Pers. Homogen

1. Jika diketahui sebuah SPL Nonhomogen sebagai berikut :

$$\boxed{X' = \underbrace{\begin{pmatrix} -3 & 0 \\ 0 & 5 \end{pmatrix}}_{A} X} + \begin{pmatrix} -15 \\ 25 \end{pmatrix} t \rightarrow \text{Extra}$$

Tentukan :

a. Persamaan Karakteristik
b. Nilai Eigen
c. Vektor Eigen
d. Solusi Homogen
e. Solusi Partikular
f. Solusi Total

a. $\det(A - \lambda I) = \left| \begin{bmatrix} -3 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = \begin{vmatrix} -3-\lambda & 0 \\ 0 & 5-\lambda \end{vmatrix}$

$= (-3-\lambda)(5-\lambda)$

$= -15 + 3\lambda - 5\lambda + \lambda^2$

Pers. Karakteristik $\rightarrow = \boxed{\lambda^2 - 2\lambda - 15 = 0}$

b. $\lambda^2 - 2\lambda - 15 = 0$

$(\lambda - 5)(\lambda + 3) = 0$

$\boxed{\lambda_1 = 5}$ $\boxed{\lambda_2 = -3}$ Nilai Eigen

c. $\lambda_1 = 5$

$(A - \lambda_1 I)(k_1) = 0$

$\left(\begin{bmatrix} -3 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \right) \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$

$\begin{bmatrix} -8 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$

$-8k_1 = 0$

$k_2 = 0$

$k_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\lambda_2 = -3$

$(A - \lambda I) K_2 = 0$

$$\left(\begin{bmatrix} -3 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \right) \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$k_1 = 0$

$8k_2 = 0$

$K_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

d. $X_h = C_1 K_1 e^{\lambda_1 t} + C_2 K_2 e^{\lambda_2 t}$

$= C_1 \begin{bmatrix} 0 \\ 0 \end{bmatrix} e^{5t} + C_2 \begin{bmatrix} 0 \\ 0 \end{bmatrix} e^{-3t}$

$ax + b$

e. $F(t) = \begin{bmatrix} -15 \\ 25 \end{bmatrix} t + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-15t + 0$

$25t + 0$

$X_p = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} t + \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$

$X_p' = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$X_p' = A \overbrace{X_p + F(t)}^{X_p}$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & 5 \end{bmatrix} \left(\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} t + \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \right) + \begin{bmatrix} -15 \\ 25 \end{bmatrix} t$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3a_2 t \\ 5b_2 t \end{bmatrix} + \begin{bmatrix} -3a_1 \\ 5b_1 \end{bmatrix} + \begin{bmatrix} -15 \\ 25 \end{bmatrix} t$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3a_2 - 15 \\ 5b_2 + 25 \end{bmatrix} t + \begin{bmatrix} -3a_1 \\ 5b_1 \end{bmatrix}$$

$$-3a_1 = 0 \qquad -3a_2 - 15 = 0 \qquad 5b_2 + 25 = 0$$

$$5b_1 = 0 \qquad -3a_2 = 15 \qquad 5b_2 = -25$$

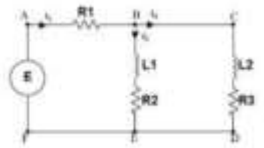
$$a_2 = -5 \qquad b_2 = -5$$

$$X_p = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} t + \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \end{bmatrix} t$$

f. $X = X_h + X_p$

$$= C_1 \begin{bmatrix} 0 \\ 0 \end{bmatrix} e^{5t} + C_2 \begin{bmatrix} 0 \\ 0 \end{bmatrix} e^{-3t} + \begin{bmatrix} -5 \\ -5 \end{bmatrix} t$$

Diketahui sistem persamaan diferensial linear untuk i_2 dan i_3 pada suatu rangkaian listrik berikut :



Tentukan :

a. Sistem persamaan diferensial untuk arus i_2 (t) dan i_3 (t)
Jika diketahui Loop 1 (ABFEA) :

$$R_1 i_1(t) + L_1 \frac{di_2(t)}{dt} + R_2 i_2(t) = E(t)$$

Loop 2 (ABCDEFA) :

$$R_1 i_1(t) + L_2 \frac{di_3(t)}{dt} + R_3 i_3(t) = E(t)$$

b. Dengan menggunakan Metode Koefisien Tak Tentu, selesaikanlah sistem tersebut jika diketahui $R_1 = 2\ \Omega$; $R_2 = 3\ \Omega$; $R_3 = 3\ \Omega$; $L_1 = 1$ H ; $L_2 = 1$ H dan $E(t) = 20$ volt, $i_2(0) = 0$ dan $i_3(0) = 0$

c. Tentukan persamaan untuk $i_2(t)$

$$\Sigma i_B = 0$$

$$i_1 = i_2 + i_3$$

2. a. Loop 1

$$R_1 i_1 + L_1 \frac{di_2}{dt} + R_2 i_2 = E$$

$$R_1 (i_2 + i_3) + L_1 \frac{di_2}{dt} + R_2 i_2 = E$$

$$R_1 i_2 + R_1 i_3 + L_1 \frac{di_2}{dt} + R_2 i_2 = E$$

$$L_1 \frac{di_2}{dt} = E - (R_1 + R_2) i_2 - R_1 i_3$$

$$\frac{di_2}{dt} = \frac{E}{L_1} - \left(\frac{R_1 + R_2}{L_1}\right) i_2 - \frac{R_1}{L_1} i_3 \quad \text{.... (1)}$$

Loop 2

$$R_1 i_1 + L_2 \frac{di_3}{dt} + R_3 i_3 = E$$

$$R_1 (i_2 + i_3) + L_2 \frac{di_3}{dt} + R_3 i_3 = E$$

$$L_2 \frac{di_3}{dt} = E - (R_1 + R_3) i_3 + R_1 i_2$$

$$\frac{di_3}{dt} = \frac{E}{L_2} - \left(\frac{R_1 + R_3}{L_2}\right) i_3 + \frac{R_1}{L_2} i_2 \quad \text{.... (2)}$$

b. $\frac{di_2}{dt} = \frac{E}{L_1} - \left(\frac{R_1+R_2}{L_1}\right) i_2 - \frac{R_1}{L_1} i_3 = \frac{20}{1} - \left(\frac{2+3}{1}\right) i_2 - \frac{2}{1} i_3$

$\frac{di_3}{dt} = \frac{E}{L_2} - \left(\frac{R_1+R_3}{L_2}\right) i_3 - \frac{R_1}{L_2} i_2 = \frac{20}{1} - \left(\frac{2+3}{1}\right) i_3 - \frac{2}{1} i_2$

$$\left.\begin{aligned} \frac{di_2}{dt} &= -5i_2 - 2i_3 + 20 \\ \frac{di_3}{dt} &= -2i_2 - 5i_3 + 20 \end{aligned}\right\} X' = \underbrace{\begin{bmatrix} -5 & -2 \\ -2 & -5 \end{bmatrix}}_{A} X + \underbrace{\begin{bmatrix} 20 \\ 20 \end{bmatrix}}_{F(t)}$$

$\det(A - \lambda I) = 0$

$$\begin{vmatrix} -5-\lambda & -2 \\ -2 & -5-\lambda \end{vmatrix} = 0$$

$(-5-\lambda)^2 - 4 = 0$

$\lambda^2 + 10\lambda + 25 - 4 = 0$

$\lambda^2 + 10\lambda + 21 = 0$

$(\lambda+7)(\lambda+3) = 0$

$\lambda_1 = -7 \quad \lambda_2 = -3$

$\lambda_1 = -7$

$(A - \lambda_1 I) K_1 = 0$

$$\begin{bmatrix} -5+7 & -2 \\ -2 & -5+7 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$k_1 = k_2 \qquad K_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = -3$

$(A - \lambda_2 I) K_2 = 0$

$$\begin{bmatrix} -5+3 & -2 \\ -2 & -5+3 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & -2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$-2k_1 - 2k_2 = 0$

$k_1 = -k_2 \qquad K_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$X_H = C_1 K_1 e^{\lambda_1 t} + C_2 K_2 e^{\lambda_2 t}$$

$$= C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-7t} + C_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t}$$

$$F(t) = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

$$X_p = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$

$$X_p' = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$X_p' = AX_p + F(t)$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ -2 & -5 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} + \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -5a_1 - 2b_1 + 20 \\ -2a_1 - 5b_1 + 20 \end{bmatrix}$$

$$\begin{aligned} 5a_1 + 2b_1 &= 20 \quad \times 5 \\ 2a_1 + 5b_1 &= 20 \quad \times 2 \end{aligned}$$

$$\begin{aligned} 25a_1 + 10b_1 &= 100 \\ 4a_1 + 10b_1 &= 40 \end{aligned} \quad -$$

$$21a_1 = 60$$

$$a_1 = \frac{60}{21} = \frac{20}{7}$$

$$b_1 = \frac{20}{7}$$

$$X_p = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 20/7 \\ 20/7 \end{bmatrix}$$

$$X = X_H + X_p = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-7t} + C_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t} + \begin{bmatrix} 20/7 \\ 20/7 \end{bmatrix}$$

$$i_2 = C_1 e^{-7t} + C_2 e^{-3t} + \frac{20}{7}$$

$$i_3 = C_1 e^{-7t} - C_2 e^{-3t} + \frac{20}{7}$$

c. $i_2(0) = 0$

$$C_1 e^{-7.0} + C_2 e^{-3.0} + \frac{20}{7} = 0$$

$$C_1 + C_2 = -\frac{20}{7}$$

$i_3(0) = 0$

$$C_1 e^{-7.0} - C_2 e^{-3.0} + \frac{20}{7} = 0$$

$$C_1 - C_2 = -\frac{20}{7}$$

$$C_1 + C_2 = -\frac{20}{7}$$

$+$

$$C_1 = -\frac{20}{7}$$

$$C_2 = 0$$

$$i_2(t) = -\frac{20}{7}e^{-7t} + \frac{20}{7}$$

$$i_3(t) = -\frac{20}{7}e^{-7t} + \frac{20}{7}$$

$$i_1 = i_2 + i_3$$

$$i_1(t) = -\frac{20}{7}e^{-7t} + \frac{20}{7} + \left(-\frac{20}{7}e^{-7t} + \frac{20}{7}\right)$$

$$i_1(t) = -\frac{40}{7}e^{-7t} + \frac{40}{7}$$

Sebuah persamaan diferensial dituliskan sebagai berikut :

$\frac{d^2x(t)}{dt^2} - 5\frac{dx(t)}{dt} + 4x(t) = 20$ dengan nilai awal x(0) = 0 dan x'(0) = 1

Tentukan :

a. Persamaan $X(s)$

b. Tentukan $x(t) = \mathcal{L}^{-1}\{X(s)\}$

a. $r^2 - 5r + 4 = 0$

$(r-1)(r-4) = 0$

$r_1 = 1 \quad r_2 = 4$

$X_H = C_1e^t + C_2e^{4t}$

$f(t) = 20$

$x_p = A$

$x_p' = 0$

$x_p'' = 0$

$x_p'' - 5x_p' + 4x_p = 20$

$0 - 5.0 + 4.A = 20$

$A = 5$

$x = x_H + A = C_1e^t + C_2e^{4t} + 5$

$x(0) = 0$

$C_1e^0 + C_2e^{4.0} + 5 = 0$

$C_1 + C_2 = -5$

$x'(0) = 1$

$C_1e^t + 4C_2e^{4t} = 1$

$C_1e^0 + 4C_2e^{4.0} = 1$

$C_1 + 4C_2 = 1$

$C_1 + C_2 = -5 \quad -$

$3C_2 = 6$

$C_2 = 2$

$C_1 = -7$

$e^x = e^x$

$e^{ax} = ae^{ax}$

$x(t) = C_1e^t + C_2e^{4t} + 5$

$x(t) = -7e^t + 2e^{4t} + 5$

b. $X(s) = \mathcal{L}\{x(t)\}$

$= \mathcal{L}\{-7e^t + 2e^{4t} + 5\}$

$= \mathcal{L}\{-7e^t\} + \mathcal{L}\{2e^{4t}\} + \mathcal{L}\{5\}$

$= -7\mathcal{L}\{e^t\} + 2\mathcal{L}\{e^{4t}\} + \mathcal{L}\{5\}$

$= -\frac{7}{s-1} + \frac{2}{s-4} + \frac{5}{s} = \frac{s+20}{s^3-5s^2+4s}$

$\boxed{\begin{aligned} \mathcal{L}\{k\} &= \frac{k}{s} \\ \mathcal{L}\{e^{at}\} &= \frac{1}{s-a} \\ \mathcal{L}\{\sin at\} &= \frac{a}{s^2+a^2} \\ \mathcal{L}\{\cos at\} &= \frac{s}{s^2+a^2} \end{aligned}}$