1. Tentukan bagian real dan bagian Imaginer dari fungsi berikut
   a. $f(z) = z^3 + 1$
   b. $f(z) = \frac{2z-4i}{z+2}$
   c. $f(z) = \frac{1}{2z}$

1. a. $z = x + iy$

$i = \sqrt{-1}$

$i^2 = -1$

$f(z) = z^3 + 1$

$f(x+iy) = (x+iy)^3 + 1$

$= (x+iy)^2 (x+iy) + 1$

$= (x^2 + 2ixy - y^2)(x+iy) + 1$

$= x^3 + ix^2y + 2ix^2y - 2xy^2 - xy^2 - iy^2 + 1$

$= x^3 - 3xy^2 + 1 + i(3x^2y - y^2)$

$Re(f) = x^3 - 3xy^2 + 1$

$Im(f) = 3x^2y - y^2$

b. $f(z) = \frac{2z - 4i}{z + 2}$

$f(x+iy) = \frac{2(x+iy) - 4i}{(x+iy) + 2}$

$f(x+iy) = \frac{2x + 2iy - 4i}{x + iy + 2}$

$f(x+iy) = \frac{2x}{x+iy+2} + \frac{(2y-4)i}{x+iy+2}$

$Re(f) = 2x$

$Im(f) = 2y - 4$

c. $f(z) = \dfrac{1}{2z}$

$f(x+iy) = \dfrac{1}{2(x+iy)}$

$f(x+iy) = \dfrac{1}{2x+2iy}$

$Re(f) = 1$

$Im(f) = 0$

2. Diberikan $f(x+iy) = U + iV = e^x \cos y + i\,(e^x \sin y + 7)$
   a. Tunjukkan U dan V harmonik
   b. Tunjukkan $f(x+iy)$ tersebut holomorfik
   c. Tentukan $f'(x+iy)$
   d. Dengan metode milne Thomson, tentukan $f(z)$

$u = e^x \cos y$

$v = e^x \sin y + 7$

a. $U_x = V_y \quad , \quad U_y = -V_x$

$e^x \cos y = e^x \cos y + 0 \quad ; \quad -e^x \sin y = -e^x \sin y + 0$

U & V harmonik

b. holomorfik → differentiable → dapat diturunkan

$f(x+iy)$ → Memenuhi PCR

PCR : I. $U_x = V_y$
II. $U_y = -V_x$

PCR → harmonik
→ holomorfik / differentiable
→ analitik

c. $$f'(x+iy) = \frac{\partial}{\partial x} f(x+iy)$$

$u'v + uv'$

$$= \frac{\partial}{\partial x}\left[\overset{u}{e^x}\,\overset{v}{\cos y} + i\,(e^x \sin y + 7)\right]$$

$$= e^x \cos y + i\,(e^x \sin y)$$

d. $f(z)$ → $\boxed{\text{Milne - Thomson : } x = z \text{ \& } y = 0}$

$f'(x+iy) = e^x \cos y + i\,(e^x \sin y)$

$f'(z) = e^z$

$f(z) = \int e^z\,dz = e^z + C$

$f(z) = e^z + c$

$f(x+iy) = \boxed{e^{x+iy}} + c$

$e^x \cos y + i(e^x \sin y + 7) = \boxed{e^x \cos y + ie^x \sin y} + c$

$c = 7i$

$f(z) = e^z + c$

$= e^z + 7i$

entire $\to$ $\boxed{\text{analitik}}$ $\to$ PCR dr semua titik pada bidang z

3. Periksa Apakah fungsi berikut entire ?
   a. $f(x+iy) = x^2 - y^2 + x + 2 + (2xy + y)i$
   b. $f(x+iy) = 2xy + i(x^2 - y^2)$
   c. $f(x+iy) = x + y + i(xy)$
   d. $f(z) = x^2 + 2z$
   e. $f(z) = ze^z$

a. $f(x+iy) = \overbrace{x^2 - y^2 + x + 2}^{u} + \overbrace{(2xy+y)}^{v}i$

$\Rightarrow u_x = v_y$ ✓ $\qquad \Rightarrow u_y = -v_x$ ✓

$2x + 1 = 2x + 1$ ✓ $\qquad -2y = -(2y)$ ✓

$\therefore f(x+iy) \to$ fungsi entire

b. $f(x+iy) = 2xy + i(x^2 - y^2)$

$\Rightarrow u_x = v_y$ ✗

$2y = 0 - 2y$ ✗

$\therefore f(x+iy) \to$ bukan fungsi entire

c. $f(x+iy) = x + y + i(xy)$

$\Rightarrow u_x = v_y$ ✗

$1 = x$ ✗

$\therefore f(x+iy) \to$ bukan fungsi entire

d. $f(z) = x^2 + 2z$

$f(x+iy) = x^2 + 2(x + iy)$

$f(x+iy) = x^2 + 2x + i(2y)$

$\Rightarrow u_x = v_y$ ✗

$2x + 2 = 2$ ✗

$\therefore f(z) = x^2 + 2z \to$ bukan fungsi entire

e. $f(z) = ze^{z}$

$f(x+iy) = (x+iy)e^{(x+iy)}$

$f(x+iy) = (x+iy)(e^{x}\cos y + i(e^{x}\sin y))$

$= xe^{x}\cos y + i(xe^{x}\sin y) + i(ye^{x}\cos y) - y(e^{x}\sin y)$

$= \underbrace{e^{x}}_{u}\underbrace{(x\cos y - y\sin y)}_{v} + i\underbrace{e^{x}}_{u}\underbrace{(x\sin y + y\cos y)}_{v}$

$\Rightarrow u_x = v_y$ ×

$e^{x}(x\cos y - y\sin y) + e^{x}(\cos y) = x\cos y + \cos y - y\sin y$ ×

$\therefore f(z) = ze^{z} \longrightarrow$ bukan fungsi entire

4. Diberikan $f(x+iy) = 3x + 5 + i(3y - 2)$. Tentukan turunannya jika ada.

Turunan → harmonik → PLR

$$f(x+iy) = \overbrace{3x+5}^{u} + i\overbrace{(3y-2)}^{v}$$

$\Rightarrow u_x = v_y$ ✓ $\qquad\qquad \Rightarrow u_y = -v_x$ ✓

$3 = 3$ ✓ $\qquad\qquad 0 = -(0)$ ✓

$$f'(x+iy) = \frac{\partial}{\partial x} f(x+iy)$$

$$f'(x+iy) = \frac{\partial}{\partial x} \left(3x + 5 + i(3y-2)\right)$$

$$f'(x+iy) = 3$$

5. Periksa $U(x,y)$ yang diberikan apakah harmonik, jika yaTentukan $\boxed{\text{sekawan harmoniknya}}$ $\rightarrow V$

a. $U(x,y) = x - y$

b. $U(x,y) = \frac{x}{x^2+y^2}$

a. $U_x = 1$ $\qquad V_x = \frac{\partial V}{\partial x}$

$U_y = -1$ $\qquad V_y = \frac{\partial V}{\partial y}$

$U_x = V_y$

$1 = \frac{\partial v}{\partial y}$

$v = \int 1 \, dy$

$v = y + g(x)$

$U_y = -V_x$

$\cancel{-}1 = \cancel{-} \frac{\partial v}{\partial x}$

$1 = \frac{\partial}{\partial x}(y + g(x))$

$1 = g'(x)$

$g(x) = \int 1 \, dx$

$g(x) = x + C$

$v = y + x + C$

b. $u = \frac{x}{x^2+y^2}$ $\begin{matrix} u \\ v \end{matrix}$ $\quad \frac{u'v - uv'}{v^2}$

$u_x = \frac{(x^2+y^2) - x(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$ $\qquad v_x = \frac{\partial v}{\partial x}$

$u_y = \frac{0 - x(2y)}{(x^2+y^2)^2} = \frac{-2xy}{(x^2+y^2)^2}$ $\qquad v_y = \frac{\partial v}{\partial y}$

$u_x = v_y$ $\qquad u_y = -v_x$

$\frac{y^2-x^2}{(x^2+y^2)^2} = \frac{\partial v}{\partial y}$ $\qquad \cancel{-}\frac{2xy}{(x^2+y^2)^2} = \cancel{-}\frac{\partial v}{\partial x}$

$\frac{y^2-x^2}{(x^2+y^2)^2} = \frac{\partial}{\partial y}\left(\frac{-y}{x^2+y^2} + g(y)\right)$

$\frac{y^2-x^2}{(x^2+y^2)^2} = \frac{-(x^2+y^2)+y(2y)}{(x^2+y^2)^2} + g'(y)$

$g'(y) = \frac{\cancel{y^2}\cancel{-x^2}+\cancel{(x^2+y^2)}-\cancel{2y^2}}{(x^2+y^2)^2}$

$g'(y) = 0$

$g'(x) = c$

$v = \int \frac{2xy}{(x^2+y^2)^2}\,dx$ $\qquad u = x^2+y^2$

$v = y\int \frac{\cancel{2x}}{u^2}\frac{du}{\cancel{2x}}$

$v = y \cdot -\frac{1}{u} + g(y)$

$v = -\frac{y}{x^2+y^2} + g(y)$

$v = -\frac{y}{x^2+y^2} + c$

6. Dengan metode milne Thomson, ubah $f(x+iy)$ holomorfik berikut menjadi $f(z)$

a. $f(x+iy) = 2x+7+i(2y-11)$

b. $f(x+iy) = e^x \cos y - i\,(e^x \sin y + 5)$

c. $f(x+iy) = x^2 - 5x + 2 - y^2 + i(2xy + 5y + i)$

a. $f(x+iy) = 2x+7+i(2y-11)$

i) $u_x = v_y$ ✓ $\qquad$ ii) $u_y = -v_x$ ✓

$2 = 2$ ✓ $\qquad$ $0 = -(0)$ ✓

$$f'(x+iy) = \frac{\partial}{\partial x}\left(2x+7+i(2y-11)\right)$$

$$f'(x+iy) = 2 \qquad x=z,\ y=0,\ x+iy \to z$$

$$f'(z) = 2$$

$$f(z) = \int 2\,dz$$

$$f(z) = 2z + c$$

$$f(x+iy) = 2(x+iy) + c$$

$$\cancel{2x}+7+i(\cancel{2y}-11) = \cancel{2x}+\cancel{2iy}+c$$

$$c = 7 - 11i$$

$$f(z) = 2z + c$$

$$f(z) = 2z + 7 - 11i$$

b. $f(x+iy) = e^x \cos y - i(e^x \sin y + 5)$

$u_x = v_y$ ✓ $\qquad$ $u_y = -v_x$ ✓

$e^x \cos y = e^x \cos y$ ✓ $\qquad$ $-e^x \sin y = -(e^x \sin y)$ ✓

$$f'(x+iy) = \frac{\partial}{\partial x}\left(e^x \cos y - i(e^x \sin y + 5)\right)$$

$$f'(x+iy) = e^x \cos y - i(e^x \sin y) \qquad x=z,\ y=0,\ x+iy \to z$$

$$f'(z) = e^z$$

$f(z) = \int e^z \, dz$

$f(z) = e^z + C$

$f(x+iy) = e^{x+iy} + C$

$\cancel{e^x \cos y} - i(e^x \sin y + 5) = \cancel{e^x \cos y} + i(e^x \sin y) + C$

$C = -i(2e^x \sin y + 5)$

$f(z) = e^z + C$

$f(z) = e^z - i(2e^x \sin y + 5)$

c. $f(x+iy) = x^2 - 5x + 2 - y^2 + i(2x + 5y + i)$

$f(x+iy) = x^2 - 5x + 2 - y^2 - 1 + i(2x + 5y)$

$f(x+iy) = x^2 - 5x + 1 - y^2 + i(2x + 5y)$

$\Rightarrow u_x = v_y$ ✗

$2x - 5 = 5$ ✗

$\therefore f(x+iy) = x^2 - 5x + 2 - y^2 + i(2x + 5y + i) \rightarrow$ tidak punya bentuk kompak

(compact)

7. Diberikan

a. $f(z) = \frac{5z}{(z-1)(z-2)(z-3)}$, periksa apakah fungsi tersebut analitik di $|z+1| < 2$, jelaskan

b. $f(z) = \frac{z^3}{(z+1)(z+i)}$, periksa apakah fungsi tersebut analitik di $|z+2i| > 3$, jelaskan

a. Titik singular : • $z - 1 = 0$ • $z - 2 = 0$ • $z - 3 = 0$

$z_1 = 1$ $z_2 = 2$ $z_3 = 3$

$|z - z_0| = r$

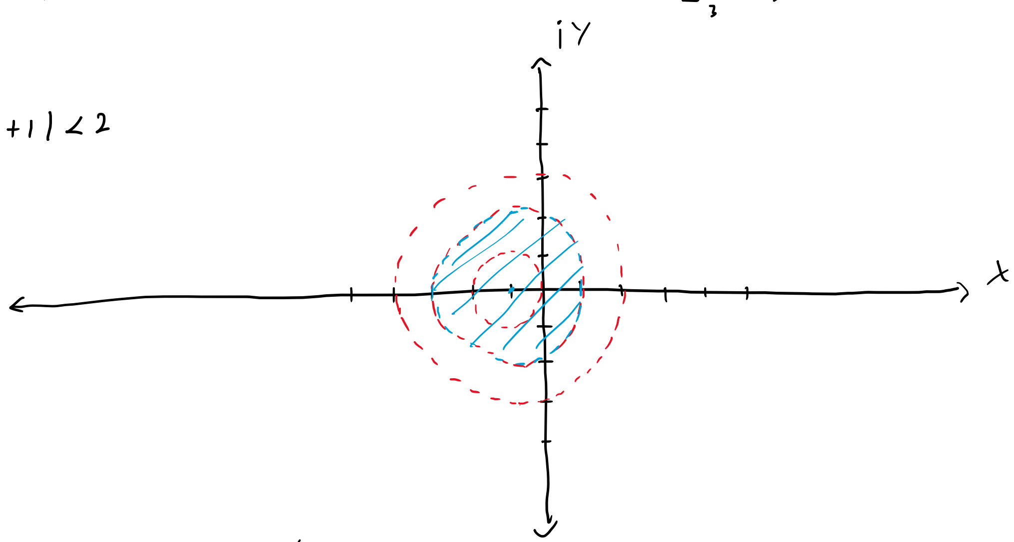
$|z+1| = 2$

$|z - (-1)| = 2$

$\underbrace{(-1)}_{TP}$ $\underbrace{2}_{r}$

TP $(-1, 0)$

$|z+1| < 2$



$\therefore f(z) = \frac{5z}{(z-1)(z-2)(z-3)}$ analitik di $|z+1| < 2$ dengan titik singular pada $z = 1$

kecuali pada titik $z = 1$

b. $f(z) = \dfrac{z^3}{(z+1)(z+i)}$ $\quad |z+2i| > 3$

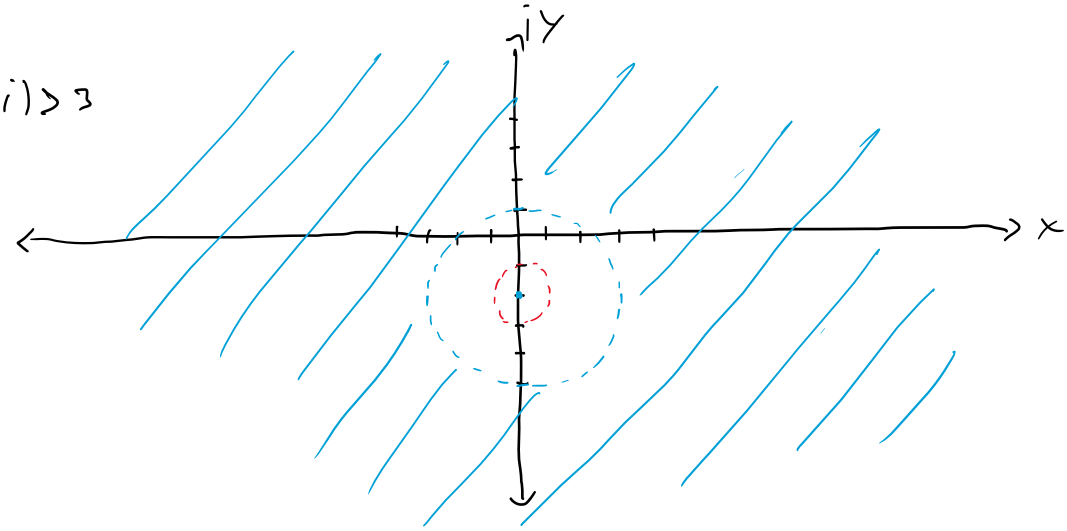
Titik singular : $z+1=0$ $\quad z+i=0$

$z_1 = -1$ $\quad z = -i$

$|z+2i| = 3$ $\quad |z+2i| > 3$

$|z - \underbrace{(-2i)}_{TP}| = \underbrace{3}_{r}$

$(0,-2)$ $\quad r=3$



$\therefore f(z) = \dfrac{z^3}{(z+1)(z+i)}$ analitik pada $|z+2i| > 3$