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TT - 43 - 11
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Saya mengerjakan ujian ini dengan jujur dan mandiri. Jika saya melakukan pelanggaran, maka saya bersedia menerima sanksi.

1. $x' = \begin{pmatrix} -6 & 2 \\ -3 & 1 \end{pmatrix} x$

$A = \begin{pmatrix} -6 & 2 \\ -3 & 1 \end{pmatrix}$

$\det(A - \lambda I) = 0$

$\left| \begin{pmatrix} -6 & 2 \\ -3 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$

$\left| \begin{pmatrix} -6 & 2 \\ -3 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$

$\begin{vmatrix} -6-\lambda & 2 \\ -3 & 1-\lambda \end{vmatrix} = 0$

$(-6-\lambda)(1-\lambda) - 2.(-3) = 0$

$\cancel{-6} + 6\lambda - \lambda + \lambda^2 \cancel{+6} = 0$

$\lambda^2 + 5\lambda = 0$

$\lambda(\lambda+5) = 0$

$\lambda_1 = 0 \quad \lambda_2 = -5$

2. $x' = \begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} x \qquad \lambda_1 = \lambda_2 = 6$

$\lambda_1 = 6$

$(A - \lambda I)K = 0$

$\left[\begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} - 6 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = 0$

$\left[\begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} \right] \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = 0$

$\begin{pmatrix} 6 & -9 \\ 4 & -6 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = 0$

$\left.\begin{matrix} 6k_1 - 9k_2 = 0 \\ 4k_1 - 6k_2 = 0 \end{matrix}\right\rangle 2k_1 - 3k_2 = 0$

$2k_1 = 3k_2 \qquad K_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

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$$P = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}$$

$$(A - \lambda I)P = K$$

$$\left[\begin{pmatrix} 12 & -9 \\ 4 & 0 \end{pmatrix} - 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}\begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$6P_1 - 9P_2 = 3$$
$$4P_1 - 6P_2 = 2$$

$$2P_1 - 3P_2 = 1$$

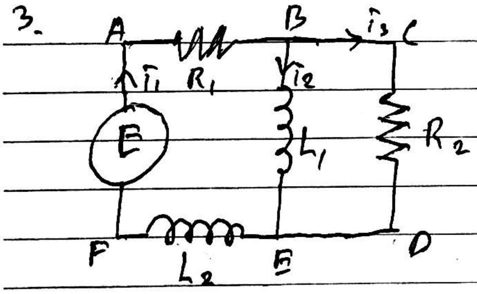
$$P_1 = \frac{1 + 3P_2}{2}$$ → ambil $P_2 = 1$ maka $P_1 = 2$

$$P = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$X = C_1 K e^{\lambda t} + C_2\left[K t e^{\lambda t} + P e^{\lambda t}\right]$$

$$X = C_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{6t} + C_2\left[\begin{pmatrix} 3 \\ 2 \end{pmatrix} t e^{6t} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{6t}\right]$$

3.



$$i_1 = i_2 + i_3$$

Loop 1 (ABEFA)

$\cancel{i_1 R_1}$

$$i_1 R_1 + L_1 \frac{di_2}{dt} + L_2 \frac{di_1}{dt} = E$$

$$(i_2 + i_3)R_1 + L_1 \frac{di_2}{dt} + L_2 \frac{d(i_2 + i_3)}{dt} = E$$

$$i_2 R_1 + i_3 R_1 + L_1 \frac{di_2}{dt} + L_2 \frac{di_2}{dt} + L_2 \frac{di_3}{dt} = E$$

$$(L_1 + L_2)\frac{di_2}{dt} + L_2 \frac{di_3}{dt} = E - i_2 R_1 - i_3 R_1$$

Loop 2 (ABCDEFA)

$$i_1 R_1 + i_3 R_2 + L_2 \frac{di_1}{dt} = E$$

$$(i_2 + i_3) R_1 + i_3 R_2 + L_2 \frac{d(i_2 + i_3)}{dt} = E$$

$$i_2 R_1 + i_3 R_1 + i_3 R_2 + L_2 \frac{di_2}{dt} + L_2 \frac{di_3}{dt} = E$$

$$L_2 \left(\frac{di_2}{dt} + \frac{di_3}{dt} \right) = E - i_2 R_1 - (R_1 + R_2) i_3$$

$$\frac{di_3}{dt} = \frac{E - i_2 R_1 - (R_1 + R_2) i_3}{L_2} - \frac{di_2}{dt}$$

$$(L_1 + L_2) \frac{di_2}{dt} + L_2 \frac{di_3}{dt} = E - i_2 R_1 - i_3 R_1$$

$$(L_1 + L_2) \frac{di_2}{dt} + L_2 \left(\frac{E - i_2 R_1 - (R_1 + R_2) i_3}{L_2} - \frac{di_2}{dt} \right) = E - i_2 R_1 - i_3 R_1$$

$$(L_1 + L_2) \frac{di_2}{dt} + \cancel{E} - \cancel{i_2 R_1} - (R_1 + R_2) i_3 - L_2 \frac{di_2}{dt} = \cancel{E} - \cancel{i_2 R_1} - i_3 R_1$$

$$L_1 \frac{di_2}{dt} = (R_1 + R_2) i_3 - i_3 R_1$$

$$\frac{di_2}{dt} = \frac{R_2}{L_1} i_3$$

$$\frac{di_3}{dt} = \frac{E - i_2 R_1 - (R_1 + R_2) i_3}{L_2} - \frac{R_2}{L_1} i_3$$

$$\frac{di_1}{dt} = \frac{di_2}{dt} + \frac{di_3}{dt}$$

$$\frac{di_1}{dt} = \cancel{\frac{R_2}{L_1} i_3} + \left(\frac{E - i_2 R_1 - (R_1 + R_2) i_3}{L_2} - \cancel{\frac{R_2}{L_1} i_3} \right)$$

$$\frac{di_1}{dt} = \frac{E - i_2 R_1 - (R_1 + R_2) i_3}{L_2}$$

4. $\frac{di_1(t)}{dt} = -\frac{R_1}{L_1} i_1(t) + \frac{R_1}{L_1} i_3(t) + \frac{V(t)}{L_1}$

$= -\frac{2}{1} i_1(t) + \frac{2}{1} i_3(t) + \frac{3}{1}$

$\frac{di_1(t)}{dt} = -2 i_1(t) + 2 i_3(t) + 3$

$\frac{di_3(t)}{dt} = \frac{R_1}{L_2} i_1(t) - \frac{(R_1 + R_2)}{L_2} i_3(t)$

$= \frac{2}{1} i_1(t) - \frac{2+3}{1} i_3(t)$

$\frac{di_3(t)}{dt} = 2 i_1(t) - 5 i_3(t)$

$\frac{di_1(t)}{dt} = -2 i_1(t) + 2 i_3(t) + 3$

$I' = \begin{pmatrix} -2 & 2 \\ 2 & -5 \end{pmatrix} I + \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ dengan $I = \begin{pmatrix} i_1 \\ i_3 \end{pmatrix}$

$\det(A - \lambda I) = 0$

$\left| \begin{pmatrix} -2 & 2 \\ 2 & -5 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$

$\begin{vmatrix} -2-\lambda & 2 \\ 2 & -5-\lambda \end{vmatrix} = 0$

$(-2-\lambda)(-5-\lambda) - 2.2 = 0$

$10 + 2\lambda + 5\lambda + \lambda^2 - 4 = 0$

$\lambda^2 + 7\lambda + 6 = 0$

$(\lambda + 6)(\lambda + 1) = 0$

$\lambda_1 = -6 \quad \lambda = -1$

$\circ) \lambda_1 = -6$

$(A - \lambda_1 I) K_1 = 0$

$\left[\begin{pmatrix} -2 & 2 \\ 2 & -5 \end{pmatrix} - \begin{pmatrix} -6 & 0 \\ 0 & -6 \end{pmatrix} \right] \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = 0$

$\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = 0$

$4k_1 + 2k_2 = 0$

$2k_1 + k_2 = 0$

$2k_1 + k_2 = 0$

$2k_1 = -k_2 \qquad K_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

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o) $\lambda_2 = -1$

$(A - \lambda_2 I) K_2 = 0$

$\left[\begin{pmatrix} -2 & 2 \\ 2 & -5 \end{pmatrix} - \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\right]\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = 0$

$\begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix} = 0$

$-k_1 + 2k_2 = 0$

$2k_1 - 4k_2 = 0$

$k_1 - 2k_2 = 0$

$k_1 = 2k_2 \to K_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$I_H = C_1 K_1 e^{\lambda_1 t} + C_2 K_2 e^{\lambda_2 t}$

$I_H = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-6t} + C_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t}$

$F(t) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$I_p' = A I_p + F(t)$

$I_p = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$I_p' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2a_1 + 2b_1 + 3 \\ 2a_1 - 5b_1 + 0 \end{pmatrix}$

$-2a_1 + 2b_1 + 3 = 0$

$2a_1 - 5b_1 = 0$

$2a_1 = 5b_1$

$-(5b_1) + 2b_1 + 3 = 0 \qquad a_1 = 5/2\, b_1$

$-3b_1 = -3 \qquad a_1 = 2,5 b_1$

$b_1 = 1 \qquad a_1 = 2,5$

$I_p = \begin{pmatrix} 2,5 \\ 1 \end{pmatrix}$

$I = I_H + I_p$

$I = C_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-6t} + C_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 2,5 \\ 1 \end{pmatrix}$

$i_1(t) = C_1 e^{-6t} + 2C_2 e^{-t} + 2,5$

$i_3(t) = -2C_1 e^{-6t} + C_2 e^{-t} + 1$

$i_1(0) = 0$

$C_1 e^{-6 \cdot 0} + 2C_2 e^{-0} + 2,5 = 0$

$C_1 + 2C_2 + 2,5 = 0$

$C_1 + 2C_2 = -2,5$

$i_3(0) = 0$

$-2C_1 e^{-6 \cdot 0} + C_2 e^{-\cdot 0} + 1 = 0$

$-2C_1 + C_2 = -1$ ... $| \times 1 |$

$C_1 + 2C_2 = -2{,}5$ $| \times 2 |$

$$\begin{array}{r} -2C_1 + C_2 = -1 \\ 2C_1 + 4C_2 = -5 \\ \hline 5C_2 = -6 \end{array} +$$

$C_2 = -\frac{6}{5} = -1{,}2$

$C_1 = -\frac{1}{10} = -0{,}1$

$i_1(t) = -0{,}1 e^{-6t} - 2{,}4 e^{-t} + 2{,}5$

$i_3(t) = 0{,}2 e^{-6t} - 1{,}2 e^{-t} + 1$

5. a. $L\{e^{-2t} \cos 3t\} = \frac{s-(-2)}{(s-(-2))^2 + 3^2}$

$= \frac{s+2}{(s+2)^2 + 9}$

$= \frac{s+2}{s^2 + 4s + 4 + 9}$

$= \frac{s+2}{s^2 + 4s + 13}$

b. $L^{-1}\left\{\frac{5}{(s-4)(s-1)}\right\}$

$\frac{5}{(s-4)(s-1)} = \frac{A}{(s-4)} + \frac{B}{(s-1)}$

$= \frac{A(s-1) + B(s-4)}{(s-4)(s-1)}$

$As + Bs - A - 4B = 5$

$A + B = 0$

$$\begin{array}{r} -A - 4B = 5 \\ \hline -3B = 5 \end{array} +$$

$B = -\frac{5}{3}$

$A = \frac{5}{3}$

$\frac{5}{(s-4)(s-1)} = \frac{5}{3} \frac{1}{s-4} - \frac{5}{3} \frac{1}{s-1}$

$$L^{-1}\left\{\frac{5}{(s-4)(s-1)}\right\} = L^{-1}\left\{\frac{5}{3}\frac{1}{s-4} - \frac{5}{3}\frac{1}{s-1}\right\}$$
$$= \frac{5}{3}\left(L^{-1}\left\{\frac{1}{s-4}\right\} - L^{-1}\left\{\frac{1}{s-1}\right\}\right)$$
$$= \frac{5}{3}\left(e^{4t} - e^{t}\right)$$

6. $\ddot{y} + 2\dot{y} + y = 2e^{-t}, \; y(0)=0, \dot{y}(0) = 6$

$$L\{\ddot{y}+2\dot{y}+y\} = L\{2e^{-t}\}$$
$$L\{\ddot{y}\}+2L\{\dot{y}\} + L\{y\} = 2L\{e^{-t}\}$$
$$\left(s^2Y(s) - s.Y(0) - \dot{y}(0)\right) + 2\left(s.Y(s) - \dot{y}(0)\right) + \ddot{y}(0) = 2\,\frac{1}{s+1}$$
$$s^2Y(s) - s.0 \cancel{-6} + 2(s.Y(s) - 6) \cancel{+6} = \frac{2}{s+1}$$
$$Y(s)(s^2+2s) - 12 = \frac{2}{s+1}$$
$$Y(s)(s^2+2s) = \frac{2}{s+1} + 12$$
$$Y(s) = \frac{2}{(s+1).(s^2+2s)} + \frac{12}{(s^2+2s)}$$
$$Y(s) = \frac{2}{s(s+1)(s+2)} + \frac{12}{s(s+2)}$$
$$Y(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2} + \frac{6}{s} - \frac{6}{s+2}$$
$$Y(s) = \frac{7}{s} - \frac{2}{s+1} - \frac{5}{s+2}$$
$$y = L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{7}{s} - \frac{2}{s+1} - \frac{5}{s+2}\right\}$$
$$= 7L^{-1}\left\{\frac{1}{s}\right\} - 2L^{-1}\left\{\frac{1}{s+1}\right\} - 5L^{-1}\left\{\frac{1}{s+2}\right\}$$
$$y = 7 - 2e^{-t} - 5e^{-2t}$$