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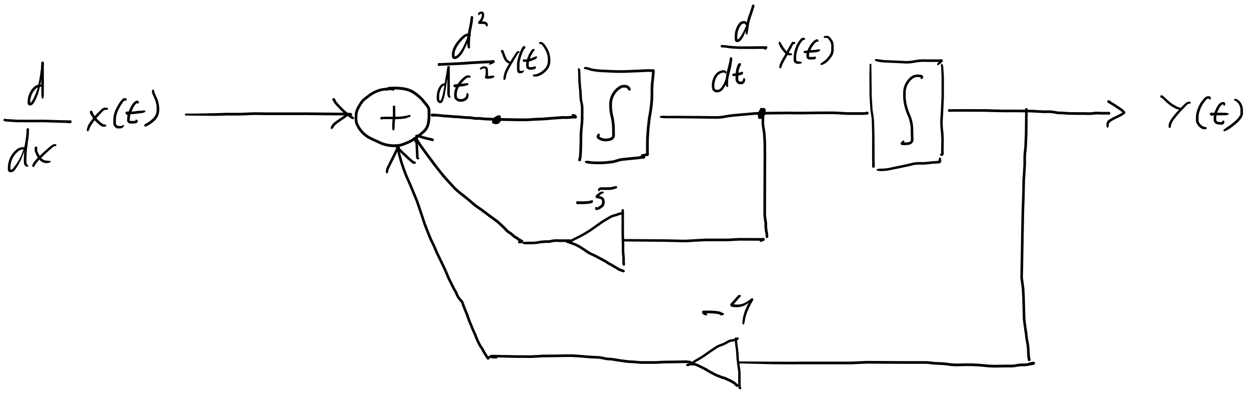
$$\frac{d^2}{dt^2} y(t) + 5 \frac{d}{dt} y(t) + 4 y(t) = \frac{d}{dt} x(t)$$

a.



b. $$\frac{d^2}{dt^2} y(t) + 5 \frac{d}{dt} y(t) + 4 y(t) = \frac{d}{dt} x(t)$$

$$(j\Omega)^2 Y(j\Omega) + 5(j\Omega) Y(j\Omega) + 4 Y(j\Omega) = (j\Omega) X(j\Omega)$$

$$\left(-\Omega^2 + 5j\Omega + 4\right) Y(j\Omega) = (j\Omega) X(j\Omega)$$

$$H(j\Omega) = \frac{Y(j\Omega)}{X(j\Omega)} = \frac{j\Omega}{-\Omega^2 + 5j\Omega + 4} = \frac{j\Omega}{4 - \Omega^2 + 5j\Omega}$$

c. $$\frac{d^2}{dt^2} y(t) + 5 \frac{d}{dt} y(t) + 4 y(t) = 0$$

$$r^2 + 5r + 4 = 0$$
$$(r+1)(r+4) = 0$$
$$r = -1 \vee r = -4$$
$$y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$
$$y_h(t) = C_1 e^{-t} + C_2 e^{-4t}$$

d. $x(t) = \cos(t)$

$$\frac{d}{dx} x(t) = -\sin(t)$$

$$y_p(t) = A \sin(t) + B \cos(t)$$

$$Y_p'(t) = A\cos(t) - B\sin(t)$$

$$Y_p''(t) = -A\sin(t) - B\cos(t)$$

$$Y_p'' + 5Y_p' + 4Y_p = -\sin(t)$$

$$-A\sin(t) - B\cos(t) + 5\big(A\cos(t) - B\sin(t)\big) + 4\big(A\sin(t) + B\cos(t)\big) = -\sin(t)$$

$$(-A - 5B + 4A)\sin(t) + (-B - 5A + 4B)\cos(t) = -\sin(t)$$

$$\begin{array}{rcl|c|rcl} 3A - 5B &=& -1 & \times 3 & 9A - 15B &=& -3 \\ -5A + 3B &=& 0 & \times 5 & -25A + 15B &=& 0 \end{array} \quad +$$

$$-16A = -3$$

$$A = \frac{3}{16}$$

$$B = \frac{5}{16}$$

$$Y_p(t) = A\sin(t) + B\cos(t)$$

$$= \frac{3}{16}\sin(t) + \frac{5}{16}\cos(t)$$

e. $Y(t) = Y_h(t) + Y_p(t)$

$$Y(t) = C_1 e^{-t} + C_2 e^{-4t} + \frac{3}{16}\sin(t) + \frac{5}{16}\cos(t)$$

$$Y(0^-) = 0$$

$$C_1 e^{-.0} + C_2 e^{-4.0} + \frac{3}{16}\sin(0) + \frac{5}{16}\cos(t) = 0$$

$$C_1 + C_2 + \frac{3}{16}\cdot 0 + \frac{5}{16}\cdot 1 = 0$$

$$C_1 + C_2 = -\frac{5}{16}$$

$$\frac{d}{dt}Y(t)\Big|_{t=0} = 0$$

$$-C_1 e^{-t} - 4C_2 e^{-4t} + \frac{3}{16}\cos(t) - \frac{5}{16}\sin(t)\Big|_{t=0} = 0$$

$$-C_1e^{-0} - 4C_2e^{-4.0} + \frac{3}{16}\cos(0) - \frac{5}{16}\sin(0) = 0$$

$$-C_1 - 4C_2 + \frac{3}{16}\cdot 1 - \frac{5}{16}\cdot 0 = 0$$

$$C_1 + 4C_2 = \frac{3}{16}$$

$$C_1 + C_2 = -\frac{5}{16} \quad -$$

$$3C_2 = \frac{8}{16}$$

$$C_2 = \frac{1}{6} \qquad C_1 = -\frac{23}{48}$$

$$Y(t) = C_1e^{-t} + C_2e^{-4t} + \frac{3}{16}\sin(t) + \frac{5}{16}\cos(t)$$

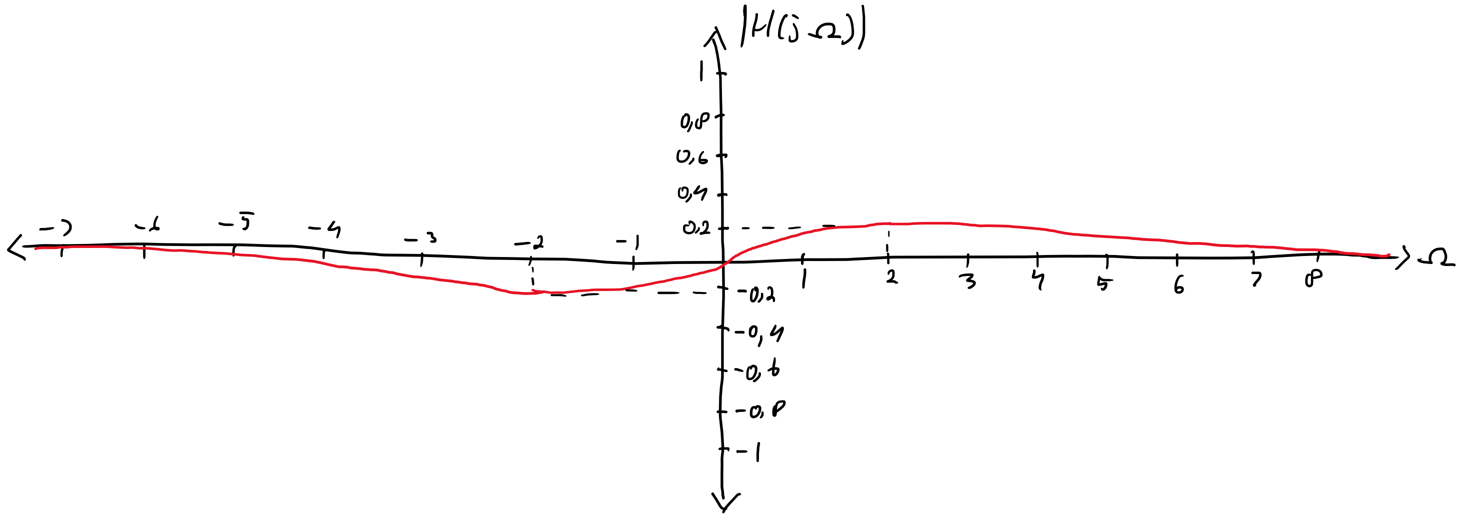
$$Y(t) = -\frac{23}{48}e^{-t} + \frac{1}{6}e^{-4t} + \frac{3}{16}\sin(t) + \frac{5}{16}\cos(t)$$

f.

$$H(j\Omega) = \frac{j\Omega}{4-\Omega^2 + j5\Omega}$$

$$|H(j\Omega)| = \frac{\sqrt{0^2+\Omega^2}}{\sqrt{(4-\Omega^2)^2+(5\Omega)^2}} = \frac{\Omega}{\sqrt{16-8\Omega^2+\Omega^4+25\Omega^2}}$$

$$|H(j\Omega)| = \frac{\Omega}{\sqrt{\Omega^4+17\Omega^2+16}}$$



9. $$H(j\Omega) = \frac{j\Omega}{4-\Omega^2+5j\Omega}\cdot\frac{4-\Omega^2-5j\Omega}{4-\Omega^2-5j\Omega}$$

$$= \frac{(4\Omega-\Omega^3)j + 5\Omega^2}{16-8\Omega^2+\Omega^4+25\Omega^2}$$

$$= \frac{5\Omega^2}{\Omega^4+17\Omega^2+16} + j\,\frac{4\Omega-\Omega^3}{\Omega^4+17\Omega^2+16}$$

$$\text{Arg } H(j\Omega) = \tan^{-1}\left(\frac{\text{Im}(H(j\Omega))}{\text{Re}(H(j\Omega))}\right) = \tan^{-1}\left(\frac{\frac{4\Omega-\Omega^3}{\Omega^4+17\Omega^2+16}}{\frac{5\Omega^2}{\Omega^4+17\Omega^2+16}}\right)$$

$$\text{Arg } H(j\Omega) = \tan^{-1}\left(\frac{4\Omega-\Omega^3}{5\Omega^2}\right)$$

$$\text{Arg } H(j\Omega) = \tan^{-1}\left(\frac{4-\Omega^2}{5\Omega}\right)$$