HOCHSCHULE RHEINMAIN

PHYSICS LAB 3

Experiment P3-4
Surface Tension of Strain Gauges

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1 Introduction

1.1 Terms and Definitions

Buoyancy force

A body submerged in a fluid experiences a force anti-parallel to the gravitational force and proportional to the mass of the fluid displaced.

Surface Tension

In a simplified manner attracting intermolecular interactions inside a fluid can be seen as isotrope. Thus, they cancel each other out. The closer a particle gets to the interfacing layer of another medium the fewer neighbors there are to exert attracting forces from that direction. The attracting force a particle along the interfacing layer experiences nets to a force normal to the interface layer and directed inside the fluid.

Surfactant

A material or compound able to alter or even nullify the intermolecular forces inside a fluid (typically a liquid).

DU NOÜY-Method to Measure the Surface Tension of a Liquid

A method to measure the surface tension of a liquid by measuring the force withdrawing a submerged ring from that liquid [4].

Other Ways to Measure the Surface Tension of a Liquid

ANDREAS-HAUSER-TUCKER method or Pendant-drop method: the shape of a droplet is solely dependent of the gravitational force and the surface energy. Knowing other parameters of the liquid in question the surface tension can be acquired by geometrically evaluating photographs of hanging droplets [1].

Capillary-tubes method: one opening of a tube of sufficiently small inner diameter is brought in contact with the specimen. The liquid will - to some extent - travel along the tube depending on the adhesive forces between the liquid and the tube wall, the gravitational force and the cohesive forces along the liquids surface i.e. the surface tension.

Parallel-plates method: similar to the capillary-tubes method, the surface tension can be determined by evaluating the rise of the liquid between two parallel plates brought in contact with the liquid.

Sessile drops and bubbles: in this method, the shape of a drop or bubble resting on a flat and horizontal surface is examined.

JAEGERS method: a tube is vertically submerged in a liquid leaving the lower and upper end open. Now, if gas pressure is applied to the top of the liquid column inside the tube until a gas bubble escapes the lower end the surface tension can be derived from the gas pressure at that moment.

Oscillating drops and bubbles: much like a pendulum, a drop or bubble is displaced from its shape of equilibrium. Released and left alone, it will oscillate about the shape of equilibrium. Measuring the frequency the surface tension can be derived [6].

Working Principle of a Strain Gauge

Stress induced change of the geometry of a strain gauge translates to a change of its resistance. The impossibility to measure electrical resistance directly it typically gets transduced to a voltage signal in order to be further processed.

Working Principle of an Instrumentation Amplifier

A instrumentation amplifier is a advanced version of the differential amplifier. A slight variation in the topology of the differential amplifier allows to adjust the gain by change of just one resistor. This is beneficial as it is likely to introduce distortion changing two resistors while maintaining balance [8, 9].

Resolution of an Analog-Digital Converter

An analog-to-digital converter (hereafter referred to as ADC) converts a continuous to a discrete, bitwise signal. The number of fractions the ADC is able to divide a given signal to gives the resolution of that ADC.

1.2 Preparation

1.2.1 Force-Time-Curve

According to fig. 1.1 the force-time-diagram is subdivided into the following 8 parts:

1. The ring is located above the interface. No force is caused.
2. The ring touches the interface, creating a small positive adhesive force between the ring and the surface.
3. The ring is about to penetrate the surface while the surface tension creates a small negative force.
4. The ring is now immersed. Its wires produce a small positive force.
5. When diving upwards, the force gradually increases.
6. The ring breaks through the interface and induces an ever-increasing force.
7. The force reaches its maximum.
8. The force decreases a bit with the raising until the lamella collapses.

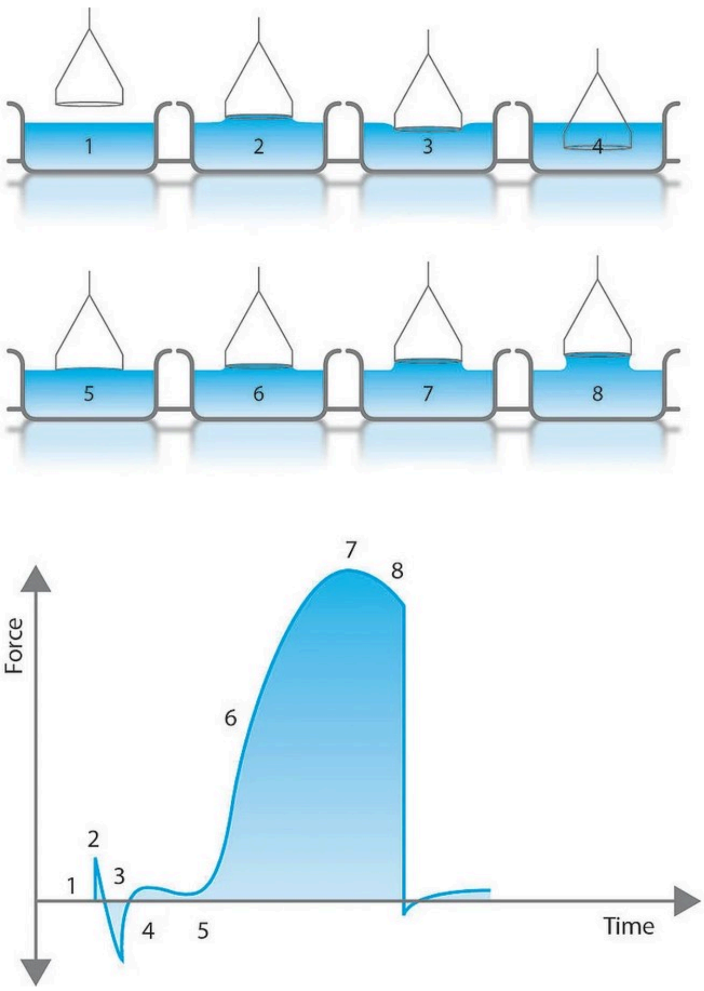


Figure 1.1: Schematic sequence and curve of the DU NOÜY method [10].

If the diagram is compared with that in fig. B.1, some differences are noted. For example, there are small periodic vibrations that probably have to do with the fluctuation of the ring and other external influences. It is particularly noticeable that after penetrating the surface, the force does not rise again into the positive, but remains in the negative and only returns into the positive from step 6. Otherwise, from a qualitative point of view, everything is similar to that in fig. 1.1, except that after the lamella has collapsed, the force oscillates around the zero position.

1.2.2 Surface Tension Formula

The change of area ΔA corresponds to the sum of inner and outer radius - r_i and r_o respectively - of the ring multiplied by the change in height Δx:

$$\Delta A = 2\pi \cdot (r_i + r_o) \cdot \Delta x \tag{1.1}$$

Since the diameter $D = 2\bar{r}$ of the ring is much bigger than the thickness $t = r_o - r_i$, eq. (1.1) can be simplified to

$$\begin{gathered} 2\bar{r} \gg r_o - r_i \quad \Rightarrow \quad r_i \approx r_o \approx \bar{r} \\ \Leftrightarrow \\ \Delta A = 4\pi \cdot \bar{r} \cdot \Delta x \end{gathered} \tag{1.2}$$

Considering, that the amount of energy ΔE is a result of the assumed constant force F_0 multiplied by the change in height, it ensues:

$$\Delta E = F_0 \cdot \Delta x \tag{1.3}$$

Since the surface tension σ is defined as the energy required to increase the surface divided by the change in surface area

$$\sigma = \frac{\Delta E}{\Delta A} \tag{1.4}$$

and eq. (1.2) as well as eq. (1.3) are inserted into eq. (1.4), it follows:

$$\begin{aligned} \sigma &= \frac{F_0 \cdot \Delta x}{4\pi \cdot \bar{r} \cdot \Delta x} \\ \sigma &= \frac{F_0}{4\pi \cdot \bar{r}} \end{aligned} \tag{1.5}$$

1.2.3 Gravitational Force of the Du Noüy-Ring in Water and Air

Air:

$$\begin{aligned} F_{Ring,Air} &= (m_{Air} - m_{Ring}) \cdot g \\ &= (p_{Air} - p_{Al}) V_{Ring} \cdot g \end{aligned}$$

Water:

$$\begin{aligned} F_{Ring,H_2O} &= (m_{H_2O} - m_{Ring}) \cdot g \\ &= (p_{H_2O} - p_{Al})\, V_{Ring} \cdot g \end{aligned} \tag{1.6}$$

where p_{H_2O} is the specific mass of water with $10^3\,\frac{\mathrm{kg}}{\mathrm{m}^3}$, p_{Al} that of aluminum with $2.7 \cdot 10^3\,\frac{\mathrm{g}}{\mathrm{cm}^3}$ and the specific mass of air at 20 °C and normal athmospheric pressure with $p_{Air} = 1.2\,\frac{\mathrm{kg}}{\mathrm{m}^3}$ [5].

1.2.4 Deriving an Equation for the Bridge Voltage

$$U_{Br} = U_2 - U_4 = U_3 - U_1 \quad \text{with} \quad U_{Br} = 0 \qquad \text{for} \qquad \frac{R_1}{R_2} = \frac{R_3}{R_4} \tag{1.7}$$

$$U_{Br} = U_2 - U_4 = U_0 \cdot \left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) \tag{1.8}$$

Defining

$$R_2 = R_3 = R + \Delta R \tag{1.9}$$

$$R_1 = R_4 = R - \Delta R \tag{1.10}$$

leads to

$$\begin{aligned} U_{Br} &= U_0 \cdot \left(\frac{R + \Delta R}{R - \Delta R + R + \Delta R} - \frac{R - \Delta R}{R + \Delta R + R - \Delta R} \right) \\ &= U_0 \cdot \left(\frac{R + \Delta R}{2R} + \frac{-R + \Delta R}{2R} \right) \\ &= U_0 \cdot \frac{\Delta R}{R} \end{aligned} \tag{1.11}$$

thus, the bridge voltage is linearly proportional to the factor $\frac{\Delta R}{R}$.
Now, with the knowledge that the resistance of a body relates with its geometry by $R = \varrho \cdot \frac{l}{b \cdot d}$ and the change of its dimensions under stress follow

$$\frac{\Delta b}{b} = -\nu \frac{\Delta l}{l}, \quad \frac{\Delta d}{d} = -\nu \frac{\Delta l}{l} \tag{1.12}$$

the bridge voltage can be expressed as

$$U_{Br} = U_0 \cdot \left(\frac{\Delta \varrho}{\varrho} + \frac{\Delta l}{l} + \nu \cdot \frac{2\Delta l}{l} \right) \tag{1.13}$$

1.2.5 Schematic Diagram of a Full Bridge

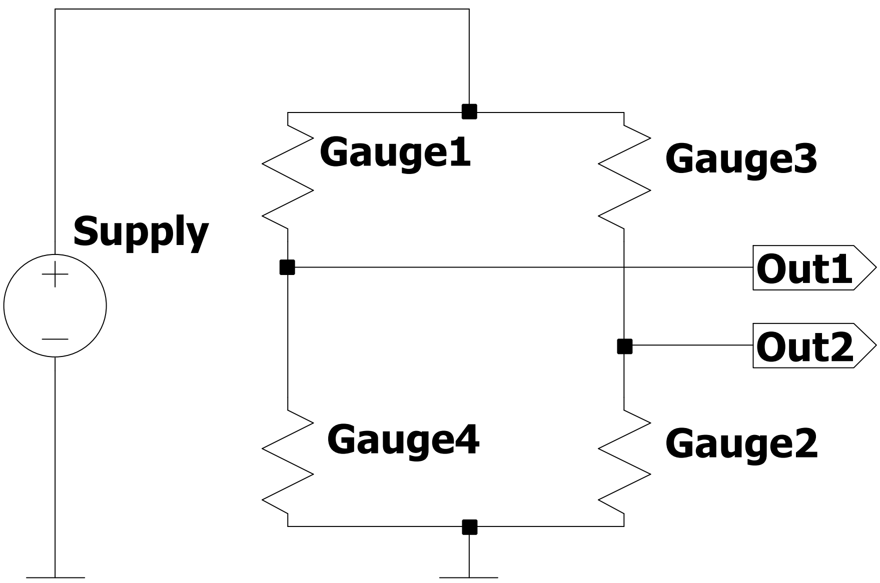


Figure 1.2: Strain gauge in full bridge configuration.

1.2.6 Characteristics of HX711

The ADC interfacing the strain gauge with the MCU is an HX711 by AVIA SEMICONDUCTOR. Referring to its datasheet [2] the relevant characteristics are

Resolution:	$n = 24\,\mathrm{bit}$	(1.14)
Full-scale range:	$U_{FSR} = \pm 20\,\mathrm{mV}$	(1.15)
Minimum voltage change:	$U_{LSB} = U_{FSR} \cdot 2^{-24} \approx 1.2\,\mathrm{nV}$	(1.16)

1.2.7 Impact of Statistical Effects on the effective Resolution

The resolution n of an ADC as per specification implies an ideal, i.e. free-of-noise DC signal. The effective resolution – or ENOB (**E**ffective **N**umber **O**f **B**its) – can be calculated by eq. (1.17)

$$ENOB = n - \log_2 \varsigma \tag{1.17}$$

where ς_{RMS} is the RMS (**R**oot **M**ean **S**quare) or standard deviation of the signals noise.

1.2.8 Step Response of an Exponential Filter

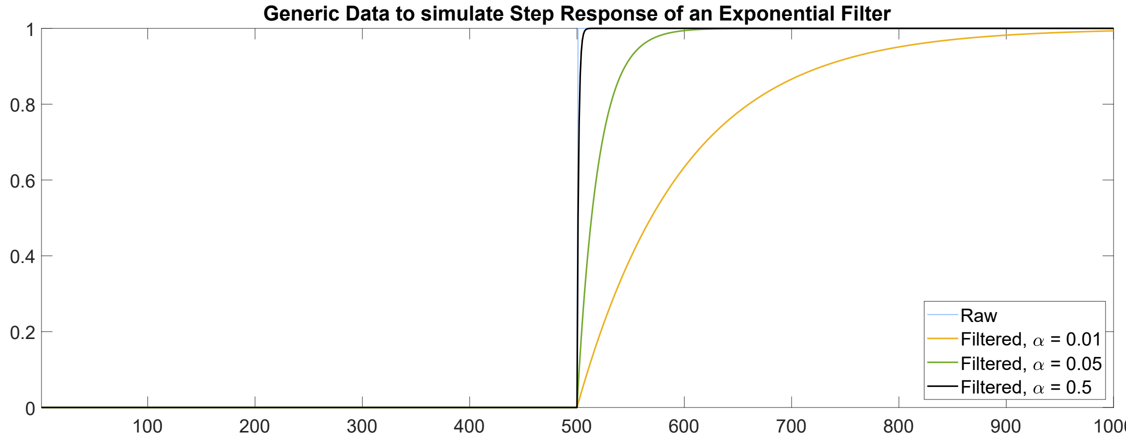
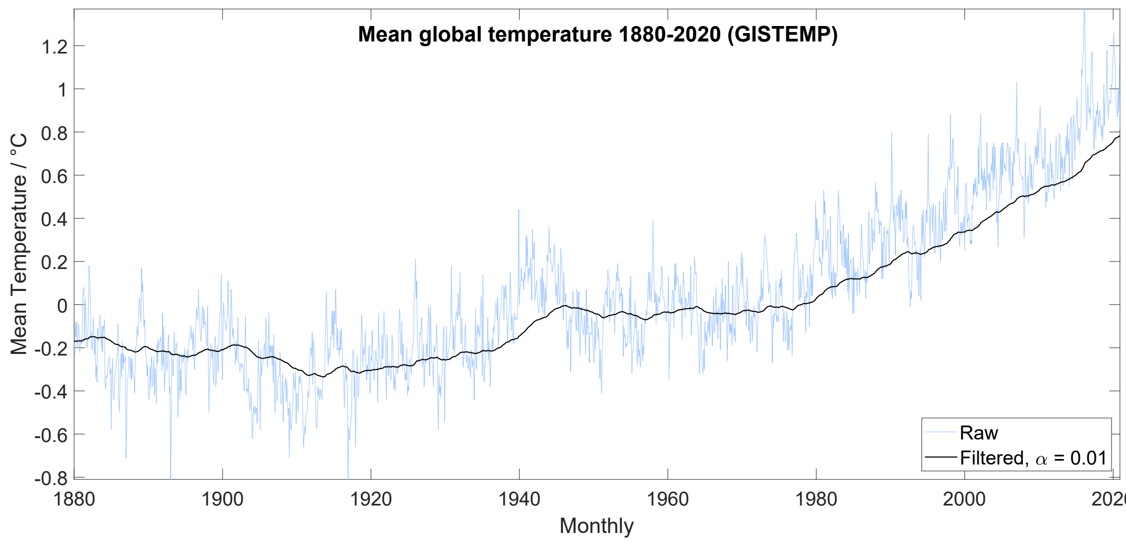


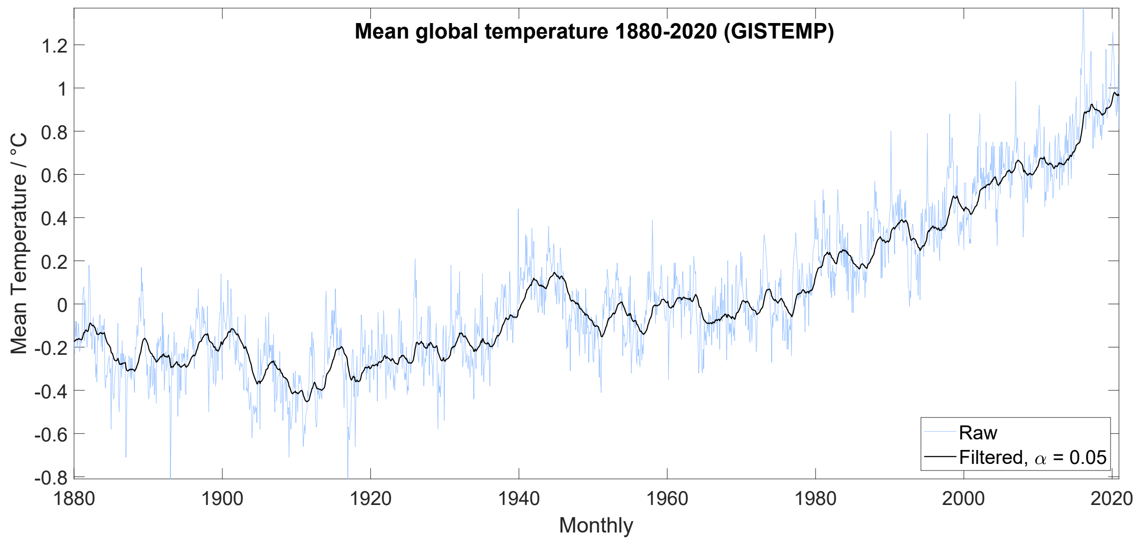
Figure 1.3: An exponential filter algorithm applied to a square step signal (light blue) and its responses for $\alpha = 0.5$ (black), $\alpha = 0.05$ (green) and $\alpha = 0.01$ (orange).

1.2.9 Simulation of the Filter Algorithm

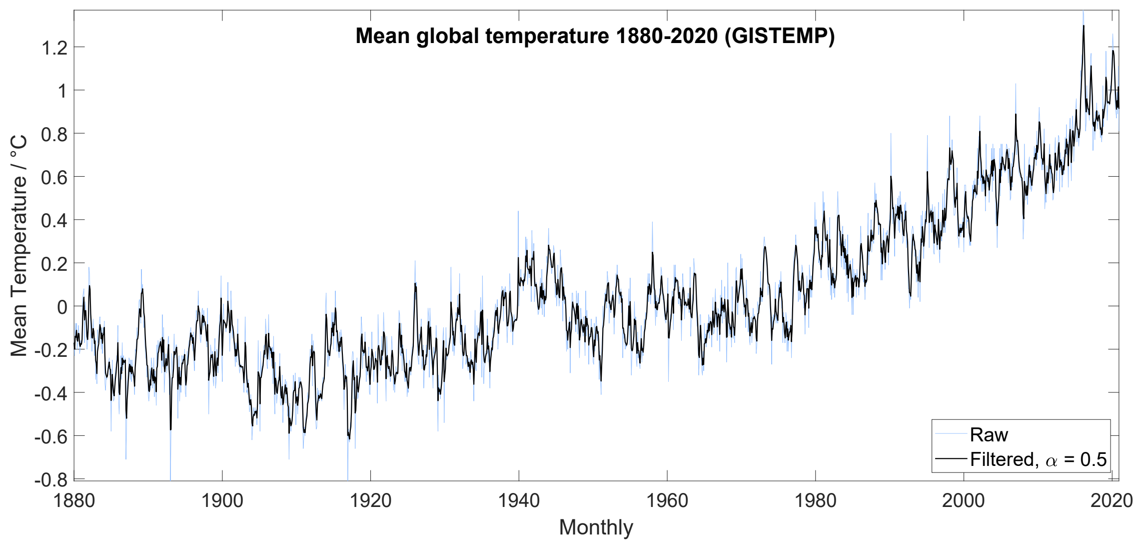
Figure 1.4 shows the effect of an exponential filter algorithm applied to a set of data to filter out statistical noise. The algorithm takes a smoothing factor $\alpha \in 0 \leq \alpha \leq 1$ as an input and outputs a smoothed representation of the raw data.



(a) Exponential filter – smoothing at $\alpha = 0.01$.



(b) Exponential filter – smoothing at $\alpha = 0.05$.



(c) Exponential filter – smoothing at $\alpha = 0.5$.

Figure 1.4: Examples of an exponential filter algorithm applied to a noisy set of data. To emphasize its effect on the output, three different values for the smoothing factor α are shown. The raw data is shown in light blue whereas black represents the filtered data. Raw data is provided by Goddard Institute for Space Studies [3]

As seen in fig. 1.4a, too aggressive filtering of the data could bias the trending of the output with respect to the raw data. Too less of smoothing on the other hand might still output too much noise (cf. fig. 1.4c). Depending on the data in question, the smoothing factor needs to be adjusted as needed.

1.2.10 Expected Forces

Relocating eq. (1.5) by the force F_0 and knowing the parameters $\bar{r}$ and σ allows for an estimation of that force.

With a radius $\bar{r} = 0.03\,\mathrm{m}$ and the surface tension of water at room temperature $\sigma_{H_2O,25} \approx 72\,\frac{\mathrm{mN}}{\mathrm{m}}$ [12] the force required to tear the lamella calculates to

$$\begin{aligned} F_{H_2O,25} &= \sigma_{H_2O,25} \cdot 4\pi \cdot \bar{r} \\ &= 72\,\frac{\mathrm{mN}}{\mathrm{m}} \cdot 4\pi \cdot 0.03\,\mathrm{m} \\ &\approx 27.1\,\mathrm{mN} \end{aligned} \tag{1.18}$$

2 Set-Up of Experiment

To perform the experiment the items shown in fig. 2.1 are needed.

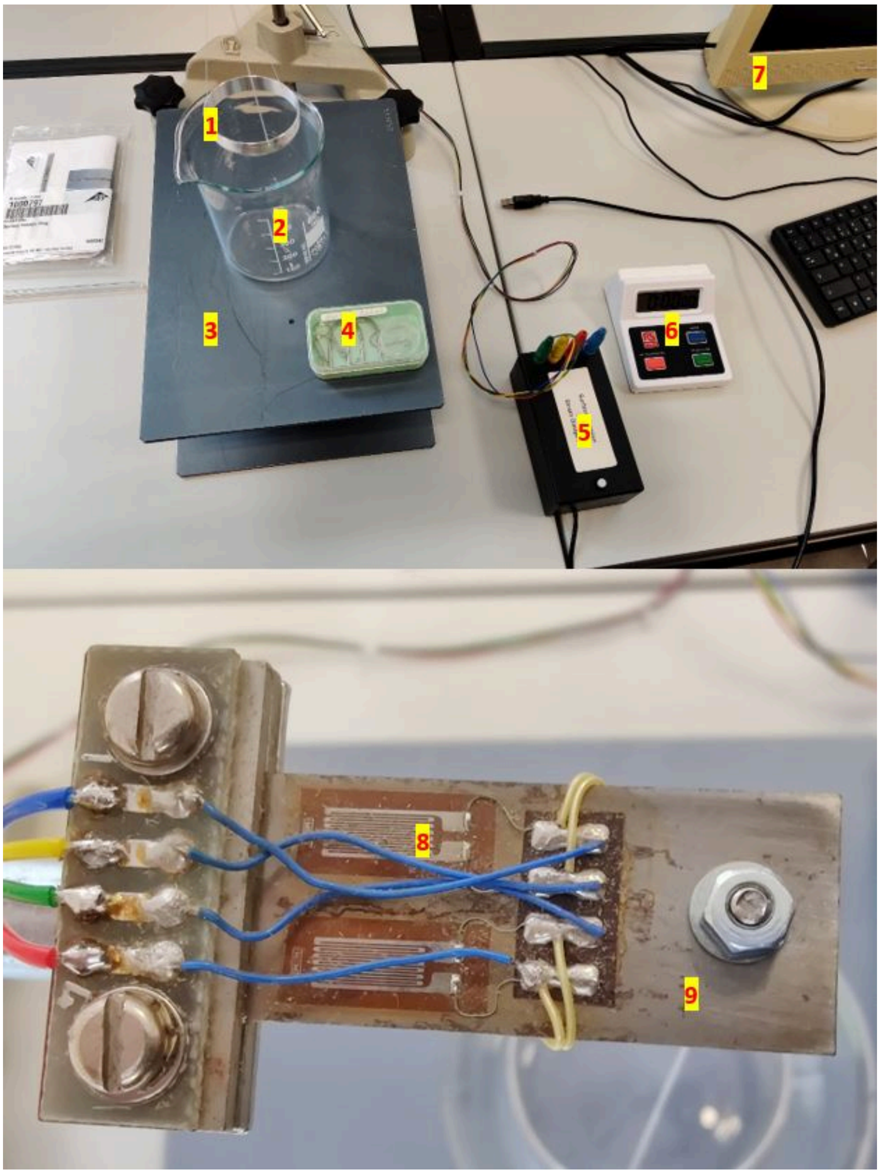


Figure 2.1: Components needed for the experiment. 1: Du Noüy-ring, 2: Glass beaker, 3: Lifting platform, 4: Calibration weights, 5: A/D converter box with offset button, 6: stopwatch, 7: Computer running RealTerm, 8: Strain gauges, 9: Cantilever.

Using a desktop computer a serial connection via USB is established. A serial monitor - RealTerm - is used to log the inbound stream send by the MCU. The relevant settings are listed below:

- 9600 Baud
- On the Display tab, *ASCII* and *New Line mode* are checked, *Direct capture* remains un-checked.
- COM-Port as assigned by the OS.

The data is now continuously sent by the microcontroller. The data is displayed on the screen. A text file is created in which the data is written and saved. Three columns are displayed. The first column contains the time in seconds, the second the raw ADC data and the third column the filtered ADC data.

3 Execution

3.1 Calibration of the Force Sensor

First, the measurement setup must be calibrated. For this purpose, three calibration weights are selected from a choice of weights. These are first weighed on the balance provided in the laboratory. They are then hung on the cantilever with the strain gauges in alternating arrangements, i.e. alone and in various combinations. The DU NOÜY-ring, which is required for later measurements, is removed in the meantime. Care is taken to ensure that the weights do not swing. For each arrangement, the offset button is first pressed and the data recorded for about 30 seconds and stored in a text file.

3.2 Determining Resolution and Acquiring Statistics

The variance of the conversion result is analyzed by repeating the procedure from section 3.1, but without additional weights. The values are recorded and stored over a period of 100 s.

3.3 Measurement of Surface Tension

To avoid unnecessary errors when measuring the surface tension due to impurities, the DU NOÜY-ring is first washed with tap water and subsequently rinsed with distilled water. A paper towel is used for drying and any skin contact is avoided not to re-contaminate the surface of the ring. After the cleaning, it is carefully hung in the hook on the boom.

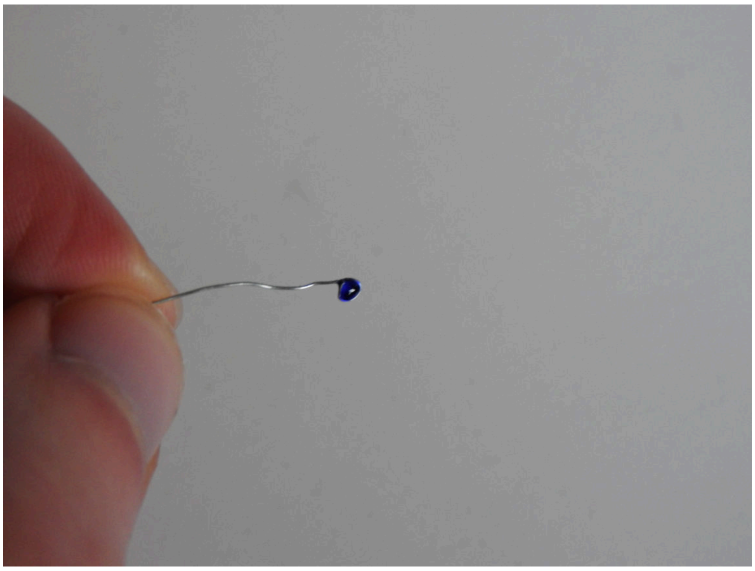


Figure 3.1: Method of dosing detergent. The tip of a thin wire is dipped in detergent to transfer a small amount to the distilled water containing beaker.

A glass beaker is filled with distilled water to the point where the ring can be completely immersed. The beaker is placed on a height-adjustable platform just below the ring. The ring is now aligned as parallel as possible to the water surface. To stabilize the alignment of the ring, a piece of thin wire is also wrapped around the holding strings. Now the platform is turned upwards until the ring almost touches the water. The offset button is pressed and the recording of the values is started with REALTERM. Now the platform is slowly moved further up until the ring is completely covered with water. Then the platform is lowered again until the ring is no longer in contact with the water and the lamella is torn. The data is stored. This procedure is repeated until 10 measurements have been recorded. After the measurement with pure water has been completed, the behavior is examined when detergent is dissolved in the water. To do this, the tip of a thin wire is bend to a little circle which can hold a small amount of detergent after dipping the tip into it (fig. 3.1). After that the tip is immersed into the water and stirred until the detergent is fully solved. Again, 10 series of measurements are recorded according to the same scheme. Then a second time detergent is stirred in with the tip of the wire and again 10 measurements are recorded. Finally, the water is tipped away and the beaker is filled with 2-isopropanol. Again, 10 measurements are recorded according to the same scheme. Finally, the diameter of the ring is measured with a caliper.

4 Evaluation

4.1 Calibration of the Force Sensor

Because of a measurement misfortune for the calibration, the common way for determining the calibration factor is inoperable. Therefore, another method is improvised. Originally, the calibration factor c was supposed to be determined by computing the slope of the correlation between the ADC values and the pulling force on the strain gauge. Thus, the weight of the different masses would have been converted into forces by

$$c = \frac{\Delta n}{\Delta F} \qquad \left[\frac{1}{\mathrm{mN}}\right] \tag{4.1}$$

Since this measurement failed, the literature value of the surface tension of distilled water was used in order to be able to carry out a backward calculation. For that, eq. (1.5) is transformed into the force F_0:

$$F_0 = \sigma \cdot 4\pi \cdot \bar{r}$$

With

$$F_0 = F_{max} = \frac{n_{max}}{c} \tag{4.2}$$

it follows for distilled water:

$$\frac{n_{max}}{c} = \sigma_{H_2O} \cdot 4\pi \cdot \bar{r}$$
$$\Leftrightarrow$$
$$c = \frac{n_{max}}{\sigma_{H_2O} \cdot 4\pi \cdot \bar{r}} \tag{4.3}$$

with $\sigma_{H_2O} = 72.8\,\frac{\mathrm{mN}}{\mathrm{m}}$ at $20\,°\mathrm{C}$ [5] and $\bar{r} = 0.03\,\mathrm{m} \pm 0.00005\,\mathrm{m}$ (approximated with the inner radius $r_i = 0.02923\,\mathrm{m}$ and outer radius $r_o = 0.03\,\mathrm{m}$)

The ADC result n_{max} can be read from Du Noüys ring method measurements with distilled water (fig. 4.1). There are 10 values for n_{max}:

$$n_{max,i} = [80321, 77600, 77085, 80827, 76744, 82423, 79322, 77319, 79543, 77873]$$

The mean value and the standard deviation are

$$\bar{n}_{max} = 78906$$
$$\Delta n_{max} = 1883$$

Equation (4.3) gives the calibration factor with

$$\boxed{c = (2875 \pm 73)\,\frac{1}{\mathrm{mN}}}$$

The deviation of the calibration factor is calculated as follows:

$$\begin{aligned}
\Delta c &= \left|\frac{\partial c}{\partial n_{max}}\right| \cdot \Delta n_{max} + \left|\frac{\partial c}{\partial \bar{r}}\right| \cdot \Delta \bar{r} \\
&= \frac{\Delta n_{max}}{\sigma_{H_2O} \cdot 4\pi \cdot \bar{r}} + \frac{n_{max} \cdot \Delta \bar{r}}{\sigma_{H_2O} \cdot 4\pi \cdot \bar{r}^2} \\
&= \frac{1883}{72.8\,\frac{\mathrm{mN}}{\mathrm{m}} \cdot 4\pi \cdot 0.03\,\mathrm{m}} + \frac{78906 \cdot 5 \cdot 10^{-5}\,\mathrm{m}}{72.8\,\frac{\mathrm{mN}}{\mathrm{m}} \cdot 4\pi \cdot (0.03\,\mathrm{m})^2} \\
&= 68.61\,\frac{1}{\mathrm{mN}} + 4.79\,\frac{1}{\mathrm{mN}} \\
&= 73.4\,\frac{1}{\mathrm{mN}} \approx 73\,\frac{1}{\mathrm{mN}} \qquad (4.4)
\end{aligned}$$

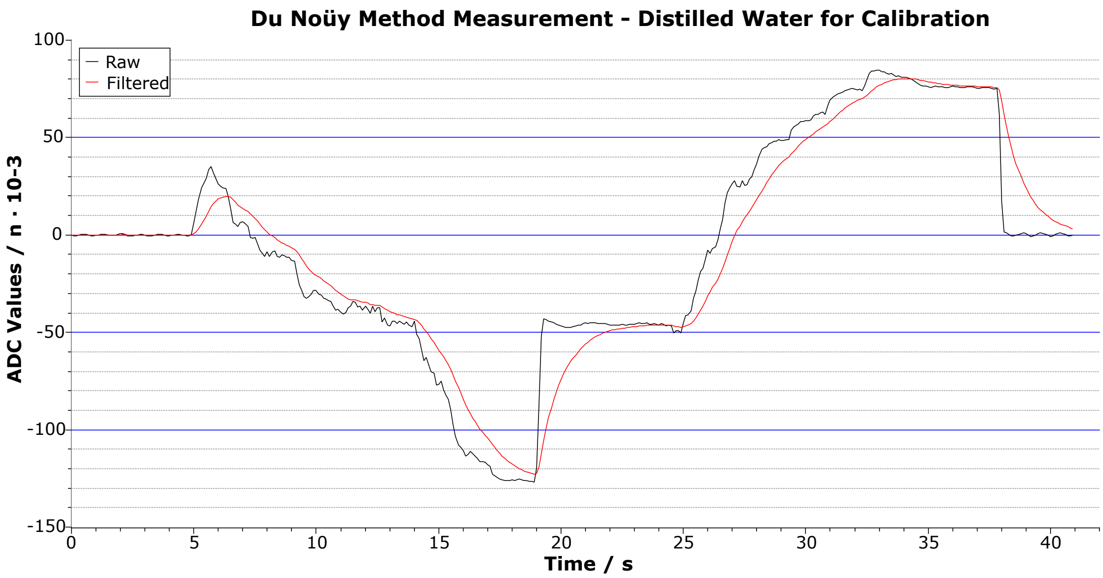


Figure 4.1: Measurement with DU NOÜY ring in distilled water for the calibration ($\vartheta \approx 20\,°\mathrm{C}$).

4.2 Resolution and Statistics

For comparison of the raw and filtered data of the ADC with regard to statistical variations, both are plotted as a function of the time in a stray diagram in fig. 4.2.

As it can be seen, the raw data has a greater scattering. This is also confirmed by the standard deviations, which are as follows:

$$\begin{aligned}
\bar{n}_{raw} &= -21 \qquad \varsigma_{raw} = 58 \\
\bar{n}_{filtered} &= -23 \qquad \varsigma_{filtered} = 39
\end{aligned}$$

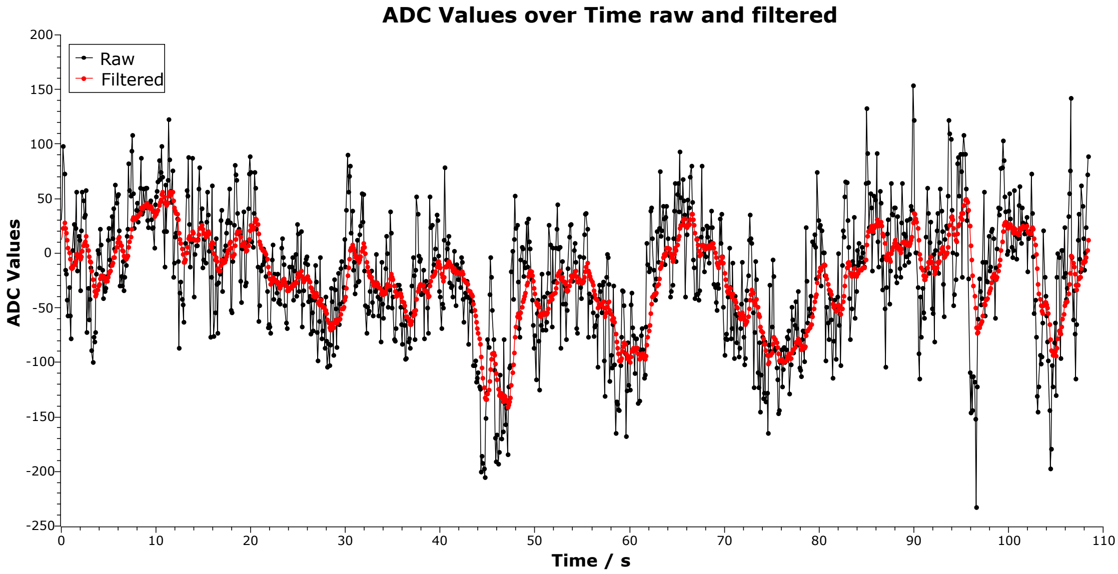


Figure 4.2: Stray diagram with no load applied to force sensor.

Based on that, the deviation of the force to be read can be determined as

$$\begin{aligned}\Delta F &= \left|\frac{\partial F}{\partial \bar{n}_{filtered}}\right| \cdot \varsigma_{filtered} + \left|\frac{\partial F}{\partial c}\right| \cdot \Delta c \\ &= \frac{1}{c} \cdot \varsigma_{filtered} + \frac{|\bar{n}_{filtered}|}{c^2} \cdot \Delta c \\ &= \frac{39}{2875\,\frac{1}{\mathrm{mN}}} + \frac{23 \cdot 73\,\frac{1}{\mathrm{mN}}}{(2875\,\frac{1}{\mathrm{mN}})^2} \cdot 73\,\frac{1}{\mathrm{mN}} \\ &= 0.0135\,\mathrm{mN} + 0.0002\,\mathrm{mN} \\ &= 13.7\,\mu\mathrm{N} \end{aligned} \tag{4.5}$$

The effective resolution of the ADC is given by means of eq. (1.17):

$$ENOB = -23 - \log_2(39) = -28$$

Further, the histograms (fig. 4.3 and fig. 4.4) show that the raw data is more widely spread than the filtered one. The raw data histogram has a binning value of ≈ 40 while the filtered datas binning value is ≈ 20.

4.3 Du Noüy Ring Method Measurement

The data is being sent to the PC during the measurement procedure. The diagram representing force over time, as it is displayed in figs. A.1 to A.4, is obtained converting the ADC data into the force by way of eq. (4.2).

4.3.1 Distilled Water

The maximum forces for distilled water are as follows:

$$F_{max,i}^{H_2O} = [27.9, 27.0, 26.8, 28.1, 26.7, 28.7, 27.6, 26.9, 27.7, 27.1] \qquad [\mathrm{mN}]$$

The forces are around the mean value

$$\bar{F}_{max}^{H_2O} = (27.5 \pm 0.7)\,\mathrm{mN}$$

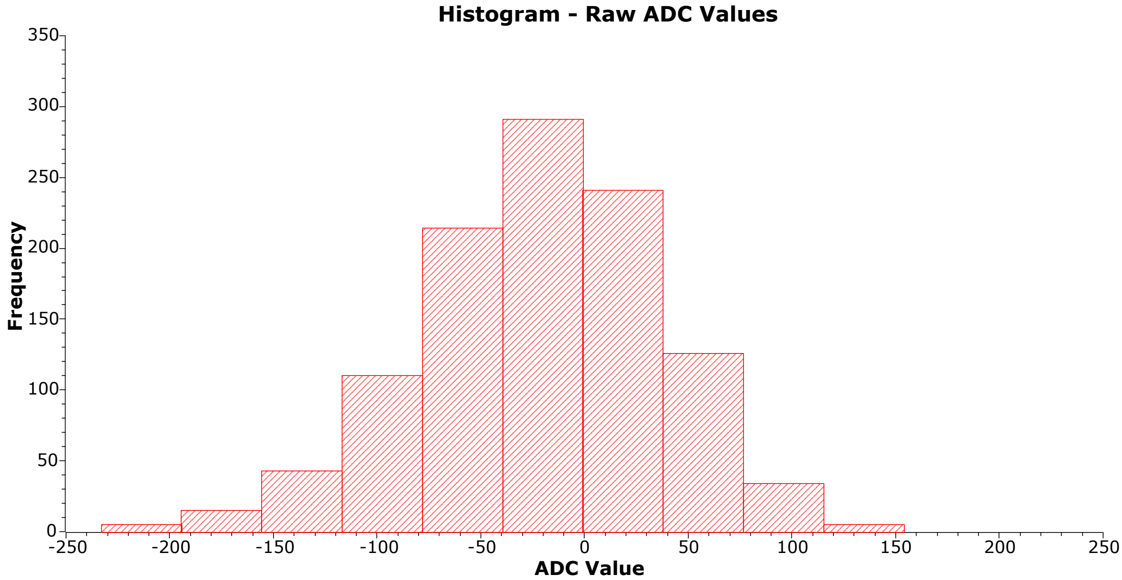


Figure 4.3: Histogram of raw data.

so the calculated value in eq. (1.18) is approximated well. By using eq. (1.5) for the surface tension of distilled water, it results:

$$\boxed{\sigma^{det}_{H_2O} = (72.95 \pm 0.15)\,\frac{\mathrm{mN}}{\mathrm{m}}}$$

with a deviation

$$\begin{aligned}\Delta\sigma^{det}_{H_2O} &= \left|\frac{\partial\sigma^{det}_{H_2O}}{\partial\bar{F}_{max,H_2O}}\right|\cdot\Delta F + \left|\frac{\partial\sigma^{det}_{H_2O}}{\partial\bar{r}}\right|\cdot\Delta\bar{r}\\ &= \frac{\Delta F}{4\pi\cdot\bar{r}} + \frac{\bar{F}^{H2O}_{max}\cdot\Delta\bar{r}}{4\pi\cdot\bar{r}^2}\\ &= \frac{0.01\,\mathrm{mN}}{4\pi\cdot 0.03\,\mathrm{m}} + \frac{27.5\,\mathrm{mN}}{4\pi\cdot(0.03\,\mathrm{m})^2}\cdot 0.00005\,\mathrm{m}\\ &= 0.0265\,\frac{\mathrm{mN}}{\mathrm{m}} + 0.122\,\frac{\mathrm{mN}}{\mathrm{m}}\\ &= 148.5\,\frac{\mu\mathrm{N}}{\mathrm{m}} \approx 150\,\frac{\mu\mathrm{N}}{\mathrm{m}}\end{aligned} \tag{4.6}$$

The determined value of $72.95\,\frac{\mathrm{mN}}{\mathrm{m}}$ fits well with the literature value of $72.8\,\frac{\mathrm{mN}}{\mathrm{m}}$. The area of error covers the literature value.

4.3.2 Detergent

The maximum forces for detergent are

$$F^{\rho_1}_{max,i} = [26.8, 25.8, 26.4, 23.9, 21.0, 21.6, 23.0, 22.6, 27.9, 23.8] \qquad [\,\mathrm{mN}]$$

and their mean value is

$$\bar{F}^{\rho_1}_{max} = (24.3 \pm 2.3)\,\mathrm{mN}$$

For the detergents surface tension it results:

$$\boxed{\sigma_{\rho_1} = (64.46 \pm 0.13)\,\frac{\mathrm{mN}}{\mathrm{m}}} \tag{4.7}$$

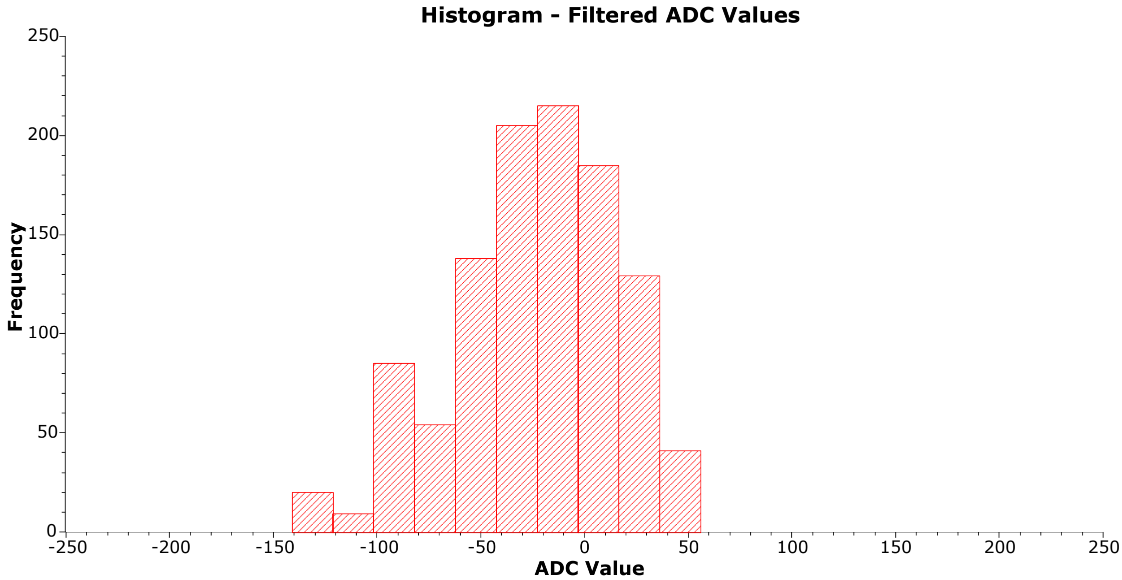


Figure 4.4: Histogram of filtered data.

The deviation is calculated equivalently to the one of distilled water.

In order to be able to compare this value with the literature value, the concentration used has to be estimated. Due to lack of preceding measurements of the masses $m_{detergent}$ of detergent applied to the water, the mass per application is approximated with $0.01\,\mathrm{g}$. Furthermore, the type of detergent was assumed to be sodium dodecyl sulfate as it is a commonly used detergent. Its molar mass M is $288.4\,\frac{\mathrm{g}}{\mathrm{mol}}$ [11]. For the amount of the substance N it follows:

$$N = \frac{m_{detergent}}{M} = \frac{0.01\,\mathrm{g}}{288.4\,\frac{\mathrm{g}}{\mathrm{mol}}} = 34.7\,\mathrm{\mu mol} \tag{4.8}$$

Thus, the concentration ρ for $V = 100\,\mathrm{mL}$ of water is estimated to

$$\rho_1 = \frac{N}{V} = \frac{34.7\,\mathrm{\mu mol}}{100\,\mathrm{mL}} \approx 0.35\,\frac{\mathrm{mmol}}{\mathrm{L}} \tag{4.9}$$

In [7] at $\rho_1 = 0.35\,\frac{\mathrm{mmol}}{\mathrm{L}}$ a surface tension of about $63\,\frac{\mathrm{mN}}{\mathrm{m}}$ can be read. So there is a good match between determined and literature value.

4.3.3 Raising the Concentration of Detergent

Following eqs. (4.8) and (4.9), the new concentration raises to

$$\rho_2 = \frac{2N}{V} \approx 0.69\,\frac{\mathrm{mmol}}{\mathrm{L}} \tag{4.10}$$

Subsequent measurements lead to maximum forces of

$$F^{\rho_2}_{max,i} = [25.8, 24.9, 20.5, 20.8, 18.5, 18.8, 20.9, 26.7, 28.0, 27.6] \qquad [\,\mathrm{mN}]$$

with a mean value of

$$\bar{F}^{\rho_2}_{max} = 23.3\,\mathrm{mN}$$

This calculates to a surface tension of

$$\boxed{\sigma_{\rho_2} = (61.81 \pm 0.13)\,\frac{\mathrm{mN}}{\mathrm{m}}}$$

The surface tension of the liquid with added detergent is a bit smaller than before. This is an expected behavior as in [7] it can be seen that the surface tension decreases by increasing its concentration.

4.3.4 Isopropanol

With isopropanol the following maximum forces are obtained:

$$F_{max,i}^{IPA} = [8.6, 8.0, 5.9, 5.9, 6.3, 7.7, 5.6, 7.9, 7.4, 7.9] \qquad [\,\mathrm{mN}]$$

They give the mean value

$$\bar{F}_{max}^{IPA} = 6.3\,\mathrm{mN}$$

The determined surface tension of isopropanol is therefore

$$\boxed{\sigma_{IPA} = (16.71 \pm 0.05)\,\frac{\mathrm{mN}}{\mathrm{m}}}$$

The deviation is also calculated as above.
Both the determined and the literature value of the surface tension of isopropanol are in the same magnitude. The literature value is $21.4\,\frac{\mathrm{mN}}{\mathrm{m}}$ [5] and thus the determined one is slightly lower. Here, the literature value is not covered by the error area as well.

5 Conclusion

When performing the experiment, there were the usual problems with REALTERM at first. However, these were soon solved. The further performing of the experiment according to the instructions did not cause any problems on the whole. A device that would have made it easier to bring the DU NOÜY-ring into the horizontal and hold it there, as well as a lifting platform that could be moved more evenly, would have been desirable. During the evaluation, unfortunately, a systematic error was found in the calibration of the force sensor. In order to enable an evaluation at all, the calibration factor had to be determined with the aid of the literature value for distilled water from the measurements with distilled water. This meant that the value for the surface tension of distilled water could not be determined retrospectively. However, the surface tensions of water with detergent and of 2-isopropanol could be determined as intended.
The stray diagrams from the measurements without additional weight showed the expected curve, i.e. the raw data have a larger scatter. This is also reflected in the histograms in the form of the lower binning for the filtered data.
The DU NOÜY-ring method gave the characteristic curve with the 8 areas for the dipping phase and the pulling out. The force calculated in the preparation section to cause the lamella to tear and the mean value from the experimentally determined value agrees in a good approximation.
The determined surface tensions fit pretty well with the literature values at all. The fact that the surface tension is decreasing with more detergent is also plausible. Except for the distilled water, the error areas do not cover the literature values. The reason for this is that the calculated deviation of the force is way too small to be assumed as the uncertainty. The deviation calculation of the surface tension implies that the ring radius uncertainty is incorrectly weighted more heavily than that of the force. For the force uncertainty the standard deviation should have been taken.
Overall, the recorded curves looked as expected. Partly there are uncleanliness which are visible in the curves. They are due to the construction and the sometimes rough execution.

List of Figures

Glossary

D	Ring diameter
$ENOB$	Effective number of bits
F	Force
M	Molar mass
N	Amount of substance
R	Bridge resistance
U	Voltage
V	Volume
b	Width
c	Calibration factor
d	Depth
g	Gravity of earth
l	Length
m	Mass
n	Resolution
p	Specific mass
r	Radius
t	Ring thickness
ΔA	Change of surface area
ΔE	Amount of energy required to increase the surface
Δx	Change in height
α	Smoothing factor
ν	Change factor
σ	Surface tension
ς_{RMS}	Root mean square / standard deviation
ρ	Concentration
ϱ	Electrical resistivity

A Appendix

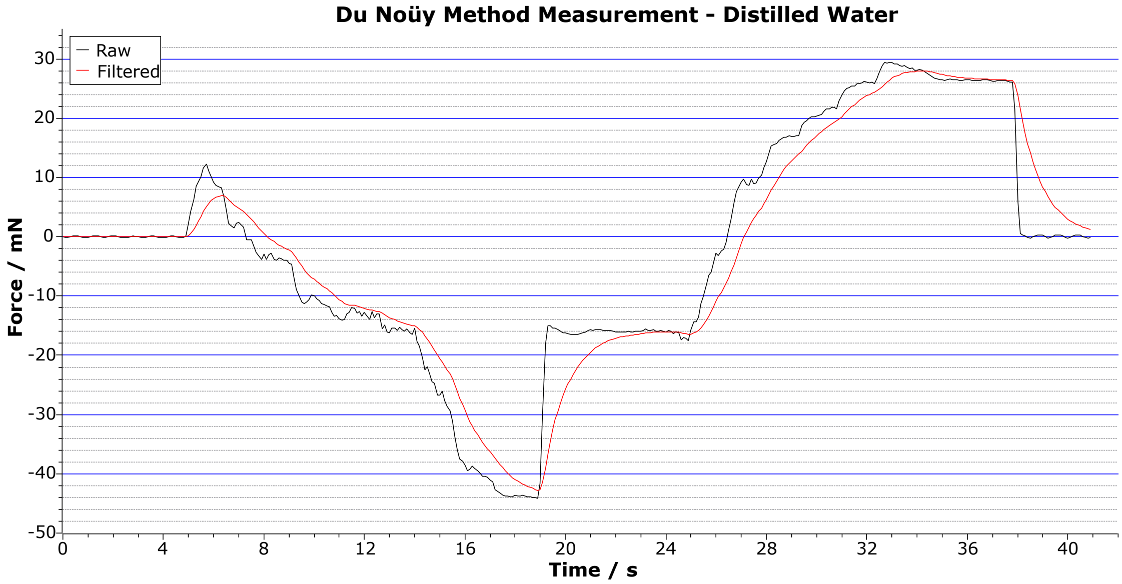


Figure A.1: Measurement with DU NOÜY ring in distilled water ($\vartheta \approx 20\,°\mathrm{C}$).

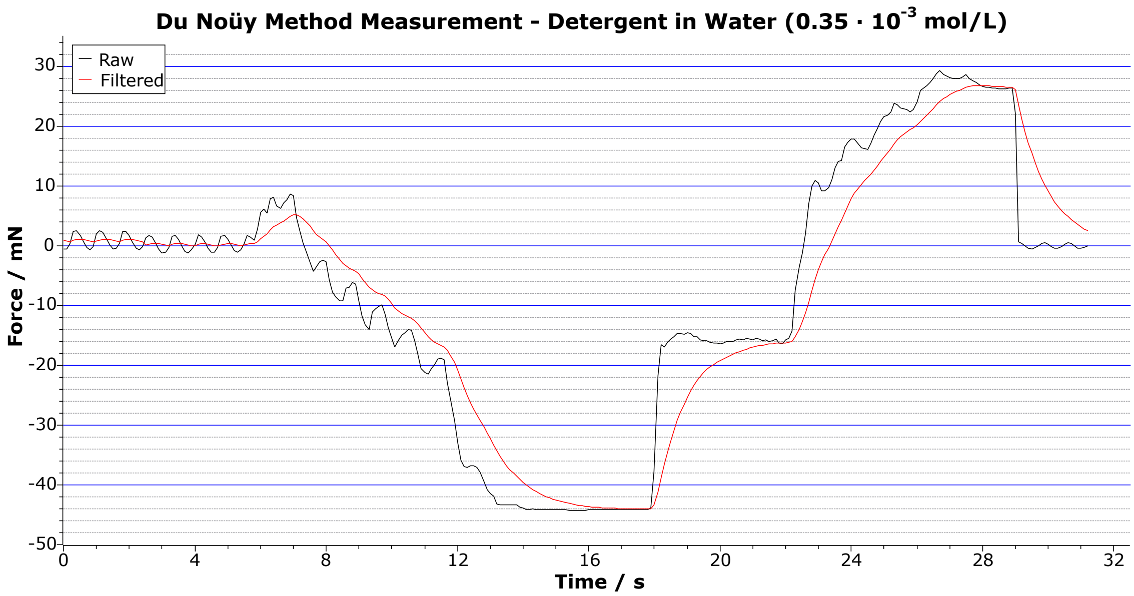


Figure A.2: Measurement with DU NOÜY ring in detergent ($\vartheta \approx 20\,°\mathrm{C}$).

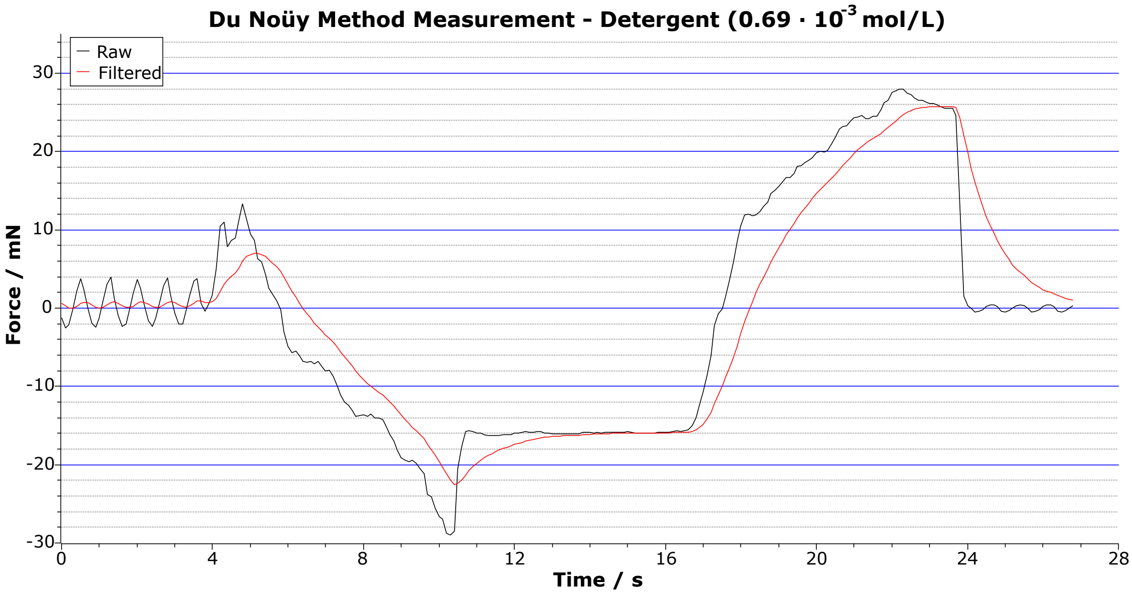


Figure A.3: Measurement with DU NOÜY ring in a larger amount of detergent ($\vartheta \approx 20\,^\circ\mathrm{C}$).

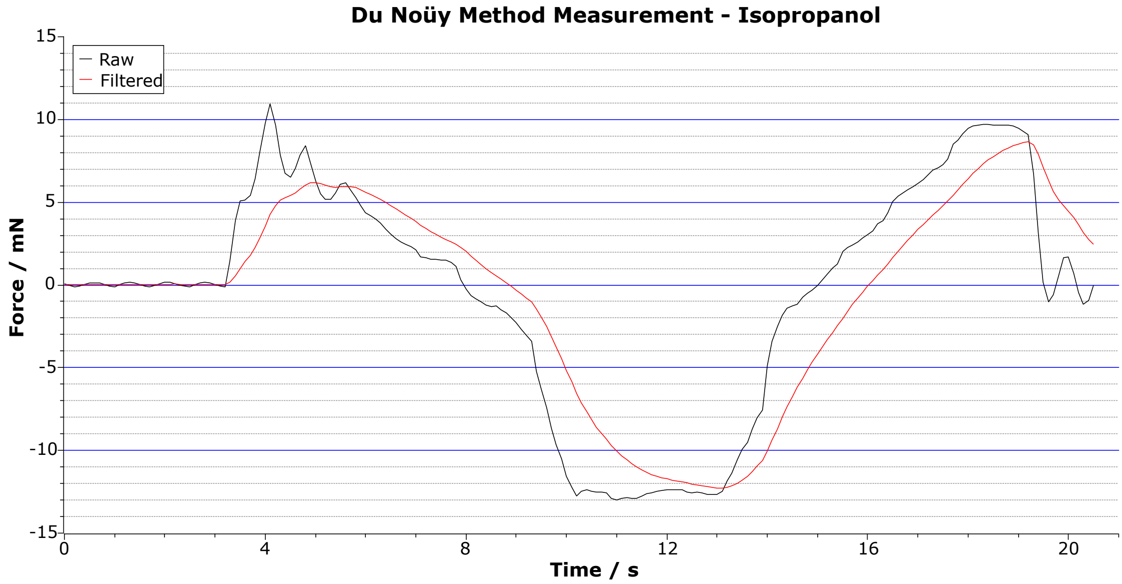


Figure A.4: Measurement with DU NOÜY ring in isopropanol ($\vartheta \approx 20\,^\circ\mathrm{C}$).

B Appendix

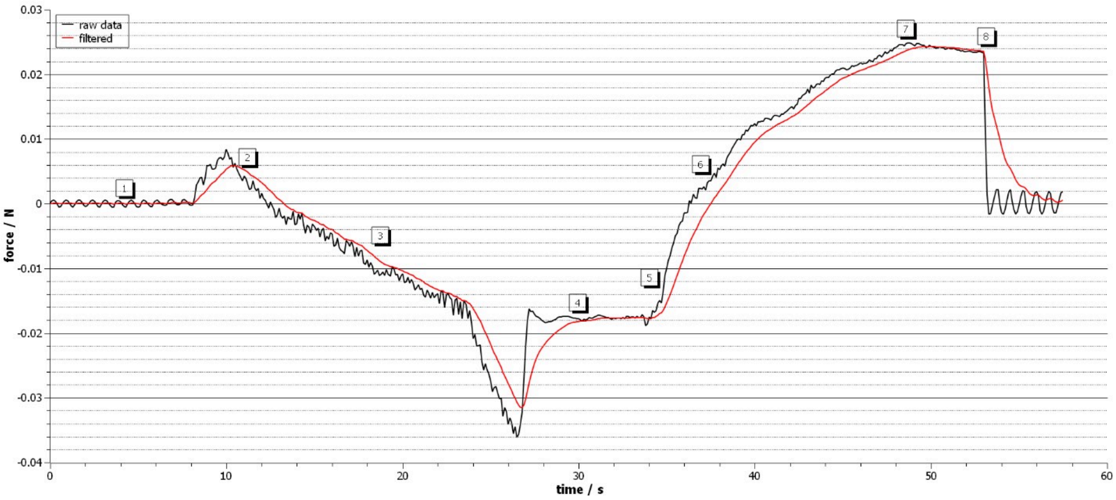


Figure B.1: Expected profile of the force-time diagram with enumerated marks on significant regions.

C Appendix

Listing C.1: MATLAB script to simulate the step response of an exponential filter algorithm.

```
clear

% generate our step function
points = 1000;
data(1:points/2) = zeros();
data((points/2)+1:points) = ones();

% defining smoothing factors
smoothing1 = 0.01;
smoothing2 = 0.05;
smoothing3 = 0.5;

% expo filter magic
data_fil(1:length(data),1:3) = zeros();
for k = 2:length(data)
    data_fil(k,1) = smoothing1 * data(k) + (1-smoothing1)*data_fil(k-1,1);
    data_fil(k,2) = smoothing2 * data(k) + (1-smoothing2)*data_fil(k-1,2);
    data_fil(k,3) = smoothing3 * data(k) + (1-smoothing3)*data_fil(k-1,3);
end

% plot stuff
p = plot(1:length(data),data,1:length(data),data_fil)
p(1).LineWidth = 1;
p(1).Color = '#99c2ff';
p(2).LineWidth = 1.5;
p(2).Color = '#EDB120';
p(3).LineWidth = 1.5;
p(3).Color = '#77AC30';
p(4).LineWidth = 1.5;
p(4).Color = 'k';
ax = gca;
ax.XAxis.FontSize = 18;
ax.YAxis.FontSize = 18;
title('Generic Data to simulate Step Response of an Exponential Filter', 'FontSize',
    20)
legend({'Raw','Filtered, \alpha = 0.01','Filtered, \alpha = 0.05','Filtered, \alpha =
    0.5'},'Location','southeast', 'FontSize', 18)
axis tight
```

Listing C.2: MATLAB script to apply an exponential filter to the GISTEMP v4 data set provided by the Goddard Institute of Space Studies [3].

```
clear
% call csv import and clear unnecessary rows and columns
generated_table_import;
GLB([1,2],:) = [];
GLB(:,[14:19]) = [];

% initialize line vectors to hold the time stamps and temperatures
gistime(1:height(GLB)*(width(GLB)-1)) = zeros();
gistemp(1:height(GLB)*(width(GLB)-1)) = zeros();

% do some math to convert the first columns numeric data to more convinient
% datetime format
k = 0;
for row = 1:height(GLB)
    for col = 2:width(GLB)
        k = k + 1;
        gistime(k) = GLB.Year(row)*10000+(col-1)*100+1;
        gistemp(k) = GLB.(col)(row);
    end
end
gistime = datetime(gistime,'ConvertFrom','yyyyMMdd','Format','yyyy-MM-dd');

% apply exp filter to the temperature data
smoothing = 0.01;

gistemp_fil(1:length(gistemp)) = zeros();
gistemp_fil(1) = gistemp(1);
for k = 2:length(gistemp)
    gistemp_fil(k) = smoothing * gistemp(k) + (1-smoothing)*gistemp_fil(k-1);
end

% plotting
p = plot(gistime,gistemp,gistime,gistemp_fil);
p(1).LineWidth = 0.2;
p(1).Color = '#99c2ff';
p(2).LineWidth = 1;
p(2).Color = 'k';
ax = gca;
ax.XAxis.FontSize = 18;
ax.YAxis.FontSize = 18;
title('Mean global temperature 1880-2020 (GISTEMP)', 'FontSize', 20)
xlabel('Monthly')
ylabel('Mean Temperature / Â°C')
legend({'Raw','Filtered, \alpha = 0.01'},'Location','southeast', 'FontSize', 18)
axis tight
```

Listing C.3: MATLAB script to import the data set.

```
%% Import data from text file
% Script for importing data from the following text file:
%
%    filename: C:\Users\Hunter\Documents\HSRM\WISE20-21\Phys Praktikum 3\Versuche\4\matlab\noisy data\GLB.Ts+dSST.csv
%
% Auto-generated by MATLAB on 27-Jan-2021 16:23:31

%% Setup the Import Options and import the data
opts = delimitedTextImportOptions("NumVariables", 19);

% Specify range and delimiter
opts.DataLines = [1, Inf];
opts.Delimiter = ",";

% Specify column names and types
opts.VariableNames = ["Year", "Jan", "Feb", "Mar", "Apr", "May", "Jun", "Jul", "Aug", "Sep", "Oct", "Nov", "Dec", "JD", "DN", "DJF", "MAM", "JJA", "SON"];
opts.VariableTypes = ["double", "double", "double", "double", "double", "double", "double", "double", "double", "double", "double", "double", "double", "double", "double", "double", "double", "double", "double"];

% Specify file level properties
opts.ExtraColumnsRule = "ignore";
opts.EmptyLineRule = "read";

% Import the data
GLB = readtable("C:\Users\Hunter\Documents\HSRM\WISE20-21\Phys Praktikum 3\Versuche\4\matlab\noisy data\GLB.Ts+dSST.csv", opts);


%% Clear temporary variables
clear opts
```

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