Responsi Varkom

① Diketahui suatu fungsi yaitu :

$$f(z) = \frac{10}{-4+2z}$$

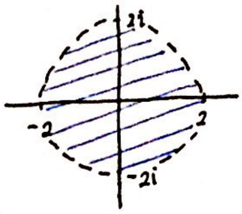
ⓐ Gambarkan nilai keanalitikan/kekonvergenan dari fungsi $f(z)$ apabila dideretkan sebagai Deret Maclaurin!

$|-4+2z| = 0$

$|2z| = 4$

$|z| = 2 \rightarrow$ titik singular

$|z| < 2$



ⓑ Tentukan deret MacLaurin dari fungsi $f(z)$!

$$f(z) = \frac{10}{-4+2z} \qquad f(x) = \frac{1}{1-kz} = 1 + kz + (kz)^2 + (kz)^3 + \dots$$

$$= 10\left(\frac{1}{-4+2z}\right)$$

$$= -\frac{10}{4}\left(\frac{1}{1-\frac{1}{2}z}\right) \rightarrow \boxed{k = \frac{1}{2}}$$

$$f(z) = 1 + \frac{1}{2}z + \left(\frac{1}{2}z\right)^2 + \left(\frac{1}{2}z\right)^3 + \dots$$

$$= 1 + \frac{1}{2}z + \frac{1}{4}z^2 + \frac{1}{8}z^3$$

ⓓ Tentukan Deret Taylor dari fungsi $f(z)$ apabila dideretkan di $z = 2i$!

$$f(z) = \frac{10}{-4+2z}$$

$$= \frac{10}{-4+2(z-2i+2i)}$$

$$= \frac{10}{-4+4i+2(z-2i)}$$

$$= \frac{10}{-4+4i}\left(\frac{1}{1+\frac{2(z-2i)}{-4+4i}}\right) \times \frac{-4-4i}{-4-4i}$$

$$= \frac{-40-40i}{32}\left(\frac{1}{1+\frac{2(z-2i)}{-4+4i}}\right) \rightarrow \boxed{\frac{1}{1-kz}}$$

$$kz = \frac{-2(z-2i)}{-4+4i}$$

$$k = \frac{-2}{-4+4i} \; ; \quad z = (z-2i)$$

$$f(z) = 1 - \frac{2(z-2i)}{-4+4i} + \left(\frac{-2(z-2i)}{-4+4i}\right)^2 + \left(\frac{-2(z-2i)}{-4+4i}\right)^3 + \dots \quad //$$

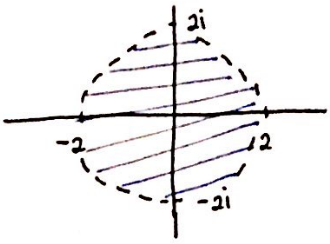
ⓒ Gambarkan daerah keanalitikan / kekonvergenan dari fungsi $f(z)$ apabila diderетkan sebagai Deret Taylor di $z=2i$!

$$\left|\frac{-2(z-2i)}{-4+4i}\right| < 1$$

$$|-2(z-2i)| < -4+4i$$

$$|z-2i| < 2-2i$$

$$|z| < 2$$



② Diketahui fungsi sebagai berikut :

$$f(z) = \frac{2z+3}{(z^2+4)(z+3i)^2}$$

ⓐ Tentukan semua titik singular dari fungsi $f(z)$ dan jenis kutub / ordenya !

$$f(z) = \frac{2z+3}{(z-2i)(z+2i)(z+3i)^2}$$

Maka :

Titik singular → $z = 2i$ (orde 1)

$z = -2i$ (orde 1)

$z = -3i$ (orde 2)

ⓑ Hitung nilai residu dari fungsi $f(z)$ untuk setiap titik singularnya !

Orde 1

• $z = 2i \rightarrow (z-2i)$

$$g(z) = \frac{2z+3}{(z+2i)(z+3i)^2}$$

$$g(2i) = \frac{2(2i)+3}{(2i+2i)(2i+3i)^2} \rightarrow \boxed{i \times i = -1}$$

$$= \frac{4i+3}{4i(-25)}$$

$$= \frac{4i+3}{-100i}$$

• $z = -2i \rightarrow (z+2i)$

$$g(z) = \frac{2z+3}{(z-2i)(z+3i)^2}$$

$$g(-2i) = \frac{2(-2i)+3}{(-2i-2i)(-2i+3i)^2}$$

$$= \frac{-4i+3}{-4i(i)^2}$$

$$= \frac{-4i+3}{4i}$$

Orde 2

• $z = -3i \longrightarrow (z+3i)^2$

$$\boxed{\text{Res } z = z_0 \ \frac{1}{(n-1)!} \ g^{n-1}(z) \Big|_{z=z_0}} \rightarrow n = \text{jumlah orde}$$

$$g(z) = \frac{2z+3}{(z^2+4)}$$

$u = 2z+3$ | $v = z^2+4$

$u' = 2$ | $v' = 2z$

$$g'(z) = \frac{u'v - uv'}{v^2}$$

$$= \frac{2(z^2+4) - 2z(2z+3)}{(z^2+4)^2}$$

$$= \frac{2z^2+8-4z^2-6z}{(z^2+4)^2}$$

$$= \frac{-2z^2-6z+8}{(z^2+4)^2}$$

$$\text{Res } z = \frac{1}{(2-1)!} \ g^{2-1}(z) \Big|_{z=-3i}$$

$$= \frac{1}{(1)} \ g'(z) \Big|_{z=-3i}$$

$$= \frac{1}{1} \cdot \frac{-2z^2-6z+8}{(z^2+4)^2} \Big|_{z=-3i}$$

$$= \frac{-2(-3i)^2 - 6(-3i) + 8}{((-3i)^2+4)^2}$$

$$= \frac{18+8+18i}{25}$$

$$= \frac{26+18i}{25}$$

ⓒ Misalkan lintasan $C : |z| = \frac{5}{2}$ dengan arah positif berlawanan jarum jam, maka hitunglah :

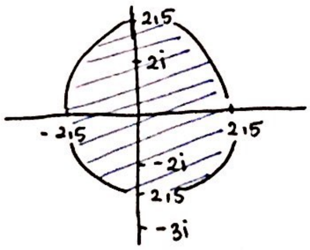
$$\oint_C \frac{2z+3}{(z^2+4)(z+3i)^2} \, dz$$

$$f(z) = \frac{2z+3}{(z+2i)(z-2i)(z+3i)^2}$$

$\hookrightarrow z = -2i$

$z = 2i$

$z = -3i$



$$\oint f(z) = 2\pi i \ (\text{Res } z = 2i + \text{Res } z = -2i)$$

$$= 2\pi \cancel{i} \left(\frac{4i+3}{-100\cancel{i}} + \left(\frac{-4i+3}{4\cancel{i}} \right) \right)$$

$$= 2\pi \left(\frac{4i+3}{-100} + \frac{3-4i}{4} \right)$$

ⓓ Dengan menggunakan hasil perhitungan 2a dan 2b hitunglah integral berikut !

$$\int_{-\infty}^{+\infty} \frac{2x+3}{(x^2+4)(x+3i)^2} \, dx$$

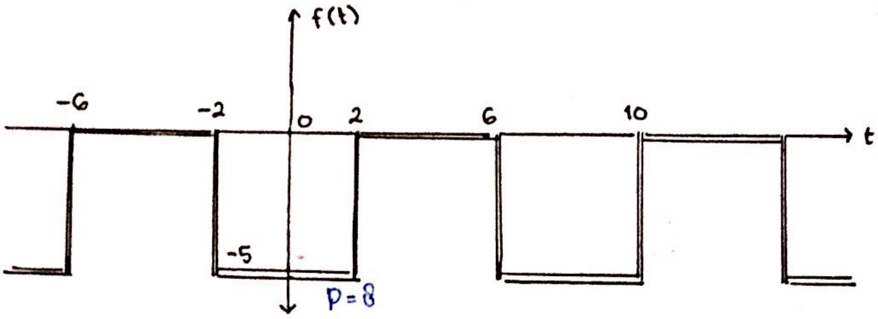
$$\int_{-\infty}^{+\infty} f(x) = \oint_C f(z) = 2\pi i \ (\text{Res } z = 2i)$$

$$= 2\pi \cancel{i} \left(\frac{4i+3}{-100\cancel{i}} \right)$$

$$= -\frac{2\pi}{100} \ (4i+3)$$

Bagian atas bidang kompleks adalah polenya (Im (z) > 0) !

(3) Diketahui suatu sinyal Periodik seperti gambar berikut ini :



→ Fungsi genap

(a) Tentukan persamaan dari sinyal periodik diatas, dan tentukan periodenya!

$$f(t)\begin{cases} -5 & u/\ -2 \le t \le 2 \\ 0 & u/\ 2 \le t \le 6 \end{cases} \qquad P = 8$$

(b) Tentukan a_0, a_n dan b_n!

- $a_0 = \frac{1}{P} \int_P f(t)\, dt$

$$= \frac{1}{8} \int_{-2}^{2} -5\, dt + \int_{2}^{6} 0\, dt$$

$$= \frac{1}{8} \left[-5t\right]_{-2}^{2}$$

$$= \frac{1}{8} \left[-10 - (10)\right]$$

$$= \frac{-20}{8} = -2{,}5$$

Fungsi genap (cermin):

$a_0 \neq 0$

$a_n \neq 0$

$b_n = 0$

Fungsi ganjil (tidak seperti cermin):

$a_0 = 0$

$a_n = 0$

$b_n \neq 0$

- $a_n = \frac{2}{P} \int_P f(t) \cos \frac{2\pi n t}{P}\, dt$

$$= \frac{2}{8} \left(\int_{-2}^{2} -5 \cos\left(\frac{2\pi n t}{8}\right) dt \right)$$

$$= \frac{1}{\cancel{4}} \left(-5 \frac{\overset{4}{\cancel{8}}}{\cancel{2}\pi n} \cdot \sin\left(\frac{\cancel{2}\pi n t}{\underset{4}{\cancel{8}}}\right) \right]_{-2}^{2}$$

$$= \left[\frac{-5}{\pi n} \sin\left(\frac{1}{4} \pi n t\right) \right]_{-2}^{2}$$

$$= \frac{-5}{\pi n} \left(\sin\left(\frac{1}{2}\pi n\right) - \sin\left(-\frac{1}{2}\pi n\right) \right)$$

$$a_n = \frac{-5}{\pi n} \left(2 \sin\left(\frac{1}{2}\pi n\right) \right)$$

Dimana :

$$\int -5 \cos\left(\frac{2\pi n t}{8}\right) dt \longrightarrow \int f \cdot g'$$

$f = -5$

$f' = 0$

$g' = \cos\left(\frac{2\pi n t}{8}\right)$

$g = \frac{8}{2\pi n} \cdot \sin\left(\frac{2\pi n t}{8}\right)$

$$\int f \cdot g' = f \cdot g - \int f' g$$

$$= -5 \left(\frac{8}{2\pi n}\right) \sin\left(\frac{2\pi n t}{8}\right) - \int 0$$

$$= -5 \left(\frac{8}{2\pi n}\right) \sin\left(\frac{2\pi n t}{8}\right)$$

$$n_1 = a_1 = \frac{-5}{\pi}\left(2\sin\overbrace{\left(\tfrac{1}{2}\pi\right)}^{1}\right) = \frac{-10}{\pi}$$

$$n_2 : a_2 = \frac{-5}{\pi 2}\left(2\sin\overbrace{\left(\tfrac{1}{\cancel{2}}\cdot\cancel{2}\pi\right)}^{0}\right) = 0$$

$$n_3 = a_3 = \frac{-5}{\pi 3}\left(2\sin\overbrace{\left(\tfrac{1}{2}\cdot\pi 3\right)}^{1}\right) = \frac{10}{\pi 3}$$

$$n_4 = a_4 = \frac{-5}{\pi 4}\left(2\sin\overbrace{\left(\tfrac{1}{2}\cdot 4\pi\right)}^{0}\right) = 0$$

maka :

$$f(t) = a_0 + a_1\cos\omega t + a_2\cos 2\omega t + a_3\cos 3\omega t + \dots$$

$$f(t) : -2{,}5 - \frac{10}{\pi}\cos\omega t + 0 + \frac{10}{\pi 3}\cos 3\omega t + 0 + \dots$$

Cara

ⓒ Tuliskanlah deret fourier sampai 4 suku pertama !

$$f(t) = -2{,}5 - \frac{10}{\pi}\cos\omega t + 0 + \frac{10}{\pi 3}\cos\omega t + 0 + \dots$$

④ Diketahui suatu sinyal berikut ini :

$$f(t) = \begin{cases} 5 & , t \geq -3 \\ 0 & , t \text{ lainnya.} \end{cases}$$

ⓐ Tentukan Fourier Transform dari fungsi f(t)

$$f(t) = 5u(t+3)$$
$$= 5\left(\frac{1}{i\omega} + \pi\,\delta(i\omega)\right)e^{j\omega\cdot 3}$$

ⓑ Tentukanlah Fourier transform dari fungsi f(5t)

$$f(5t) = \frac{1}{|5|}\cdot\cancel{5}\left(e^{j\omega\cdot 3}\left(\frac{1}{\frac{i\omega}{5}} + \pi\,\delta\left(\frac{i\omega}{5}\right)\right)\right)$$

$$= e^{j\omega\cdot 3}\left(\frac{1}{i\omega/5} + \pi\delta\left(\frac{i\omega}{5}\right)\right)$$

•) $u(t)$

$u(t-t_0) \longrightarrow f$ bergeser kekanan (+)

$u(t+t_0) \longrightarrow f$ bergeser kekiri (-)

$u(t) \longrightarrow \dfrac{1}{i\omega} + \pi\,\delta(i\omega)$

$f(t-t_0) \rightarrow f(i\omega)\,e^{-j\omega t_0}$

•) $f(at)$

$\dfrac{1}{|a|}F\left(\dfrac{i\omega}{a}\right)$

⑤ Tentukan invers Transformasi fourier dan sinyal domain frekuensi berikut ini :

ⓐ $F(i\omega) = \dfrac{-12}{(i\omega+81)^2}$

$$e^{-81t}u(t) = \frac{1}{(i\omega+81)}$$

$$te^{-81t}u(t) = i\,\frac{d\left(\frac{1}{i\omega+81}\right)}{d\omega}$$

$$= i\,\frac{-i}{(i\omega+81)^2}$$

$$= -12\left(\frac{1}{(i\omega+81)^2}\right)$$

$$= -12te^{-81t}u(t)$$

ⓑ $F(i\omega) = \underbrace{\pi\delta(\omega-4\pi) + \pi\,\delta(\omega+4\pi)}_{\cos at \,\rightarrow\, a = 4\pi} + 5 \rightarrow \delta(t)$

$$f(t) = \cos 4\pi t + 5\,\delta(t)$$