1.) $\frac{dy}{dx} = \frac{y(1-x)}{x^2}$, $y(-1) = -1$

$dy = \frac{y(1-x)}{x^2}\, dx$

$\frac{1}{y}\, dy = \frac{1-x}{x^2}\, dx$

$\int \frac{1}{y}\, dy = \int \frac{1-x}{x^2}\, dx$

$\ln y + c = \int \frac{1}{x^2} - \frac{\cancel{x}}{x^{\cancel{2}}}\, dx$

$\ln y + c = \int x^{-2} dx - \int \frac{1}{x}\, dx$

$\ln y + c = -x^{-1} - \ln x + c$

$\ln y = -\frac{1}{x} - \ln x + c$

$y = e^{-\frac{1}{x} - \ln x + c}$ $\quad e^{-\ln x} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$

$y = e^{-\frac{1}{x}} \cdot e^{-\ln x} \cdot e^{c}$

$y = e^{-\frac{1}{x}} \cdot \frac{1}{x} \cdot c$

$y = \frac{c}{x} e^{-x^{-1}}$ → SU

$y(-1) = -1$

$\frac{c}{-1} e^{-(-1)^{-1}} = -1$

$\cancel{-} c e = \cancel{-} 1$

$c = \frac{1}{e}$

∴ Solusi khusus $y = \frac{c}{x} e^{-x^{-1}}$

$y = \frac{e^{-x^{-1}}}{ex}$

2.) $\overset{M}{6xy}\,dx + (\overset{N}{4y + 9x^2})\,dy = 0$ → bkn PD ehsak

$M_y = N_x$

$6x \neq 18x$

$$\mu = e^{\int \frac{M_y - N_x}{N} dx} = e^{\int \frac{6x - 18x}{4y + 9x^2} dx} = e^{\int \frac{-12x}{4y+9x^2} dx}$$

$$= e^{\int \frac{N_x - M_y}{M} dy} = e^{\int \frac{18x - 6x}{6xy} dy} = e^{\int \frac{\cancel{12x}}{\cancel{6x}y} dy} = e^{\int \frac{2}{y} dy} = e^{2\int \frac{1}{y} dy}$$

$$= e^{2 \ln y}$$

$$\mu = e^{\ln y^2} = y^2$$

$\mu \left(6xy\,dx + (4y + 9x^2)\,dy \right) = 0 \cdot \mu$

$\underset{M}{6xy^3}\,dx + (\underset{N}{4y^3 + 9x^2y^2})\,dy = 0 \rightarrow$ SU $\Rightarrow F(x,y) = c$

$F_x(x,y) = M$

$F(x,y) = \int M \, dx$

$F(x,y) = \int 6xy^3 \, dx$

$F(x,y) = 3x^2y^3 + c \rightarrow g(x)$

$F(x,y) = 3x^2y^3 + g(y)$

$F_y(x,y) = N$

$\frac{\partial F}{\partial y} = N$

$\frac{\partial}{\partial y} \left(3x^2y^3 + g(y) \right) = 4y^3 + 9x^2y^2$

$\cancel{9x^2y^2} + g'(y) = 4y^3 + \cancel{9x^2y^2}$

$$g'(y) = 4y^3$$

$$g(y) = \int 4y^3 \, dy$$

$$g(y) = y^4$$

$\therefore$ Solusi umum : $F(x,y) = C$

$$3x^2y^3 + g(y) = C$$

$$3x^2y^3 + y^4 = C$$

3.) $y'' - 2y' + 2y = 0, \quad y(0) = -1, \quad y'(\pi) = 1$

$$r^2 - 2r + 2 = 0$$

$$r_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4.1.2}}{2.1}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{\cancel{2} \pm \cancel{2}\sqrt{-1}}{\cancel{2}}$$

$$= 1 \pm i \quad \begin{array}{l} r_1 = 1 + i \longrightarrow \alpha = 1, \ \beta = 1 \\ r_2 = 1 - i \end{array}$$

$$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$$

$y = \underbrace{e^x}_{u}\underbrace{(c_1 \cos x + c_2 \sin x)}_{v} \rightarrow$ Solusi umum

$$y' = u'v + uv'$$

$$y' = e^x(c_1 \cos x + c_2 \sin x) + e^x(c_1 \cdot -\sin x + c_2 \cos x)$$

$$y' = e^x((c_1 + c_2)\cos x + (c_2 - c_1)\sin x)$$

$$y(0) = -1$$

$$e^0(c_1 \cos 0 + c_2 \sin 0) = -1$$

$$c_1 + 0 = -1$$

$$c_1 = -1$$

$$y'(\pi) = 1$$

$$e^\pi((-1 + c_2)\cos \pi + (c_2 - (-1))\sin \pi) = 1$$

$$e^\pi(1 - c_2 + 0) = 1$$

$$e^\pi - e^\pi c_2 = 1$$

$$e^\pi c_2 = e^\pi - 1$$

$$c_2 = \frac{e^\pi - 1}{e^\pi} = 1 - \frac{1}{e^\pi}$$

$$y = e^x (c_1 \cos x + c_2 \sin x)$$

$$y = e^x \left(-1 \cdot \cos x + \left(1 - \frac{1}{e^\pi}\right) \sin x\right)$$

$$y = e^x \left(\sin x - \cos x - \frac{1}{e^\pi} \sin x\right)$$

4. a.) $y'' - 4y' + 5y = \boxed{2x^2 - x} \neq 0$

$y_T = y_u + y_h$

$y_u : y'' - 4y' + 5y = 0$

$r^2 - 4r + 5 = 0$

$r_{1,2} = \dfrac{-b \pm \sqrt{D}}{2a}$

$= \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4.1.5}}{2.1}$

$= \dfrac{4 \pm \sqrt{-4}}{2}$

$= \dfrac{\cancel{4}^{2} \pm \cancel{2}^{1}\sqrt{-1}}{\cancel{2}_{1}}$

$= 2 \pm i$

$r_1 = 2 + i \rightarrow \alpha = 2$

$\beta = 1$

$r_2 = 2 - i$

$y_u = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

$y_u = e^{2x}(c_1 \cos x + c_2 \sin x)$

$y_h = Ax^2 + Bx + C$

$y_h' = 2Ax + B$

$y_h'' = 2A$

$y'' - 4y' + 5y = 2x^2 - x$

$2A - 4(2Ax + B) + 5(Ax^2 + Bx + C) = 2x^2 - x$

$2A - 8Ax - 4B + 5Ax^2 + 5Bx + 5C = 2x^2 - x$

$5Ax^2 + (5B - 8A)x + 2A - 4B + 5C = 2x^2 - x + 0$

$5A = 2$

$A = \dfrac{2}{5}$

$5B - 8A = -1$

$5B - \dfrac{16}{5} = -1$

$5B = \dfrac{11}{5}$

$B = {}^{11}/_{25}$

$2A - 4B + 5C = 0$

$\dfrac{4}{5} - \dfrac{44}{25} + 5C = 0$

$5C = \dfrac{24}{25}$

$C = \dfrac{24}{125}$

$$y_h = Ax^2 + Bx + C$$

$$y_h = \frac{2}{5}x^2 + \frac{11}{25}x + \frac{24}{125}$$

$$y_T = y_u + y_h$$

$$= e^{2x}(c_1 \cos x + c_2 \sin x) + \frac{2}{5}x^2 + \frac{11}{25}x + \frac{24}{125}$$

4.b.) $\underline{y'' - 6y' + 9y = 2e^{3x}} \longrightarrow f(x) = 2e^{3x}$

$y_T = y_u + y_h$

$y'' - 6y' + 9y = 0$

$r^2 - 6r + 9 = 0$

$(r-3)^2 = 0$

$r_1 = r_2 = 3$

$y_u = (c_1 + c_2 x)e^{r_1 x}$

$y_u = (c_1 + c_2 x)e^{3x}$

$y_u = c_1 e^{3x} + c_2 x e^{3x} \longleftrightarrow y_h = v_1 g_1 + v_2 g_2$

$g_1 = e^{3x}$ $\qquad$ $g_2 = \underset{u}{x}\,\underset{v}{e^{3x}}$

$g_1' = 3e^{3x}$ $\qquad$ $g_2' = e^{3x} + 3xe^{3x}$

$$W = \begin{vmatrix} g_1 & g_2 \\ g_1' & g_2' \end{vmatrix} = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & e^{3x} + 3xe^{3x} \end{vmatrix} = e^{6x} + \cancel{3xe^{6x}} - \cancel{3xe^{6x}} = e^{6x}$$

$$v_1 = \int \frac{g_2 \cdot f(x)}{W}\,dx = \int \frac{x\cancel{e^{3x}} \cdot 2\cancel{e^{3x}}}{\cancel{e^{6x}}}\,dx = x^2$$

$$v_2 = \int \frac{g_1 \cdot f(x)}{W}\,dx = \int \frac{\cancel{e^{3x}} \cdot 2\cancel{e^{3x}}}{\cancel{e^{6x}}}\,dx = 2x$$

$y_h = v_1 g_1 + v_2 g_2$

$y_h = x^2 \cdot e^{3x} + 2x \cdot xe^{3x}$

$y_h = x^2 e^{3x} + 2x^2 e^{3x}$

$y_h = 3x^2 e^{3x}$

$y_T = y_u + y_h$

$= (c_1 + c_2 x)e^{3x} + 3x^2 e^{3x}$

$= (3x^2 + c_2 x + c_1)e^{3x}$

5.) $L = 4\,H$, $R = 5\,\Omega$, $E = 8\,V$, $I(0) = 0$, $I(0{,}5) = \ldots ?$

$$I(t) = \frac{E_0}{R} + k e^{-\frac{R}{L}t}$$

$$I(t) = \frac{8}{5} + k e^{-\frac{5}{4}t}$$

$$I(t) = 1{,}6 + k e^{-1{,}25t}$$

$$I(0) = 0$$

$$1{,}6 + k e^{-1{,}25.0} = 0$$

$$1{,}6 + k = 0$$

$$k = -1{,}6$$

$$I(t) = 1{,}6 - 1{,}6 e^{-1{,}25t}$$

$$I(t) = 1{,}6(1 - e^{-1{,}25t})$$

$$I(0{,}5) = 1{,}6(1 - e^{-1{,}25.\,0{,}5})$$

$$I(0{,}5) = 1{,}6(1 - 0{,}54)$$

$$I(0{,}5) = 1{,}6 . 0{,}46 = 0{,}736\ A$$

6. → 5 dr file word

$$Y = Ce^{kt}$$

$$Y_0 = Ce^{k.0} = C$$

$$Y_4 = Ce^{4k}$$

$$Y_4 = 2Y_0$$

$$\cancel{C}e^{4k} = 2.\cancel{C}$$

$$e^{4k} = 2$$

$$4k = \ln 2$$

$$k = \frac{1}{4}\ln 2$$

$$Y = Ce^{(\frac{1}{4}\ln 2)t}$$

$$3Y_0 = Y_x$$

$$3\cancel{C} = \cancel{C}e^{(\frac{1}{4}\ln 2).x}$$

$$3 = e^{\frac{1}{4}\ln 2 . x}$$

$$\ln 3 = \frac{x}{4}\ln 2$$

$$\ln 3 = \ln 2^{x/4}$$

$$(3)^4 = \left(2^{x/4}\right)^4$$

$$81 = 2^x$$

$$x = {}^2\log 81 = 6,34 \text{ jam}$$

6. a. $V_R = iR$ $\quad i = \dfrac{dQ}{dt}$ $\quad V_s = \cos 2t$

$V_L = L\dfrac{di}{dt}$ $\quad R = 10\ \Omega$

$L = 1\ H$

$V_C = \dfrac{Q}{C}$ $\quad C = \dfrac{1}{12}\ F$

$$V_s = V_R + V_L + V_C$$

$$V_s = iR + L\frac{di}{dt} + \frac{Q}{C}$$

$$V_s = R\frac{dQ}{dt} + L\frac{d}{dt}\cdot\frac{dQ}{dt} + \frac{Q}{C}$$

$$V_s = L\cdot\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C}$$

$$\frac{d^2Q}{dt^2} + 10\frac{dQ}{dt} + 12Q = \cos 2t$$

b. $Q(0) = 0$

$Q'' + 10Q' + 12Q = \cos 2t \longrightarrow g(t) = \cos 2t$

$Y_T = Y_H + Y_P$

Y_H: $r^2 + 10r + 12 = 0$

$$r_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-10 \pm \sqrt{10^2 - 4.1.12}}{2.1}$$

$$= \frac{-10 \pm \sqrt{52}}{2}$$

$$= \frac{\cancel{-10} \pm \cancel{2}\sqrt{13}}{\cancel{2}}$$

$$= -5 \pm \sqrt{13}$$

$r_1 = -5 + \sqrt{13}$

$r_2 = -5 - \sqrt{13}$

$$Y_H = C_1e^{r_1x} + C_2e^{r_2x}$$

$$= C_1e^{(-5+\sqrt{13})x} + C_2e^{(-5-\sqrt{13})x} \longleftrightarrow Y_H = V_1y_1 + V_2y_2$$

$$y_1 = e^{(-5+\sqrt{13})x} \qquad y_2 = e^{(-5-\sqrt{13})x}$$

$$y_1' = (-5+\sqrt{13})e^{--} \qquad y_2' = (-5-\sqrt{13})\,e^{--}$$

$$w = \begin{vmatrix} e^{(-5+\sqrt{13})x} & e^{(-5-\sqrt{13})x} \\ (-5+\sqrt{13})e^{-} & (-5-\sqrt{13})e^{-} \end{vmatrix} = (-5-\sqrt{13})e^{-10x} - (-5+\sqrt{13})e^{-10x}$$

$$w = -2\sqrt{13}\,e^{-10x} \longrightarrow -2\sqrt{13}\,e^{-10t}$$

$$V_1 = \int \frac{y_2 \cdot f(t)}{w}\,dt = \int \frac{e^{(-5-\sqrt{13})t} \cdot \cos 2t}{-2\sqrt{13}\,e^{-10t}}\,dt = \_\_\_\_$$

$$V_2 = \int \frac{y_1 \cdot f(t)}{w}\,dt = \int \frac{e^{(-5+\sqrt{13})t} \cdot \cos 2t}{-2\sqrt{13}\,e^{-10t}}\,dt = \_\_\_\_$$

$$y_h = V_1 y_1 + V_2 y_2$$

$$y_T = y_u + y_h$$

$$f(t) = \cos 2t$$

$$Y_h = A \sin 2t + B \cos 2t$$

$$Y_h' = 2A \cos 2t - 2B \sin 2t$$

$$Y_h'' = -4A \sin 2t - 4B \cos 2t$$

$$y'' + 10y' + 12y = \cos 2t$$

$$(-4A \sin 2t - 4B \cos 2t) + 10(2A \cos 2t - 2B \sin 2t) + 12(A \sin 2t + B \cos 2t) = \cos 2t$$

$$(8A - 20B) \sin 2t + (20A + 8B) \cos 2t = \cos 2t + 0 \cdot \sin 2t$$

$8A - 20B = 0 \qquad 20A + 8B = 1$

$2A - 5B = 0 \qquad 20\left(\frac{5}{2}B\right) + 8B = 1$

$2A = 5B$

$A = \frac{5}{2}B$

$58B = 1$

$B = \frac{1}{58}$

$A = \frac{5}{116}$

$$Y_h = \frac{5}{116} \sin 2t + \frac{1}{58} \cos 2t$$

$$Y_T = Y_n + Y_h$$

$$Y_T = c_1 e^{(-5+\sqrt{13})x} + c_2 e^{(-5-\sqrt{13})x} + \frac{5}{116} \sin 2t + \frac{1}{58} \cos 2t$$