

Assignment No.: 01

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Topic: FDS (PROBABILITY)

Course: MSc DSAT PART 1 2020-21 (SEM 1)

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- (1) Alice has 2 kids and one of them is a girl. What is the probability that the other child is also a girl?

You can assume that there are an equal number of male and females in the world.

⇒ As per given assumption there are an equal number of male & females in world.

∴ Universal set for having 2 kids

$$U = \{ (B, B), (B, G), (G, B), (G, G) \}$$

Where

B stands for Boy

G stands for Girl

As per given information one of them is a girl

Hence sample space is

$$S = \{ (G, B), (B, G), (G, G) \}$$

Let's consider, Event A is, the other child is also girl, i.e. Both the kids are girls.

$$E_A = \{ (G, G) \}$$

$$\therefore n(S) = 3 \quad \& \quad n(E_A) = 1$$

∴ The probability of the other child is also a girl is $= \frac{1}{3} = 0.333$

(2) A fair six-sided die is rolled twice. What is the probability of getting 2 on first roll and not getting 4 on the second roll?

⇒ Let's,

E_A (Event A) : Getting 2 on first roll.

E_B (Event B) : Getting 4 on the second roll.

Universal set

$$U = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

$$n(U) = 36$$

Sample set for Event A,

$$E_A = \{ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \}$$

$$n(E_A) = 6$$

Sample set for Event B,

$$E_B = \{ (1,1), (1,2), (1,3), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,5), (6,6) \}$$

$$n(E_B) = 30$$

The given Event A & Event B are independent.

$$\therefore P(E_A) = 6/36 = 1/6$$

$$\therefore P(E_B) = 30/36 = 5/6$$

∴ Given 40 events are independent.

$$\therefore P(\text{getting 2 on first roll \& not 4 on second roll})$$

$$= \frac{1}{6} \times \frac{5}{6}$$

$$= \frac{5}{36}$$

$$P = 0.139$$

(3) Amita randomly picks 4 cards from a deck of 52-cards and places them back into the deck (Any set of 4 cards is equally likely). Then Bobita randomly choose 8 cards out of the same deck (Any set of 8 cards is equally likely). Assume that the choice of 4 cards by Amita and the choice of 8 cards by Bobita are independent. What is the probability that all 4 cards chosen by Amita are in the set of 8 cards chosen by Bobita?

⇒

Total possible combination for selecting 4 cards by Amita = ${}^{52}C_4$

Total possible combination for selecting 8 cards by Bobita = ${}^{52}C_8$

∴ Total possible combination for selecting 4 cards by Amita & 8 cards by Bobita is

$$n(s) = {}^{52}C_4 \times {}^{52}C_8$$

∴ As per given event, all 4 cards chosen by Bobita, Since the 4 cards selected by Amita are common, so all possible combination

For the event is ${}^{48}C_4$

∴ For given Event number of all possible combination are $n(E) = {}^{52}C_4 \times {}^{48}C_4$

$$\begin{aligned} \therefore P(E) &= \frac{n(E)}{n(S)} = \frac{({}^{52}C_4 \times {}^{48}C_4)}{({}^{52}C_8 \times {}^{48}C_8)} \\ &= \frac{48!}{4! \times 44!} \times \frac{8! \times 44!}{52 \times 51 \times 50 \times 49 \times 48!} \\ &= \frac{1680}{6497400} \end{aligned}$$

$$\therefore \boxed{P(E) = 0.000259}$$

∴ Probability that all 4 cards chosen by Amita are in the set of 8 cards chosen by Babita is 0.000259

44) Cross-fertilizing a red and a white flowers produces red flowers 25% of the time. Now we cross-fertilize five pairs of red and white flowers and produce five offspring? What is the probability that there are no red flower plants in the five offspring?

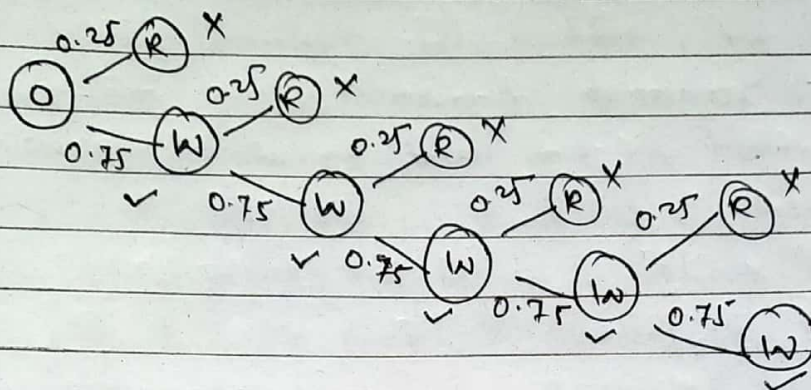
⇒ Let's consider,

Event A (E_A) = cross-fertilizing produce red flowers

$$P(E_A) = 0.25$$

Event B (E_B) = Cross-fertilizing produce white flowers

$$P(E_B) = 0.75$$



\therefore Probability of no red flower plant in the five offspring is $= 0.75 \times 0.75 \times 0.75 \times 0.75 \times 0.75$

$$P(R=0) = 0.237$$

By Binomial Distribution

$$p = 0.25$$

$$n = 5$$

$\therefore P(R=0)$ = No red flower plant in the five offspring

$$= \frac{n!}{r!(n-r)!} p^r q^{n-r}$$

$$= \frac{5!}{0!(5)!} \times (0.25)^0 (0.75)^5$$

$$= (0.75)^5$$

$$= 0.237$$

$$\therefore P(R=0) = 0.237$$

\therefore Probability of no red flower plant in the five offspring is $= 0.237$.

- 15) Ahmed is playing matches the 2 as a lottery game where he must pick 2 numbers from 0 and 9 followed by an English alphabet (from 26-letters). He may choose the same number both times.

If his ticket matches the 2 numbers and 1 letter drawn in order, he wins the grand prize and receives \$10405. If just his letter matches but one or both of the numbers do not match, he wins \$100. Under any other circumstance, he wins nothing. The game costs him \$5 to play. Suppose he has chosen 04R to play. What is the expected net profit from playing this ticket?

⇒

Let's consider

$$P(\text{Number chose from 0-9}) = \frac{1}{10}$$

$$P(\text{Letter chose}) = \frac{1}{26}$$

$$P(\text{grand prize}) = \left(\frac{1}{10}\right) * \left(\frac{1}{10}\right) * \left(\frac{1}{26}\right)$$

$$\text{The game cost} = \$5$$

∴ Need to match both Numbers as well as Letter, to win grand prize.

$P(\text{small prize}) = \text{just letter matches but one or both of the number do not match}$

$$\therefore P(\text{small prize}) = \left(\frac{1}{26}\right) - \left(\frac{1}{2600}\right)$$

(∴ Excluding the case where where both the numbers also match [grand prize winning condition])

∴ Probability of losing the prize is

$$\begin{aligned} P(\text{Losing Prize}) &= 1 - P(\text{Winning}) \\ &= 1 - \left(\frac{1}{26}\right) - \frac{1}{26 \times 10 \times 10} \end{aligned}$$

The game cost = \$5

Grand Prize = \$10405

Small prize = \$100

∴ Expected Net profit

$$= P(\text{grand prize}) * (\text{grand prize} - \text{game cost}) \\ + P(\text{small prize}) * (\text{small prize} - \text{game cost}) \\ - P(\text{Losing}) * (\text{game cost})$$

$$= \left(\frac{1}{2600} \right) * (10405 - 5) + \left(\frac{1}{26} - \frac{1}{2600} \right) \\ (100 - 5) - \left(1 - \frac{1}{26} - \frac{1}{2600} \right) 5$$

$$= \boxed{2.81}$$

∴ Expected net profit = \$2.81