3rd Material Subject: Introduction To Probability

Undergraduate of Telecommunication Engineering

MUH1F3 - PROBABILITY AND STATISTICS

Telkom University
Center of eLearning & Open Education Telkom University
Jl. Telekomunikasi No.1, Bandung - Indonesia
http://www.telkomuniversity.ac.id

Lecturer: Nor Kumalasari Caecar Pratiwi, S.T., M.T. (caecarnkcp@telkomuniversity.ac.id)

السلام عليكم ورحمة الله وبركاته

WELCOME

TABLE OF CONTENTS:

LEARNING OBJECTIVES:

After careful study of this chapter, student should be able to do the following:

1. **Understand and describe sample spaces and events for random experiments with graphs, lists, or tree diagrams**
2. **Determine the MEE and NON-MEE of events and use to calculate probabilities**
3. **Determine the Independent Event and use to calculate system Reliability**

SET REVIEW

Our interest in the study of a probability (random phenomena) is closely tied to the theory of sets, and we give in this section some of its basic concepts and algebraic operations of set.

- **SET** is acollection of objects possessing some common properties, denoted by capital letters **A**, **B**, $\cdots$
- Members of set **A** called **ELEMENT**, if **x** is an element of **A** then it can be written down as **x** $\in$ **A** otherwise if **y** not an element of **A** then it can be written down as **y** $\notin$ **A**.
- Element are enclosed in curly braces.
  Example:
  1. $A = \{x \mid x \in \mathfrak{B}, 2 < x < 9\} = \{3, 4, 5, 6, 7, 8\}$
  2. $B = \{x \mid x \in \mathfrak{B}, x^2 + 1 \leq 10\} = \{-3, -2, -1, 1, 2, 3\}$
  3. $C = \{x \mid x \in \mathfrak{B}, x \text{ Odd Numbers}, -5 < x < 5\} = \{-3, -1, 1, 3\}$
- The **UNIVERSAL SET** is notated by $\mathcal{U}$.
- The **NULL SET** is notated by $\emptyset$, where $\emptyset = \{\ \ \}$
- The **POWER SET** of **A** is notated by $\wp(A)$, dimana $\wp(A) = 2^A$

SET REVIEW

If every element of a set **A** is also an element of a set **B**, the set **A** is called a **SUBSET** of **B** and this is represented symbolically by:

$$\mathbf{A} \subset \mathbf{B} \quad \text{or} \quad \mathbf{B} \supset \mathbf{A} \tag{1}$$

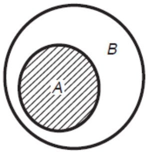


Figure 1: Venn Diagram of $\mathbf{A} \subset \mathbf{B}$

Example: Let $\mathbf{A} = \{\mathbf{2}, \mathbf{4}\}$ and $\mathbf{B} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}\}$ Then $\mathbf{A} \subset \mathbf{B}$ since every element of **A** is also an element of **B**. This relationship can also be presented graphically by using a Venn diagram, as shown in Figure 1. The set occupies the interior of the larger circle and the shaded area in the figure.

SET REVIEW

If A and B is a subset of the universal $\mathcal{U}$, then:

- **COMPLEMENT** of **A** is element of universal set $\mathcal{U}$ but is not an element of **A** and represented by $\overline{\mathbf{A}}$.

$$\overline{\mathbf{A}} = \{\mathbf{x} \mid \mathbf{x} \in \mathcal{U}, \mathbf{x} \notin \mathbf{A}\} \tag{2}$$

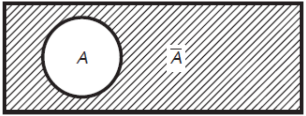


Figure 2: Venn Diagram of $\overline{\mathbf{A}}$

Example:If $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$ and $C = \{1, 3, 5, 7\}$, then:
$\overline{A} = \{4, 5, 6, 7\}$, $\overline{B} = \{1, 3, 7\}$, and $\overline{C} = \{2, 4, 6\}$

In Probability, the word mark used for COMPLEMENT operations is "NO" or "NOT".

- **INTERSECTION** or product of **A** and **B**, written as $\mathbf{A} \cap \mathbf{B}$ or simply **AB**, is the set of all elements that are common to A and B.

$$\mathbf{A} \cap \mathbf{B} = \{\mathbf{x} \mid \mathbf{x} \in \mathbf{A} \text{ dan } \mathbf{x} \in \mathbf{B}\} \quad (3)$$

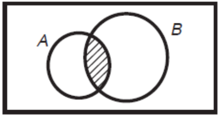


Figure 3: Venn Diagram of $\mathbf{A} \cap \mathbf{B}$

Example:If $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$ and $C = \{1, 3, 5, 7\}$, then:
$\mathbf{A} \cap \mathbf{B} = \{\mathbf{2}\}$, $\mathbf{A} \cap \mathbf{C} = \{\mathbf{1}, \mathbf{3}\}$, and $\mathbf{B} \cap \mathbf{C} = \{\ \} = \emptyset$

In Probability, the word mark used for INTERSECTION operations is "AND".

SET REVIEW

- **UNION** or sum of **A** and **B**, denoted by **A** $\cup$ **B** is the set of all elements belonging to **A** or **B** or both.

$$\mathbf{A} \cup \mathbf{B} = \left\{\mathbf{x} \,\middle|\, \mathbf{x} \in \mathbf{A} \text{ or } \mathbf{x} \in \mathbf{B}\right\} \tag{4}$$

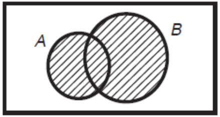


Figure 4: Venn Diagram of **A** $\cup$ **B**

Example:If $\mathcal{U} = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$ and $C = \{1, 3, 5, 7\}$, then: $\mathbf{A} \cup \mathbf{B} = \{\mathbf{1, 2, 3, 4, 6}\}$, $\mathbf{A} \cup \mathbf{C} = \{\mathbf{1, 2, 3, 5, 7}\}$, and $\mathbf{B} \cup \mathbf{C} = \{\mathbf{1, 2, 3, 4, 5, 6, 7}\} = \mathcal{U}$

In Probability, the word mark used for UNION operations is "OR".

SET REVIEW

The laws that apply to the Association include:

1. **IDENTITY LAWS**
   - $A \cup \emptyset = A$
   - $A \cup \mathcal{U} = \mathcal{U}$
   - $A \cap \emptyset = \emptyset$
   - $A \cap \mathcal{U} = A$
2. **DE' MORGAN LAWS**
   - $\overline{A \cup B} = \overline{A} \cap \overline{B}$
   - $\overline{A \cap B} = \overline{A} \cup \overline{B}$
3. **ASSOCIATIVE LAWS**
   - $A \cup (B \cup C) = (A \cup B) \cup C$
   - $A \cap (B \cap C) = (A \cap B) \cap C$
4. **DISTRIBUTIVE LAWS**
   - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

SET REVIEW

General rules of addition and multiplication in a set:

1. **ADDITION**

$$A \cup B = A + B - (A \cap B) \tag{5}$$

then the same rules apply in looking for probability:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{6}$$

2. **MULTIPLICATION**

$$P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B) \tag{7}$$

RANDOM EXPERIMENT, SAMPLE SPACE & EVENT

- **Random Experiment**, An experiment that can result in different outcomes, even though it is repeated in the same manner every time. Example: Roll a dice
- **Sample Space**, is the set of all possible outcomes of a random experiment, denoted by **S**. A sample space is **discrete** if it consists of a finite or countable infinite set of outcomes. A sample space is **continuous** if it contains an interval (either finite or infinite) of real numbers. Example: The sample space of random experiment Roll a dice:

$$\mathbf{S} = \{1, 2, 3, 4, 5, 6\}$$

- **Event**, is a subset of the sample space of a random experiment. Example: If **B** is event occurrence of odd numbers when throwing a dice:

$$\mathbf{B} = \{1, 3, 5\}$$

Its appears that $\mathbf{B} \subset \mathbf{S}$

RANDOM EXPERIMENT, SAMPLE SPACE & EVENT

Furthermore, an **Event** can be considered as a **Set**, so the operation $\cap$ and $\cup$ can apply. The **Event** is divided into

- **Mutually Exclusive Event (MEE)**
- **Non Mutually Exclusive Event (NON MEE)**
  - **Independent Event (IE)**
  - **Dependent Event (DE)**

MUTUALLY EXCLUSIVE EVENT (MEE)

If there are two events or more, namely **A** and **B** associated with the results of a random experiment, if the intersection are empty sets, then event said to be **Mutually Exclusive Event**.

$$\mathbf{A} \cap \mathbf{B} = \emptyset \tag{8}$$

So the probability value will be:

$$\mathbf{P}(\mathbf{A} \cap \mathbf{B}) = \mathbf{0} \tag{9}$$

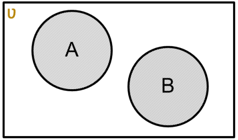


Figure 5: Venn Diagram of Mutually Exclusive Event $\mathbf{P}(\mathbf{A} \cap \mathbf{B}) = \mathbf{0}$

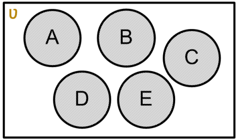
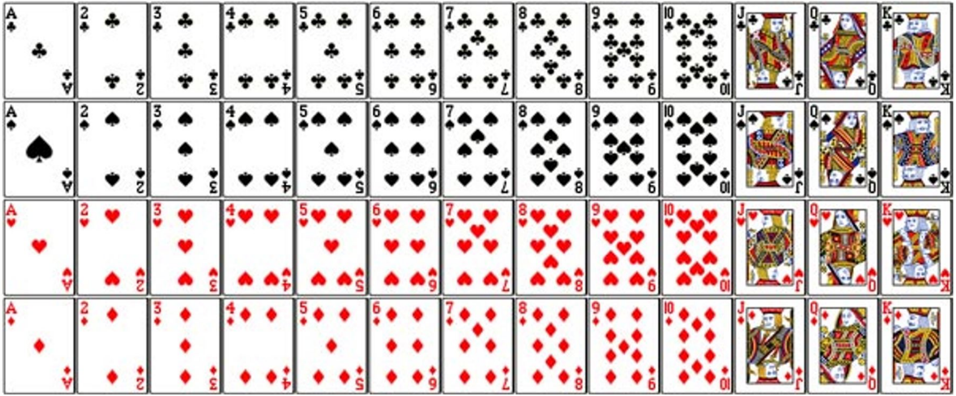


Figure 6: Venn Diagram of Mutually Exclusive Event $\mathbf{P}(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C} \cap \mathbf{D} \cap \mathbf{E}) = \mathbf{0}$

MUTUALLY EXCLUSIVE EVENT (MEE)



♣ (Club) ♠ (Spade) ♥ (Heart) ♦ (Diamond)

MUTUALLY EXCLUSIVE EVENT (MEE)

If one card is taken at random, event **A** states the selected card is ♣ event **B** states the selected card is ♠, then **A** and **B** is Mutually Exclusive event, $\mathbf{P(A \cap B) = 0}$.

If $A \perp B$ (read: **A** and**B** are**Mutually Exclusive Event**), then the general rule of addition:

$$\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Previously known if $P(A \cap B) = 0$, so:

$$\mathbf{P(A \cup B) = P(A) + P(B)} \tag{10}$$

Generally it can be said:

$$\mathbf{P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum_{i=1}^{n} P(A_i)} \tag{11}$$

MUTUALLY EXCLUSIVE EVENT (MEE)

Example: In a shopping center there are 9 coffee shops, which are 3 tubruk coffee shops, 2 civet coffee shops and 4 charcoal coffee shops. In one time, Budi will visit one random coffee shop. Determine:

1. Probability Budi visited the Tubruk Coffee shop and Luwak Coffee Shop at one time
2. Probability Budi visited the Tubruk Coffee shop or Luwak Coffee Shop at one time
3. Probability Budi visited the Tubruk Coffee shop or Charcoal Coffee Shop at one time

Answer: If **A** is an event that states that the coffee shop was selected, **B** is an event that states selected Luwak Coffee shop, as well as **C** are the stated events selected Charcoal Coffee shop, then:

1. Probability Budi visited the Tubruk Coffee shop and Luwak Coffee Shop at one time

$$P(A \cap B) = 0$$

2. Probability Budi visited the Tubruk Coffee shop or Luwak Coffee Shop at one time

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{9} + \frac{2}{9} - 0 = \frac{5}{9}$$

3. Probability Budi visited the Tubruk Coffee shop or Charcoal Coffee Shop at one time

$$P(A \cup C) = P(A) + P(C) - P(A \cap C) = \frac{3}{9} + \frac{4}{9} - 0 = \frac{7}{9}$$

NON - MUTUALLY EXCLUSIVE EVENT (NON-MEE)

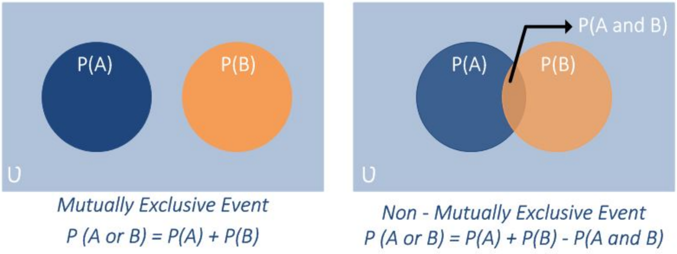


Figure 7: Venn Diagram of MEE and NON-MEE

In the MEE known that:

$$\mathbf{P}\big(\mathbf{A}\text{ and }\mathbf{B}\big) = \mathbf{P}\big(\mathbf{A} \cap \mathbf{B}\big) = \mathbf{0}$$

Then, the opposite happened to Non-MEE:

$$\mathbf{P}\big(\mathbf{A}\text{ and }\mathbf{B}\big) = \mathbf{P}\big(\mathbf{A} \cap \mathbf{B}\big) \neq \mathbf{0}$$

NON-MEE is divided into **Independent Event (IE)** and **Dependent Event (DE)**.

NON - MUTUALLY EXCLUSIVE EVENT (NON-MEE)

$A \perp\!\!\!\perp B$ read as an Event **A** and **B** is **Independent Event (IE)**, not affect each other. The occurrence of **A** is not affected by **B** and vice versa, so:

$$\mathbf{P}(\mathbf{A} \mid \mathbf{B}) = \mathbf{P}(\mathbf{A}) \quad \text{and} \quad \mathbf{P}(\mathbf{B} \mid \mathbf{A}) = \mathbf{P}(\mathbf{B}) \tag{12}$$

So it affects the general rules of multiplication:

$$\mathbf{P}(\mathbf{A} \cap \mathbf{B}) = \mathbf{P}(\mathbf{A}) \cdot \mathbf{P}(\mathbf{B} \mid \mathbf{A}) = \mathbf{P}(\mathbf{A}) \cdot \mathbf{P}(\mathbf{B})$$

$$\mathbf{P}(\mathbf{A} \cap \mathbf{B}) = \mathbf{P}(\mathbf{B}) \cdot \mathbf{P}(\mathbf{A} \mid \mathbf{B}) = \mathbf{P}(\mathbf{B}) \cdot \mathbf{P}(\mathbf{A})$$

So in general:

$$\mathbf{P}(\mathbf{A}_1 \cap \mathbf{A}_2 \cap \cdots \cap \mathbf{A}_n) = \mathbf{P}(\mathbf{A}_1) \cdot \mathbf{P}(\mathbf{A}_2) \cdot \cdots \cdot \mathbf{P}(\mathbf{A}_n) \tag{13}$$

The effect on the general rule of addition:

$$\mathbf{P}(\mathbf{A} \cup \mathbf{B}) = \mathbf{P}(\mathbf{A}) + \mathbf{P}(\mathbf{B}) - \mathbf{P}(\mathbf{A} \cap \mathbf{B}) = \mathbf{P}(\mathbf{A}) + \mathbf{P}(\mathbf{B}) - \mathbf{P}(\mathbf{A}) \cdot \mathbf{P}(\mathbf{B}) \tag{14}$$

NON - MUTUALLY EXCLUSIVE EVENT (NON-MEE)

Example: Known **A**, **B** and **C** are independent each other. If $\mathbf{P(A) = 0.2}$, $\mathbf{P(B) = 0.1}$ and $\mathbf{P(C) = 0.4}$, calculate the $\mathbf{P(A \cup B \cup C)}$!

Answer: Due to **A**, **B** and **C** are independent each other, then:

$$\mathbf{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}$$

where:

$$\mathbf{P(A \cap B) = P(A) \cdot P(B) = 0.02} \qquad \mathbf{P(A \cap C) = P(A) \cdot P(C) = 0.08}$$

$$\mathbf{P(B \cap C) = P(B) \cdot P(B) = 0.04} \quad \text{and} \quad \mathbf{P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = 0.008}$$

So that:

$$\mathbf{P(A \cup B \cup C) = 0.2 + 0.1 + 0.4 - 0.02 - 0.08 - 0.04 + 0.008 = 0.568}$$

NON - MUTUALLY EXCLUSIVE EVENT (NON-MEE)

The used of **Independent Event (IE)** concept is to calculate the system **Reliability**.

- In **Serial System**, it is said to be successful if all components are in good condition, on the contrary, the system is said to be failed if at least one component does not work.

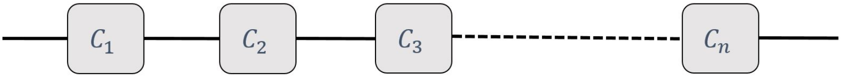


Figure 8: Serial System

$$\text{Reliability} = \mathbf{P}(\text{System Work}) = \mathbf{P_s}(\mathbf{C_1} \cap \mathbf{C_2} \cap \cdots \cap \mathbf{C_n}) = \mathbf{P_s}(\mathbf{C_1}) \cdot \mathbf{P_s}(\mathbf{C_2}) \cdots \mathbf{P_s}(\mathbf{C_n}) \quad (15)$$

Note: $\mathbf{P_s}(\mathbf{C_i})$ denoted the Probability of component $\mathbf{C_i}$ **Success**.

- In **Parallel System**, it is said to be successful if at least one component in a good condition and failed if all component does not work.

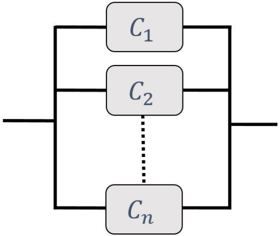


Figure 9: Paralel System

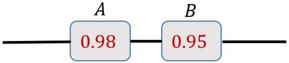
$$\mathbf{P}(\text{System Failed}) = \mathbf{P_f}(\mathbf{C_1} \cap \mathbf{C_2} \cap \cdots \cap \mathbf{C_n}) = \mathbf{P_f}(\mathbf{C_1}) \cdot \mathbf{P_f}(\mathbf{C_2}) \cdot \cdot \mathbf{P_f}(\mathbf{C_n}) \tag{16}$$

$$\text{Reliabilitas Sistem} = \mathbf{1} - \mathbf{P}(\text{System Failed}) \tag{17}$$

Note: $\mathbf{P_f}(\mathbf{C_i})$ denoted the Probability of component $\mathbf{C_i}$ **Failed**.

NON - MUTUALLY EXCLUSIVE EVENT (NON-MEE)

Example: If numbers in the boxes are the probability of component success, calculate the reliability of system below:



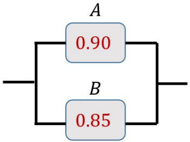
Answer:

Known $P_s(A) = 0.98$ and $P_s(B) = 0.95$,

then Reliability $= P_s(A) \cdot P_s(B) = 0.931$

Obtained system reliability of 0.931 or 93.1%

Example: If numbers in the boxes are the probability of component success, calculate the reliability of system below:



Answer: Known $P_f(A) = 1 - P_s(A) = 0.1$ and

$P_f(B) = 1 - P_s(B) = 0.15$ and

$P(\text{System Failed}) = P_f(A) \cdot P_f(B) = 0.015.$

Reliability $= 1 - P(\text{System Failed}) = 0.985$

Obtained system reliability of 0.985 or 98.5%

Thank You