5. $f(x,y) = (x^2 - y^2 + B) + i(2xy)$

$f(x,y) = (x^2 - y^2 + 95) + i(2xy)$

a. $u_x = 2x$ $\qquad v_x = 2y$

$u_y = -2y$ $\qquad v_y = 2x$

$u_x = v_y$ $\qquad u_y = -v_x$

$2x = 2x$ $\qquad -2y = -(2y)$

$f'(x,x) = \frac{\partial}{\partial x}\left(x^2 - y^2 + 95) + i(2xy)\right)$

$f'(x,y) = 2x + i2y$

$f'(x,y) = 2(x + iy)$

$f'(z) = 2z$

$f(z) = \int 2z \, dz$

$f(z) = z^2 + c$

$f(x,y) = (x+iy)^2 + c$

$(x^2 - y^2 + 95) + i(2xy) = x^2 - y^2 + i2xy + c$

$c = 95$

$f(z) = z^2 + c$

$f(z) = z^2 + 95$

b. $\int_c f(z)\,dz$, $\qquad z = t + it^2$, $\quad 0 \le t \le 1$

$\int_c f(z)\,dz = \int_c (z^2 + 95)\,dz$

$= \int_0^1 \left[(t + it^2)^2 + 95\right] d(t + it^2)$

$$= \int_0^1 (t^2 - t^4 + i2t^3 + 95)(dt + idt^2)$$

$$= \int_0^1 (t^2 - t^4 + i2t^3 + 95)dt + i\int_0^1 (t^2 - t^4 + i2t^3 + 95)dt^2$$

$$= \frac{1}{3}t^3 - \frac{1}{5}t^5 + \frac{it^4}{2} + 95t \Big|_0^1 + i\int_0^1 (u - u^2 + i2u^{3/2} + 95)du$$

$$= \left(\frac{1}{3} - \frac{1}{5} + \frac{1}{2}i + 95\right) + i\left(\frac{1}{2}u^2 - \frac{1}{3}u^3 + \frac{i4u^{5/2}}{5} + 95u \Big|_0^1\right)$$

$$= \frac{1427}{15} + \frac{1}{2}i + i\left(\frac{1}{2}t^4 - \frac{1}{3}t^6 + \frac{i4t^5}{5} + 95t^2 \Big|_0^1\right)$$

$$= \frac{1427}{15} + \frac{1}{2}i + i\left(\frac{1}{2} - \frac{1}{3} + \frac{4}{5}i + 95\right)$$

$$= \frac{1427}{15} + \frac{1}{2}i + \frac{571}{6}i - \frac{4}{5}$$

$$= \frac{283}{3} + \frac{287}{3}i$$