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Rangkuman Transformasi Fourier

Fungsi-Fungsi Dasar

1. Fungsi Impulse (Delta Dirac) : $f(t) = \delta(t)$
2. Fungsi unit step : $f(t) = u(t)$
3. Fungsi ramp : $f(t) = t$
4. Fungsi eksponen : $f(t) = e^{at}$
5. Fungsi sinus : $f(t) = \sin at$
6. Fungsi cosinus : $f(t) = \cos at$

Transformasi Fourier, $f(t) \rightarrow F(iw)$:

$\delta(t) \rightarrow 1$

$1 \rightarrow 2\pi\,\delta(w)$

$u(t) \rightarrow \frac{1}{iw} + \pi\,\delta(w)$

$t\,u(t) \rightarrow -\frac{1}{w^2} + \pi\,\delta'(w)$

$e^{at}\,u(t) \rightarrow \frac{1}{iw - a}$

$\sin at \rightarrow i\pi\left[\delta(w+a) - \delta(w-a)\right]$

$\sin at\,u(t) \rightarrow \frac{a}{(iw)^2 + a^2}$

$\cos at \rightarrow \pi\left[\delta(w+a) + \delta(w-a)\right]$

$\cos at\,u(t) \rightarrow \frac{iw}{(iw)^2 + a^2}$

Sifat-sifat Transformasi Fourier:

$a f_1(t) + b f_2(t) = a F_1(iw) + b F_2(iw)$

$f(at) = \frac{1}{|a|} F\left(\frac{iw}{a}\right)$

$f(t - t_0) = F(w - t_0)$

$e^{at} f(t) = F(iw - a)$

$t\,f(t) = i \cdot \frac{dF(iw)}{dw}$

$\frac{df(t)}{dt} = iw\,F(iw)$

$f(t)\cos at = \frac{1}{2}\left[F(iw + a) + F(iw - a)\right]$

$f_1(t) * f_2(t) = F_1(iw)\,F_2(iw)$

Contoh soal Slide Transformasi Fourier (Bagian III)

Contoh 3: $f(t) = \sin 20\pi t\, u(t)$

$$F(iw) = \frac{20\pi}{(iw)^2 + (20\pi)^2} = \frac{20\pi}{(iw)^2 + 400\pi^2}$$

Contoh 4: $f(t) = \cos 20\pi t\, u(t)$

$$F(iw) = \frac{iw}{(iw)^2 + (20\pi)^2} = \frac{iw}{(iw)^2 + 400\pi^2}$$

Contoh 6: $f(t) = e^{2t} \cos \pi t\, u(t)$

$$\cos \pi t\, u(t) \rightarrow \frac{iw}{(iw)^2 + \pi^2}$$

$$e^{2t} \cos \pi t\, u(t) \rightarrow \frac{iw}{(iw-2)^2 + \pi^2} \quad \text{(Sifat 4)}$$

Contoh 7: $f(t) = t\, e^{2t} u(t)$

$$e^{2t} u(t) \rightarrow \frac{1}{iw - 2}$$

$$t\, e^{2t} u(t) \rightarrow i \cdot \frac{d}{dw}\left(\frac{1}{iw-2}\right) = i \cdot \frac{-i}{(iw-2)^2} = \frac{1}{(iw-2)^2}$$

Contoh 8: $f(t) = t \sin 5\, u(t)$

$$\sin 5\, u(t) \rightarrow \frac{5}{(iw)^2 + 25} = \frac{5}{-w^2 + 25}$$

$$t \sin 5\, u(t) \rightarrow i \frac{d}{dw}\left(\frac{5}{-w^2+25}\right) = i \cdot \frac{10w}{(-w^2+25)^2} = \frac{10iw}{(-w^2+25)^2}$$

Contoh Soal Transformasi Fourier (Bagian IV)

Contoh 4: $F(iw) = \frac{i2w + 15}{(iw)^2 + 10} = \frac{2iw}{(iw)^2 + (\sqrt{10})^2} + \frac{15}{(iw)^2 + (\sqrt{10})^2}$

$$= 2 \cdot \frac{iw}{(iw)^2 + (\sqrt{10})^2} + \frac{15}{\sqrt{10}} \cdot \frac{\sqrt{10}}{(iw)^2 + (\sqrt{10})^2}$$

$$= 2 \cos \sqrt{10}t\, u(t) + \frac{15}{\sqrt{10}} \cdot \sin \sqrt{10}t\, u(t)$$

Contoh 6 & 7 : $F(iw) = \dfrac{10}{(iw)^2 + 6iw + 10} = \dfrac{10}{(iw)^2 + 6iw + 9 + 1}$

$$= \frac{10}{(iw+3)^2 + 1}$$

$$= 10 \cdot \frac{1}{(iw+3)^2 + 1}$$

$$= 10 \cdot e^{-3t} \cdot \sin t \; u(t)$$

Contoh 8 : $F(iw) = \dfrac{2}{iw + 9} = 2 \cdot \dfrac{1}{iw - (-9)}$

$$= 2 e^{-9t} u(t)$$

Contoh 9 : $F(iw) = \dfrac{2}{(iw+9)(iw-3)} = -\frac{1}{6} \cdot \frac{1}{iw+9} + \frac{1}{6} \frac{1}{iw-3}$

$$= -\frac{1}{6} \cdot \frac{1}{iw-(-9)} + \frac{1}{6} \cdot \frac{1}{iw-3}$$

$$= -\frac{1}{6} \cdot e^{-9t} u(t) + \frac{1}{6} \cdot e^{3t} u(t)$$

Contoh 10 : $F(iw) = \dfrac{4}{(iw)^2 - 1} = \dfrac{4}{(iw+1)(iw-1)}$

$$= \frac{-2}{iw+1} + \frac{2}{iw-1}$$

$$= -2 \cdot \frac{1}{iw+1} + 2 \cdot \frac{1}{iw-1}$$

$$= -2 e^{-t} u(t) + 2 e^{t} u(t)$$

Contoh 12 : $F(iw) = \dfrac{5}{(iw+3)^2} = 5 \cdot \dfrac{1}{(iw+3)^2}$

$$= 5 \cdot i \frac{d}{dw} \left(\frac{1}{iw-(-3)} \right)$$

$$= 5 \cdot i \frac{d}{dw} \left(e^{-3t} u(t) \right)$$

$$= 5t \, e^{-3t} u(t)$$