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1. $T : R^3 \to R^2$

$$T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a-2b \\ a+c \end{pmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \underset{\sim}{-b_1+b_2} \begin{bmatrix} 1 & -2 & 0 \\ 0 & 2 & 1 \end{bmatrix} \underset{\sim}{b_2+b_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \underset{\sim}{\tfrac{1}{2}b_2} \begin{array}{c} \begin{matrix} a & b & c \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1/2 \end{bmatrix} \end{array}$$

$a + c = 0$

$a = -c$

$b + \frac{1}{2}c = 0$

$b = -\frac{1}{2}$

Solusi : $\left\{ \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ -1/2 \end{bmatrix} p \right\}$; p = parameter

Basis $Ker(T) = \left\{ \begin{bmatrix} -1 \\ -1/2 \end{bmatrix} \right\}$

Dimensi $ker(T) = 1$

Basis $R(t) = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

Dimensi $R(t) = 3$

2. $T : R^2 \to R^3$; $T\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$; $T\begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$$A\begin{bmatrix} 1 & -3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 5 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -5 & -3 \\ -2 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} -17 & -10 \\ 1 & 1 \\ -3 & -2 \end{bmatrix} \underset{\sim}{\substack{3b_2+b_3 \\ 17b_2+b_1}} \begin{bmatrix} 0 & 7 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \underset{\sim}{\substack{-b_3+b_2 \\ -7b_3+b_1}} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \sim \begin{array}{c} \begin{matrix} a & b \end{matrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \end{array}$$

Solusi : $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

a. $A = \begin{bmatrix} -17 & -10 \\ 1 & 1 \\ -3 & -2 \end{bmatrix}$

b. $T\begin{pmatrix} 1 \\ 3 \end{pmatrix} = A\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -17 & -10 \\ 1 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} -47 \\ 4 \\ -9 \end{bmatrix}$

c. $\mathrm{Ker}(T) = \{\ \}$

$R(T) = \left\{ \begin{bmatrix} -17 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -10 \\ 1 \\ -2 \end{bmatrix} \right\}$

3. $A = \begin{bmatrix} -1 & 0 \\ 4 & -3 \end{bmatrix}$; $B = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 4 & 2 \\ -2 & 2 & 4 \end{bmatrix}$

a. $\det(A - \lambda I) = \left| \begin{bmatrix} -1 & 0 \\ 4 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right|$

$= \begin{vmatrix} -1-\lambda & 0 \\ 4 & -3-\lambda \end{vmatrix} = (-1-\lambda)(-3-\lambda) = (\lambda+1)(\lambda+3)$

$\det(B - \lambda I) = \left| \begin{bmatrix} 4 & 2 & -2 \\ 2 & 4 & 2 \\ -2 & 2 & 4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right|$

$= \begin{vmatrix} 4-\lambda & 2 & -2 \\ 2 & 4-\lambda & 2 \\ -2 & 2 & 4-\lambda \end{vmatrix} = (4-\lambda)^3 + (-8) + (-8) - 3 \cdot 4(4-\lambda)$

$= -\lambda^3 + 12\lambda^2 - 36\lambda$

$= (-\lambda^2 + 12\lambda - 36)\lambda$

b. Nilai Eigen matriks A :

$(\lambda+1)(x+3) = 0$

$\lambda_1 = -1 \quad \lambda_2 = -3$

Nilai Eigen matriks B :

$(-\lambda^2 + 12\lambda - 36)\lambda = 0$

$-(\lambda^2 - 12\lambda + 36)\lambda = 0$

$-(\lambda - 6)^2 \lambda = 0$

$\lambda_1 = 6 \qquad \lambda_2 = 0$

c. Matriks A

$\lambda_1 = -1$

$(A - \lambda_1 I)\vec{v} = 0$

$$\left(\begin{bmatrix} -1 & 0 \\ 4 & -3 \end{bmatrix} - (-1)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\vec{v} = 0$$

$$\begin{bmatrix} 0 & 0 \\ 4 & -2 \end{bmatrix}\vec{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\downarrow$

$$\left[\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \to \vec{v} = \left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\} \to \vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\lambda_2 = -3$

$(A - \lambda I)\vec{v} = 0$

$$\left(\begin{bmatrix} -1 & 0 \\ 4 & -3 \end{bmatrix} - (-3)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\vec{v} = 0$$

$$\begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix}\vec{v} = 0$$

$\downarrow$

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \to \vec{v} = \left\{\begin{bmatrix} 0 \\ x \end{bmatrix}, x \in R\right\} \to \vec{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Matriks B

$\lambda_1 = 6$

$(B - \lambda_1 I)\vec{v} = 0$

$$\left(\begin{bmatrix} 4 & 2 & -2 \\ 2 & 4 & 2 \\ -2 & 2 & 4 \end{bmatrix} - 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\vec{v} = 0$$

$$\begin{bmatrix} -2 & 2 & -2 \\ 2 & -2 & 2 \\ -2 & 2 & -2 \end{bmatrix}\vec{v} = 0$$

$\downarrow$

$$\begin{bmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \rightarrow \vec{v} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$\lambda_2 = 0$

$(B - \lambda_2 I)\vec{v} = 0$

$$\left(\begin{bmatrix} 4 & 2 & -2 \\ 2 & 4 & 2 \\ -2 & 2 & 4 \end{bmatrix} - 0.I \right) \vec{v} = 0$$

$$\begin{bmatrix} 4 & 2 & -2 \\ 2 & 4 & 2 \\ -2 & 2 & 4 \end{bmatrix} \vec{v} = 0$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & 0 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \rightarrow \vec{v} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

d. Matriks A

$\vec{P_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{P_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \rightarrow$ Matriks pendiagonal matriks A

$P^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

Matriks B

$\vec{P_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{P_2} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad \vec{P_3} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow$ Matrik pendiagonal matriks B

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{-b_2 + b_1} \left[\begin{array}{ccc|ccc} 0 & -1 & 2 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{b_3 + b_1}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 3 & 1 & -1 & 1 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{\tfrac{1}{3} b_1} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1/3 & -1/3 & 1/3 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{\begin{matrix} b_1 + b_2 \\ -b_1 + b_3 \end{matrix}}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1/3 & -1/3 & 1/3 \\ 1 & 0 & 0 & 1/3 & 2/3 & 1/3 \\ 0 & 1 & 0 & -1/3 & 1/3 & 2/3 \end{array}\right] \sim P^{-1} = \begin{bmatrix} 1/3 & 2/3 & 1/3 \\ -1/3 & 1/3 & 2/3 \\ 1/3 & -1/3 & 1/3 \end{bmatrix}$$

e. Matriks A:

$D = P^{-1} . AP$

$$D = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$$

Matriks B:

$D = P^{-1} B P$

$$D = \begin{bmatrix} 1/3 & 2/3 & 1/3 \\ -1/3 & 1/3 & 2/3 \\ 1/3 & -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 4 & 2 & -2 \\ 2 & 4 & 2 \\ -2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 & 2/3 & 1/3 \\ -1/3 & 1/3 & 2/3 \\ 1/3 & -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 6 & -6 & 0 \\ 6 & 0 & 0 \\ 0 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4. $\lambda_1 = -3 \rightarrow \vec{v}_1 = \left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ $\qquad$ $\lambda_2 = 1 \rightarrow \vec{v}_2 = \left\{ \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$

$A\vec{v}_1 = \lambda_1 \vec{v}_1$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = -3 \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} a + 3b \\ c + 3d \end{bmatrix} = \begin{bmatrix} -3 \\ -9 \end{bmatrix}$$

$A\vec{v}_2 = \lambda_2 \vec{v}_2$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = 1 \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -a + 2b \\ -c + 2d \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{array}{r} a + 3b = -3 \\ -a + 2b = 1 \\ \hline 5b = -2 \end{array} \; +$$

$$b = -\frac{2}{5}$$

$$a + 3 \cdot -\frac{2}{5} = -3$$

$$a = -\frac{9}{5}$$

$$\begin{array}{r} c + 3d = -9 \\ -c + 2d = 2 \\ \hline 5d = -7 \end{array} \; +$$

$$d = -\frac{7}{5}$$

$$c + 3 \cdot -\frac{7}{5} = -9$$

$$c = -\frac{24}{5}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -9/5 & -2/5 \\ -24/5 & -7/5 \end{bmatrix}$$