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5.3 NORMAL-INCIDENCE PLANE-WAVE REFLECTION AT PERFECTLY CONDUCTING PLANE

This is just an important special case of the general analysis presented in the last section. We assume that region 2 is a perfect conductor $\sigma_2 \to \infty$, and the wave impedance in this region would then be

$$\hat{\eta}_2 = \sqrt{\frac{\mu_2}{\epsilon_2 - j\frac{\sigma_2}{\omega}}} = 0 \qquad \text{as } \sigma_2 \to \infty \tag{5.11}$$

To simplify the standing wave analysis to be described next, we shall further assume that region 1 is a perfect dielectric $\sigma_1 = 0$.

Substituting equation 5.11 in the reflection and transmission coefficient expressions in equations 5.8 and 5.10, we obtain

$$\hat{T} = 0, \qquad \hat{\Gamma} = -1$$

The zero value of the transmission coefficient simply means that the amplitude of the transmitted field in region 2—that is, $\hat{E}^+_{m2} = 0$. This can be explained in terms of the depth of penetration parameter, which is zero in a perfectly conducting region. In other words, there would be no transmitted wave in the perfectly conducting region because of the inability of time-varying fields to penetrate media with conductivities approaching ∞. With the absence of a transmitted wave, the incident and reflected fields in region 1 will be the only present ones in our special case. For $\hat{\Gamma} = -1$, the amplitude of the

reflected wave $\hat{E}^-_{m1} = -\hat{E}^+_{m1}$. The reflected wave thus is equal in amplitude and is opposite in phase to the incident wave. In other words, all the incident energy is reflected back by the perfect conductor. The incident and reflected fields also combine with their equal magnitudes and opposite phases to satisfy the boundary condition at the surface of the perfect conductor. This can be illustrated by examining the expression for the total electric field $\mathbf{E}^{tot}(z)$ in region 1, which is assumed to be a perfect dielectric (i.e., $\alpha_1 = 0$)

$$\hat{\mathbf{E}}^{tot}(z) = \hat{\mathbf{E}}^i(z) + \hat{\mathbf{E}}^r(z) = \hat{E}^+_{m1} e^{-j\beta_1 z}\,\mathbf{a}_x + \hat{E}^-_{m1} e^{j\beta_1 z}\,\mathbf{a}_x$$

Substituting $\hat{E}^-_{m1} = -\hat{E}^+_{m1}$, we obtain

$$\begin{aligned}\hat{\mathbf{E}}^{tot}(z) &= \hat{E}^+_{m1}(e^{-j\beta_1 z} - e^{j\beta_1 z})\,\mathbf{a}_x \\ &= -2j\,\hat{E}^+_{m1} \sin\beta_1 z\,\mathbf{a}_x\end{aligned} \tag{5.12}$$

From equation 5.12, it is clear that the total electric field is zero at the perfectly conducting surface ($z = 0$), which satisfies the required boundary condition.

To study the propagation characteristics of the compound wave in front of the perfect conductor, we need to obtain the real-time form of the electric field. We routinely multiply the complex form of the field in equation 5.12 by $e^{j\omega t}$ and take the real part of the resulting expression, hence,

$$\begin{aligned}\mathbf{E}^{tot}(z,t) &= Re\left[e^{j\omega t}\,\hat{\mathbf{E}}^{tot}(z)\right] \\ &= 2E^+_{m1} \sin(\beta_1 z) \sin\omega t\,\mathbf{a}_x\end{aligned} \tag{5.13}$$

In equation (5.13) the amplitude of the electric field was assumed real E_{m1}^{+}. It is our objective to show next that this total field in region 1 is not a traveling wave, although it was obtained by combining two traveling waves of the same frequency, equal amplitudes, and are propagating in opposite directions. To show this, let us sketch the variation of the total electric field in equation 5.13 as a function of z at various time intervals. Figure 5.2 shows these variations from which we may make the following observations:

1. The amplitude of the total electric field is always zero at the surface of the perfect conductor. This simply indicates that the total field satisfies the boundary condition at all times.
2. The maximum amplitude of the total electric field is double that of the incident wave. This maximum amplitude occurs at the specific locations ($z = \lambda/4, 3\lambda/4$, etc.) and at specific times ($\omega t = \pi/2, 3\pi/2$, etc.) at which both the incident and reflected waves constructively interfere.
3. There are locations in front of the perfect conductor ($z = \lambda/2, \lambda, 3\lambda/2$, etc.) at which the total electric field is always zero. These are the locations at which the incident and reflected fields go through a destructive interference process for all values of ωt. These locations are known as the null locations of the total electric field.
4. The null locations as well as the locations at which the constructive interferences occur do not change with time (i.e., as a function of ωt). All that actually changes

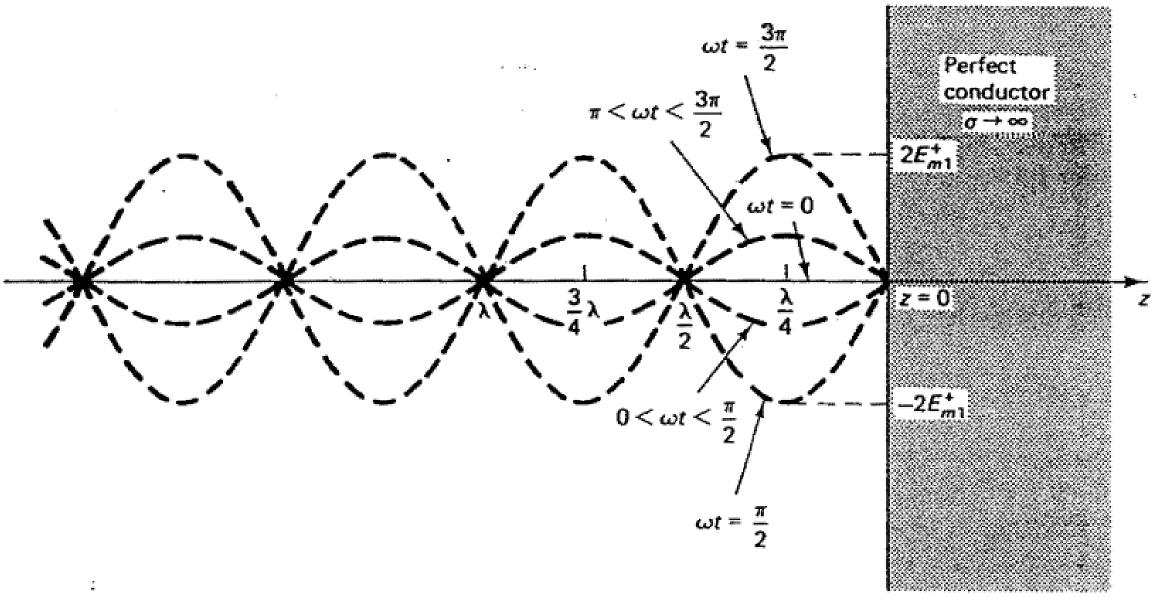

Figure 5.2 The variation of the total electric field in front of the perfect conductor as a function of z and at various time intervals ωt.

with time is the amplitude of the total field at the nonnull locations. This is why the wave resulting from the interference of the incident and reflected waves is called "standing wave" or nonpropagating wave.

We should also emphasize the difference between the electric field expressions for the traveling and standing waves. For a traveling wave, the electric field is given by

$$\mathbf{E}(z,t) = E_{m1}^{+} \cos(\omega t - \beta_1 z)\,\mathbf{a}_x$$

where the term $(\omega t - \beta_1 z)$ or $\omega(t - z/v_1)$ emphasizes the coupling between the location as a function of time of a specific point (constant phase) propagating along the wave. The constant phase term $(t - z/v_1)$ indicates that with the increase in t, z should also increase to maintain a constant value of $(t - z/v_1)$, which characterizes a specific point on the wave. This simply means that a wave with an electric field expression which includes $\cos(\omega t - \beta_1 z)$ is a propagating wave in the positive z direction. From equation 5.13, conversely, the time t and location z variables are uncoupled. In other words, the electric field distribution as a function of z in front of the perfect conductor follows a $\sin(\beta_1 z)$ variation, with the locations of the field nulls being those values of z at which $\sin(\beta_1 z) = 0$. The effect of the time term $\sin(\omega t)$ is simply to modify the amplitude of the field so as to vary as a function of time at the nonzero field locations as shown in Figure 5.2.

The permanent locations of the electric field nulls are determined by finding the values of $\beta_1 z$, which would make the value of the field zero. Thus, from equation 5.12, it may be seen that

$$\hat{\mathbf{E}}^{\omega t}(z) = 0 \text{ at } \beta_1 z = n\pi \qquad (n = 0, \pm 1, \pm 2, \ldots)$$

Hence,

$$\frac{2\pi}{\lambda_1} z = n\pi$$

or

$$z = n\frac{\lambda_1}{2} \tag{5.14}$$

where λ_1 is the wavelength in region 1. Equation 5.14 shows that $\hat{\mathbf{E}}^{tot}(z)$ is zero at the boundary $z = 0$ and at every half wavelength distance away from the boundary in region 1 as shown in Figure 5.2.

Let us also obtain an expression for the total magnetic field,

$$\hat{\mathbf{H}}^{tot}(z) = \hat{\mathbf{H}}^{i}(z) + \hat{\mathbf{H}}^{r}(z) = \left(\frac{E_{m1}^{+}}{\eta_1} e^{-j\beta_1 z} - \frac{E_{m1}^{-}}{\eta_1} e^{j\beta_1 z}\right)\mathbf{a}_y$$

The minus sign in the reflected magnetic field expression is simply because for a negative z-propagating wave the amplitude of the reflected magnetic field is related to that of the reflected electric field by $(-\eta_1)$. Substituting $E_{m1}^{-} = -E_{m1}^{+}$, we obtain

$$\begin{aligned} \hat{\mathbf{H}}^{tot}(z) &= \frac{E_{m1}^{+}}{\eta_1}\left(e^{-j\beta_1 z} + e^{j\beta_1 z}\right)\mathbf{a}_y \\ &= 2\frac{E_{m1}^{+}}{\eta_1} \cos\beta_1 z \, \mathbf{a}_y \end{aligned} \tag{5.15}$$

The time-domain form of the magnetic field expression is obtained from equation 5.15 as

$$\mathbf{H}^{tot}(z,t) = 2\frac{E_{m1}^{+}}{\eta_1} \cos\beta_1 z \, \cos\omega t \, \mathbf{a}_y \tag{5.16}$$

This is also a standing wave as shown in Figure 5.3, with the maximum amplitude of the magnetic field occurring at the perfect conductor interface ($z = 0$) where the total electric field is zero. The locations of the nulls in the magnetic field are at the values of z at which $\cos\beta_1 z = 0$, hence,

$$\beta_1 z = \text{odd number of } \frac{\pi}{2} = (2m+1)\frac{\pi}{2} \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

or

$$z = (2m+1)\frac{\lambda_1}{4} \tag{5.17}$$

The magnetic field distribution in front of a perfectly conducting boundary is shown in Figure 5.3, where it is clear that its first null occurs at $z = \lambda_1/4$, which is the location of the maximum electric field (see Figure 5.2). Comparing equation 5.13 with equation 5.16 also shows that the electric and magnetic fields of a standing wave are 90° out of time phase. This is simply because equation 5.13 contains a $\sin(\omega t)$ term, whereas 5.16

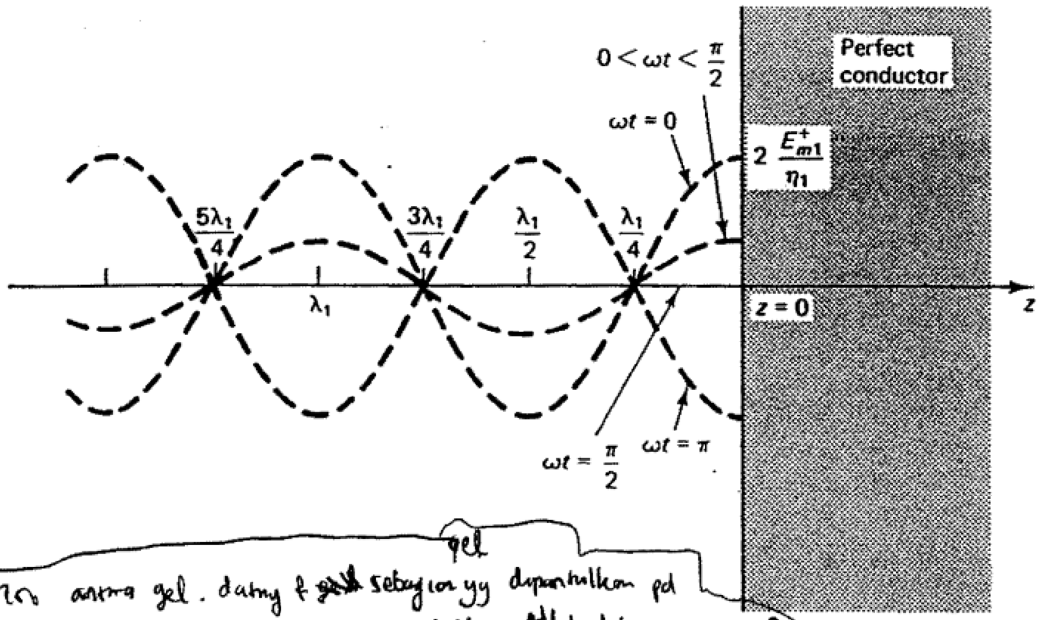

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Figure 5.3 The magnetic field distribution in front of a perfect conductor as a function of time.

includes a $\cos(\omega t)$ time variation. The fact that these fields are 90° out of time phase results in a zero average power being transmitted in either direction by the standing wave. This can be illustrated by using the complex forms of the fields to calculate the time-average Poynting vector $\mathbf{P}_{av}(z)$

$$\begin{aligned}\mathbf{P}_{av}(z) &= \frac{1}{2}Re[\hat{\mathbf{E}}(z) \times \hat{\mathbf{H}}^*(z)] \\ &= \frac{1}{2}Re\left[-2j\,E_{m1}^+ \sin\beta_1 z\,\mathbf{a}_x \times 2\frac{E_{m1}^+}{\eta_1}\cos\beta_1 z\,\mathbf{a}_y\right] = 0\end{aligned} \tag{5.18}$$

The zero value of $\mathbf{P}_{av}(z)$ is obtained because the result of the vector product of $\hat{\mathbf{E}}(z) \times \hat{\mathbf{H}}^*(z)$ is an imaginary number. This zero value of average power transmitted by this wave is yet another reason for calling the total wave in front of a perfect conductor a "standing wave."

Exercise

Use the time-domain forms of the electric and magnetic fields of a standing wave to show that by integrating the instantaneous Poynting vector over a complete period, the time-average Poynting vector is also zero, thus confirming the result in equation 5.18.

EXAMPLE 5.3

A plane wave of amplitude $\hat{E}_{m1}^+ = 100e^{j0^\circ}$ and frequency = 150 MHz is propagating in a medium of $\mu_1 = \mu_o$ and of a wave impedance $\eta_1 = 100\ \Omega$. If this wave is reflected by a perfectly conducting plane boundary, determine the following:

1. The location of the first two consecutive nulls of the total electric field in front of the perfect conductor.
2. The location of the first null of the total magnetic field.

Also obtain time-domain expressions for the total electric and magnetic fields and determine the amplitude of the magnetic field at the surface of the conductor and at distance $z = -2$ m from it.

Solution

To determine the locations of the nulls as well as the required expressions for the electric and magnetic fields, we need the properties of the propagation medium. Because η_1 is given as a real number, it is clear that the medium is nonconductive—that is, $\sigma_1 = 0$. Substituting σ_1 and $\mu_1 = \mu_o$ in the expression of η_1, we obtain

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1 - j\dfrac{\sigma_1}{\omega}}} = \sqrt{\frac{\mu_o}{\epsilon_o \epsilon_{r_1}}} = \frac{120\pi}{\sqrt{\epsilon_{r_1}}} = 100$$

$$\therefore \epsilon_{r_1} = 14.21$$

The propagation constant β_1 is then

$$\beta_1 = \omega\sqrt{\mu_o \epsilon_o \epsilon_{r_1}} = 11.84 \text{ rad/m}$$

$$\lambda_1 = \frac{2\pi}{\beta_1} = 0.53 \text{ m}$$

1. Locations of the first two nulls of the electric field are thus

$$z = 0, \frac{\lambda_1}{2}$$

$$= 0, 0.27 \text{ m}$$

2. Location of the first null of the magnetic field is

$$z = \frac{\lambda_1}{4} = 0.135 \text{ m}$$

The time-domain expressions of the electric and magnetic fields are obtained by substituting the appropriate constants in equations 5.13 and 5.16, hence,

$$\mathbf{E}(z,t) = 200 \sin(11.84z) \sin(9.4 \times 10^8 t)\, \mathbf{a}_x \text{ V/m}$$

$$\mathbf{H}(z,t) = 2 \cos(11.84z) \cos(9.4 \times 10^8 t)\, \mathbf{a}_y \text{ A/m}$$

The amplitude of the magnetic field at the surface of the conductor $z = 0$ is

$$|\mathbf{H}(0,t)| = 2 \cos(9.4 \times 10^8 t) \text{ A/m}$$

At $z = -2m$,

$$|\mathbf{H}(-2,t)| = 2 \cos(-23.68) \cos(9.4 \times 10^8 t) = 1.83 \cos(9.4 \times 10^8 t) \text{ A/m}$$

EXAMPLE 5.4

A uniform plane wave is propagating in a lossless medium and is normally incident on a plane perfect conductor at $f = 400$ MHz. If the measured distance between any two successive zeros of the total electric field in front of the conductor is 12.5 cm, determine the following:

1. The relative permittivity of the lossless medium, assuming $\mu = \mu_o$.
2. The shortest distance from the conductor at which the total magnetic field is zero.
3. If the amplitude of the incident electric field $\hat{E}^+_{m1} = 120e^{j\pi}$ V/m, calculate the magnitude of the magnetic field at a distance $z = 0.8$ m from the surface of the conductor, and also find the magnitude and direction of the induced surface current.

Solution

1. The distance between successive zeros is $\lambda_1/2$ where λ_1 is the wavelength in the medium. Hence, from the given information we note that,

0,25 m

$$\lambda_1 = 0.25 \text{ m}$$

β_1 for a lossless medium is given by $\beta_1 = \omega\sqrt{\mu_o \epsilon_o \epsilon_r}$. Relating β_1 to λ_1, we obtain

$$\lambda_1 = \frac{2\pi}{\beta_1} = \frac{2\pi}{\omega\sqrt{\mu_o \epsilon_o \epsilon_r}} = 0.25$$

Therefore the dielectric constant of the medium ϵ_r is given by

$$\epsilon_r = 9$$

2. The magnetic field is zero at distance $z = -\lambda_1/4 = -0.0625$ m.

3. $H_y(z,t) = \dfrac{2E^+_{m1}}{\eta_1} \cos \beta_1 z \cos \omega t$

η_1 for a lossless medium is given by

$$\eta_1 = \sqrt{\frac{\mu_o}{\epsilon_o \epsilon_r}} = \frac{120\pi}{\sqrt{9}} = 40\pi$$

$$\therefore H_y(-0.8,t) = \frac{2 \times 120}{40\pi} \cos(-6.4\pi) \cos \omega t$$

$$= 1.91 \cos(-6.4\pi) \cos \omega t \text{ A/m}$$

The current induced on the surface of the perfect conductor may be obtained from the boundary condition on the tangential component of the magnetic field. Hence,

$$\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

Because the magnetic field inside the perfect conductor $\mathbf{H}_2$ is zero, we have

$$\mathbf{n} \times \mathbf{H}_1 = \mathbf{J}_s$$

or

$$\mathbf{J}_s = -\mathbf{a}_z \times \frac{2E_{m1}^{+}}{\eta_1} \cos \beta_1 z \cos \omega t \, \mathbf{a}_y \Big|_{z=0}$$

$$= \frac{2E_{m1}^{+}}{\eta_1} \cos \omega t \, \mathbf{a}_x = 1.91 \cos \omega t \, \mathbf{a}_x \text{ A/m}$$