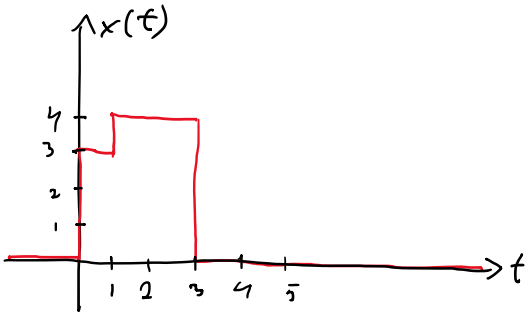
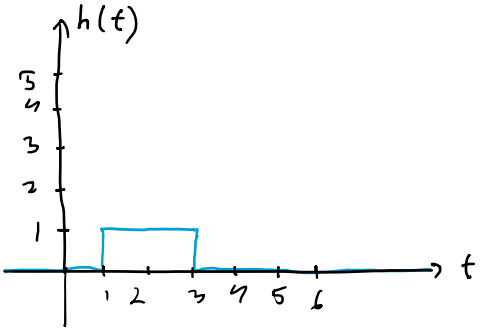
M. Masyim Abdillah P. (1101910095) TT-43-11

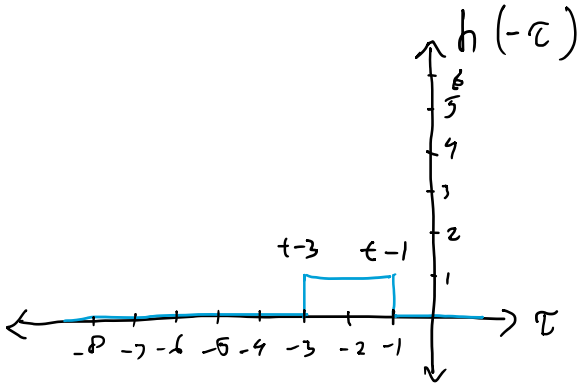
32.

1. $x(t) = 3u(t) + u(t-1) - 4u(t-3)$

$h(t) = u(t-1) - u(t-3)$







$0 \le t < 1$

$Y(t) = 0$

$1 \le t < 2$

$$Y(t) = \int_0^{t-1} 1.3 \, d\tau$$

$$= 3\tau \Big|_0^{t-1}$$

$$= 3t - 3$$

$2 \le t < 3$

$$Y(t) = \int_0^1 1.3 \, d\tau + \int_1^{t-1} 1.4 \, d\tau$$

$$= 3\tau \Big|_0^1 + 4\tau \Big|_1^{t-1}$$

$$= 3 + 4t - 8$$

$$= 4t - 5$$

$3 \le t < 4$

$$Y(t) = \int_{t-3}^1 1.3 \, d\tau + \int_1^{t-1} 1.4 \, d\tau$$

$$= 3\tau \Big|_{t-3}^1 + 4\tau \Big|_1^{t-1}$$

$$= 3 - 3(t-3) + 4(t-1-1)$$

$$= t + 4$$

$4 \le t < 6$

$$Y(t) = \int_{t-3}^3 1.4 \, d\tau$$

$$= 4\tau \Big|_{t-3}^3$$

$$= 24 - 4t$$

$t \ge 6$

$Y(t) = 0$

$$y(t) \cdot \begin{cases} 0, & t < 1 \\ 3t-3, & 1 \le t < 2 \\ 4t-5, & 2 \le t < 3 \\ t+4, & 3 \le t < 4 \\ 2y-4t, & 4 \le t < 6 \\ 0, & \ge 6 \end{cases}$$

2. $\frac{d^2}{dy^2} y(t) + 2\frac{d}{dy} y(t) - 3y(t) = x(t)$; $x(t) = \sin(t)$

a. $\frac{d^2}{dy^2} y(t) + 2\frac{d}{dy} y(t) - 3y(t) = 0$

$r^2 + 2r - 3 = 0$

$(r-1)(r+3) = 0$

$r_1 = 1 \vee r_2 = -3$

$y_h = c_1 e^{r_1 t} + c_2 e^{r_2 t}$

$y_h = c_1 e^{t} + c_2 e^{-3t}$

b. $x(t) = \sin(t)$

$y_p = A\sin(t) + B\cos(t)$

$\frac{d}{dy} y_p = A\cos(t) - B\sin(t)$

$\frac{d^2}{dy^2} y_p = -A\sin(t) - B\cos(t)$

$\frac{d^2}{dy^2} y + 2\frac{d}{dy} y - 3y = \sin(t)$

$-A\sin(t) - B\cos(t) + 2\left(A\cos(t) - B\sin(t)\right) - 3\left(A\sin(t) + B\cos(t)\right) = \sin(t)$

$-A\sin(t) - B\cos(t) + 2A\cos(t) - 2B\sin(t) - 3A\sin(t) + 3B\cos(t) = \sin(t)$

$(-A - 2B - 3A)\sin(t) + (-B + 2A + 3B)\cos(t) = \sin(t)$

$-4A - 2B = 1 \quad \Rightarrow \quad A = -\frac{1}{2}$

$2A + 2B = 0 \quad \Rightarrow \quad B = \frac{1}{2}$

$$y_p = A\sin(t) + B\cos(t)$$

$$y_p = -\frac{1}{2}\sin(t) + \frac{1}{2}\cos(t)$$ //

c. $y_T = y_h + y_p$

$$y_T = c_1 e^{t} + c_2 e^{-3t} - \frac{1}{2}\sin(t) + \frac{1}{2}\cos(t)$$

$$y(0^-) = 0$$

$$c_1 e^{0} + c_2 e^{-3\cdot 0} - \frac{1}{2}\sin(0) + \frac{1}{2}\cos(0) = 0$$

$$c_1 + c_2 - 0 + \frac{1}{2} = 0$$

$$c_1 + c_2 = -\frac{1}{2}$$

$$\frac{d}{dt} y_T = c_1 e^{t} - 3c_2 e^{-3t} - \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t)$$

$$\frac{d}{dt} y_T \Big|_{t=0} = 0$$

$$c_1 e^{0} - 3c_2 e^{-3\cdot 0} - \frac{1}{2}\cos(0) - \frac{1}{2}\sin(0) = 0$$

$$c_1 - 3c_2 - \frac{1}{2} - 0 = 0$$

$$c_1 - 3c_2 = \frac{1}{2}$$

$$c_1 + c_2 = -\frac{1}{2}$$

$$-4c_2 = 1$$

$$c_2 = -\frac{1}{4}$$

$$c_1 = -\frac{1}{4}$$

$$y_T = c_1 e^{t} + c_2 e^{-3t} - \frac{1}{2}\sin(t) + \frac{1}{2}\cos(t)$$

$$y_T = -\frac{1}{4} e^{t} - \frac{1}{4} e^{-3t} - \frac{1}{2}\sin(t) + \frac{1}{2}\cos(t)$$ //