1 Find all basic solutions for the system of simultaneous equations:

$$2x_1 + 3x_2 + 4x_3 = 5 \qquad \text{and} \qquad 3x_1 + 4x_2 + 5x_3 = 6.$$

(Hint. The maximum number of possible basic solutions is

$$C_2^3 = \frac{3!}{2!\,(3-2)!} = 3.$$

And no feasible solution)

2 Determine all the basic feasible solutions of the system of equations:

$3x_1 + 5x_2 + x_3 = 15$and $5x_1 + 2x_2 + x_4 = 10.$

Further, discuss that whether the solution is degenerate or non-degenerate.

(Hint. The maximum number of possible basic solutions will be $C_2^4 = \frac{4!}{2!2!} = \frac{4\times3\times2}{2\times2} = 6.$

And the possible solutions are

(0, 0, 15, 10), (0, 3, 0, 4), (0, 5, -10, 0), (5, 0, 0, -15), (2, 0, 9, 0) and $(\frac{50}{19}, \frac{45}{19}, 0, 0)$.)

20

3- Suppose an Industry is manufacturing tow types of products P1 and P2.The profits per Kg of the two products are Rs.30 and Rs.40 respectively. These two products require processing in three types of machines. The following table shows the available machine hours per day and the time required on each machine to produce one Kg of P1 and P2. Formulate the problem in the form of linear programming model

UG_31202094@ics.tanta.edu.eg UG_31202094@ics.tanta.edu.eg

Profit/Kg	P_1 (Rs.30)	P_2 (Rs.40)	Total available Machine (hours/day)
Machine 1	3	2	600
Machine 2	3	5	800
Machine 3	5	6	1100

Solution (Maximize : 30X1 + 40X2 Subject to : 3X1 + 2X2 ≤ 600 3X1 + 5X2 ≤ 800

5X1 + 6X2 ≤ 1100 X1 ≥ 0, X2 ≥ 0).

4-

Examine convexity of the sets in the following equations.

1. $S=\{(x_1,x_2):x_1+2x_2\leq 5\}\subset R^2$
2. $S=\{(x_1,x_2):x_1-2x_2=2\}\subset R^2$.
3. $S=\{(x,y):x^2+y^2\leq 4\}$.
4. $S=\{(x_1,x_2):x_1\geq 2,\quad x_2\leq 4\}\subset R^2$.
5. $S=\{(x,y):|x|\leq 2,\quad |y|\leq 1\}$.
6. $S=\{(x_1,x_2):x_1^2+x_2^2\leq 1,\quad x_1+x_2\geq 1\}$

Therefore, a slack variable is to be added to the left side of each of these inequalities. The second inequality is of the type "more than or equal to (≥) " . So, a surplus variable is to be subtracted from the left side of this inequality.

Thus, a standard form of the given LPP is

Max.Z' = = $-2x1 - x2 - 4x3 + 0s1 + 0s2 + 0s3$, where Z' = -Z

subject to the constraints:

$-2x1 + 4x2 + s1 = 4$

$x1 + 2x2 + x3 - s2 = 5$

$-2x1 - 3x3 + s3 = 2$

$x1 \geq 0$, $x2 \geq 0$, $x3 \geq 0$, $s1 \geq 0$, $s2 \geq 0$, $s3 \geq 0$.

Exercises.

1- Maximize

$1170x1 + 1110x2$

Subject to:

$9x1 + 5x2 \geq 500$

$7x1 + 9x2 \geq 300$

$5x1 + 3x2 \leq 1500$

$7x1 + 9x2 \leq 1900$

$2x1 + 4x2 \leq 1000$

$x1, x2 \geq 0$

Find graphically the feasible region and the optimal solution.

2- Solve the following LP problem graphically

Minimize

$2x1 + 1.7x2$

Subject to:

$0.15x1 + 0.10x2 \geq 1.0$

$0.75x1 + 1.70x2 \geq 7.5$

$1.30x1 + 1.10x2 \geq 10.0$

$x1, x2 \geq 0$

3- Solve the following LP problem graphically

Maximize

$2x1 + 3x2$

Subject to:

$x1 - x2 \leq 1$

$x1 + x2 \geq 3$

$x1, x2 \geq 0$

4- Graphically solve the following problem of LP

Maximize

$3x1 + 2x2$

Subject to:

$2x1 - 3x2 \geq 0$

$3x1 + 4x2 \leq -12$

$x1, x2 \geq 0$

5- Solve the following problem graphically

Maximize

$4x1 + 4x2$

Subject to:

$-2x1 + x2 \leq 1$

$x1 \leq 2$

$x1 + x2 \leq 3$

$x1, x2 \geq 0$

6-A bed mart company is in the business of manufacturing beds and pillows. The company has 40 hours for assembly and 32 hours for finishing work per day. Manufacturing of a bed requires 4 hours for assembly and 2 hours in finishing. Similarly a pillow requires 2 hours for assembly and 4 hours for finishing. Profitability analysis indicates that every bed would contribute Rs.80, while a pillow contribution is Rs.55 respectively. Find out the daily production of the company to maximize the contribution (profit).

7- Solve graphically the given linear programming problem. (Minimization Problem).

Minimize $Z = 3a + 5b$ S.T

$-3a + 4b \leq 12$

$2a - 1b \geq -2$

$2a + 3b \geq 12$

$1a + 0b \geq 4$

$0a + 1b \geq 2$

And both a and b are ≥ 0.

8- Solve graphically the given linear programming problem

Maximise $Z = 0.75\, a + 1b$ *S.T.*

$1a + 1b \geq 0$

$-0.5\,a + 1b \leq 1$

and both a and b are ≥ 0.

9- A company manufactures two products X and Y on two facilities A and B. The data collected by the analyst is presented in the form of inequalities. Find the optimal product mix for maximising the profit.

Maximise $Z = 6x - 2y$ S.T.

$2x - 1y \leq 2$

$1x + 0y \leq 3$ and both x and y are ≥ 0.

10- Solve graphically the given linear programming problem

Maximise Z $3x + 4y$ S.T.

$$5x + 4y \leq 200$$
$$3x + 5y \leq 150$$
$$5x + 4y \leq 100$$
$$8x + 4y \leq 80$$

And both x and y are ≥ 0

11- Solve graphically the given linear programming problem

Maximise $Z = 3a + 4b$ S.T.

$$1a - 1b \leq -1.$$
$$-1a + 1b \leq 0$$

And both a and b are ≥ 0

12- Solve the l.p.p. by graphical method.

Maximise $Z = 3a + 2b$ S.T.

$1a + 1b \leq 4$

$1a - 1b \leq 2$ and both a and b are ≥ 0.

13- Solve the l.p.p. by graphical method.

Maximise $Z = 0.5\ y - 0.1\ \times$ S.T.

$2\ x + 5\ y \leq 80$

$1x + 1y \leq 20$ and both x and y are ≥ 0

Solutions

1-Optimum variables values are: x1=271.4, x2=0

The maximum value is: 317573

2-Optimum variables values are: x1=6.32, x2=1.63

The minimum values is: 15.4

3-The solution is unbounded

4-The problem has no feasible solution

5-The problem has multiple solutions with the following optimum variable values:

x1=2, x2 =1 or x1=2/3, x2=7/3

The Maximum objective function value is: 12

6-

Beds = 8

Pillows = 4

Maximum Profits is: Rs.860

7- The solution for the problem is the point P (3,2,) and the

Minimum cost is Rs. 3 × 3 + 2 × 5 =

Rs. 19/-

8- The polygon is not closed one *i.e.*, the feasible area is unbound. Thus there is no finite maximum value of Z. That the value of Z can be increased indefinitely. When the value of Z can be increased indefinitely, the problem is said to have an UNBOUND solution.

9- The coordinates of M are (3, 4) and the **maximum Z = Rs. 10/-**

10- The optimal solution $Z(32,10) = 3 \times 32 + 4 \times 10 =$ Rs. 136/.

11- We see that there is no point, which satisfies both the constraints simultaneously. Hence there is no feasible solution. **Given l.p.p. has no feasible solution.**

12- The optimal solution is at the point N (3,1). Hence **optimal Profit $Z = 3 \times 3 + 2 \times 1 =$ Rs.11 /-**

13- Z (0,16) = Rs.8.00

Table 2: Complete Initial Simplex Table

	$c_j \rightarrow$		3	9	0	0	
Basic Variables	Profit/unit	Qty	x_1	x_2	s_1	s_2	R.R.
s_1	0	8	1	4	1	0	2, ¼
$\leftarrow$ s_2	0	4	1	$\boxed{2}$	0	1	2, 0$\leftarrow$
	Z = 0	$z_j \rightarrow$ $c_j - z_j \rightarrow$	0 3	0 9	0 0	0 0	
				$\uparrow$			

Now we apply simply our simplex method procedure to obtain the optimum value of the objective function. It will be Max. Z =18 at $x_1 = 0$, $x_2 = 2$.

Exercises.

1. Maximise $Z = 2x1+4x2$

subject to the constraints:

$x1+2x2\leq 5$

$x1+x2\leq 4$

$x1\geq 0,2\geq 0$

Answer. Max. Z = 10 at the two vertices (0,5/2) and (3, 1).Max. Z = 10at many other points also which are given as (3 – 3t, 1+3t/2), 0≤t≤1.

2. Maximise $Z = 100x1+60x2+40x3$

Subject to the constraints:

$x1+x2+x3\leq 100$

$10x1+4x2+5x3\leq 500$

$x1+x2+3x3\leq 150$,

$x1$, $x2$,$x3$ ≥0

Answer. Z=22000/3 at $x1$= 100/3, $x2$= 200/3,3=0**.**

3-Solve the following linear programming problem using simplex method.

Maximize: 60X1 + 70X2

Subject to: 2X1 + X2 ≤ 300;

3X1 + 4X2 ≤ 509;

4X1 + 7X2 ≤ 812;

X1, X2 ≥ 0

Answer. The objective function is maximized for x1 = 691/5 and x2=118/5 and

The maximum value of the objective function is 9944**.**

4- Solve the following problem with Two phase method

Minimize 12.5X1 + 14.5X2

Subject to :

X1 + X2 ≥ 2000

0.4X1 + 0.75X2 ≥ 1000

0.075X1 + 0.1X2 ≤ 200

X1, X2 ≥ 0

Answer:- The solution of the problem is:

X1 = 10000/7 = 1428 X2 = 4000/7 = 571.4

and The Minimum Value of the objective function is: 26135.3

5- Solve the following problem with **Big –M Method**

Maximize: -12.5X1 – 14.5X2

Subject to:

$X1 + X2 - S3 = 2000$

$40X1 + 75X2 - S4 = 100000$

$75X1 + 100X2 + S5 = 200000$ and

$X1, X2, S3, S4, S5 \geq 0.$

Answer :- The optimum solution of the problem is

$X1 = 10000/7$

$X2 = 4000/7$ and The Minimum Value of the Objective Function is: 26135.3.

6- A soft drinks company has a two products viz. Coco-cola and Pepsi with profit of \$2 an \$1 per unit. The following table illustrates the labour, equipment and materials to produce per unit of each product.Determine suitable product mix which maximizes the profit using simplex method.

	Pepsi	Coco-cola	Total Resources
Labour	3	2	12
Equipment	1	2.3	6.9
Material	1	1.4	4.9

Answer Coco-Cola = 20/9, Pepsi = 161/90

Maximum Profit = \$6.23

7- A factory produces three using three types of ingredients viz. A, B and C in different proportions. The following table shows the requirements o various ingredients as inputs per kg of the products.

Products	Ingredients A	B	C
1	4	8	8
2	4	6	4
3	8	4	0

The three profits coefficients are 20, 20 and 30 respectively. The factory has 800 kg of ingredients A, 1800 kg of ingredients B and 500 kg of ingredient C.

Determine the product mix which will maximize the profit and also find out maximum profit.

Answer: $x1 = 0$, $x2 = 125$, $x3 = 75/2$

Maximum Profit = 5375

8- Solve the following linear programming problem using two phase and M method.

Maximize

$12x1 + 15x2 + 9x3$

Subject to:

$8x1 + 16x2 + 12x3 \leq 250$

$4x1 + 8x2 + 10x3 \geq 80$

$7x1 + 9x2 + 8x3 = 105$

$x1, x2, x3 \geq 0$

Answer:- $x1 = 6$, $x2 = 7$, $x3 = 0$

Maximum Profit = 177

9- Solve the following linear programming problem using simplex method.

Maximize

$3x1 + 2x2$

Subject to:

$x1 - x2 \leq 1$

$x1 + x2 \geq 3$

$x1, x2 \geq 0$

Answer :- Unbounded Solution

10- Solve the following linear programming problem using simplex method.

Maximize

$x1 + x2$

Subject to:

$-2x1 + x2 \leq 1$

$x1 \leq 2$

$x1 + x2 \leq 3$

$x1, x2, x3 \geq 0$

Answer :- $x1 = 2, x2 = 1$ or $x1 = 2/3, x2 = 7/3$

Maximum Profit = 3.

11- Maximize

$P = 3x1 + 4x2 + x3$

Subject to:

$x1 + 2x2 + x3 \leq 6$

$2x1 + 2x3 \leq 4$

$3x1 + x2 + x3 \leq 9$

$x1, x2, x3 \geq 0$

Answer:-

$x1 = 2, x2 = 2, x3 = 0$

Maximum P = 14

12-Solve the following linear programming problem using simplex method.

Maximize $Z = 3a + 9b$ s.t.

$1a + 4b \leq 8$

$1a + 2b \leq 4$ both a and b are ≥ 0

Answer:- Optimal solution is $b = 2$ and Profit is 2×9 = Rs. 18/–

13- Solve the following linear programming problem using simplex method.

Maximize $Z = 2x + 1y$ s.t

$4x + 3y \leq 12$

$4x + 1y \leq 8$

$4x - 1y \leq 8$

Both x and y are ≥ 0

Answer :- the solution is optimal.

$x = 3/2$ and $y = 2$, and Z = Rs. $2 \times 3/2 + 2 \times 1$ = Rs. 5/–

Shadow price = $\frac{1}{4} \times 12 + \frac{1}{4} \times 8$ = Rs. 5/–.

14- Solve the following linear programming problem using M method.

Minimize Z =$4x1+2x2$

Subject to the constraints

$3x1+x2 \geq 27$

$x1+x2 \geq 21$

$x1+2x2 \geq 30$

$x1,2 \geq 0$

Answer. Max. Z' = ─48, i.e. Max. ─Z = ─48i.e. Min. Z = 48 when $x1 = 3$, $x2 = 18$

15- Solve the following linear programming problem using M method.

Maximise Z = $x1+2x2$

Subject to the constraints:

$x1+x2 \leq 4$

$x1+x2 \geq 6$

$x1,2 \geq 0$

Answer. No feasible solution.

16- Solve the following linear programming problem using M method.

Maximise Z = $10x1+2x2$

Subject to the constraints:

$-x1+x2 \leq 2$

$x1+x2 \geq 4$

$x1,2 \geq 0$

Answer. Unbounded solution.