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1. $x[n] = \{1, 0, 1\}, \quad 0 \le n \le 2$

$$W_N^k = e^{-j2\pi k/N}$$

$$W_N = \begin{bmatrix} W_3^0 & W_3^0 & W_3^0 \\ W_3^0 & W_3^1 & W_3^2 \\ W_3^0 & W_3^2 & W_3^4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^1 & W_3^2 \\ 1 & W_3^2 & W_3^4 \end{bmatrix}$$

$$W_3^1 = e^{-j2\pi \cdot 1/3} = \cos\frac{2\pi}{3} - j\sin\frac{2\pi}{3} = -\frac{1}{2} - j\frac{1}{2}\sqrt{3}$$

$$W_3^2 = e^{-j2\pi \cdot 2/3} = \cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3} = -\frac{1}{2} + j\frac{1}{2}\sqrt{3}$$

$$W_3^4 = e^{-j2\pi \cdot 4/3} = \cos\frac{8\pi}{3} - j\sin\frac{8\pi}{3} = -\frac{1}{2} - j\frac{1}{2}\sqrt{3}$$

$$W_N = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \left(-\frac{1}{2} - j\frac{1}{2}\sqrt{3}\right) & \left(-\frac{1}{2} + j\frac{1}{2}\sqrt{3}\right) \\ 1 & \left(-\frac{1}{2} + j\frac{1}{2}\sqrt{3}\right) & \left(-\frac{1}{2} - j\frac{1}{2}\sqrt{3}\right) \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$X_3 = W_N \cdot x_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \left(-\frac{1}{2} - j\frac{1}{2}\sqrt{3}\right) & \left(-\frac{1}{2} + j\frac{1}{2}\sqrt{3}\right) \\ 1 & \left(-\frac{1}{2} + j\frac{1}{2}\sqrt{3}\right) & \left(-\frac{1}{2} - j\frac{1}{2}\sqrt{3}\right) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 2 \\ \frac{1}{2} + j\frac{1}{2}\sqrt{3} \\ \frac{1}{2} - j\frac{1}{2}\sqrt{3} \end{bmatrix}$$

2. $x[n] = \{1, 1, 0, 1\}, \quad 0 \leq n \leq 3$

$$W_N = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$X_4 = W_N . x_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} . \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

3. $x[n] = \{1, 1, 1, 1\}, \quad 0 \leq n \leq 7$

$x[n] = \{1, 1, 1, 1, 0, 0, 0, 0\}, \quad 0 \leq n \leq 7$

$f[n] = x[2n] = \{x[0], x[2], x[4], x[6]\} = \{1, 1, 0, 0\}$

$g[n] = x[2n+1] = \{x[1], x[3], x[5], x[7]\} = \{1, 1, 0, 0\}$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\begin{bmatrix} G[0] \\ G[1] \\ G[2] \\ G[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} . \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix} = G_4$$

karena $f[n] = g[n]$, maka $F_4 = G_4 \rightarrow \begin{bmatrix} F[0] \\ F[1] \\ F[2] \\ F[3] \end{bmatrix} = \begin{bmatrix} G[0] \\ G[1] \\ G[2] \\ G[3] \end{bmatrix} = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$

$$W_8^0 = e^{-j2\pi \cdot 0/8} = \cos\frac{0 \cdot \pi}{8} - j\sin\frac{0\pi}{8} = 0$$

$$W_8^1 = e^{-j2\pi \cdot 1/8} = \cos\frac{2\pi}{8} - j\sin\frac{2\pi}{8} = \frac{1}{2}\sqrt{2} - j\frac{1}{2}\sqrt{2}$$

$$W_8^2 = e^{-j2\pi \cdot 2/8} = \cos\frac{4\pi}{8} - j\sin\frac{4\pi}{8} = j$$

$$W_8^3 = e^{-j2\pi \cdot 3/8} = \cos\frac{6\pi}{8} - j\sin\frac{6\pi}{8} = -\frac{1}{2}\sqrt{2} - j\frac{1}{2}\sqrt{2}$$

$$X[0] = F[0] - W_8^0 G[0] = 2 - 0.2 = 2$$

$$X[1] = F[1] - W_8^1 . G[1] = (1-j) - \left(\frac{1}{2}\sqrt{2} - j\frac{1}{2}\sqrt{2}\right)(1-j) = 1 + j(-1+\sqrt{2})$$

$$X[2] = F[2] - W_8^2 . G[2] = 0 - 0.j = 0$$

$$X[3] = F[3] - W_8^3 G[3] = (1+j) - \left(-\frac{1}{2}\sqrt{2} - j\frac{1}{2}\sqrt{2}\right)(1+j) = 1 + j(1+\sqrt{2})$$

$$X[4] = G[0] - W_8^0 . F[0] = 2 - 0.2 = 2$$

$$X[5] = G[1] - W_8^1 F[1] = (1-j) - \left(\frac{1}{2}\sqrt{2} - j\frac{1}{2}\sqrt{2}\right)(1-j) = 1 + j(-1+\sqrt{2})$$

$$X[6] = G[2] - W_8^2 F[2] = 0 - 0.j = 0$$

$$X[7] = G[3] - W_8^3 F[3] = (1+j) - \left(-\frac{1}{2}\sqrt{2} - j\frac{1}{2}\sqrt{2}\right)(1+j) = 1 + j(1+\sqrt{2})$$

$$X_8 = \begin{bmatrix} 2 \\ 1 + j(-1+\sqrt{2}) \\ 0 \\ 1 + j(1+\sqrt{2}) \\ 2 \\ 1 + j(-1+\sqrt{2}) \\ 0 \\ 1 + j(1+\sqrt{2}) \end{bmatrix}$$

4. $x[n] = \{1, 1, 0, 1, 1\}, \quad 0 \le n \le 4$

$h[n] = \{1, 1\}, \quad 0 \le n \le 1$

$h[n] = \{1, 1, 0, 0, 0\}, \quad 0 \le n \le 4$

$Y[n] = x[n] \otimes h[n]$

h \ x	1	1	0	1	1
1	1	1	0	1	1
1	1	1	0	1	1
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$Y[0] = 1 + 1 + 0 + 0 + 0 = 2$

$Y[1] = 1 + 0 + 0 + 0 + 0 = 1$

$Y[2] = 0 + 1 + 0 + 0 + 0 = 1$

$Y[3] = 1 + 1 + 0 + 0 + 0 = 2$

$Y[4] = 1 + 1 + 0 + 0 + 0 = 2$

$Y[n] = \{2, 1, 1, 2, 2\}, \quad 0 \le n \le 5$

5. $x[n] = \{1, 2, 3, 4, 5\}, \quad 0 \le n \le 4$

$h[n] = \{2, 2\}, \quad 0 \le n \le 1$

$h[n] = \{2, 2, 0, 0, 0\}, \quad 0 \le n \le 4$

h \ x	1	2	3	4	5
2	2	4	6	8	10
2	2	4	6	8	10
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$Y[0] = 2 + 4 + 0 + 0 + 0 = 6$

$Y[1] = 4 + 6 + 0 + 0 + 0 = 10$

$Y[2] = 6 + 8 + 0 + 0 + 0 = 14$

$Y[3] = 8 + 10 + 0 + 0 + 0 = 18$

$Y[4] = 10 + 2 + 0 + 0 + 0 = 12$

$Y[n] = \{6, 10, 14, 18, 12\}, \quad 0 \le n \le 4$

6. $x[n] = \{6, 5, 4, 3, 2, 1\}, \quad 0 \le n \le 5$

$h[n] = \{1, 0, 1\}, \quad 0 \le n \le 2$

h \ x	6	5	4	3	2	1
1	6	5	4	3	2	1
0	0	0	0	0	0	0
1	6	5	4	3	2	1

$Y[0] = 6 + 0 + 4 = 10$

$Y[1] = 5 + 0 + 3 = 8$

$Y[2] = 4 + 0 + 2 = 6$

$Y[3] = 3 + 0 + 1 = 4$

$Y[4] = 2 + 0 + 6 = 8$

$Y[5] = 1 + 0 + 5 = 6$

$Y[n] = \{10, 8, 6, 4, 8, 6\}, \quad 0 \le n \le 5$