12th Material Subject: De-Moivre Laplace Theorem

Undergraduate of Telecommunication Engineering

MUH1F3 - PROBABILITY AND STATISTICS

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LEARNING OBJECTIVES:

After careful study of this chapter, student should be able to do the following:

1. **Approximate probabilities for some binomial**

DE MOIVRE LAPLACE THEOREM

We began our section on the normal distribution as an approximation to a random binomial probabilities for cases in which **n** is large.

- Suppose **X** is a binomial random variable with $\Rightarrow$ **BIN** $(\mathbf{n}, \mathbf{p})$, then $\mu_{\mathbf{X}} = \mathbf{np}$ dan $\sigma^2_{\mathbf{X}} = \mathbf{npq}$.
- If it met the conditions which $\mathbf{n} \to \infty$, such that $\mathbf{np} > \mathbf{5}$ and $\mathbf{p} \leq \frac{1}{2}$ or ketika $\mathbf{nq} > \mathbf{5}$ and $\mathbf{p} > \frac{1}{2}$.
- If the binomial distribution meets the above conditions, then in 1973, a scientist named De-Moivre and Laplace presented a theorem to solve the problem with an approach to the Standard Normal distribution or known as **DE MOIVRE LAPLACE THEOREM** .

DE MOIVRE LAPLACE THEOREM

Consequently, a modified interval is used to better compensate for the difference between the continuous normal distribution and the discrete binomial distribution. This modification is called a **Continuity Correction**.

Binomial Distribution	The Continuity Correction	Standard Normal Distribution
$P(X = x)$	$P([x - 0.5] \leq X \leq [x + 0.5])$	$P\left(\left[\frac{x - 0.5 - np}{\sqrt{npq}}\right] \leq Z \leq \left[\frac{x + 0.5 - np}{\sqrt{npq}}\right]\right)$
$P(X \leq x)$	$P(X \leq [x + 0.5])$	$P\left(X \leq \left[\frac{x + 0.5 - np}{\sqrt{npq}}\right]\right)$
$P(X \geq x)$	$P(X \geq [x - 0.5])$	$P\left(X \geq \left[\frac{x - 0.5 - np}{\sqrt{npq}}\right]\right)$
$P(a \leq X \leq b)$	$P([a - 0.5] \leq X \leq [b + 0.5])$	$P\left(\left[\frac{a - 0.5 - np}{\sqrt{npq}}\right] \leq Z \leq \left[\frac{b + 0.5 - np}{\sqrt{npq}}\right]\right)$

DE MOIVRE LAPLACE THEOREM

Example: Suppose $\mathbf{X} \Rightarrow \mathbf{BIN}\left(\mathbf{100}, \mathbf{0.8}\right)$, determine:

a. $\mathbf{P}\left(\mathbf{X} \leq \mathbf{80}\right)$

b. $\mathbf{P}\left(\mathbf{70} < \mathbf{X} \leq \mathbf{90}\right)$

c. $\mathbf{P}\left(\mathbf{X} = \mathbf{80}\right)$

d. $\mathbf{P}\left(\mathbf{70} \leq \mathbf{X}\right)$

Answer:

$$\mu_{\mathbf{X}} = \mathbf{np} = \mathbf{100} \cdot \mathbf{0.8} = \mathbf{80}$$

$$\sigma_{\mathbf{X}}^{\mathbf{2}} = \mathbf{npq} = \mathbf{100} \cdot \mathbf{0.8} \cdot \mathbf{0.2} = \mathbf{16}$$

DE MOIVRE LAPLACE THEOREM

a. $\mathbf{P(X \leq 80)}$

$$P\left(X_B \leq 80\right) = P\left(X_N \leq 80.5\right) = P\left(Z \leq \frac{80.5 - 80}{4}\right)$$

$$P\left(Z \leq 0.125\right) = \phi(0.125) = 0.54972$$

b. $\mathbf{P(70 < X \leq 90)}$

$$P\left(70 < X \leq 90\right) = P\left(71 \leq X_B \leq 90\right)$$

$$P\left(70.5 \leq X_N \leq 90.5\right) = P\left(\frac{70.5 - 80}{4} \leq Z \leq \frac{90.5 - 80}{4}\right)$$

$$P\left(-2.375 \leq Z \leq 2.625\right) = P\left(Z \leq 2.625\right) - P\left(Z \leq -2.375\right)$$

$\phi(2.625) - \phi(-2.375) = \ldots\ldots$**(see Standart Normal Table)**

DE MOIVRE LAPLACE THEOREM

c. $\mathbf{P(X = 80)}$

$$P\left(X_B = 80\right) = P\left(79.5 \leq X_N \leq 80.5\right)$$

$$P\left(\frac{79.5-80}{4} \leq Z \leq \frac{80.5-80}{4}\right) = P\left(-0.125 \leq Z \leq 0.125\right)$$

$$P\left(-0.125 \leq Z \leq 0.125\right) = \phi(0.125) - \phi(-0.125) = \ldots\ldots\textbf{(see Standart Normal Table)}$$

d. $\mathbf{P(70 \leq X)}$

$$P\left(70 \leq X_B\right) = P\left(X_B \geq 70\right) = 1 - P\left(X_B \leq 69\right)$$

$$1 - P\left(X_N \leq 69.5\right) = 1 - P\left(Z \leq \frac{69.5-80}{4}\right)$$

$$1 - \phi\left(-2.625\right) = \phi\left(2.625\right) = \ldots\ldots\textbf{(see Standart Normal Table)}$$

Thank You