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1.) $x_1 + 2x_2 + x_3 = 1$

$-x_1 + 4x_2 + 3x_3 = 2$

$2x_1 - 2x_2 + kx_3 = 3$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 2 & -2 & k & 3 \end{bmatrix} \overset{2b_2 + b_3}{\sim} \begin{bmatrix} 1 & 2 & 1 & 1 \\ -1 & 4 & 3 & 2 \\ 0 & 6 & k+6 & 7 \end{bmatrix} \overset{b_1 + b_2}{\sim} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 6 & 4 & 3 \\ 0 & 6 & k+6 & 7 \end{bmatrix} \overset{-b_2 + b_3}{\sim}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 6 & 4 & 3 \\ 0 & 0 & k+2 & 4 \end{bmatrix}$$

a. Agar memiliki solusi tunggal maka :

$k + 2 \neq 0$

$k \neq -2$

b. Agar memiliki solusi tak hingga banyak :

$k + 2 = 0$ dan $4 = 0$

karena $4 \neq 0$, maka k tidak dapat di tentukan.

c. Agar SPL tidak memiliki solusi :

$k + 2 = 0$ dan $4 \neq 0$

$k = -2$

2.)

$$\begin{aligned} x_1 + x_2 - x_3 + x_4 &= -1 \\ 2x_1 + 3x_2 + 3x_3 + 2x_4 &= 3 \\ x_1 - 2x_3 + 3x_4 &= -2 \\ x_2 + 2x_4 &= 0 \end{aligned}$$

a. $$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 3 & 2 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

b. $$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 1 & -1 \\ 2 & 3 & 3 & 2 & 3 \\ 1 & 0 & -2 & 3 & -2 \\ 0 & 1 & 0 & 2 & 0 \end{array}\right] \underset{\sim}{b_2 \leftrightarrow b_4} \begin{bmatrix} 1 & 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 2 & 0 \\ 1 & 0 & -2 & 3 & -2 \\ 2 & 3 & 3 & 2 & 3 \end{bmatrix} \underset{\sim}{-b_1 + b_3}$$

$$\begin{bmatrix} 1 & 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & -1 & -1 & 2 & -1 \\ 2 & 3 & 3 & 2 & 3 \end{bmatrix} \begin{array}{c} -b_3 \\ -2b_1 + b_4 \\ \sim \end{array} \begin{bmatrix} 1 & 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 1 & 5 & 0 & 5 \end{bmatrix} \begin{array}{c} -b_2 + b_3 \\ -b_2 + b_4 \\ \sim \end{array}$$

$$\begin{bmatrix} 1 & 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 5 & -2 & 5 \end{bmatrix} \begin{array}{c} -b_2 + b_1 \\ -5b_3 + b_4 \\ \sim \end{array} \begin{bmatrix} 1 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 18 & 0 \end{bmatrix} \begin{array}{c} b_3 + b_1 \\ \frac{1}{18} b_5 \\ \sim \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & -5 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{array}{c} 5b_4 + b_1 \\ -2b_4 + b_2 \\ 4b_4 + b_3 \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} ; \quad \begin{aligned} x_1 &= 0 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \end{aligned}$$

c.

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 3 & 2 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 3 & 2 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & 2 \end{vmatrix} \quad b_2 \leftrightarrow b_4$$

$$-\det(A) = \begin{vmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & -2 & 3 \\ 2 & 3 & 3 & 2 \end{vmatrix} \quad \begin{matrix} -b_1 + b_3 \\ -2b_1 + b_4 \end{matrix}$$

$$-\det(A) = \begin{vmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -1 & 2 \\ 0 & 1 & 5 & 0 \end{vmatrix} \quad \begin{matrix} b_2 + b_3 \\ -b_2 + b_4 \end{matrix}$$

$$-\det(A) = \begin{vmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 5 & -2 \end{vmatrix} \quad 5b_3 + b_4$$

$$\det(A) = -\begin{vmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 18 \end{vmatrix}$$

$$\det(A) = -(1 \cdot 1 \cdot -1 \cdot 18) = 18$$

d.
$$\left[\begin{array}{cccc|cccc} 1 & 1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 3 & 3 & 2 & 0 & 1 & 0 & 0 \\ 1 & 0 & -2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} b_2 \leftrightarrow b_4 \\ \sim \end{array} \left[\begin{array}{cccc|cccc} 1 & 1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 0 & 1 \\ 1 & 0 & -2 & 3 & 0 & 0 & 1 & 0 \\ 2 & 3 & 3 & 2 & 0 & 1 & 0 & 0 \end{array}\right] \begin{array}{c} -b_1 + b_3 \\ -2b_1 + b_4 \\ \sim \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & 1 & 5 & 0 & -2 & 1 & 0 & 0 \end{array}\right] \begin{array}{c} -b_2 + b_1 \\ b_2 + b_3 \\ -b_2 + b_4 \\ \sim \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & -1 & -1 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 4 & -1 & 0 & 1 & 1 \\ 0 & 0 & 5 & -2 & -2 & 1 & 0 & -1 \end{array}\right] \begin{array}{c} 5b_3 + b_4 \\ -b_3 + b_1 \\ \sim \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -5 & 2 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 4 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 18 & -7 & 1 & 5 & 4 \end{array}\right] \begin{array}{c} -b_3 \\ \frac{1}{18} b_4 \\ \sim \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -5 & 2 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -4 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & \frac{-7}{18} & \frac{1}{18} & \frac{5}{18} & \frac{4}{18} \end{array}\right] \begin{array}{c} 5b_4 + b_1 \\ -2b_4 + b_2 \\ 4b_4 + b_3 \\ \sim \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{18} & \frac{5}{18} & \frac{7}{18} & -\frac{8}{9} \\ 0 & 1 & 0 & 0 & \frac{7}{9} & -\frac{1}{9} & -\frac{5}{9} & \frac{5}{9} \\ 0 & 0 & 1 & 0 & -\frac{5}{9} & \frac{2}{9} & \frac{1}{9} & -\frac{1}{9} \\ 0 & 0 & 0 & 1 & -\frac{7}{18} & \frac{1}{18} & \frac{5}{18} & \frac{2}{9} \end{array}\right] \qquad A^{-1} = \begin{bmatrix} \frac{1}{18} & \frac{5}{18} & \frac{7}{18} & -\frac{8}{9} \\ \frac{7}{9} & -\frac{1}{9} & -\frac{5}{9} & \frac{5}{9} \\ -\frac{5}{9} & \frac{2}{9} & \frac{1}{9} & -\frac{1}{9} \\ -\frac{7}{18} & \frac{1}{18} & \frac{5}{18} & \frac{2}{9} \end{bmatrix}$$

e.
$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \longleftarrow A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 3 & 2 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$$C_A = \begin{bmatrix} 1 & 14 & -10 & -7 \\ 5 & -2 & 4 & 1 \\ 7 & -10 & 2 & 5 \\ -16 & 10 & -2 & 4 \end{bmatrix}$$

5.

$$\operatorname{adj}(A) = C_A^T = \begin{bmatrix} 1 & 5 & 7 & -16 \\ 14 & -2 & -10 & 10 \\ -10 & 4 & 2 & -2 \\ -7 & 1 & 5 & 4 \end{bmatrix}$$

9.

$$A^{-1} = \begin{bmatrix} 1/18 & 5/18 & 7/18 & -8/9 \\ 7/9 & -1/9 & -5/9 & 5/9 \\ -5/9 & 2/9 & 1/9 & -1/9 \\ -7/18 & 1/18 & 5/18 & 2/9 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} -1 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1/18 & 5/18 & 7/18 & -8/9 \\ 7/9 & -1/9 & -5/9 & 5/9 \\ -5/9 & 2/9 & 1/9 & -1/9 \\ -7/18 & 1/18 & 5/18 & 2/9 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} (-1/18 + 15/18 - 14/18 + 0) \\ (-7/9 - 3/9 + 10/9 + 0) \\ (5/9 + 6/9 - 2/9 + 0) \\ (7/18 + 3/18 - 10/18 + 0) \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$