15.) $S = \{1, 2-x^2, 4-4x+x^2\}$

A.) $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & -4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right] \xrightarrow[-\frac{1}{4}b_2]{2b_3+b_1} \left[\begin{array}{ccc|c} 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[-b_2+b_3]{-6b_2+b_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \xrightarrow{-b_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$\therefore$ S Bebas linear

B.) $\vec{u} \in P^2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & u_1 \\ 0 & 0 & -4 & u_2 \\ 0 & -1 & 1 & u_3 \end{array}\right] \xrightarrow[-\frac{1}{4}b_2]{2b_3+b_1} \left[\begin{array}{ccc|c} 1 & 0 & 6 & u_1+2u_3 \\ 0 & 0 & 1 & -\frac{1}{4}u_2 \\ 0 & -1 & 1 & u_3 \end{array}\right]$$

$$\xrightarrow{-6b_2+b_1} \xrightarrow{-b_2+b_3} -b_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & u_1+2u_3+\frac{3}{2}u_2 \\ 0 & 0 & 1 & -\frac{1}{4}u_2 \\ 0 & 1 & 0 & -u_3-\frac{1}{4}u_2 \end{array}\right]$$

$\therefore$ Maka S membangun P^2

C.) Karena S Bebas linear & membangun $P^2 \longrightarrow$ S Basis P^2

16.) $\langle \vec{u}, \vec{v} \rangle = u_1 v_1 + u_2 v_2 + 2u_3 v_3$

A.) $\langle (1,1,-1), (2,2,1) \rangle = 2 + \cancel{2} - \cancel{2} = \underline{2}$

B.) $S = \{ \vec{w_1} = (1,0,1), \vec{w_2} = (0,1,1) \}$

$\therefore \langle \vec{w_1}, \vec{w_2} \rangle = 0 + 0 + 1 = 1$

Bukan Himp. Orthogonal.

$\therefore$ Misal, A adalah Himp. Orthonomal hasil konversi S.

$\therefore A = \{ \vec{A_1}, \vec{A_2} \}$

$\therefore \vec{A_1} = \dfrac{\vec{w_1}}{|\vec{w_1}|} = \dfrac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

$\because \ \vec{W_2} - \langle \vec{W_2}, \vec{A_1} \rangle \vec{A_1}$

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{\sqrt{2}} \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1/2 \\ 0 \\ 1/2 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 1 \\ 1/2 \end{pmatrix}$$

$$\therefore \ \vec{A_2} = \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 + 1^2 + \left(\frac{1}{2}\right)^2}} \begin{pmatrix} -1/2 \\ 1 \\ 1/2 \end{pmatrix}$$

$$= \sqrt{2/3} \begin{pmatrix} -1/2 \\ 1 \\ 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{\sqrt{2}}{2\sqrt{3}} \\ \sqrt{2/3} \\ \frac{\sqrt{2}}{2\sqrt{3}} \end{pmatrix}$$

$$\therefore \ A = \left\{ \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} -\frac{\sqrt{2}}{2\sqrt{3}} \\ \sqrt{2/3} \\ \frac{\sqrt{2}}{2\sqrt{3}} \end{pmatrix} \right\}$$

$\boxed{7.}$ $T: R^3 \longrightarrow R^2$

$$T\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \qquad T\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$T\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\therefore \quad A \cdot \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 3 \\ 3 & -1 & 1 \end{bmatrix}$$

$$\therefore \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[b_3+b_1]{b_2+b_1}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 3 & 1 & 1 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{3}b_1}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-b_2]{-b_1+b_2} \xrightarrow[-b_3]{-b_1+b_3}$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \\ 0 & 1 & 0 & 1/3 & -2/3 & 1/3 \\ 1 & 0 & 0 & 1/3 & 1/3 & -2/3 \end{array}\right]$$

$$\therefore \quad A = \begin{bmatrix} 0 & 0 & 3 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 1/3 & -2/3 \\ 1/3 & -2/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix}$$

A.) $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

B.) $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 2 & -2 & 0 \end{array}\right] \xrightarrow{-b_1+b_2}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] \xrightarrow{-b_2+b_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right]$$

$\therefore \quad x = -y - z = -4t$

$y = 3z = 3t$

$z = t$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} t$$

$\therefore$ Basis $\ker(T) = \left\{ \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix} \right\}$

Nulitas $= 1$

$R(T) = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$

Rank $= 2$

18. $T : R^3 \to R^2$

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + y - z \\ y - z \end{pmatrix}$$

A.) $T\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

B.) $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

$$\therefore \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \xrightarrow{-b_2+b} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \Longrightarrow \begin{array}{l} x = 0 \\ y = z \\ z = t \end{array}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} t$$

Basis $Ker(T) = \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$

Nulitas $= 1$

c.) $R(T) = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$

Rank $= 2$

19.

$$A = \begin{pmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{pmatrix}$$

A.) $\therefore$

$$\begin{vmatrix} 5-\lambda & -3 & 2 \\ 15 & -9-\lambda & 6 \\ 10 & -6 & 4-\lambda \end{vmatrix} = 0$$

$$(5-\lambda) \cdot \begin{vmatrix} -9-\lambda & 6 \\ -6 & 4-\lambda \end{vmatrix} + 3 \begin{vmatrix} 15 & 6 \\ 10 & 4-\lambda \end{vmatrix} + 2 \begin{vmatrix} 15 & -9-\lambda \\ 10 & -6 \end{vmatrix}$$

$$(5-\lambda)(\cancel{-36} + 5\lambda + \lambda^2 + \cancel{36}) + 3(\cancel{60} - 15\lambda - \cancel{60}) + 2(-\cancel{90} + \cancel{90} + 10\lambda)$$

$$(5-\lambda) \cdot \lambda(\lambda+5) \quad - 45\lambda + 20\lambda = 0$$

$$(5-\lambda)(\lambda+5)\lambda - 25\lambda = 0$$

$$[\cancel{5\lambda} + \cancel{25} - \lambda^2 - \cancel{5\lambda} - \cancel{25}]\lambda = 0$$

$$\lambda^3 = 0$$

$$\lambda = 0$$

B.) $\left[\begin{array}{ccc|c} 5 & -3 & 2 & 0 \\ 15 & -9 & 6 & 0 \\ 10 & -6 & 4 & 0 \end{array}\right] \xrightarrow[-2b_1+b_2]{-3b_1+b_2}$

$\left[\begin{array}{ccc|c} 5 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow$ $5x = 3y - 2z$

$x = \frac{3}{5}y - \frac{2}{5}z$

$y = s$

$z = t$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3/5 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -2/5 \\ 0 \\ 1 \end{bmatrix} t$

Basis Ruang Eigen $\rightarrow \left\{ \begin{pmatrix} 3/5 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2/5 \\ 0 \\ 1 \end{pmatrix} \right\}$

C.) A tidak dapat didiagonalkan karena P tidak memiliki invers

$\boxed{10}$ $y_1' = 2y_1 - 2y_2$

$y_2' = -y_1 + 2y_2$

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2 & -2 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\therefore \begin{vmatrix} 2-\lambda & -2 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$$4 - 4\lambda + \lambda^2 - 2 = 0$$

$$\lambda^2 - 4\lambda + 2 = 0$$

$$\lambda_{1,2} = \frac{4 \pm \sqrt{16-8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$\therefore \quad \underline{\lambda = 2 + \sqrt{2}}$

$$\begin{bmatrix} -\sqrt{2} & -2 \\ -1 & -\sqrt{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \left.\begin{aligned} x &= -\sqrt{2}\,y \\ y &= t \end{aligned}\right\} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\sqrt{2} \\ 1 \end{bmatrix} t$$

$\therefore \quad \underline{\lambda = 2 - \sqrt{2}}$

$$\begin{bmatrix} \sqrt{2} & -2 \\ -1 & \sqrt{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \left.\begin{aligned} x &= \sqrt{2}\,y \\ y &= t \end{aligned}\right\} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ 1 \end{bmatrix} t$$

$$\therefore \quad P = \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} -\frac{\sqrt{2}}{4} & \frac{1}{2} \\ \frac{\sqrt{2}}{4} & \frac{1}{2} \end{bmatrix}$$

$$D = \begin{bmatrix} -\frac{\sqrt{2}}{4} & \frac{1}{2} \\ \frac{\sqrt{2}}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-\sqrt{2}-1}{2} & \frac{2+\sqrt{2}}{2} \\ \frac{\sqrt{2}-1}{2} & \frac{-\sqrt{2}+2}{2} \end{bmatrix} \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2+\sqrt{2}}{2} & 0 \\ 0 & \frac{2-\sqrt{2}}{2} \end{bmatrix}.$$

$$\therefore \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} \frac{2+\sqrt{2}}{2} & 0 \\ 0 & \frac{2-\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} c_1 e^{\frac{2+\sqrt{2}}{2}t} \\ c_2 e^{\frac{2-\sqrt{2}}{2}t} \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} & \sqrt{2} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 e^{\frac{2+\sqrt{2}}{2}t} \\ c_2 e^{\frac{2-\sqrt{2}}{2}t} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} \cdot c_1 e^{\frac{2+\sqrt{2}}{2}t} + \sqrt{2} c_2 e^{\frac{2-\sqrt{2}}{2}t} \\ c_1 e^{\frac{2+\sqrt{2}}{2}t} + c_2 e^{\frac{2-\sqrt{2}}{2}t} \end{bmatrix}$$