How to Combine Two Relative Rankings of Credit Risk into One Ranking?

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Motivation

- There are two groups of Borrowers:
  - Group 1, Financial firms, which has 8848 firms,
  - Group 2, Oil and Gas, which has 1968 firms.
- Each borrower is ranked based on expert opinions. A rank is a number between 1 and 16 (default banks have rank 17). The bank has monthly data on ranks over a period of 10 years for each borrower.

Challenge

If there are two borrowers coming from different groups, how can the bank know which borrower is riskier?

Available Data and Plan

Available Data

- Group A_i has N_i firms, $i = 1,\ 2$,
- The ranks of borrower a at time t is $R_t^a, t = 0, \ldots T, R_t^a \in \{1, \ldots, 17\}$.

Our Plan

To find a global ranking scale to compare the riskiness between borrowers from two groups.

First Approach

- Local rank $\overline{R^a}$ for borrower $a \in A_1,\ A_2$:

$$\overline{R^a} = \sum_{t=1}^{T} w_t R_t^a, \tag{1}$$

where (for example)

$$w_t = \frac{1}{T}, \quad t = 1, \ldots, T.$$

- $(\overline{R^a})_{a \in A_i}$: realizations of the same random variable R^i with distribution function F_i for $i = 1,\ 2$.

- If a given borrower comes from group 1 with local score $\overline{R^a} = x$, then we define the Relative Distance to Default as

$$RDD^1(x) := P(R^1 > 16 | R^1 \geq x) = \frac{P(R^1 > 16)}{P(R^1 \geq x)}.$$

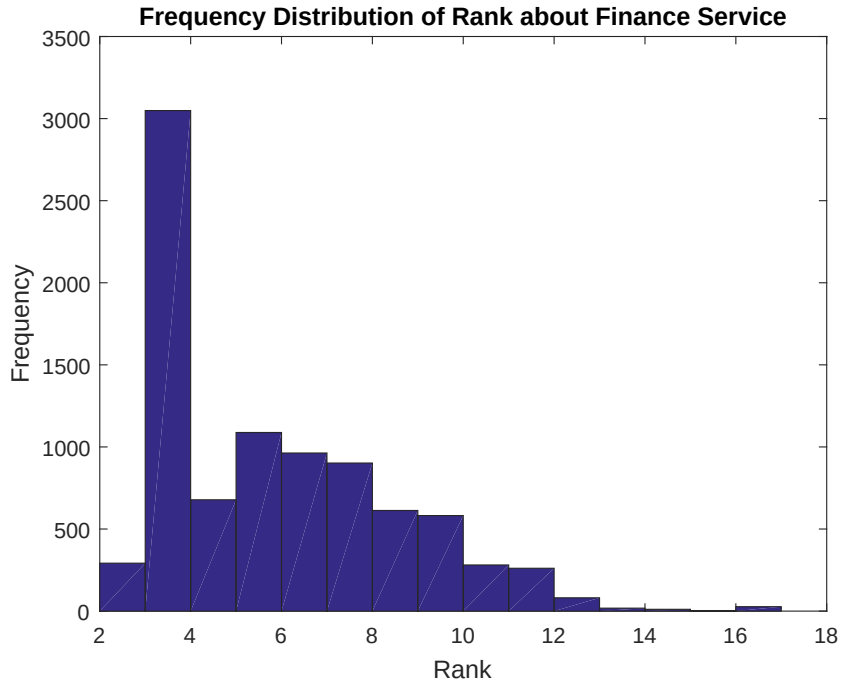
- We repeat the previous calculation for group 2. Now, if a second borrower comes from group 2 with local score of y, the bank has to compare $RDD^1(x)$ and $RDD^2(y)$ to decide which borrower is riskier.

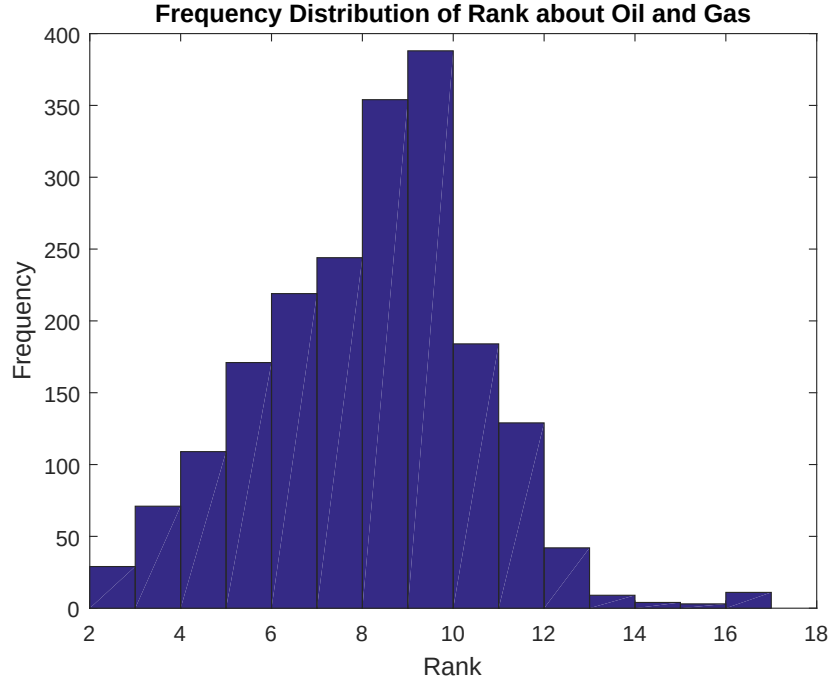
- We go one step further by establishing a function to relate ranks in the group 1 into group 2:

$$i \rightarrow \varphi(i), \quad i = \{1, 2, \ldots, 17\}$$

where φ is given by the following equation,

$$RDD^1(x) = RDD^2(\varphi(x)). \tag{2}$$


Frequency Distribution of Rank about Finance Service
Frequency
Rank
0
500
1000
1500
2000
2500
3000
3500
2
4
6
8
10
12
14
16
18


Frequency Distribution of Rank about Oil and Gas
Frequency
Rank
0
50
100
150
200
250
300
350
400
2
4
6
8
10
12
14
16
18

The Relation Between the Rank of Oil and Finance

Rank of Oil and Gas	Rank of Finance Service
1	4.92
2	5.00
3	5.07
4	5.23
5	5.47
6	5.86
7	6.40
8	7.02
9	7.98
10	9.56
11	10.69
12	11.88
13	12.90
14	13.77
15	14.51
16	No data

Second Approach: Markov Chain

For each company a, we model the random process R_t^a $(t = 1, \ldots, T)$ as a Markov chain on the state space $\{1, 2, \ldots, 16, D\}$. Assume that there is one transition probability matrix for the Oil & Gas companies, and another for the Financial Services companies.

For states i and j, we estimate the transition probability $P(i, j)$ by

$$\frac{\text{number of observed transitions from } i \text{ to } j}{\text{number of observations in state } i}$$

(within each sector).

Financial Probability Matrix

$$
\begin{bmatrix}
0.961 & 0.026 & 0.000 & 0.013 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00000 \\
0.001 & 0.984 & 0.010 & 0.003 & 0.000 & 0.001 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00000 \\
0.000 & 0.001 & 0.985 & 0.012 & 0.001 & 0.001 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00000 \\
0.000 & 0.000 & 0.001 & 0.994 & 0.003 & 0.001 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00004 \\
0.000 & 0.000 & 0.000 & 0.007 & 0.980 & 0.010 & 0.002 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00003 \\
0.000 & 0.000 & 0.000 & 0.002 & 0.006 & 0.980 & 0.009 & 0.002 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00014 \\
0.000 & 0.000 & 0.000 & 0.001 & 0.001 & 0.008 & 0.978 & 0.009 & 0.001 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00008 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.002 & 0.009 & 0.978 & 0.008 & 0.002 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.002 & 0.013 & 0.972 & 0.010 & 0.002 & 0.001 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00008 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.001 & 0.003 & 0.012 & 0.971 & 0.009 & 0.002 & 0.001 & 0.000 & 0.000 & 0.000 & 0.00024 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.002 & 0.004 & 0.015 & 0.967 & 0.010 & 0.002 & 0.000 & 0.000 & 0.000 & 0.00009 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.001 & 0.003 & 0.010 & 0.980 & 0.005 & 0.001 & 0.000 & 0.000 & 0.00038 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.001 & 0.002 & 0.003 & 0.002 & 0.020 & 0.964 & 0.006 & 0.001 & 0.000 & 0.00163 \\
0.000 & 0.000 & 0.000 & 0.003 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.002 & 0.000 & 0.002 & 0.006 & 0.965 & 0.017 & 0.000 & 0.00471 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.002 & 0.000 & 0.007 & 0.000 & 0.005 & 0.968 & 0.000 & 0.01855 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.00000 \\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.002 & 0.001 & 0.000 & 0.000 & 0.99524
\end{bmatrix}
$$

Oil & Gas Probability Matrix

0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00000
0.000	0.995	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00000
0.000	0.001	0.988	0.008	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00000
0.000	0.000	0.003	0.984	0.008	0.001	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00000
0.000	0.000	0.000	0.006	0.977	0.011	0.004	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.00000
0.000	0.000	0.000	0.001	0.006	0.979	0.012	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.00000
0.000	0.000	0.000	0.001	0.002	0.010	0.965	0.017	0.003	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.00009
0.000	0.000	0.000	0.000	0.001	0.002	0.011	0.963	0.016	0.005	0.001	0.000	0.000	0.000	0.000	0.000	0.00000
0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.013	0.960	0.018	0.004	0.001	0.000	0.000	0.000	0.000	0.00008
0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.011	0.959	0.017	0.004	0.001	0.001	0.001	0.000	0.00015
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.007	0.018	0.935	0.027	0.007	0.002	0.001	0.000	0.00038
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.007	0.014	0.944	0.021	0.006	0.004	0.000	0.00113
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.008	0.014	0.020	0.904	0.030	0.017	0.000	0.00464
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.023	0.005	0.897	0.059	0.000	0.00651
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.004	0.006	0.914	0.000	0.06953
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.00000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.026	0.026	0.000	0.92308

Remarks

- We see some transitions $D \to i$ (e.g. companies emerging from bankruptcy protection), but we ignore these and treat D as an absorbing state (i.e., $P(D, D) = 1$).
- There are no 16's in the Financial sector. There are a few 16's in Oil & Gas, but we never see them change. So we ignore state 16.
- Variant: Using a continuous-time interpretation, we can use $\exp(P - I)$ for the transition matrix (should be smoother).

Given a transition probability matrix P, there are different ways to assess default probabilities of states for comparison purposes.

- Single month: Look at $P(i, D)$ for each $i = 1, 2, 3, \ldots, 17$. These numbers are small (sometimes 0), with substantial statistical uncertainty (via the *Clopper-Pearson* confidence intervals for binomial distributions).
- One year: Compute P^{12}, the 12-month transition matrix, and look at $P^{(12)}(i, D)$ for each i.
- "Half-life": Starting in state i, let $H(i)$ be the number of periods needed for the probability of default to exceed 0.5:

$$H(i) := \min\{t : P^{(t)}(i, D) \geq 0.5\}.$$

12-Month Default Probabilities

Rank in Financial Industry	Probability of default	Rank in Oil and Gas Industry	Probability of default
1	3.1E-05	1	0E+00
2	2.5E-05	2	6E-09
3	5.9E-05	3	5E-07
4	4.5E-04	4	7E-06
5	4.6E-04	5	1E-05
6	1.6E-03	6	4E-05
7	1.1E-03	7	2E-04
8	2.4E-04	8	3E-04
9	1.3E-03	9	1E-03
10	2.9E-03	10	4E-03
11	1.6E-03	11	9E-03
12	5.0E-03	12	2E-02
13	1.9E-02	13	5E-02
14	6.2E-02	14	1E-01
15	1.7E-01	15	4E-01
16	0.0E+00	16	0E+00
17	9.4E-01	17	1E+00

The Half-Life $H(i)$

i	Financial	Oil & Gas
1	3001	No Data
2	2963	937
3	2909	745
4	2849	654
5	2769	600
6	2730	543
7	2650	458
8	2555	371
9	2443	301
10	2343	225
11	2286	169
12	2119	115
13	1890	66
14	885	33
15	153	10
16	No Data	No Data

Thank you!