1.
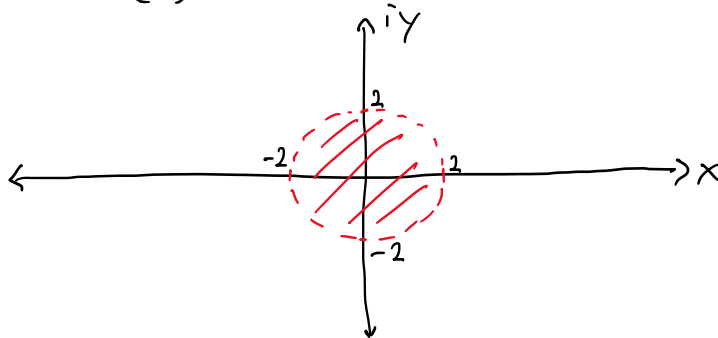
$$f(z) = \frac{5}{2+z}$$

a.

$$f(z) = \frac{5}{2+z} = \frac{5}{2} \cdot \frac{1}{1+\left(\frac{z}{2}\right)} = \frac{5}{2} \cdot \frac{1}{1-\left(-\frac{z}{2}\right)} \qquad k\,z = -\frac{z}{2}$$

$$|k\,z| < 1$$

$$\left|-\frac{z}{2}\right| < 1$$

$$|z| < 2$$

b.

$$f(z) = \frac{5}{2+z} = \frac{5}{2} \cdot \frac{1}{1-\left(-\frac{z}{2}\right)}$$

$$= \frac{5}{2} \sum_{n=0}^{\infty} \left(-\frac{z}{2}\right)^n$$

$$= \frac{5}{2} \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{2^n} = \sum_{n=0}^{\infty} (-1)^n \frac{5\,z^n}{2^{n+1}}$$

c. $z_0 = i$

$$f(z) = \frac{5}{2+(z-z_0)} = \frac{5}{2+(z-i)} = \frac{5}{2-i+z}$$
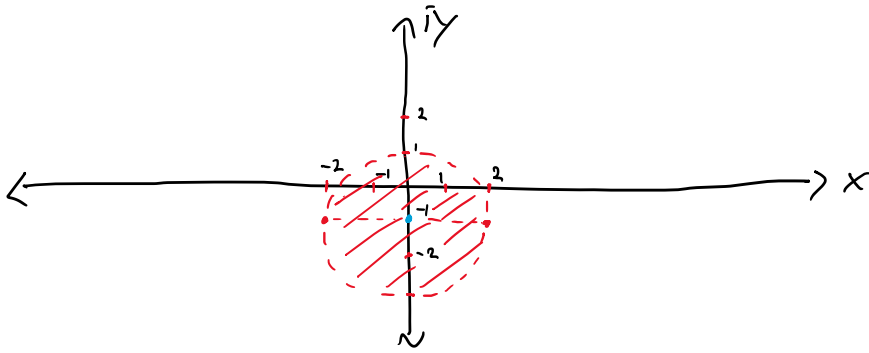
$$= \frac{5}{2-i} \cdot \frac{1}{1+\frac{z}{2-i}}$$

$$= \frac{5}{2-i} \cdot \frac{1}{1-\left(-\frac{z}{2-i}\right)} \qquad k\,z = -\frac{z}{2-i}$$

$$|k\,z| < 1$$

$$\left|-\frac{z}{2-i}\right| < 1$$

$$|z| < 2-i \;\longrightarrow\; |z+i| < 2 \;\longrightarrow\; |z-(-i)| < 2$$

Titik pusat : $-i$, $r = 2$

d.

$$f(z) = \frac{5}{2+(z-i)} = \frac{5}{2-i} \cdot \frac{1}{1-\left(-\frac{z}{2-i}\right)}$$

$$= \frac{5}{2-i} \cdot \sum_{n=0}^{\infty} \left(-\frac{z}{2-i}\right)^n$$

$$= \frac{5}{2-i} \sum_{n=0}^{\infty} (-1)^n \frac{z^n}{(2-i)^n}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{5z^n}{(2-i)^{n+1}}$$