2. $f(z) = \dfrac{3z+2}{(z^2+9)(z+i)^2(z-2i)^2} = \dfrac{3z+2}{(z+3i)(z-3i)(z+i)^2(z-2i)^2}$

a. $z + 3i = 0 \rightarrow z_1 = -3i$, orde 1

$z - 3i = 0 \rightarrow z_2 = 3i$, orde 1

$z + i = 0 \rightarrow z_3 = -i$, orde 2

$z - 2i = 0 \rightarrow z_4 = 2i$, orde 2

b. $q_1(z) = \dfrac{3z+2}{(z-3i)(z+i)^2(z-2i)^2}$

$$Res_{z=z_1} = \frac{3(-3i)+2}{(-3i-3i)(-3i+i)^2(-3i-2i)^2}$$

$$Res_{z=-3i} = \frac{-9i+2}{-6i.(-2i)^2.(-5i)^2}$$

$$Res_{z=z_1} = \frac{2-9i}{-600i}$$

$q_2(z) = \dfrac{3z+2}{(z+3i)(z+i)^2(z-2i)^2}$

$$Res_{z=z_2} = \frac{3(3i)+2}{(3i+3i)(3i+i)^2(3i-2i)^2}$$

$$= \frac{9i+2}{6i(4i)^2(i)^2}$$

$$Res_{z=3i} = \frac{9i+2}{64i}$$

$$q_3(z) = \frac{3z+2}{(z^2+9)(z-2i)^2}$$

$$\text{Res}_{z=z_3} = z_3 \cdot \frac{1}{(n-1)!}\, q_3^{n-1}(z)\Big|_{z=z_3}$$

$$= -i \cdot \frac{1}{(2-1)!} \cdot q_3^{2-1}(z)\Big|_{z=-i}$$

$$= -i\, \frac{1}{1!}\, \frac{d}{dz}\left(\frac{3z+2}{(z^2+9)(z-2i)^2}\right)\Big|_{z=i}$$

$$= -i \cdot \frac{3(z^2+9)(z-2i)^2 - (3z+2)\left(2z(z-2i)^2 + (z^2+9)\cdot 2(z-2i)\right)}{(z^2+9)^2(z-2i)^4}\Bigg|_{z=-i}$$

$$= -i \cdot \frac{3((-i)^2+9)(-i-2i)^2 - (3\cdot -i+2)\left(-2i(-i-2i)^2 + ((-i)^2+9)\cdot 2(-i-2i)\right)}{((-i)^2+9)^2(-i-2i)^4}$$

$$= -i \cdot \frac{3\cdot 8(-3i)^2 - (-3i+2)\left(-2i(-3i)^2 + 8\cdot 2(-3i)\right)}{8^2\cdot(-3i)^4}$$

$$= -i \cdot \frac{24\cdot -9 - (-3i+2)(-2i\cdot -9 + 16\cdot -3i)}{64\cdot 81}$$

$$= -i \cdot \frac{-216 - (-3i+2)(18i - 48i)}{5184}$$

$$= -i \cdot \frac{-216 - (-90-60i)}{5184}$$

$$= -i \cdot \frac{-126+60i}{5184}$$

$$\text{Res}_{z=-i} = \frac{60+126i}{5184}$$

$$q_4(z) = \frac{3z+2}{(z^2+9)(z+i)^2}$$

$$\operatorname{Res}_{z=z_4} = z_4 \cdot \frac{1}{(n-1)!}\, q_4^{n-1}(z)\Big|_{z=z_4}$$

$$= 2i \cdot \frac{1}{(2-1)!}\, q_4^{2-1}(z)\Big|_{z=2i}$$

$$= 2i \cdot \frac{1}{1!} \cdot \frac{d}{dz}\left(\frac{3z+2}{(z^2+9)(z+i)^2}\right)\Big|_{z=2i}$$

$$= 2i \cdot \frac{3(z^2+9)(z+i)^2 - (3z+2)\left[2z(z+i)^2 + (z^2+9)2(z+i)\right]}{(z^2+9)^2(z+i)^4}\Bigg|_{z=2i}$$

$$= 2i \cdot \frac{3((2i)^2+9)(2i+i)^2 - (3.2i+2)\left[2.2i(2i+i)^2 + ((2i)^2+9).2(2i+i)\right]}{((2i)^2+9)^2(2i+i)^4}$$

$$= 2i \cdot \frac{3.(-4+9)(3i)^2 - (6i+2)(4i(3i)^2 + (-4+9).2(3i))}{(-4+9)^2(3i)^4}$$

$$= 2i \cdot \frac{3.5.(-9) - (6i+2)(4i.-9 + 5.2.3i)}{5^2 . 81}$$

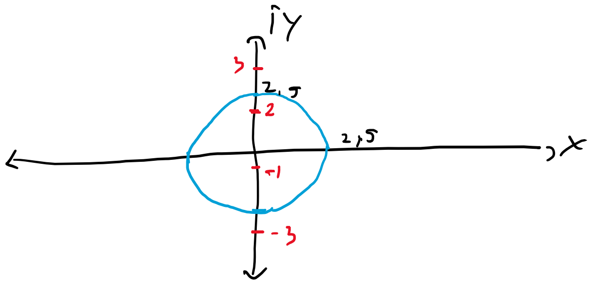
$$= 2i \cdot \frac{-135 - (6i+2)(-36i+30i)}{25.81}$$

$$= 2i \cdot \frac{-135 - (36 - 12i)}{2025}$$

$$= 2i \cdot \frac{-171 + 12i}{2025}$$

$$\operatorname{Res}_{z=2i} = \frac{-24 - 242i}{2025}$$

c. $C : |z| = \frac{5}{2}$



Nilai yang memenuhi $z = 2i$ & $z = -i$

$$\oint_C f(z)\,dz = 2\pi i \left(Res_{z=2i} + Res_{z=-i} \right)$$

$$= 2\pi i \left(\frac{60 + 126i}{5184} + \frac{-24 - 242i}{2025} \right)$$

$$= 2\pi i \left(\frac{-1}{3600} - \frac{6169}{64800} i \right)$$

$$= \frac{6169\pi}{32400} - \frac{\pi}{1800} i$$

d. $$\int_{-\infty}^{\infty} \frac{3x+2}{(x^2+9)(x+i)^2(x-2i)^2}\,dx$$

Titik singular : $x_1 = -3i$, $x_2 = 3i$, $x_3 = -i$, $x_4 = 2i$

Nilai x yang memenuhi $x > 0.i$

$x = 3i$

$x = 2i$

$$\int_{-\infty}^{\infty} \frac{3x+2}{(x^2+9)(x+i)^2(x-2i)^2}\,dx = 2\pi i \left(Res_{x=3i} + Res_{x=2i} \right)$$

$$= 2\pi i \left(\frac{9i+2}{64i} + \frac{60+126i}{5184} \right)$$

$$= 2\pi i \left(\frac{263}{1728} - \frac{1}{144} i \right)$$

$$= \frac{\pi}{72} + \frac{263\pi}{864} i$$