13th Material Subject: Central Limit Theorem

Undergraduate of Telecommunication Engineering

MUH1F3 - PROBABILITY AND STATISTICS

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WELCOME

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LEARNING OBJECTIVES:

After careful study of this chapter, student should be able to do the following:

1. **Understand the central limit theorem**
2. **Explain the important role of the normal distribution as a sampling distribution**
3. **Explain the general concepts of estimating the parameters of a population or a probability distribution**

CENTRAL LIMIT THEOREM

- The constellation of **n** random variables derived from the parent random variables **X**, that is: $\mathbf{X_1}$, $\mathbf{X_2}$, $\mathbf{X_3}$,... , $\mathbf{X_n}$ which is *Independent Identically Distributed* (IID) called ***Random Sample***.
- The probability distribution of ***Random Sample*** called ***Sampling Distribution***.
- **Sampling Distribution** of population / parent distributed **Normal** random variables
- **Sampling Distribution** of population / parent distributed **Normal** random variables, used **CENTRAL LIMIT THEOREM**.
- **Sampling Distribution** divided into **Sampling Total** dan **Sampling Mean**

$$\mathbf{T} = \sum_{i=1}^{n} \mathbf{X_i} \tag{1}$$

$$\overline{\mathbf{X}} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{X_i} \tag{2}$$

CENTRAL LIMIT THEOREM

If random sample X_1, X_2 , ... , X_n taken from the **NORMAL** distribution with *Mean* $= \mu$ dan *Variance* $= Var(x) = \sigma^2$, then:

1. Parent random variable

$$X \rightarrow NOR(\mu, \sigma^2)$$

2. Random variable of sample distribution is:

   - *Sample* Total

$$\mathbf{T \rightarrow NOR(n\mu, n\sigma^2)} \tag{3}$$

   - *Sample* Mean

$$\mathbf{\overline{X} \rightarrow NOR(\mu, \frac{\sigma^2}{n})} \tag{4}$$

CENTRAL LIMIT THEOREM

If random sample $X_1, X_2, \ldots, X_n$ taken from the **NON-NORMAL** distribution with *Mean* $= \mu$ dan *Variance* $= Var(x) = \sigma^2$, then:

1. Random variable of sample distribution is:
   - *Sample* Total
   $$\mathbf{T} \rightarrow \mathbf{NOR}(\mathbf{n}\mu, \mathbf{n}\sigma^2) \tag{5}$$
   - *Sample* Mean
   $$\bar{\mathbf{X}} \rightarrow \mathbf{NOR}(\mu, \frac{\sigma^2}{\mathbf{n}}) \tag{6}$$

CENTRAL LIMIT THEOREM

Example: From an airline's log-book data, data is obtained that baggage weight per passenger is known to have normal distribution with an average of 18 kg and a variance of 4 kg.

a. Of the 25 baggage being queued for weighing, determine the probability that the **AVERAGE** luggage weight will be less than 17 kg

b. Out of a total of 400 passenger baggage, determine the probability that the **TOTAL** weight will exceed 7150 kg

Answer: Parent random variables are Normal with an average of 18 kg and Variance of 4 kg:

$$X \rightarrow NOR\left(\mu, \sigma^2\right)$$

$$X \rightarrow NOR\left(18, 4\right)$$

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a. Of the 25 baggage being queued for weighing, determine the probability that the **AVERAGE** luggage weight will be less than 17 kg

The sampling distribution become:

$$\overline{X} \rightarrow NOR(\mu, \frac{\sigma^2}{n})$$

$$\overline{X} \rightarrow NOR(18, \frac{4}{25})$$

$$P(\overline{X} < 17) = P\left(\overline{X} < \frac{17-18}{\sqrt{\frac{4}{25}}}\right) = P(Z < -2.5)$$

$$\phi(-2.5) = 0.00621$$

b. Out of a total of 400 passenger baggage, determine the probability that the **TOTAL** weight will exceed 7150 kg ...

Distribution Total Become:

$$T \to NOR\left(n\mu, n\sigma^2\right)$$

$$T \to NOR\left(\left(400 \cdot 18\right), \left(400 \cdot 4\right)\right)$$

$$T \to NOR\left(7200, 1600\right)$$

$$P\left(T > 7150\right) = 1 - P\left(T \leq 7150\right) = 1 - P\left(Z \leq \frac{7150 - 7200}{\sqrt{1600}}\right)$$

$$1 - P\left(Z \leq -1.25\right) = 1 - \phi\left(-1.25\right) = \phi\left(1.25\right) = 0.89435$$

Thank You