$$h[n] = 0{,}5\,\delta[n] + \delta[n-1] + \delta[n-2] + 0{,}5\,\delta[n-3]$$

a. $H(z) = 0{,}5 + z^{-1} + z^{-2} + 0{,}5\,z^{-3}$

$$H(z) = 0{,}5 + \frac{1}{z} + \frac{1}{z^2} + \frac{0{,}5}{z^3}$$

$$= \frac{0{,}5\,z^3 + z^2 + z + 0{,}5}{z^3}$$

b. Pole: $z^3 = 0$

$P = 0$

Zero: $0{,}5\,z^3 + z^2 + z + 0{,}5 = 0$

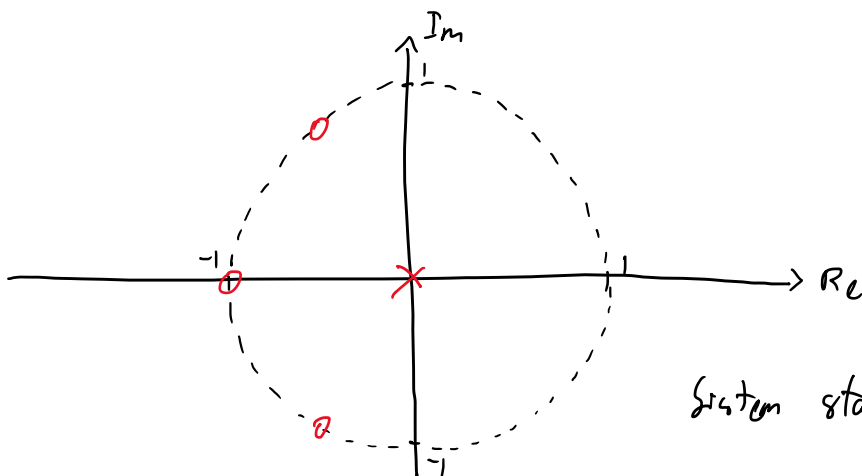$$0{,}5\left(z^3 + 2z^2 + 2z + 1\right) = 0$$

$$0{,}5(z+1)(z^2 + z + 1) = 0$$

$z + 1 = 0 \;\rightarrow\; z_1 = -1$

$z^2 + z + 1 = 0 \;\rightarrow\; \dfrac{-1 \pm \sqrt{1^2 - 4\cdot 1\cdot 1}}{2} \,;\, \dfrac{-1 \pm \sqrt{-3}}{2} \,;\, \dfrac{-1 \pm j\sqrt{3}}{2}$

$z_2 = -\dfrac{1}{2} + j\boxed{\dfrac{\sqrt{3}}{2}} \rightarrow 0{,}87$

$z_3 = -\dfrac{1}{2} - j\dfrac{\sqrt{3}}{2}$

System stabil marginal

c. $H(e^{-j\omega}) = 0{,}5 + e^{-j\omega} + e^{-j2\omega} + 0{,}5\,e^{-j3\omega}$

$$= 0{,}5\left(1 + e^{-j3\omega}\right) + e^{-j\omega}\left(1 + e^{-j\omega}\right)$$

$$= 0{,}5\,e^{-j\frac{3}{2}\omega}\left(e^{j\frac{3}{2}\omega} + e^{-j\frac{3}{2}\omega}\right) + e^{-j\omega}\cdot e^{-j\frac{1}{2}\omega}\left(e^{j\frac{1}{2}\omega} + e^{-j\frac{1}{2}\omega}\right)$$

$$= 0{,}5\,e^{-j\frac{3}{2}\omega}\left(\cos\tfrac{3}{2}\omega + j\sin\tfrac{3}{2}\omega + \cos\tfrac{3}{2}\omega - j\sin\tfrac{3}{2}\omega\right) +$$

$$e^{-j\frac{3}{2}\omega}\left(\cos\tfrac{1}{2}\omega + j\sin\tfrac{1}{2}\omega + \cos\tfrac{1}{2}\omega - j\sin\tfrac{1}{2}\omega\right)$$

$$= 0{,}5\,e^{-j\frac{3}{2}\omega}\left(2\cos\tfrac{3}{2}\omega\right) + e^{-j\frac{3}{2}\omega}\left(2\cos\tfrac{1}{2}\omega\right)$$

$$= e^{-j\frac{3}{2}\omega} \cos\frac{3}{2}\omega + 2e^{-j\frac{3}{2}\omega} \cos\frac{1}{2}\omega$$

$$H(e^{-j\omega}) = e^{-j\frac{3}{2}\omega}\left(\cos\frac{3}{2}\omega + 2\cos\frac{1}{2}\omega\right)$$

$$H(e^{-j\omega}) = \underbrace{e^{-j\theta}}_{} \cdot |H(e^{-j\omega})| \xrightarrow{\quad} \arg\{H(e^{-j\omega})\}$$

d. $|H(e^{-j\omega})| = \cos\frac{3}{2}\omega + 2\cos\frac{1}{2}\omega$

e. $e^{-j\frac{3}{2}\omega} = \cos\frac{3}{2}\omega - j\sin\frac{3}{2}\omega$

$$\arg\{H(e^{-j\omega})\} = \tan^{-1}\left(\frac{-\sin\frac{3}{2}\omega}{\cos\frac{3}{2}\omega}\right) = \tan^{-1}\left(-\tan\frac{3}{2}\omega\right) = -\frac{3}{2}\omega, \quad 0 \le \omega \le 2\pi$$

f. $x[n] = 2\cos 0{,}5\pi n \longrightarrow \omega = 0{,}5\pi$

$$y[n] = h[n] * x[n]$$

$$y[n] = 2|H(e^{-j\omega})|\cos\left(0{,}5\pi n + \arg\{H(e^{-j\omega})\}\right)$$

$$= 2\left(\cos\frac{3}{2}\omega + 2\cos\frac{1}{2}\omega\right)\cos\left(0{,}5\pi n - \frac{3}{2}\omega\right)$$

$$= 2\cos\left(0{,}5\pi n - \frac{3}{2}\omega\right)\left(\cos\frac{3}{2}\omega + 2\cos\frac{1}{2}\omega\right)$$

$$= 2\cos\left(0{,}5\pi n - \frac{3}{2}\cdot 0{,}5\pi\right)\left(\cos\frac{3}{2}\cdot 0{,}5\pi + 2\cos\frac{1}{2}\cdot 0{,}5\pi\right)$$

$$= 2\cos\left(0{,}5\pi n - 0{,}75\pi\right)\left(\cos 0{,}75\pi + 2\cdot\cos 0{,}25\pi\right)$$

$$= 2\cos\left(0{,}5\pi n - 0{,}75\pi\right)\cdot\frac{1}{2}\sqrt{2}$$

$$= \sqrt{2}\cos\left(0{,}5\pi n - 0{,}75\pi\right)$$

9. $x[n] = u[n]$

$$X(z) = \frac{z}{z-1}$$

$Y(z) = H(z) \cdot X(z)$

$$= \left(0,5 + \frac{1}{z} + \frac{1}{z^2} + \frac{0,5}{z^3} \right) \cdot \frac{z}{z-1}$$

$$= 0,5 \frac{z}{z-1} + z^{-1} \frac{z}{z-1} + z^{-2} \frac{z}{z-1} + \frac{z}{z-1} \cdot 0,5 \, z^{-3}$$

$Y[n] = 0,5 \, u[n] + u[n-1] + u[n-2] + 0,5 \, u[n-3]$