$\quad\quad\quad\quad\quad\quad A_r \quad\; A_\theta \quad\; A_\phi$

1. Diketahui vektor $\vec{A} = 3a_r - 7a_\theta + 2a_\phi$ memiliki titik pangkal pada titik **(1, π/2, 0)** pada koordinat bola dan

$\quad\quad\quad\quad B_r \quad B_\theta \quad B_\phi$

vektor $\vec{B} = -2a_r - 4a_\theta + 2a_\phi$ dengan titik pangkal pada titik **(3, π/2, π/2)**.

Tentukan $\vec{A} - \vec{B}$.

* Vektor $\vec{A}$

$(r, \theta, \phi) = (1, \pi/2, 0)$

$A_x = A_r \sin\theta \cos\phi + A_\theta \cos\theta \cos\phi - A_\phi \sin\phi$

$A_x = 3 . \sin\left(\frac{\pi}{2}\right) \cos(0) + (-7) . \cos\left(\frac{\pi}{2}\right) \cos(0) - 2 \sin(0)$

$A_x = 3 . 1 . 1 - 7 . 0 . 1 - 2 . 0$

$A_x = 3$

$A_y = A_r \sin\theta . \sin\phi + A_\theta \cos\theta . \sin\phi + A_\phi \cos\phi$

$A_y = 3 . \sin\left(\frac{\pi}{2}\right) . \sin(0) + (-7) \cos\left(\frac{\pi}{2}\right) . \sin(0) + 2 . \cos(0)$

$A_y = 3 . 1 . 0 + (-7) . 0 . 0 + 2 . 1$

$A_y = 2$

$A_z = A_r \cos\theta - A_\theta \sin\theta$

$A_z = 3 . \cos\left(\frac{\pi}{2}\right) - (-7) . \sin\frac{\pi}{2}$

$A_z = 3 . 0 + 7 . 1 = 7$

$\vec{A} = 3\hat{a}_x + 2\hat{a}_y + 7\hat{a}_z$

* Vektor $\vec{B}$

$(r, \theta, \phi) = (3, \pi/2, \pi/2)$

$B_x = B_r \sin\theta \cos\phi + B_\theta \cos\theta \cos\phi - B_\phi \sin\phi$

$B_x = -2 . \sin(\pi/2) . \cos(\pi/2) + (-4) \cos(\pi/2) \cos(\pi/2) - 2 . \sin(\pi/2)$

$B_x = -2 . 1 . 0 - 4 . 0 . 0 - 2 . 1 = -2$

$$B_y = B_r \sin\theta \cdot \sin\phi + B_\theta \cos\theta \sin\phi + B_\phi \cos\phi$$

$$B_y = -2 \cdot \sin(\pi/2) \sin(\pi/2) + (-4) \cos(\pi/2) \sin(\pi/2) + 2 \cos(\pi/2)$$

$$B_y = -2 \cdot 1 \cdot 1 - 4 \cdot 0 \cdot 1 + 2 \cdot 0 = -2$$

$$B_z = B_r \cos\theta - B_\theta \sin\theta$$

$$B_z = -2 \cdot \cos(\pi/2) - (-4) \sin(\pi/2)$$

$$B_z = -2 \cdot 0 + 4 \cdot 1 = 4$$

$$\vec{B} = -2\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z$$

$$\vec{A} - \vec{B} = (3\hat{a}_x + 2\hat{a}_y + 7\hat{a}_z) - (-2\hat{a}_x - 2\hat{a}_y + 4\hat{a}_z)$$

$$\vec{A} - \vec{B} = 5\hat{a}_x + 4\hat{a}_y + 3\hat{a}_z$$

$\vec{A} \rightarrow Q_1$

$\vec{B} \rightarrow Q_2$

2. Dua buah muatan listrik terletak pada bidang XY. Muatan Q_1 sebesar 10^{-9} C terletak pada titik koordinat (0,0) m dan muatan Q_2 sebesar 4×10^{-9} C terletak pada titik (3,0) m. Tentukan total intensitas medan listrik pada titik (1,0) m dan (1,2) m yang disebabkan oleh kedua muatan listrik tersebut.

$r = \sqrt{A_x^2 + A_y^2 + A_z^2}$

* Titik (1,0)

$\vec{A} = (1-0)\hat{a}_x + (0-0)\hat{a}_y = \hat{a}_x \; ; \; r_A = 1$

$\vec{B} = (1-3)\hat{a}_x + (0-0)\hat{a}_y = -2\hat{a}_x \; ; \; r_B = 2$

$$\vec{E} = \vec{E_A} + \vec{E_B}$$

$$= \frac{k\,Q_1}{r_A^2}\vec{a} + \frac{k\,Q_2}{r_B^2}\vec{b}$$

$$= \frac{9\times\cancel{10^9}\cdot\cancel{10^{-9}}}{1^2}(\hat{a}_x) + \frac{9\times\cancel{10^9}\cdot \cancel{4}\times\cancel{10^{-9}}}{\cancel{2^2}}\left(\frac{-\cancel{2}\hat{a}_x}{\cancel{2}}\right)$$

$$= 9\hat{a}_x - 9\hat{a}_x = 0 \text{ N/C}$$

* Titik (1,2)

$\vec{A} = (1-0)\hat{a}_x + (2-0)\hat{a}_y = \hat{a}_x + 2\hat{a}_y \; ; \; r_A = \sqrt{1^2+2^2} = \sqrt{5}$

$\vec{B} = (1-3)\hat{a}_x + (2-0)\hat{a}_y = -2\hat{a}_x + 2\hat{a}_y \; ; \; r_B = \sqrt{(-2)^2+2^2} = 2\sqrt{2}$

$$\vec{E} = \vec{E_A} + \vec{E_B} = \frac{k\,Q_1}{r_A^2}\vec{a} + \frac{k\,Q_2}{r_B^2}\vec{b}$$

$$\vec{E} = \frac{9\times\cancel{10^9}\cdot\cancel{10^{-9}}}{(\sqrt{5})^2}\cdot\left(\frac{\hat{a}_x+2\hat{a}_y}{\sqrt{5}}\right) + \frac{9\times\cancel{10^9}\cdot 4\times\cancel{10^{-9}}}{(2\sqrt{2})^2}\left(\frac{-\cancel{2}\hat{a}_x + \cancel{2}\hat{a}_y}{\cancel{2}\sqrt{2}}\right)$$

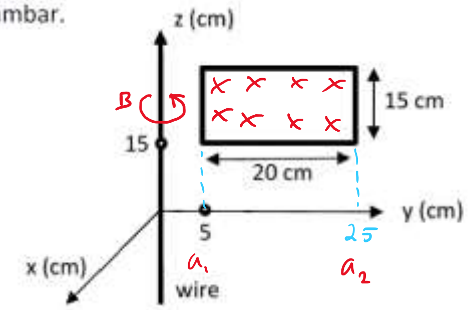
$$= \frac{9}{5}\cdot\left(\frac{\hat{a}_x+2\hat{a}_y}{\sqrt{5}}\right) + \frac{\overset{9}{\cancel{36}}}{\cancel{8}_2}\left(\frac{-\hat{a}_x+\hat{a}_y}{\sqrt{2}}\right)$$

$$= \frac{9\hat{a}_x + 18\hat{a}_y}{5\sqrt{5}} + \frac{-9\hat{a}_x + 9\hat{a}_y}{2\sqrt{2}}$$

$$= -2{,}37\,\hat{a}_x + 4{,}79\,\hat{a}_y \text{ N/C}$$

$I = I\cos(\omega t)$ A

3. Jika sebuah kawat lurus yang terletak pada sumbu-Z dialiri arus listrik sebesar $\boxed{\text{I cos ωt Ampere,}}$ maka tentukan induced electromagnetic force (emf) atau gaya gerak listrik induksi yang dihasilkan di sekitar loop segi empat yang terletak pada bidang ZY seperti gambar.



$\mu_0 = 4\pi \times 10^{-7}$

JAWAB:

$$B = \frac{\mu_0 I}{2\pi a} = \int_{a_1}^{a_2} \frac{\mu_0 I}{2\pi a}\, da = \frac{\mu_0 I}{2\pi} \ln(a)\Big|_{a_1}^{a_2}$$

$$= \frac{\overset{2}{\cancel{4\pi}} \times 10^{-7} . I}{\cancel{2\pi}} \ln(a)\Big|_{0,05}^{0,25} \quad \text{dalam meter}$$

$$= 2\times 10^{-7} . I \left(\ln(0,25) - \ln(0,05)\right)$$

$$= 2\times 10^{-7} . I (\ln 5)$$

$$= 2\times 10^{-7} . I . 1,6 = 3,2\times 10^{-7}\, I \ T$$

$I \rightarrow I\cos(\omega t)$

$$A = 0,2 . 0,15 = 0,03 \ m^2$$

$$\varepsilon = \frac{d\phi}{dt} = \frac{d(BA)}{dt} = \frac{d}{dt}\left(3,2\times 10^{-7} . I\cos(\omega t) . 0,03\right)$$

$$= \frac{d}{dt}\, 9,6\times 10^{-9} . I\cos(\omega t)$$

$$= \omega . 9,6 \times 10^{-9} . I . -\sin(\omega t) \quad \text{nano}$$

$$= -9,6\, I\omega \sin(\omega t) \ nV$$

$E \to$ silinder; $(h_1, h_2, h_3) = (1, \rho, 1)$; $(u_1, u_2, u_3) = (\rho, \phi, z) = (0.5, \pi/3, 0)$

$\varepsilon \nabla \cdot E$

4. Kuat medan listrik statis dinyatakan dalam bentuk vektor $\vec{E} = \overset{E_1}{3\rho^2 \hat{a}_\rho} + \overset{E_2}{\rho \cos\phi\, \hat{a}_\phi} + \overset{E_3}{\rho^3 \hat{a}_z}$ pada koordinat silinder. Tentukan rapat muatan volume yang terkait dengan medan listrik tersebut pada titik $(0{,}5, \pi/_3, 0)$.

$\rho \quad \phi \quad z$

$$\rho_v = \nabla \cdot E = \frac{1}{h_1 h_2 h_3}\left(\frac{\partial E_1 . h_2 h_3}{\partial u_1} + \frac{\partial h_1 E_2 h_3}{\partial u_2} + \frac{\partial h_1 h_2 E_3}{\partial u_3}\right)$$

$$= \frac{1}{1.\rho.1}\left(\frac{\partial 3\rho^2.\rho.1}{\partial \rho} + \frac{\partial 1.\rho \cos\phi.1}{\partial \phi} + \frac{\partial 1.\rho.\rho^3}{\partial z}\right)$$

$$= \frac{1}{\cancel{\rho}}\left(9\rho^{\cancel{2}} + \cancel{\rho}. - \sin\phi + 0\right)$$

$$= 9\rho - \sin\phi$$

$$= 9(0,5) - \sin(\pi/3)$$

$$= 4,5 - \frac{1}{2}\sqrt{3}$$

$$= 3,6\ C/m^3$$