5th Material Subject: Counting Technique

Undergraduate of Telecommunication Engineering

MUH1F3 - PROBABILITY AND STATISTICS

Telkom University

Center of eLearning & Open Education Telkom University

Jl. Telekomunikasi No.1, Bandung - Indonesia

http://www.telkomuniversity.ac.id

Lecturer: Nor Kumalasari Caecar Pratiwi, S.T., M.T. (caecarnkcp@telkomuniversity.ac.id)

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WELCOME

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LEARNING OBJECTIVES:

After careful study of this chapter, student should be able to do the following:

1. **Use Bayes theorem to calculate conditional probabilities**
2. **Interpret and calculate conditional probabilities of events**

MULTIPLICATION RULE (FOR COUNTING TECHNIQUE)

In more complicated examples, determining the outcomes in the sample space becomes more difficult. Instead, counts of the numbers of outcomes in the sample space and various events are used to analyze the random experiments. These methods are referred to as **Counting Techniques**. Some simple rules can be used to simplify the calculations.

Examples: An automobile manufacturer provides vehicles with selected options. Each vehicle is ordered:

- With or without an automatic transmission
- With or without a sunroof
- With one of three choices of a stereo system
- With one of four exterior colors

If the sample space consists of the set of all possible vehicle types, what is the number of outcomes in the sample space?

Answer: The sample space contains 48 outcomes. The tree diagram for the different types of vehicles is displayed in Figure below.

MULTIPLICATION RULE (FOR COUNTING TECHNIQUE)

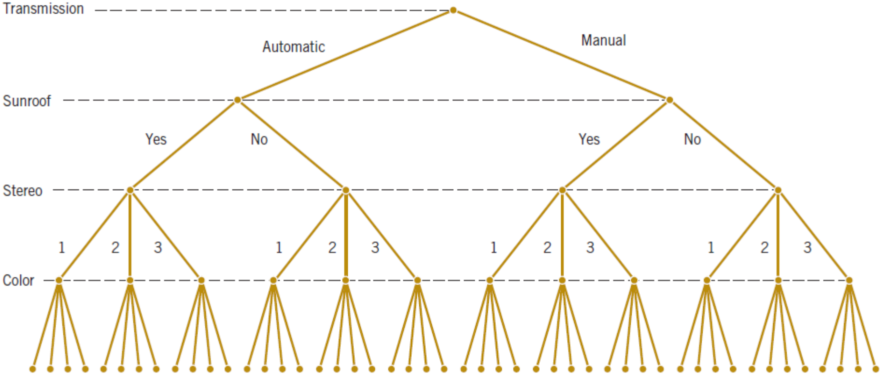

Figure 1: Tree diagram for different types of vehicles with 48 outcomes in the sample space

MULTIPLICATION RULE (FOR COUNTING TECHNIQUE)

The tree diagram in Figure 1 describes the sample space of all possible vehicle types. The size of the sample space equals the number of branches in the last level of the tree, and this quantity equals $\mathbf{2} \times \mathbf{2} \times \mathbf{3} \times \mathbf{4} = \mathbf{48}$. This leads to the following useful result, called **Multiplication Rule for Counting Techniques**.

Assume an operation can be described as a sequence of **k** steps, and:

- the number of ways of completing **step 1** is $\mathbf{n_1}$, and
- the number of ways of completing **step 2** is $\mathbf{n_2}$ for each way of completing step 1, and
- the number of ways of completing **step 3** is $\mathbf{n_3}$ for each way of completing step 2, and so forth.

The total number of ways of completing the operation is:

$$\mathbf{n_1 \times n_2 \times n_3 \times \cdots \times n_k} \tag{1}$$

MULTIPLICATION RULE (FOR COUNTING TECHNIQUE)

Example: The design for a Website is to consist of four colors, three fonts, and three positions for an image. From the multiplication rule, $\mathbf{4} \times \mathbf{3} \times \mathbf{3} = \mathbf{36}$ different designs are possible. Practical Interpretation: The use of the multiplication rule and other counting techniques enables one to easily determine the number of outcomes in a sample space or event and this, in turn, allows probabilities of events to be determined.

PERMUTATION

Another useful calculation finds the number of ordered sequences of the elements of a set. Consider a set of elements, such as $\mathbf{S} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$. A **Permutation** of the elements is an ordered sequence of the elements. For example, **abc**, **acb**, **bac**, **bca**, **cab**, and **cba** are all of the permutations of the elements of **S**. The number of permutations of **n** different elements is $\mathbf{n}!$ where:

$$\mathbf{n}! = \mathbf{n} \times (\mathbf{n}-\mathbf{1}) \times (\mathbf{n}-\mathbf{2}) \times (\mathbf{n}-\mathbf{3}) \times \cdots \times \mathbf{2} \times \mathbf{1} \tag{2}$$

This result follows from the multiplication rule. A permutation can be constructed by selecting the element to be placed in the first position of the sequence from the **n** elements, then selecting the element for the second position from the remaining $\mathbf{n}-\mathbf{1}$ elements, then selecting the element for the third position from the remaining $\mathbf{n}-\mathbf{2}$ elements, and so forth. The number of permutations of subsets of **r** elements selected from a set of **n** different elements is:

$$\mathbf{P}_{\mathbf{r}}^{\mathbf{n}} = \frac{\mathbf{n}!}{(\mathbf{n}-\mathbf{r})!} \tag{3}$$

PERMUTATION

Example: A printed circuit board has eight different locations in which a component can be placed. If three different components are to be placed on the board, how many different designs are possible?

Answer: Each design consists of selecting a location from the eight locations for the first component, a location from the remaining seven for the second component, and a location from the remaining six for the third component. Therefore,

$$P_3^8 = 8 \times 7 \times 6 = \frac{8!}{(8-3)!} = 336 \text{ different designs are possible}$$

COMBINATION

Another counting problem of interest is the number of subsets of **r** elements that can be selected from a set of **n** elements. Here, order is not important. These are called **Combination**. The number of combinations, subsets of **r** elements that can be selected from a set of **n** elements, is denoted as $\binom{n}{r}$ or $\mathbf{C}^{\mathbf{n}}_{\mathbf{r}}$.

$$\mathbf{C}^{\mathbf{n}}_{\mathbf{r}} = \binom{n}{r} = \frac{\mathbf{n}!}{\mathbf{r}! \cdot (\mathbf{n} - \mathbf{r})!} \tag{4}$$

Example: A printed circuit board has eight different locations in which a component can be placed. If fi ve identical components are to be placed on the board, how many different designs are possible?

Answer: Each design is a subset of size fi ve from the eight locations that are to contain the components. From Equation 4, the number of possible designs is

$$\mathbf{C}^{8}_{5} = \frac{8!}{5!(8-5)!} = \mathbf{56}\ \textbf{ different designs are possible}$$

COMBINATION

Example: A bin of 50 manufactured parts contains 3 defective parts and 47 non defective parts. A sample of 6 parts is selected from the 50 parts without replacement. That is, each part can be selected only once, and the sample is a subset of the 50 parts. How many different samples are there of size 6 that contain exactly 2 defective parts?

Answer: A subset containing exactly 2 defective parts can be formed by first choosing the 2 defective parts from the three defective parts. Using Equation 4, this step can be completed in

$$C_2^3 = \frac{3!}{2!(3-2)!} = 3 \text{ different ways}$$

Then, the second step is to select the remaining 4 parts from the 47 acceptable parts in the bin. The second step can be completed in:

$$C_4^{47} = \frac{47!}{4!(47-4)!} = 178.365 \text{ different ways}$$

COMBINATION

Therefore, from the multiplication rule, the number of subsets of size 6 that contain exactly 2 defective parts is:

$$\mathbf{3 \times 178365 = 535.095 \ \ different\ ways}$$

As an additional computation, the total number of different subsets of size 6 is found to be:

$$\mathbf{C_6^{50} = \frac{50!}{6!(50-6)!} = 15.890.700 \ \ different\ ways}$$

Thank You