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1.a. $T: V \to R$; V adalah RHD; $T(\vec{u}) = \|\vec{u}\|$

$T(\vec{u}) = \langle \vec{u}, \vec{u} \rangle = u_1^2 + u_2^2 + \dots + u_n^2$

$\Rightarrow T(\vec{u}+\vec{v}) = \langle (\vec{u}+\vec{v}), (\vec{u}+\vec{v}) \rangle = (u_1+v_1)^2 + (u_2+v_2)^2 + \dots + (u_n+v_n)^2$

$= (u_1^2 + 2u_1v_1 + v_1^2) + (u_2^2 + 2u_2v_2 + v_2^2) + \dots + (u_n^2 + 2u_nv_n + v_n^2)$

Jadi $T(\vec{u}+\vec{v}) \neq T(\vec{u}) + T(\vec{v})$

$\therefore$ $T: V \to R$ dengan $T(\vec{u}) = \|\vec{u}\|$ bukan merupakan transformasi linear

b. $T: M_{mn} \to M_{nm}$; $T(A) = A^T$

$\Rightarrow$ $T(A+B) = (A+B)^T$

$T(A+B) = A^T + B^T$

$T(A+B) = T(A) + T(B)$

$\Rightarrow$ $\alpha \in R$

$T(\alpha A) = (\alpha A)^T$

$= \alpha \cdot A^T$

$= \alpha . T(A)$

$\therefore$ $T: M_{mn} \to M_{nm}$ dengan $T(A) = A^T$ merupakan transformasi linear

c. $T: P_2 \to P_2$; $T(a_0 + a_1x + a_2x^2) = a_0 + a_1(x+1) + a_2(x+1)^2$

$\Rightarrow T((a_0 + a_1x + a_2x^2) + (b_0 + b_1x + b_2x^2))$

$\hookrightarrow T((a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2) = (a_0+b_0) + (a_1+b_1)(x+1) + (a_2+b_2)(x+1)^2$

$= a_0 + a_1(x+1) + a_2(x+1)^2 + b_0 + b_1(x+1) + b_2(x+1)^2$

$= T(a_0 + a_1x + a_2x^2) + T(b_0 + b_1x + b_2x^2)$

$\Rightarrow$ $\alpha \in R$

$T(\alpha a_0 + \alpha a_1 x + \alpha a_2 x^2) = \alpha a_0 + \alpha a_1(x+1) + \alpha a_2(x+1)^2$

$= \alpha(a_0 + a_1(x+1) + a_2(x+1)^2)$

$= \alpha . T(a_0 + a_1x + a_2x^2)$

$\therefore$ $T: P_2 \to P_2$ dengan $T(a_0 + a_1x + a_2x^2) = a_0 + a_1(x+1) + a_2(x+1)^2$ merupakan transformasi linear

d. $T: P_2 \to P_2$; $T(a_0 + a_1x + a_2x^2) = (a_0+1) + (a_1+1)x + (a_2+1)x^2$

$\Rightarrow T((a_0 + a_1x + a_2x^2) + (b_0 + b_1x + b_2x^2))$

$\hookrightarrow T((a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2) = (a_0+b_0+1) + (a_1+b_1+1)x + (a_2+b_2+1)x^2$

$= (a_0+1) + (a_1+1)x + (a_2+1)x^2 + b_0 + b_1x + b_2x^2$

$= T(a_0 + a_1x + a_2x^2) + b_0 + b_1x + b_2x^2$

$\therefore$ $T: P_2 \to P_2$ dengan $T(a_0 + a_1x + a_2x^2) = (a_0+1) + (a_1+1)x + (a_2+1)x^2$ bukan transformasi linear

2. $\vec{u}_1 = (1,1)$, $\vec{u}_2 = (1,0)$, $T(\vec{u}_1) = (1,-2)$, $T(\vec{u}_2) = (-4,1)$

$T: R^2 \to R^2$

$T(\vec{u}) = A(\vec{u})$

$T\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = A\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

$\begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{-1}$

$A = \begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$A = \begin{bmatrix} -4 & 5 \\ 1 & -3 \end{bmatrix}$

a. $T(x_1, x_2) = A(x_1, x_2)$

$= \begin{bmatrix} -4 & 5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= \begin{bmatrix} -4x_1 + 5x_2 \\ x_1 - 3x_2 \end{bmatrix}$

$= (-4x_1 + 5x_2, x_1 - 3x_2)$

b. $T(5,-3) = A(5,-3)$

$= \begin{bmatrix} -4 & 5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 5 \\ -3 \end{bmatrix}$

$= \begin{bmatrix} -35 \\ 14 \end{bmatrix}$

$= (-35, 14)$

3. $\vec{v}_1 = (1,1,1)$, $\vec{v}_2 = (1,1,0)$, $\vec{v}_3 = (1,0,0)$, $T: R^3 \to R^3$, $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

$T(\vec{v}_1) = (2,-1,4)$, $T(\vec{v}_2) = (3,0,1)$, $T(\vec{v}_3) = (-1,5,1)$

$T(\vec{v}_1, \vec{v}_2, \vec{v}_3) = A(\vec{v}_1, \vec{v}_2, \vec{v}_3)$

$$\begin{bmatrix} 2 & 3 & -1 \\ -1 & 0 & 5 \\ 4 & 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & -1 \\ -1 & 0 & 5 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} 2 & 3 & -1 \\ -1 & 0 & 5 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 4 & -2 \\ 5 & -5 & -1 \\ 1 & 0 & 3 \end{bmatrix}$$

a. $T(x_1, x_2, x_3) = A(x_1, x_2, x_3)$

$$= \begin{bmatrix} -1 & 4 & -2 \\ 5 & -5 & -1 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} -x_1 + 4x_2 - 2x_3 \\ 5x_1 - 5x_2 - x_3 \\ x_1 + 3x_3 \end{bmatrix}$$

b.

$$T(2,4,-1) = \begin{bmatrix} -.2 + 4.4 - 2.(-1) \\ 5.2 - 5.4 - (-1) \\ 2 + 3.(-1) \end{bmatrix} = \begin{bmatrix} 16 \\ -9 \\ -1 \end{bmatrix} = (16, -9, -1)$$

4. $T: V \to R^3, \quad T(\vec{v}_1) = (1,-1,2), \; T(\vec{v}_2) = (0,3,2), \; T(\vec{v}_3) = (-3,1,2)$

$$T(2\vec{v}_1 - 3\vec{v}_2 + 4\vec{v}_3) = T(2\vec{v}_1) + T(-3\vec{v}_2) + T(4\vec{v}_3)$$

$$= 2T(\vec{v}_1) - 3T(\vec{v}_2) + 4T(\vec{v}_3)$$

$$= 2(1,-1,2) - 3(0,3\,2) + 4(-3,1,2)$$

$$= (2,-2,4) - (0,9,6) + (-12,4,8)$$

$$= (-10, -7, 6)$$