M. Hasyim Abdillah P.
1101191095 / TT-43-11

1.) a. $(1-x)y'' + 4xy' + 5y = \cos x$

$\therefore$ PD Linear orde 2

b. $\dfrac{d^2y}{dx^2} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$

$\therefore$ PD nonlinear orde 2

2.) a. $y = -\cos x \ln(\sec x + \tan x)$

$$y = -\cos x \ln\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$$

$$y = -\cos x \ln\left(\frac{1 + \sin x}{\cos x}\right)$$

$$\frac{dy}{dx} = \sin x . \ln\left(\frac{1+\sin x}{\cos x}\right) - \cos x . \frac{1}{\frac{1+\sin x}{\cos x}} . \frac{\cos x . \cos x - (1+\sin x) . -\sin x}{\cos^2 x}$$

$$y' = \sin x . \ln\left(\frac{1+\sin x}{\cos x}\right) - \frac{\cancel{\cos^2 x}}{1+\sin x} . \frac{\cos^2 x + \sin^2 x + \sin x}{\cancel{\cos^2 x}}$$

$$y' = \sin x \ln\left(\frac{1+\sin x}{\cos x}\right) - \frac{1+\sin x}{1+\sin x}$$

$$y' = \sin x \ln\left(\frac{1+\sin x}{\cos x}\right) - 1$$

$$y'' = \cos x \ln\left(\frac{1+\sin x}{\cos x}\right) + \sin x \cdot \frac{1}{\frac{1+\sin x}{\cos x}} \cdot \frac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x}$$

$$y'' = \cos x \ln\left(\frac{1+\sin x}{\cos x}\right) + \frac{\sin x \cancel{\cos x}}{\cancel{1+\sin x}} \cdot \frac{\cancel{1+\sin x}}{\cos^{\cancel{2}} x}$$

$$y'' = \cos x \ln\left(\frac{1+\sin x}{\cos x}\right) + \tan x$$

$$y'' + y = \cos x \ln\left(\frac{1+\sin x}{\cos x}\right) + \tan x - \cos x \ln\left(\frac{1+\sin x}{\cos x}\right)$$

$$y'' + y = \tan x$$

$\therefore\ y = -\cos x \ln(\sec x + \tan x)$ penyelesaian eksplisit dari $y'' + y = \tan x$

b. $-2x^2 y + y^2 = 1$

$$\frac{d}{dx}(-2x^2 y) + \frac{d}{dx} y^2 = \frac{d}{dx} 1$$

$$\frac{d}{dx} -2x^2 \cdot y + -2x^2 \frac{d}{dx} y + \frac{d}{dy} \cdot \frac{dy}{dx} y^2 = 0$$

$$-4xy - 2x^2 \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$\cancel{2}(-x^2 + y) \frac{dy}{dx} = \cancel{4}^{2} xy$$

$$(-x^2+y)\,dy = 2xy\,dx$$

$$2xy\,dx + (x^2-y)\,dy = 0$$

$\therefore\ -2x^2y + y^2 = 1$ penyelesaian implisit dari PD

$$2xy\,dx + (x^2-y)\,dy = 0$$

3.) a. $y = c_1 \cos 2x + c_2 \sin 2x$

$y(0) = 1 = c_1 \cos 2.0 + c_2 \sin 2.0$

$1 = c_1 + 0$

$c_1 = 1$

$y' = -2c_1 \sin 2x + 2c_2 \cos 2x$

$y'(\pi) = 5 = -2c_1 \sin 2\pi + 2c_2 \cos 2\pi$

$5 = 0 + 2c_2$

$c_2 = 2,5$

$\therefore\ c_1 = 1\ ;\ c_2 = 2,5$

b $y = \cos 2x + 2,5 \sin 2x$

4.) $y = x - \dfrac{2}{x}\ ;\ y(x_0) = 1$

$$1 = x_0 - \frac{2}{x_0}$$

$$1 = \frac{x_0^2 - 2}{x_0}$$

$$x_0 = x_0^2 - 2$$

$$x_0^2 - x_0 - 2 = 0$$

$$(x_0 - 2)(x_0 + 1) = 0$$

$$x_0 = 2 \lor x_0 = -1$$

$$y = x - \frac{2}{x}$$

$$y' = 1 + \frac{2}{x^2}, \quad x \neq 0$$

$$xy' + y = x\left(1 + \frac{2}{x^2}\right) + x - \frac{2}{x}$$

$$= x + \frac{2}{x} + x - \frac{2}{x} = 2$$

$$\therefore x_0 = 2 \lor x_0 = -1 \; ; \; I = (-\infty, 0)(0, \infty)$$