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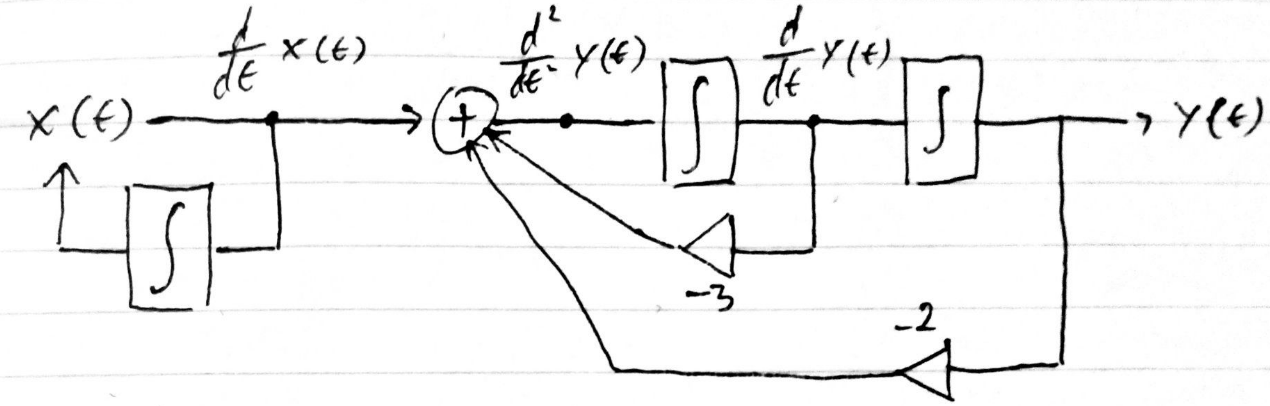
2. $\frac{d^2}{dt^2} y(t) + 3\frac{d}{dt} y(t) + 2y(t) = \frac{d}{dt} x(t) + x(t)$

a.



b. $y_h(t) = \dots ?$

$$\frac{d^2}{dt^2} y(t) + 3\frac{d}{dt} y(t) + 2y(t) = 0$$

$$r^2 + 3r + 2 = 0$$

$$(r+1)(r+2) = 0$$

$$r_1 = -1 \qquad r_2 = -2$$

$$y_h(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

$$= c_1 e^{-t} + c_2 e^{-2t}$$

c. $x(t) = \cos(t)$ $\qquad$ $\frac{d}{dt} x(t) = -\sin t$

$$y_p(t) = A\sin(t) + B\cos(t)$$

$$y_p'(t) = A\cos t - B\sin t$$

$$y_p''(t) = -A\sin t - B\cos t$$

$$Y_p''(t) + 3Y_p'(t) + 2Y_p(t) = X'(t) + X(t)$$

$$-A\sin t - B\cos t + 3(A\cos t - B\sin t) + 2(A\sin t + B\cos t) = -\sin t + \cos t$$

$$(-A - 3B + 2A)\sin t + (-B + 3A + 2B)\cos t = -\sin t + \cos t$$

$$\begin{array}{ll} A - 3B = -1 & \times 1 \\ 3A + B = 1 & \times 3 \end{array} \qquad \begin{array}{l} A - 3B = -1 \\ 9A + 3B = 3 \\ \hline 10A = 2 \end{array} \; +$$

$$A = 0,2 \rightarrow B = 0,4$$

$$Y_p(t) = A\sin t + B\cos t$$
$$= 0,2\sin t + 0,4\cos t$$

d. $Y(t) = Y_h(t) + Y_p(t)$

$$Y(t) = C_1 e^{-t} + C_2 e^{-2t} + 0,2\sin t + 0,4\cos t$$

$$Y(0) = 0$$

$$C_1 + C_2 + 0 + 0,4 = 0$$

$$C_1 + C_2 = -0,4$$

$$\left.\frac{d}{dt} Y(t)\right|_{t=0} = 0$$

$$\left. -C_1 e^{-t} - 2C_2 e^{-2t} + 0,2\cos t - 0,4\sin t \right|_{t=0} = 0$$

$$-C_1 - 2C_2 + 0,2 - 0 = 0$$

$$\begin{array}{l} C_1 + 2C_2 = -0,2 \\ C_1 + C_2 = -0,4 \\ \hline C_2 = 0,2 \end{array} \; -$$

$$C_1 = -0,6$$

$$Y(t) = C_1 e^{-t} + C_2 e^{-2t} + 0,2\sin t + 0,4\cos t$$
$$= -0,6e^{-t} + 0,2e^{-2t} + 0,2\sin t + 0,4\cos t$$

2 $$\frac{d^2}{dt^2}y(t)+3\frac{d}{dt}y(t)+2y(t)=\frac{d}{dt}x(t)+x(t)$$

$$(j\Omega)^2Y(j\Omega)+3(j\Omega)Y(j\Omega)+2Y(j\Omega)=(j\Omega)X(j\Omega)+X(j\Omega)$$

$$H(j\Omega)=\frac{Y(j\Omega)}{X(j\Omega)}$$

$$=\frac{1+j\Omega}{2-\Omega^2+3j\Omega}$$

5 $$|H(j\Omega)|=\frac{\sqrt{1^2+\Omega^2}}{\sqrt{(2-\Omega^2)^2+(3\Omega)^2}}$$

$$=\frac{\sqrt{1+\Omega^2}}{\sqrt{4-4\Omega^2+\Omega^4+9\Omega^2}}$$

$$=\frac{\sqrt{\Omega^2+1}}{\sqrt{\Omega^4+5\Omega^2+4}}$$

g. $$H(j\Omega) = \frac{1 + j\Omega}{2 - \Omega^2 + 3j\Omega} \times \frac{2 - \Omega^2 - 3j\Omega}{2 - \Omega^2 - 3j\Omega}$$

$$= \frac{2}{\Omega^2 + 4} + i\,\frac{(-\Omega - \Omega^3)}{\Omega^4 + 5\Omega^2 + 4}$$

$$\operatorname{Arg} H(j\Omega) = \tan^{-1}\left(\frac{Im}{Re}\right)$$

$$= \tan^{-1}\left(\frac{\dfrac{-\Omega - \Omega^3}{\Omega^4 + 5\Omega^2 + 4}}{\dfrac{2}{\Omega^2 + 4}}\right)$$

$$= \tan^{-1}\left(-\frac{\Omega}{2}\right)$$