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1. A (1, 1) D (1, 1, 1)
   B (6, 5) E (1, 1, 9)
   C (2, 4) F (1, 14, 5)

a. $\vec{CA} = \hat{A} - \hat{C}$
$= (1\hat{i} + 1\hat{j}) - (2\hat{i} + 4\hat{j})$
$= -\hat{i} - 3\hat{j}$

$\vec{DF} = F - D$
$= (\hat{i} + 14\hat{j} + 5\hat{k}) - (\hat{i} + \hat{j} + \hat{k})$
$= 13\hat{j} + 4\hat{k}$

b. $\vec{AB} = B - A$
$= (6\hat{i} + 5\hat{j}) - (\hat{i} + \hat{j})$
$= 5\hat{i} + 4\hat{j}$

$\vec{AC} = C - A$
$= (2\hat{i} + 4\hat{j}) - (\hat{i} + \hat{j})$
$= \hat{i} + 3\hat{j}$

$$\text{proy}_{\vec{AC}} \vec{AB} = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AC}|^2} \cdot \vec{AC}$$

$$= \frac{(5\hat{i} + 4\hat{j}) \cdot (\hat{i} + 3\hat{j})}{1^2 + 3^2} \cdot (\hat{i} + 3\hat{j})$$

$$= \frac{5 + 12}{10} (\hat{i} + 3\hat{j})$$

$$= \frac{17}{10} (\hat{i} + 3\hat{j}) = 1{,}7\hat{i} + 5{,}1\hat{j}$$

c. $$\text{proy}_{\vec{AB}} \vec{AC} = \frac{\vec{AC} \cdot \vec{AB}}{|AB|^2} \cdot \vec{AB}$$

$$= \frac{5 + 12}{5^2 + 4^2} (5\hat{i} + 4\hat{j})$$

$$= \frac{17}{41} (5\hat{i} + 4\hat{j}) = \frac{85}{41}\hat{i} + \frac{68}{41}\hat{j}$$

d. $\vec{BC} = C - B$ $\qquad\qquad$ $\vec{u} = x\hat{i} + y\hat{j}$

$$= (2\hat{i} + 4\hat{j}) - (6\hat{i} + 5\hat{j})$$
$$= -4\hat{i} - \hat{j}$$

$$\vec{BC} \cdot \vec{u} = 0$$
$$(-4\hat{i} - \hat{j}) \cdot (x\hat{i} + y\hat{j}) = 0$$
$$-4x - y = 0$$
$$y = -4x$$

$$\vec{u} = x\hat{i} + y\hat{j}$$
$$= x\hat{i} - 4x\hat{j}$$

$\vec{u}_1 = \hat{i} - 4\hat{j}$, $\vec{u}_2 = 2\hat{i} - 8\hat{j}$, $\vec{u}_3 = 3\hat{i} - 12\hat{j}$

e. $\vec{DE} = E - D$ $\qquad\qquad$ $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$$= (\hat{i} + \hat{j} + 9\hat{k}) - (\hat{i} + \hat{j} + \hat{k})$$
$$= 8\hat{k}$$

$$\vec{DE} \cdot \vec{u} = 0$$
$$(8\hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k}) = 0$$
$$8z = 0$$
$$z = 0$$

$\vec{u} = x\hat{i} + y\hat{j} + z\hat{k} = x\hat{i} + y\hat{j} + 0\hat{k}$ $\quad x, y \in \mathbb{R}$; $x, y \neq 0$

$\vec{u}_1 = \hat{i} + \hat{j}$, $\vec{u}_2 = 2\hat{i} - 2\hat{j}$, $\vec{u}_3 = -3\hat{i} + 3\hat{j}$

f. $\vec{EF} = F - E$ $\qquad\qquad$ $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$$= (\hat{i} + 14\hat{j} + 5\hat{k}) - (\hat{i} + \hat{j} + 9\hat{k})$$
$$= 13\hat{j} - 4\hat{k}$$

$$\vec{EF} \cdot \vec{u} = 0$$
$$(13\hat{j} - 4\hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k}) = 0$$
$$13y - 4z = 0$$
$$13y = 4z$$

$$Y = \frac{4}{13} Z$$

Tegak lurus dengan $\vec{DE}$ & $\vec{EF}$

$Z = 0 \; (\perp \vec{DE})$ & $Y = \frac{4}{13} Z \; (\perp \vec{EF})$

$\therefore Y = Z = 0, \; X \in R, \; x \neq 0$

$\vec{u} = x\hat{i}$

g. $L_{DEF} \rightarrow$ Orientasi titik D

$\vec{DE} = E - D = (\hat{i} + \hat{j} + 9\hat{k}) - (\hat{i} + \hat{j} + \hat{k}) = 8\hat{k}$

$\vec{DF} = F - D = (\hat{i} + 14\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + \hat{k}) = 13\hat{j} + 4\hat{k}$

$$\vec{DE} \times \vec{DF} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 8 \\ 0 & 13 & 4 \end{vmatrix} = -104\hat{i} - 0\hat{j} + 0\hat{k} = -104\hat{i}$$

$L_{DEF} = \frac{1}{2} |\vec{DE} \times \vec{DF}| = \frac{1}{2} \sqrt{(-104)^2} = \frac{1}{2} 104 = 52$

2. $X = (-1, 2, 0)$

$Y = (-1, 7, 12)$

$Z = (-1, 2, 4)$

a. $\vec{XY} = Y - X = (-\hat{i} + 7\hat{j} + 12\hat{k}) - (-\hat{i} + 2\hat{j} + 0\hat{k}) = 5\hat{j} + 12\hat{k}$

$\vec{XZ} = Z - X = (-\hat{i} + 2\hat{j} + 4\hat{k}) - (-\hat{i} + 2\hat{j} + 0\hat{k}) = 4\hat{k}$

b. $\vec{XY} \cdot (\vec{XY} - 3\vec{XZ}) = (5\hat{j} + 12\hat{k}) \cdot (5\hat{j} + 12\hat{k} - 12\hat{k})$

$= (5\hat{j} + 12\hat{k}) \cdot (5\hat{j}) = 25$

c. $\text{proy}_{\vec{XZ}} \vec{XY} = \frac{\vec{XY} \cdot \vec{XZ}}{|\vec{XZ}|^2} \cdot \vec{XZ}$

$= \frac{(5\hat{j} + 12\hat{k}) \cdot (4\hat{k})}{4^2} \cdot 4\hat{k}$

$= \frac{48}{16} \cdot 4\hat{k} = 12\hat{k}$

d. L_{XYZ} → Orientasi titik X

$$\vec{XY} \times \vec{XZ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & 12 \\ 0 & 0 & 4 \end{vmatrix} = 20\,\hat{i}$$

$$L_{XYZ} = \frac{1}{2}\left|\vec{XY} \times \vec{XZ}\right| = \frac{1}{2} \cdot \sqrt{20^2}$$

$$= 10$$