5. $J = \{a + bx + cx^2 \mid a^2 = b^2 + c^2\}$

o) $a + bx + cx^2 \rightarrow 0 = 0 + 0.x + 0.x^2$

$J \neq \{\}$

o) $a^2 = b^2 + c^2$

$a = \sqrt{b^2 + c^2}$

$a + bx + cx^2 \Rightarrow cx^2 + bx + \sqrt{b^2 + c^2}$

$J \subset P_2$

o) $u = \{a_1 + b_1 x + c_1 x^2 \mid a_1^2 = b_1^2 + c_1^2\}$

$v = \{a_2 + b_2 x + c_2 x^2 \mid a_2^2 = b_2^2 + c_2^2\}$

$u, v \in J$

$u + v = (a_1 + b_1 x + c_1 x^2) + (a_2 + b_2 x + c_2 x^2)$

$= (a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2$

$= (\sqrt{b_1^2 + c_1^2} + \sqrt{b_2^2 + c_2^2}) + (b_1 + b_2)x + (c_1 + c_2)x^2$

$a \qquad b \qquad c$

$u + v \notin J$

$(\sqrt{b_1^2 + c_1^2} + \sqrt{b_2^2 + c_2^2})^2 = (b_1 + b_2)^2 + (c_1 + c_2)^2$

$b_1^2 + c_1^2 + b_2^2 + c_2^2 + 2(\sqrt{b_1^2 + c_1^2} \cdot \sqrt{b_2^2 + c_2^2}) = b_1^2 + b_2^2 + 2b_1 b_2 + c_1^2 + c_2^2 + 2c_1 c_2$

~~o) $k \in R$~~

$ku = k(a_1 + b_1 x + c_1 x^2)$

$= ka_1 + kb_1 x + kc_1 x^2$

$= k\sqrt{b_1^2 + c_2^2} + kb_1 x + kc_1 x^2$

$= \sqrt{k^2 b_1^2 + k^2 c_2^2} + kb_1 x + kc_1 x^2$

$ku \in J$

$\therefore J$ merupakan subruang P_2

$\therefore J$ bukan merupakan subruang P_2

Menentukan basis

$\partial = \{a + bx + cx^2 \mid a^2 = b^2 + c^2\}$

$k\partial = u$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} k = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \to \begin{matrix} ka = p \\ kb = q \\ kc = r \end{matrix} \to \begin{matrix} k\sqrt{b^2+c^2} = p \\ kb = q \\ kc = r \end{matrix} \to p^2 = \sqrt{q^2 + r^2}$$

Untuk $p, q, r \in R$ sembarang, ∂ tidak membangun P_2

Untuk $p = q = r = 0$, ∂ merupakan bebas linear