec{u}, \vec{v} \rangle$

$\circ$ $\langle \vec{u}, \vec{u} \rangle = 5u_1^2 - u_1u_2 - u_2u_1 + 10u_2^2$

$= 5u_1^2 - 2u_1u_2 + 10u_2^2$

Saat $2u_1u_2 > 5u_1^2 + 10u_2^2$ maka $\langle \vec{u}, \vec{u} \rangle < 0$

Tidak memenuhi positivitas

$\therefore$ $\langle \vec{u}, \vec{v} \rangle = 5u_1v_1 - u_1v_2 - u_2v + 10u_2v_2$ bukan merupakan RHD

1.c. $\langle \vec{u}, \vec{v} \rangle = u_1v_1^2 - u_2^2v_2$ di R^2

$\circ$ $\langle \vec{v}, \vec{u} \rangle = v_1u_1^2 - v_2^2u_2$

$\therefore$ Tidak simetris karena $\langle \vec{u}, \vec{v} \rangle \neq \langle \vec{v}, \vec{u} \rangle$

Syarat vektor orthogonal $\langle \vec{u}, \vec{v} \rangle = 0$

o) $\langle \vec{a}, \vec{b} \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$

$$0 = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{3}} + 0 \cdot \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}} \cdot \frac{-1}{\sqrt{3}}$$

$$0 = \frac{1}{\sqrt{6}} + 0 - \frac{1}{\sqrt{6}} \quad \checkmark$$

o) $\langle \vec{a}, \vec{c} \rangle = a_1 c_1 + a_2 c_2 + a_3 c_3$

$$0 = \frac{1}{\sqrt{2}} \cdot \frac{-1}{\sqrt{2}} + 0 . 0 + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$$

$$0 = -\frac{1}{2} + 0 + \frac{1}{2} \quad \checkmark$$

o) $\langle \vec{b}, \vec{c} \rangle = b_1 c_1 + b_2 c_2 + b_3 c_3$

$$0 = \frac{1}{\sqrt{3}} \cdot \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \cdot 0 + \frac{-1}{\sqrt{3}} \cdot \frac{1}{\sqrt{2}}$$

$$0 = \frac{-1}{\sqrt{6}} + 0 - \frac{1}{\sqrt{6}} \quad \times$$

$\therefore$ $S = \{\vec{a}, \vec{b}, \vec{c}\}$ bukan himpunan vektor orthogonal

b. $S = \{\vec{a}, \vec{b}, \vec{c}\}$

$\vec{a} = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$ $\quad \vec{b} = \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ $\quad \vec{c} = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

Syarat orthogonal : $\langle \vec{u}, \vec{v} \rangle = 0$

o) $\langle \vec{a}, \vec{b} \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$

$$0 = \frac{2}{3} \cdot \frac{2}{3} + \frac{-2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{-2}{3}$$

$$0 = \frac{4}{9} - \frac{2}{9} - \frac{2}{9} \quad \checkmark$$

o) $\langle \vec{a}, \vec{c} \rangle = a_1 c_1 + a_2 c_2 + a_3 c_3$

$$0 = \frac{2}{3} \cdot \frac{1}{3} + \frac{-2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{3}$$

$$0 = \frac{2}{\ } - \frac{4}{\ } + \frac{2}{\ } \quad \checkmark$$

$\circ \; \langle \vec{b}, \vec{c} \rangle = b_1 c_1 + b_2 c_2 + b_3 c_3$

$$0 = \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} + \frac{-2}{3} \cdot \frac{2}{3}$$

$$0 = \frac{2}{9} + \frac{2}{9} - \frac{4}{9} \checkmark$$

$\therefore \; S = \{\vec{a}, \vec{b}, \vec{c}\}$ merupakan himpunan vektor orthogonal

c. $S = \{\vec{a}, \vec{b}, \vec{c}\}$

$\vec{a} = (1,0,0) \quad \vec{b} = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) \quad \vec{c} = (0,0,1)$

Syarat vektor orthogonal $\langle \vec{u}, \vec{v} \rangle = 0$

$\circ \; \langle \vec{a}, \vec{b} \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3$

$$0 = 1 \cdot 0 + 0 \cdot \frac{1}{\sqrt{2}} + 0 \cdot 1$$

$$0 = 0 + 0 + 0 \checkmark$$

$\circ \; \langle \vec{a}, \vec{c} \rangle = a_1 c_1 + a_2 c_2 + a_3 c_3$

$$0 = 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1$$

$$0 = 0 + 0 + 0 \checkmark$$

$\circ \; \langle \vec{b}, \vec{c} \rangle = b_1 c_1 + b_2 c_2 + b_3 c_3$

$$0 = 0 \cdot 0 + \frac{1}{\sqrt{2}} \cdot 0 + \frac{1}{\sqrt{2}} \cdot 1$$

$$0 = 0 + 0 + \frac{1}{\sqrt{2}} \times$$

$\therefore \; S = \{\vec{a}, \vec{b}, \vec{c}\}$ bukan merupakan himpunan vektor orthogonal

5. b. $\vec{u} = (k, -6, -7) \quad \vec{v} = (k, -k, 1)$

Syarat agar vektor saling orthogonal ($\vec{u} \perp \vec{v}$):

$\langle \vec{u}, \vec{v} \rangle = 0$	$k^2 + 6k - 7 = 0$
	$(k-1)(k+7) = 0$

6. $\vec{u}_1 = (0,1,2) \qquad \vec{u}_2 = (-1,0,1) \qquad \vec{u}_3 = (-1,1,4)$

$U = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ basis bagi R^3

$$\vec{v}_1 = \frac{\vec{u}_1}{\|\vec{u}_1\|} = \frac{(0,1,2)}{\sqrt{0^2+1^2+2^2}} = \frac{(0,1,2)}{\sqrt{0+1+4}} = \left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$\vec{u}_2 - proy_{\vec{v}_1}\vec{u}_2 = \vec{u}_2 - \langle \vec{u}_2, \vec{v}_1 \rangle \vec{v}_1$$

$$= (-1,0,1) - \left(-1.0 + 1\,\frac{1}{\sqrt{5}} + 1.\frac{2}{\sqrt{5}}\right).\left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$= (-1,0,1) - \frac{3}{\sqrt{5}}\left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$= (-1,0,1) - \left(0, \frac{3}{5}, \frac{6}{5}\right)$$

$$= \left(-1, -\frac{3}{5}; -\frac{1}{5}\right)$$

$$\vec{v}_2 = \frac{\vec{u}_2 - proy_{\vec{v}_1}\vec{u}_2}{\|\vec{u}_2 - proy_{\vec{v}_1}\vec{u}_2\|} = \frac{(-1, -3/5, -1/5)}{\sqrt{(-1)^2 + (-3/5)^2 + (-1/5)^2}}$$

$$= \frac{(-1, -3/5, -1/5)}{\sqrt{1 + \frac{9}{25} + \frac{1}{25}}} = \frac{(-1, -3/5, -1/5)}{\sqrt{\frac{35}{25}}}$$

$$= \frac{5(-1, -3/5, -1/5)}{\sqrt{35}}$$

$$= \left(-\frac{5}{\sqrt{35}}, -\frac{3}{\sqrt{35}}, -\frac{1}{\sqrt{35}}\right)$$

$$\vec{u}_3 - proy_w \vec{u}_3 = \vec{u}_3 - \langle \vec{u}_3, \vec{v}_1 \rangle v_1 - \langle \vec{u}_3, \vec{v}_2 \rangle \vec{v}_2$$

$$= (-1,1,4) - \left(-1\,0 + 1.\frac{1}{\sqrt{5}} + 4.\frac{2}{\sqrt{5}}\right)\left(0, \frac{1}{\sqrt{5}}; \frac{2}{\sqrt{5}}\right) - \left(-1\,\frac{-5}{\sqrt{35}} + 1.\frac{-3}{\sqrt{35}} + 4.\frac{-1}{\sqrt{35}}\right)\left(-\frac{5}{\sqrt{35}}, \frac{-3}{\sqrt{35}}, \frac{-1}{\sqrt{35}}\right)$$

$$= (-1,1,4) - \frac{9}{\sqrt{5}}\left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) - \frac{-12}{\sqrt{35}}\left(-\frac{5}{\sqrt{35}}, -\frac{3}{\sqrt{35}}, \frac{-1}{\sqrt{35}}\right)$$

$$\|\vec{u}_3 - \text{proy}_w \vec{u}_3\| = \sqrt{\left(\frac{-95}{35}\right)^2 + \left(\frac{-64}{35}\right)^2 + \left(\frac{2}{35}\right)^2}$$

$$= \frac{5\sqrt{3}}{\sqrt{7}} = \frac{5\sqrt{21}}{7}$$

$$\vec{v}_3 = \frac{\vec{u}_3 - \text{proy}_w \vec{u}_3}{\|\vec{u}_3 - \text{proy}_w \vec{u}_3\|} = \frac{\left(\frac{-95}{35}, \frac{-64}{35}, \frac{2}{35}\right)}{\frac{5\sqrt{21}}{7}}$$

$$= \left(-\frac{95}{25\sqrt{21}}, -\frac{64}{25\sqrt{21}}, \frac{2}{25\sqrt{21}}\right)$$

$$= \left(-\frac{19}{5\sqrt{21}}, -\frac{64}{25\sqrt{21}}, \frac{2}{25\sqrt{21}}\right)$$

V merupakan basis orthonormal dari U dengan:

$$V = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$$

$$V = \left\{\left(0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right), \left(-\frac{5}{\sqrt{35}}, -\frac{3}{\sqrt{35}}, -\frac{1}{\sqrt{35}}\right), \left(-\frac{19}{5\sqrt{21}}, -\frac{64}{25\sqrt{21}}, \frac{2}{25\sqrt{21}}\right)\right\}$$