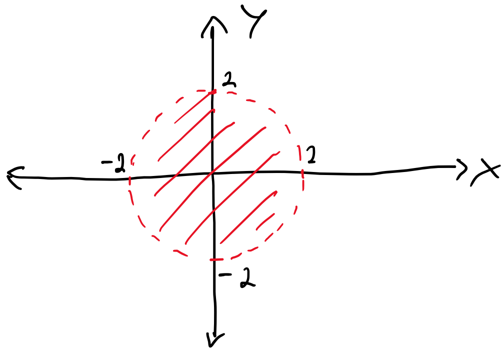
M. Masyim Abdillah P. (1101192095) TT-43-11

1.

a. $f(z) = \dfrac{1}{2+z} = \dfrac{1}{2\left(1+\frac{z}{2}\right)} = \dfrac{1}{2} \cdot \dfrac{1}{1-\left(-\frac{z}{2}\right)}$

$\left|-\frac{z}{2}\right| < 1 \longrightarrow |z| < 2$



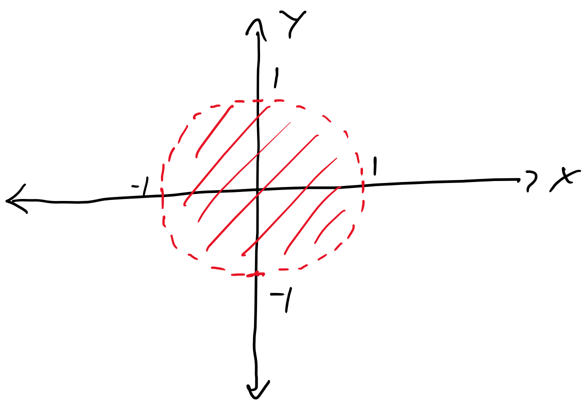
b. $\dfrac{1}{2} \cdot \dfrac{1}{1-\left(-\frac{z}{2}\right)} = \dfrac{1}{2} \cdot 1 + \dfrac{1}{2}\left(-\dfrac{z}{2}\right) + \dfrac{1}{2}\left(-\dfrac{z}{2}\right)^2 + \dfrac{1}{2} \cdot \left(-\dfrac{z}{2}\right)^3$

$= \dfrac{1}{2} \sum_{n=0}^{\infty} \left(-\dfrac{z}{2}\right)^n$

$= \dfrac{1}{2} \sum_{n=0}^{\infty} (-1)^n \dfrac{z^n}{2^n} = \sum_{n=0}^{\infty} (-1)^n \dfrac{z^n}{2^{n+1}}$

c. $f(z) = \dfrac{1}{2+(z-1)} = \dfrac{1}{1+z} = \dfrac{1}{1-(-z)}$

$|-z| < 1 \rightarrow |z| < 1$



d. $\dfrac{1}{1-(-z)} = 1 + (-z) + (-z)^2 + (-z)^3$

$= \sum_{n=0}^{\infty} (-z)^n = \sum_{n=0}^{\infty} (-1)^n z^n$

2. a. $f(z) = \dfrac{2z-1}{(z^2+16)(z-1)^2}$

Titik singular :

$(z^2+16)(z-1)^2 = 0$

$(z+4i)(z-4i)(z-1)^2 = 0$

∘> $z = -4i$, orde 1

∘> $z = 4i$, orde 1

∘> $z = 1$, orde 2

b. $\mathrm{Res}_{z=-4i}(f(z)) = \dfrac{2z-1}{(z-4i)(z-1)^2} = \dfrac{2(-4i)-1}{(-4i-4i)(-4i-1)^2}$

$= \dfrac{-8i-1}{-8i(-15+8i)}$

$= \dfrac{8i+1}{-120i-64}$

$\mathrm{Res}_{z=4i}(f(z)) = \dfrac{2z-1}{(z+4i)(z-1)^2} = \dfrac{2(4i)-1}{(4i+4i)(4i-1)^2}$

$= \dfrac{8i-1}{8i(-15-8i)} = \dfrac{8i-1}{-120i+64}$

$\mathrm{Res}_{z=1}(f(z)) = \dfrac{1}{(2-1)!}\dfrac{d}{dz}\left(\dfrac{2z-1}{z^2+16}\right)\Bigg|_{z=1} = \dfrac{1}{1!}\cdot\dfrac{2(z^2+16)-(2z-1)2z}{(z^2+16)^2}\Bigg|_{z=1}$

$= \dfrac{2z^2+32-4z^2+2z}{(z^2+16)^2}\Bigg|_{z=1}$

$= \dfrac{-2(1)^2+2(1)+32}{(1^2+16)^2}$

$= \dfrac{32}{17^2} = \dfrac{32}{289}$

c. Untuk $C : |z| = 1,5$

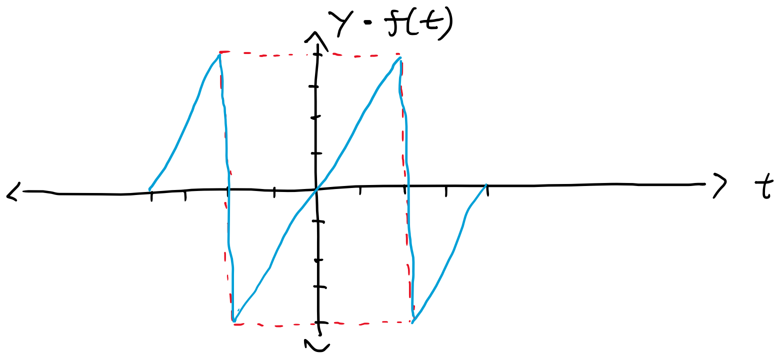
nilai z yang memenuhi : $z = 1$, orde 2

$$\oint_C f(z)\,dz = 2\pi i \left(\mathrm{Res}_{z=1} f(z) \right)$$

$$= 2\pi i \left(\frac{32}{289} \right) = \frac{64\pi i}{289}$$

3. $f(t) = 2t$; $-2 \leq t \leq 2$; Periode 4

a.



b. $f(t) = 2t$

$f(-t) = 2(-t)$

$f(-t) = -2t$

$f(-t) = -f(t) \rightarrow$ Fungsi ganjil

c. $f(t) \rightarrow$ Fungsi ganjil

$a_0 = 0$

$a_n = 0$

$$b_n = \frac{2}{p} \int_p f(t) \sin \frac{2\pi n t}{p}\, dt = \frac{2}{4} \int_{-2}^{2} 2t \cdot \sin \frac{2\pi n t}{4}\, dt$$

$$= \frac{1}{2} \int_{-2}^{2} 2t \sin \frac{\pi n t}{2}\, dt$$

$$= \int_{-2}^{2} t \cdot \sin \frac{\pi n t}{2}\, dt$$

$$= t \cdot \cos\left(\frac{\pi n t}{2}\right) \cdot \frac{\pi n}{2} - \int \frac{\pi n}{2} \cos \frac{\pi n t}{2}\, dt$$

$$= \frac{\pi n t}{2} \cos \frac{\pi n t}{2} - \sin \frac{\pi n t}{2} \Big|_{-2}^{2}$$

$= (\pi n \cos \pi n - \sin \pi n) - (-\pi n \cos(-\pi n) - \sin(-\pi n))$

$= \pi n \cos(\pi n) - \sin(\pi n) + \pi n \cos(\pi n) - \sin(\pi n)$

$= 2\pi n \cos(\pi n) - 2 \sin(\pi n)$

$b_1 = 2\pi \cos(\pi) - 2 \sin(\pi) = -2\pi$

$b_2 = 2\pi . 2 \cos(2\pi) - 2 \sin(2\pi) = 4\pi$

$b_3 = 2\pi . 3 \cos(3\pi) - 2 \sin(3\pi) = -6\pi$

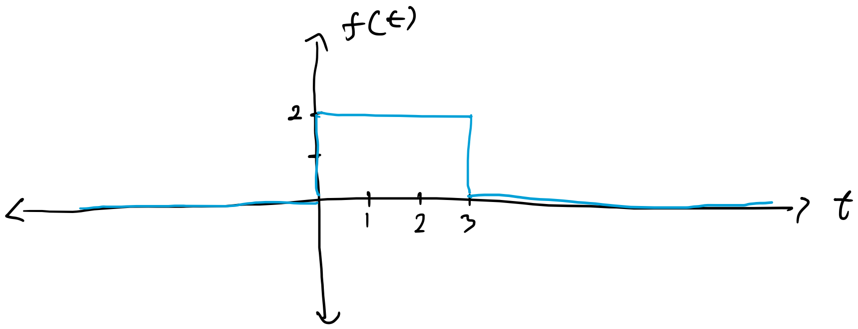
$\vdots \qquad \vdots$

d. $f(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \dots + a_n \cos n\omega t + b_1 \sin \omega t +$
$b_2 \sin 2\omega t + \dots + b_n \sin n\omega t$

$= 0 + 0 + -2\pi \sin \frac{2\pi}{4} t + 4\pi \sin 2 . \frac{2\pi}{4} t + -6\pi \sin 3 \frac{2\pi}{4} t + \dots$

$= -2\pi \sin \frac{\pi}{2} t + 4\pi \sin \pi t - 6 \sin \frac{3\pi}{2} t + \dots$

4. $f(t) \begin{cases} 2 \ ; \ 0 < t < 3 \\ 0 \ ; \ \text{selain } t \text{ di atas} \end{cases}$



$f(t) = 2u(t).u(t-3)$

5. a. $F(i\omega) = \frac{3}{(i\omega)^2 + 16} = \frac{3}{4} \frac{4}{(i\omega)^2 + 4^2}$

$f(t) = \frac{3}{4} \sin(4t).u(t)$

b. $F(i\omega) = \frac{1}{2} \frac{i\omega}{(i\omega)^2 + (-1)} = \frac{1}{2} \frac{i\omega}{(i\omega) + i^2}$

$f(t) = \frac{1}{2} \cos(it).u(t)$

c. $F(i\omega) = \dfrac{i^2\omega + 2}{(i\omega)^2 + 9} = \dfrac{i^2\omega}{(i\omega)^2+9} + \dfrac{2}{(i\omega^2)+9}$

$$= i\,\frac{i\omega}{(i\omega)^2+3^2} + \frac{2}{3}\cdot\frac{3}{(i\omega)^2+3^2}$$

$$= i\cos(3t)\,u(t) + \frac{2}{3}\sin(3t)\cdot u(t)$$