

| | | |
|--------------------|---------------|---------------------------------|
| 1. $f(x) = \sin x$ | $f(0) = 0$ | $a_0 = 0$ |
| $f'(x) = \cos x$ | $f'(0) = 1$ | $a_1 = 1$ |
| $f''(x) = -\sin x$ | $f''(0) = 0$ | $a_2 = 0$ |
| $f^3(x) = -\cos x$ | $f^3(0) = -1$ | $a_3 = \frac{1}{3!} \cdot (-1)$ |
| $f^4(x) = \sin x$ | $f^4(0) = 0$ | $a_4 = 0$ |

$$f(x) = \sin x = 0 + 1 \cdot x + 0 + \left(-\frac{1}{3!} \cdot x^3\right) + 0 + \frac{1}{5!} x^5 + \dots$$

$$= x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 - \frac{1}{7!} x^7 + \dots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1}$$

| | | |
|---------------------|--------------|----------------------|
| 2. $f(z) = \sinh z$ | $f(0) = 0$ | $a_0 = 0$ |
| $f'(z) = \cosh z$ | $f'(0) = 1$ | $a_1 = 1$ |
| $f''(z) = \sinh z$ | $f''(0) = 0$ | $a_2 = 0$ |
| $f^3(z) = \cosh z$ | $f^3(0) = 1$ | $a_3 = \frac{1}{3!}$ |

$$f(z) = \sinh z = 0 + 1 \cdot z + 0 + \frac{1}{3!} z^3 + 0 + \frac{1}{5!} z^5 + \dots$$

$$= z + \frac{1}{3!} z^3 + \frac{1}{5!} z^5 + \frac{1}{7!} z^7$$

$$= \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$$