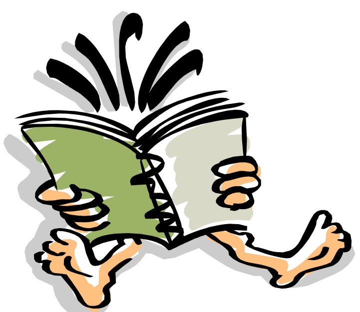
CS1101
Discrete Mathematics 1

Chapter 01
The Foundations: Logic



Today's Topics

- 1.4 Predicates and Quantifiers
- 1.5 Nested Quantifiers
- 1.6 Rules of Inference

Predicates and Quantifiers (1/22)

Predicate:

x is greater than 3

Predicates and Quantifiers (1/22)

Predicate:

$\boxed{x}$ $\boxed{\text{is greater than 3}}$

The subject The predicate

We can denote the statement "x *is greater than* 3" by $\boldsymbol{P(x)}$

where $\boldsymbol{P}$ denotes the predicate "*is greater than* 3" and $\boldsymbol{x}$ is the variable.

The statement $\boldsymbol{P(x)}$ is also said to be the value of the **propositional function** $\boldsymbol{P}$ at $\boldsymbol{x}$. Once a value has been assigned to the variable $\boldsymbol{x}$, the statement $\boldsymbol{P(x)}$ becomes a proposition and has a truth value.

Predicates and Quantifiers (2/22)

Example1:

Let $P(x)$ denote the statement "$x > 3$."

What are the truth values of $P(4)$ and $P(2)$?

Solution

We obtain the statement *P(4)* by setting x = 4 in the statement "x > 3." Hence, *P(4)*, which is the statement "4 > 3," is true. However, *P(2)*, which is the statement "2 > 3," is false.

Predicates and Quantifiers (2/22)

Example1:

Let $P(x)$ denote the statement "$x > 3$."

What are the truth values of $P(4)$ and $P(2)$?

T F

Predicates and Quantifiers (3/22)

Example2:

Let $Q(x, y)$ denote the statement "$x = y + 3$."

What are the truth values of the propositions

$Q(1, 2)$ and $Q(3, 0)$?

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F **T**

Predicates and Quantifiers (4/22)

Example3:

1. Let $P(x)$ denote the statement "$x \leq 4$." What are the truth values?

a) $P(0)$ **b)** $P(4)$ **c)** $P(6)$

2. Let $P(x)$ be the statement "the word x contains the letter a." What are the truth values?

a) $P(\text{orange})$ **b)** $P(\text{lemon})$

c) $P(\text{true})$ **d)** $P(\text{false})$

Predicates and Quantifiers (4/22)

Example3:

1. Let $P(x)$ denote the statement "$x \leq 4$." What are the truth values?

   a) $P(0)$ **T** b) $P(4)$ **T** c) $P(6)$ **F**

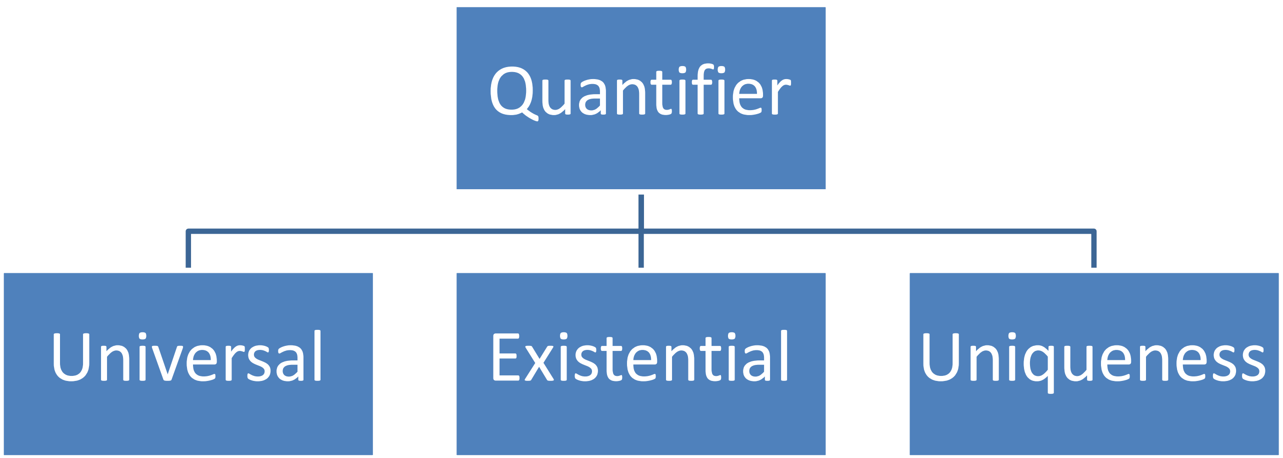
2. Let $P(x)$ be the statement "the word x contains the letter a." What are the truth values?

   a) $P(\text{orange})$ **T** b) $P(\text{lemon})$ **F**
   c) $P(\text{true})$ **F** d) $P(\text{false})$ **T**

Predicates and Quantifiers (5/22)

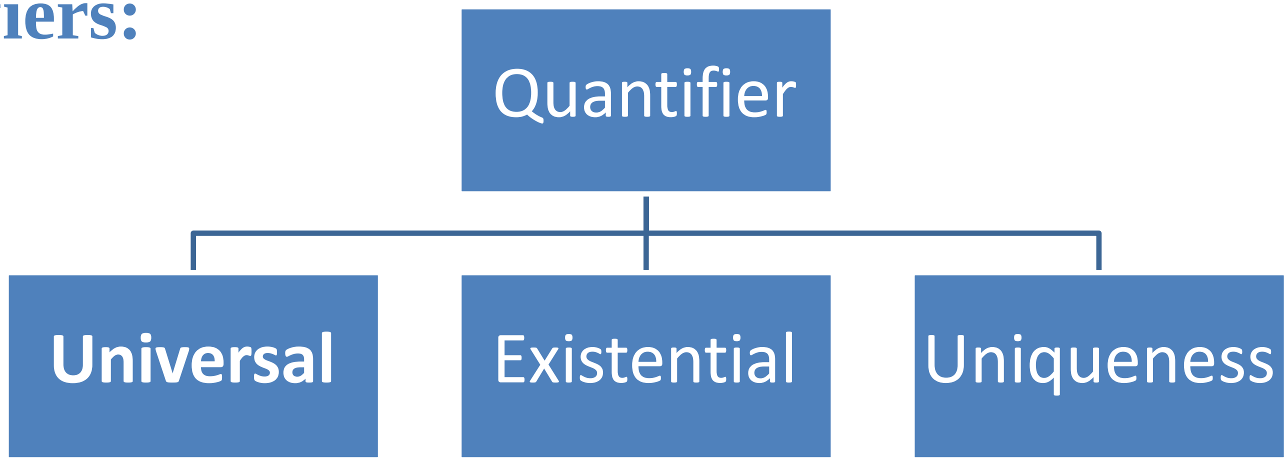
Quantifiers:

Expresses the extent to which a predicate is true over a **range** of elements.



Predicates and Quantifiers (5/22)

Quantifiers:



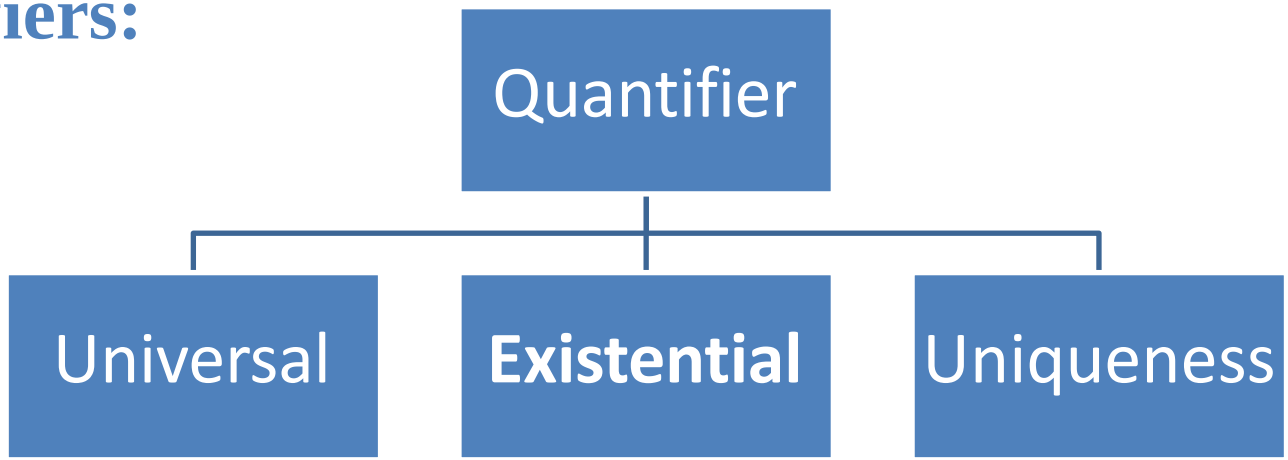
$\forall x P(x)$

The *universal quantification* of $P(x)$ is the statement

"$P(x)$ for all values of x in the domain."

Predicates and Quantifiers (6/22)

Quantifiers:

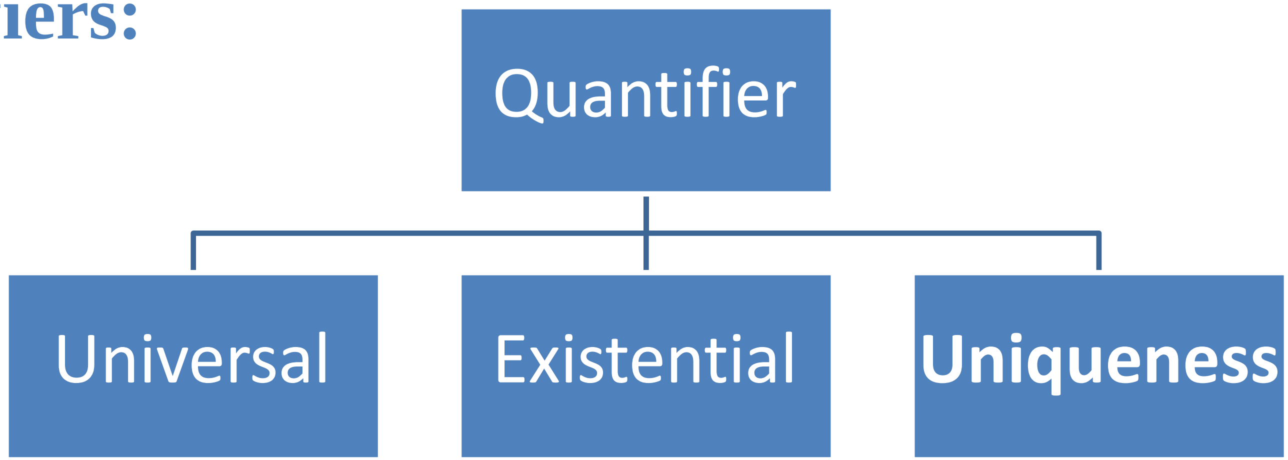


$$\exists x\, P(x)$$

The *existential quantification* of $P(x)$ is the proposition

"There exists an element x in the domain such that $P(x)$."

Quantifiers:



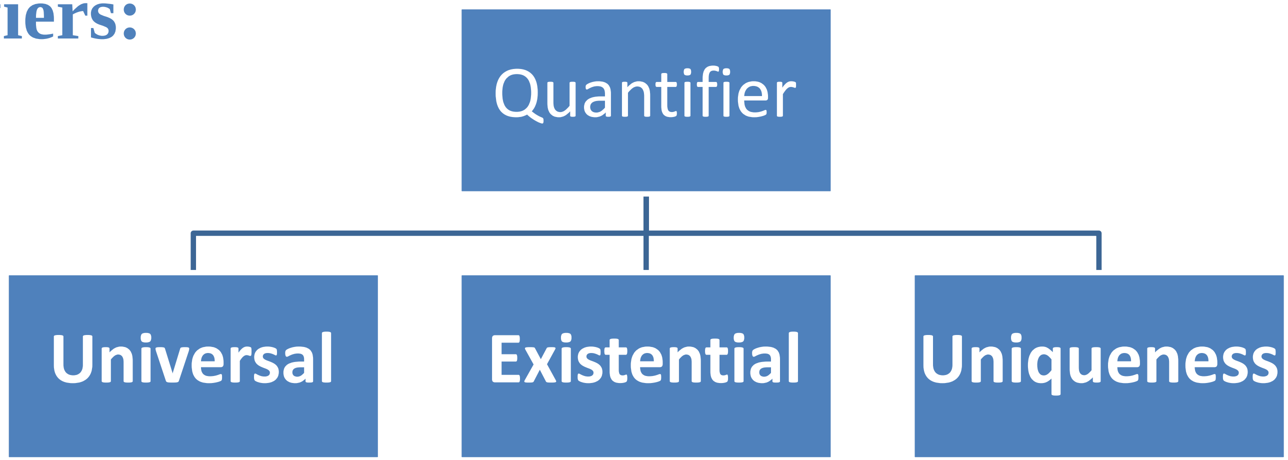
$$\exists! x P(x)$$

$$\exists_1 x P(x)$$

"There exists a unique x such that $P(x)$ is true."

Predicates and Quantifiers (8/22)

Quantifiers:



$\forall x\, P(x)$ $\exists x\, P(x)$ $\exists!\, xP(x),\ \exists_1 xP(x)$

TABLE 1 Quantifiers.		
Statement	***When True?***	***When False?***
$\forall x\, P(x)$	$P(x)$ is true for every x.	There is an x for which $P(x)$ is false.
$\exists x\, P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for every x.

Translate into English – Example1:

Express the statement “Every student in this class has studied calculus.

Solution $P(x)$: x has studied calculus.

$S(x)$: x is in this class.

The statement can be expressed as $\boxed{\forall x(S(x) \rightarrow P(x))}$

Example2:

Let $P(x)$ be the statement "$x + 1 > x$."

What is the truth value of the quantification $\forall x P(x)$,

where the domain consists of all real numbers?

Predicates and Quantifiers (10/22)

Example2:

Let $P(x)$ be the statement "$x + 1 > x$."

What is the truth value of the quantification $\forall x P(x)$,

where the domain consists of all real numbers?

Solution: Because $P(x)$ is true for all real numbers x, the quantification

$$\forall x P(x)$$

is true.

Predicates and Quantifiers (10/22)

Example3:

Let $Q(x)$ be the statement "$x < 2$."

What is the truth value of the quantification $\forall x\, Q(x)$, where the domain consists of all real numbers?

Example3:

Let $Q(x)$ be the statement "$x < 2$."

What is the truth value of the quantification $\forall x\, Q(x)$, where the domain consists of all real numbers?

Solution: $Q(x)$ is not true for every real number x, because, for instance, $Q(3)$ is false. That is, $x = 3$ is a counterexample for the statement $\forall x\, Q(x)$. Thus $\forall x\, Q(x)$ is false.

Predicates and Quantifiers (11/22)

Example3:

Let $P(x)$ denote the statement "$x > 3$."

What is the truth value of the quantification $\exists x P(x)$,

where the domain consists of all real numbers?

Predicates and Quantifiers (11/22)

Example3:

Let $P(x)$ denote the statement "$x > 3$."

What is the truth value of the quantification $\exists x P(x)$,

where the domain consists of all real numbers?

Solution: Because "$x > 3$" is sometimes true—for instance,

when $x = 4$—the existential quantification of $P(x)$,

which is $\exists x P(x)$, is true.

Example4:

What is the truth value of $\exists x P(x)$,

where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4?

Example5:

What is the truth value of $\exists x P(x)$,

where $P(x)$ is the statement "$x^2 > 10$" and the universe of discourse consists of the positive integers not exceeding 4?

Solution: Because the domain is $\{1, 2, 3, 4\}$,

the proposition $\exists x P(x)$ is the same as the disjunction

$P(1) \vee P(2) \vee P(3) \vee P(4)$.

Because $P(4)$, which is the statement "$4^2 > 10$," is true,

it follows that $\exists x P(x)$ is true.

Example6:

Let $P(x)$ be the statement "$x = x^2$." If the domain consists of the integers, what are the truth values?

a) $P(0)$ **b)** $P(1)$ **c)** $P(2)$

d) $P(-1)$ **e)** $\exists x P(x)$ **f)** $\forall x P(x)$

Predicates and Quantifiers (13/22)

Example6:

Let $P(x)$ be the statement "$x = x^2$." If the domain consists of the integers, what are the truth values?

a) $P(0)$ **T** **b)** $P(1)$ **T** **c)** $P(2)$ **F**

d) $P(-1)$ **F** **e)** $\exists x P(x)$ **T** **f)** $\forall x P(x)$ **F**

Predicates and Quantifiers (15/22)

Translate into English – Example2:

Translate the statement $\forall x(C(x) \vee \exists y(C(y) \wedge F(x, y)))$ into English, where $C(x)$ is "x has a computer", $F(x, y)$ is "x and y are friends," and both x and y is the set of all students in your school.

Solution

Every student in your school has a computer or has a friend who has a computer.

Translate into English – Example3:

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \to F(x))$

Answer

a) Every comedian is funny.

Translate into English – Example3:

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

b) $\forall x(C(x) \wedge F(x))$

Answer

b) Every person is a funny comedian.

Translate into English – Example3:

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

c) $\exists x(C(x) \rightarrow F(x))$

Answer

c) There exists a person such that if she or he is a comedian, then she or he is funny.

Translate into English – Example3:

Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

d) $\exists x(C(x) \wedge F(x))$

Answer

d) Some comedians are funny.

Translate into Logical Expression – Example1:

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

Let $P(x)$ be "x is perfect" let $F(x)$ be "x is your friend"

the domain be all people

a) No one is perfect.

Answer

a) $\forall x \neg P(x)$

Translate into Logical Expression – Example1:

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

Let $P(x)$ be "x is perfect" let $F(x)$ be "x is your friend"

the domain be all people

b) Not everyone is perfect.

Answer

b) $\neg\forall x\, P(x)$

Translate into Logical Expression – Example1:

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

Let $P(x)$ be "x is perfect" let $F(x)$ be "x is your friend"

the domain be all people

c) All your friends are perfect.

Answer

c) $\forall x(F(x) \rightarrow P(x))$

Predicates and Quantifiers (17/22)

Translate into Logical Expression – Example1:

Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

Let $P(x)$ be "x is perfect" let $F(x)$ be "x is your friend"

the domain be all people

d) At least one of your friends is perfect.

Answer

d) $\exists x(F(x) \wedge P(x))$

Predicates and Quantifiers (18/22)

Precedence of Quantifiers:

The quantifiers $\forall$ and $\exists$ have higher precedence then all logical operators from propositional calculus.

For example, $\forall x P(x) \vee Q(x)$ is the disjunction of $\forall x P(x)$ and $Q(x)$.

In other words,

it means $(\forall x P(x)) \vee Q(x)$ rather than $\forall x(P(x) \vee Q(x))$.

Predicates and Quantifiers (18/22)

Logical Equivalences Involving Quantifiers:

Show that $\forall x(P(x) \wedge Q(x))$ and $\forall x P(x) \wedge \forall x Q(x)$ are logically equivalent

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Show that $\forall x(P(x) \wedge Q(x))$ and $\forall x P(x) \wedge \forall x Q(x)$ are logically equivalent

1) We assume that $\forall x\left(P(x) \wedge Q(x)\right)$ is true for all values x in the domain.
2) Then, $P(x)$ is true for all values x in the domain. And $Q(x)$ is true for all values x in the domain.
3) Then, $\forall x P(x)$ is true. And $\forall x Q(x)$ is true. $\left(\forall x P(x) \wedge \forall x Q(x)\right)$ is true.

----

1. We assume that $\forall x P(x) \wedge \forall x Q(x)$ is true for all values x in the domain.
2. Then,$\forall x P(x)$ is true. And $\forall x Q(x)$ is true. Then, $P(x)$ is true for all values x in the domain. And $xQ(x)$ is true for all values x in the domain.
3. Then, $P(x) \wedge Q(x)$ is true for all values x in the domain $\forall x\left(P(x) \wedge Q(x)\right)$ is true.

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\forall x \boldsymbol{P}(\boldsymbol{x})$ **:**

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$\forall \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$ **:**

"Every student in your class has taken a course in calculus"

Predicates and Quantifiers (19/22)

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"Every student in your class has taken a course in calculus"

<u>The negation of this statement is</u>

"There is at least one student in your class who has not taken a course in calculus"

$$\neg\forall x P(x) \equiv \exists x \neg P(x)$$

Predicates and Quantifiers (20/22)

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"At least one student in your class has taken a course in calculus"

Predicates and Quantifiers (20/22)

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Predicates and Quantifiers (20/22)

Negating Quantified Expressions:

$P(x)$ is the statement "x has taken a course in calculus" and the domain consists of the students in your class.

$\exists \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$ **:**

"At least one student in your class has taken a course in calculus"

<u>The negation of this statement is</u>

"Every student in this class has not taken calculus"

Predicates and Quantifiers (20/22)

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“At least one student in your class has taken a course in calculus"

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$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$

Predicates and Quantifiers (21/22)

Example1:

What are the negations of the statements

$$\forall \boldsymbol{x}(\boldsymbol{x}^{2} > \boldsymbol{x})$$

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What are the negations of the statements

$$\forall \boldsymbol{x}(\boldsymbol{x}^2 > \boldsymbol{x})$$

$$\neg\forall \boldsymbol{x}(\boldsymbol{x}^2 > \boldsymbol{x}) \equiv \exists \boldsymbol{x}\neg(\boldsymbol{x}^2 > \boldsymbol{x})$$

$$\exists\ \boldsymbol{x}\,(\boldsymbol{x}^2 \leq \boldsymbol{x})$$

Example2:

What are the negations of the statements

$$\exists x(x^2 = 2)$$

Example2:

What are the negations of the statements

$$\exists x(x^2 = 2)$$

$$\neg\exists x(x^2 = 2) \equiv \forall x\neg(x^2 = 2)$$

$$\forall\, x\,(x^2 \neq 2)$$

1.5 Nested Quantifiers

Nested Quantifiers

- One quantifier is within the scope of another

Example: “Every real number has an inverse” is

$$\forall x \exists y(x + y = 0)$$

where the domains of x and y are the real numbers.

- We can also think of nested propositional functions:

$\forall x \exists y(x + y = 0)$ can be viewed as

$\forall x\, Q(x)$ where $Q(x)$ is $\exists y\, P(x, y)$

where $P(x, y)$ is $(x + y = 0)$

Nested Quantifiers

- EXAMPLE : Assume that the domain for the variables x and y consists of all real numbers.

The statement

$$\forall x \forall y (x + y = y + x)$$

says that $x + y = y + x$ for all real numbers x and y. (the commutative law for addition)

- the statement

$$\forall x \exists y (x + y = 0)$$

says that for every real number x there is a real number y such that $x + y = 0$.

(every real number has an additive inverse)

Order of Quantifiers

Examples:

1. Let P(x,y) be the statement "x + y = y + x." Assume that U is the real numbers.

□x □yP(x,y) and □y □xP(x,y) have the same truth value.

"For all real numbers x, for all real numbers y, x + y = y + x."

2. Let Q(x,y) be the statement "x + y = 0." Assume that U is the real numbers. Then □x □yQ(x,y) is true,

"For every real number x there is a real number y such that Q(x, y)."

But ∃y∀xQ(x, y) is false

"There is a real number y such that for every real number x, Q(x, y)."

Question on Order of Quantifiers

Example 1: Let U be the real numbers,

Define $P(x,y) : x \cdot y = 0$

What is the truth value of the following:

1. $\forall x \forall y P(x,y)$

   Answer: False

2. $\forall x \exists y P(x,y)$

   Answer: True

3. $\exists x \forall y P(x,y)$

   Answer: True

4. $\exists x \exists y P(x,y)$

   Answer: True

Nested Quantifiers

Example 2: Let U be the real numbers,

Define $P(x,y) : x / y = 0$

What is the truth value of the following:

1. $\forall x \forall y P(x,y)$

   Answer: False

2. $\forall x \exists y P(x,y)$

   Answer: False

3. $\exists x \forall y P(x,y)$

   Answer: False

4. $\exists x \exists y P(x,y)$

   Answer: True

Quantifications of two Variables

Statement	When True?	When False
$\forall x \forall y P(x,y)$ $\forall y \forall x P(x,y)$	$P(x,y)$ is true for every pair x,y.	There is a pair x, y for which $P(x,y)$ is false.
$\forall x \exists y P(x,y)$	For every x there is a y for which $P(x,y)$ is true.	There is an x such that $P(x,y)$ is false for every y.
$\exists x \forall y P(x,y)$	There is an x for which $P(x,y)$ is true for every y.	For every x there is a y for which $P(x,y)$ is false.
$\exists x \exists y P(x,y)$ $\exists y \exists x P(x,y)$	There is a pair x, y for which $P(x,y)$ is true.	$P(x,y)$ is false for every pair x,y

1.6 Rules of Inference

Valid Arguments in Propositional Logic

Consider the following argument involving propositions (which, by definition, is a sequence of propositions):

"If you have a current password, then you can log onto the network."

"You have a current password."

Therefore,

"You can log onto the network."

Rules of Inference (1/9)

Valid Arguments in Propositional Logic

Consider the following argument involving propositions (which, by definition, is a sequence of propositions):

"If you have a current password, then you can log onto the network."

"You have a current password."

Premises

Therefore,

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Conclusion

Rules of Inference (1/9)

Valid Arguments in Propositional Logic

Consider the following argument involving propositions (which, by definition, is a sequence of propositions):

$p \rightarrow q$

p **Premises**

$\therefore q$ **Conclusion**

Rules of Inference (1/9)

Valid Arguments in Propositional Logic

Consider the following argument involving propositions (which, by definition, is a sequence of propositions):

$p \rightarrow q$

p **Premises**

$\therefore q$ **Conclusion**

This argument is valid if $((p \rightarrow q) \land p) \rightarrow q$ is a tautology.

Rules of Inference (1/9)

Valid Arguments in Propositional Logic

An **argument** in propositional logic is a sequence of propositions. All the proposition in the argument are called **premises** and the final proposition is called the **conclusion**.

$$p \rightarrow q$$

$$p$$

Premises

$$\therefore q$$

Conclusion

This argument is valid if $((p \rightarrow q) \land p) \rightarrow q$ is a tautology.

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

$p \rightarrow q$

p

$\therefore q$

Rules of Inference (2/9)

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

Premise 1

p	q	$p \rightarrow q$				
T	T	T				
T	F	F				
F	T	T				
F	F	T				

$p \rightarrow q$

p

$\therefore q$

Rules of Inference (2/9)

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

p	q	$p \to q$ (Premise 1)	p (Premise 2)			
T	T	T	T			
T	F	F	T			
F	T	T	F			
F	F	T	F			

$$p \to q$$
$$p$$
$$\underline{\qquad\qquad}$$
$$\therefore q$$

Rules of Inference (2/9)

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

Premise 1 Premise 2

p	q	$p \rightarrow q$	p	$(p \rightarrow q) \wedge p$		
T	T	T	T	T		
T	F	F	T	F		
F	T	T	F	F		
F	F	T	F	F		

$p \rightarrow q$

p

$\therefore q$

Rules of Inference (2/9)

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

		Premise 1	Premise 2		Conclusion	
p	q	$p \rightarrow q$	p	$(p \rightarrow q) \wedge p$	q	
T	**T**	**T**	**T**	**T**	**T**	
T	**F**	**F**	**T**	**F**	**F**	
F	**T**	**T**	**F**	**F**	**T**	
F	**F**	**T**	**F**	**F**	**F**	

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Valid Arguments in Propositional Logic

We can always use a truth table to show that an argument form is valid.

Premise 1 Premise 2 Conclusion

p	q	$p \to q$	p	$(p \to q) \wedge p$	q	$((p \to q) \wedge p) \to q$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	F	F	F	T

$$
\begin{array}{l}
p \to q \\
p \\
\hline
\therefore q
\end{array}
$$

$((p \to q) \wedge p) \to q$ is a tautology

Rules of Inference (3/9)

TABLE 1 **Rules of Inference.** **Part 1**

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism

Rules of Inference (3/9)

TABLE 1 Rules of Inference. **Part 2**

Rule of Inference	*Tautology*	*Name*
$\begin{array}{rl} & p \\ \hline \therefore & p \vee q \end{array}$	$p \to (p \vee q)$	Addition
$\begin{array}{rl} & p \wedge q \\ \hline \therefore & p \end{array}$	$(p \wedge q) \to p$	Simplification
$\begin{array}{rl} & p \\ & q \\ \hline \therefore & p \wedge q \end{array}$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$\begin{array}{rl} & p \vee q \\ & \neg p \vee r \\ \hline \therefore & q \vee r \end{array}$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

Example1

Using the truth table to show that the hypotheses

$p \vee q$

$\neg p \vee r$

lead to the conclusion

$q \vee r$

$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Rules of Inference (4/9)

Example1

Using the truth table to show that the hypotheses

$p \vee q$

$\neg p \vee r$

$q \vee r$

			Premise 1		Premise 2		Conclusion
p	q	r	$p \vee q$	$\neg p$	$\neg p \vee r$	$(p \vee q) \wedge (\neg p \vee r)$	$q \vee r$
T	T	T	T	F	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	F	T	T	T
T	F	F	T	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	F

Rules of Inference (5/9)

Example2

Using the rules of inference to show that the hypotheses

$\neg p \land q$

$r \rightarrow p$

$\neg r \rightarrow s$

$s \rightarrow t$

lead to the conclusion

t

Rules of Inference (5/9)

Example2

$\neg p \wedge q$

$\therefore \neg p$

$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$	$(p \wedge q) \rightarrow p$	Simplification

$\cancel{\neg p \wedge q}$

$r \rightarrow p$

$\neg r \rightarrow s$

$s \rightarrow t$

Rules of Inference (5/9)

Example2

$\neg p \wedge q$

$\therefore \neg p$

$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification

$\neg p$

$r \to p$

$\therefore \neg r$

$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus tollens

$\cancel{\neg p \wedge q}$

$\cancel{r \to p}$

$\neg r \to s$

$s \to t$

Rules of Inference (5/9)

Example2

$\neg r$

$\neg r \rightarrow s$

$\therefore s$

p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens

$\cancel{\neg p \wedge q}$

$\cancel{r \rightarrow p}$

$\cancel{\neg r \rightarrow s}$

$s \rightarrow t$

Rules of Inference (5/9)

Example2

$\neg r$

$\neg r \rightarrow s$

$\therefore s$

p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens

s

$s \rightarrow t$

$\boxed{\therefore t}$ conclusion

$\cancel{\neg p \wedge q}$

$\cancel{r \rightarrow p}$

$\cancel{\neg r \rightarrow s}$

$\cancel{s \rightarrow t}$

Rules of Inference (6/9)

Example3

Show that the premises “If you send me an e-mail message, then I will finish writing the program,” “If you do not send me an e-mail message, then I will go to sleep early,” and “If I go to sleep early, then I will wake up feeling refreshed” *lead to the conclusion* “If I do not finish writing the program, then I will wake up feeling refreshed.”

Rules of Inference (6/9)

Example3

Show that the premises "If you send me an e-mail message (p), then I will finish writing the program (q)," "If you do not send me an e-mail message ($\neg p$), then I will go to sleep early (r)," and "If I go to sleep early (r), then I will wake up feeling refreshed (s)" *lead to the conclusion* "If I do not finish writing the program ($\neg q$), then I will wake up feeling refreshed (s)."

Example3 $p \rightarrow q$

Show that the premises "If you send me an e-mail message (p), then I will finish writing the program (q)," "If you do not send me an e-mail message ($\neg p$), then I will go to sleep early (r)," and "If I go to sleep early (r), then I will wake up feeling refreshed (s)" *lead to the conclusion* "If I do not finish writing the program ($\neg q$), then I will wake up feeling refreshed (s)."

Rules of Inference (6/9)

Example3 $p \to q$, $\neg p \to r$

Show that the premises "If you send me an e-mail message (p), then I will finish writing the program (q)," "If you do not send me an e-mail message ($\neg p$), then I will go to sleep early (r)," and "If I go to sleep early (r), then I will wake up feeling refreshed (s)" *lead to the conclusion* "If I do not finish writing the program ($\neg q$), then I will wake up feeling refreshed (s)."

Rules of Inference (6/9)

Example3 $p \to q$, $\neg p \to r$, $r \to s$

p

Show that the premises “If you send me an e-mail message,

q $\neg p$

then I will finish writing the program,” “If you do not send

r

me an e-mail message, then I will go to sleep early,” and

r s

“If I go to sleep early, then I will wake up feeling

$\neg q$

refreshed” *lead to the conclusion* “If I do not finish writing

s

the program, then I will wake up feeling refreshed.”

Rules of Inference (6/9)

Example3 $p \to q$, $\neg p \to r$, $r \to s$, con: $\neg q \to s$

Show that the premises "If $\overset{p}{\text{you send me an e-mail message}}$, then $\overset{q}{\text{I will finish writing the program}}$," "If $\overset{\neg p}{\text{you do not send me an e-mail message}}$, then $\overset{r}{\text{I will go to sleep early}}$," and "If $\overset{r}{\text{I go to sleep early}}$, then $\overset{s}{\text{I will wake up feeling refreshed}}$" *lead to the conclusion* "If $\overset{\neg q}{\text{I do not finish writing the program}}$, then $\overset{s}{\text{I will wake up feeling refreshed}}$."

Rules of Inference (6/9)

Example3

$p \to q$

$\neg p \to r$

$r \to s$

lead to the conclusion

$\neg q \to s$

Rules of Inference (6/9)

Example3

$p \to q$

$\neg p \to r$

$r \to s$

\- - - - - - - -

$\neg q \to s$

1.	$p \to q$	Premise 1
2.	$\neg q \to \neg p$	Contrapositive of (1)

Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$

$\boxed{p \to q \equiv \neg q \to \neg p}$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(p \to \neg q)$

Rules of Inference (6/9)

Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

- - - - - - - -

$\neg q \rightarrow s$

1.	$\cancel{p \rightarrow q}$	Premise 1
2.	$\neg q \rightarrow \neg p$	Contrapositive of (1)
3.	$\neg p \rightarrow r$	Premise 2

$p \rightarrow q$ $q \rightarrow r$ $\therefore \overline{p \rightarrow r}$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism

Rules of Inference (6/9)

Example3

$p \to q$

$\neg p \to r$

$r \to s$

$\neg q \to s$

1. $\cancel{p \to q}$ Premise 1
2. $\cancel{\neg q \to \neg p}$ Contrapositive of (1)
3. $\cancel{\neg p \to r}$ Premise 2
4. $\neg q \to r$ Hypothetical syllogism

$p \to q$ $q \to r$ $\therefore \overline{p \to r}$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism

Rules of Inference (6/9)

Example3

$p \to q$

$\neg p \to r$

$r \to s$

$\neg q \to s$

1.	$\cancel{p \to q}$	Premise 1
2.	$\neg q \to \neg p$	Contrapositive of (1)
3.	$\neg p \to r$	Premise 2
4.	$\neg q \to r$	Hypothetical syllogism
5.	$r \to s$	Premise 3

Rules of Inference (6/9)

Example3

$p \to q$

$\neg p \to r$

$r \to s$

$\neg q \to s$

1. $\cancel{p \to q}$ Premise 1
2. $\neg q \to \cancel{\neg p}$ Contrapositive of (1)
3. $\cancel{\neg p} \to r$ Premise 2
4. $\neg q \to r$ Hypothetical syllogism
5. $r \to s$ Premise 3

$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism

Rules of Inference (6/9)

Example3

$p \to q$

$\neg p \to r$

$r \to s$

$\neg q \to s$

1. $p \to q$ Premise 1
2. $\neg q \to \neg p$ Contrapositive of (1)
3. $\neg p \to r$ Premise 2
4. $\neg q \to r$ Hypothetical syllogism
5. $r \to s$ Premise 3
6. $\neg q \to s$ Hypothetical syllogism

$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism

Rules of Inference (6/9)

Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

$\boxed{\neg q \rightarrow s}$

1.	$\cancel{p \rightarrow q}$	Premise 1
2.	$\cancel{\neg q \rightarrow \neg p}$	Contrapositive of (1)
3.	$\cancel{\neg p \rightarrow r}$	Premise 2
4.	$\cancel{\neg q \rightarrow r}$	Hypothetical syllogism
5.	$\cancel{r \rightarrow s}$	Premise 3
6.	$\boxed{\neg q \rightarrow s}$	Hypothetical syllogism

Rules of Inference (7/9)

Example4 – Same as Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

lead to the conclusion

$\neg q \rightarrow s$

Rules of Inference (7/9)

Example4 – Same as Example3

$p \to q$

$\neg p \to r$

$r \to s$

- - - - - - - -

$\neg q \to s$

1. $p \to q$ Premise 1
2. $\neg p \lor q$ Logical Equivalence

Logical Equivalences Involving Conditional Statements.

$\boxed{p \to q \equiv \neg p \lor q}$

$p \to q \equiv \neg q \to \neg p$

$p \lor q \equiv \neg p \to q$

$p \land q \equiv \neg(p \to \neg q)$

Rules of Inference (7/9)

Example4 – Same as Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

- - - - - - - -

$\neg q \rightarrow s$

1.	$\cancel{p \rightarrow q}$	Premise 1
2.	$\neg p \vee q$	Logical Equivalence
3.	$\neg p \rightarrow r$	Premise 2

Logical Equivalences Involving Conditional Statements.

$\boxed{p \rightarrow q \equiv \neg p \vee q}$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

Rules of Inference (7/9)

Example4 – Same as Example3

$p \to q$

$\neg p \to r$

$r \to s$

$\neg q \to s$

1.	$\cancel{p \to q}$	Premise 1
2.	$\neg p \vee q$	Logical Equivalence
3.	$\cancel{\neg p \to r}$	Premise 2
4.	$p \vee r$	Logical Equivalence

Logical Equivalences Involving Conditional Statements.

$\boxed{p \to q \equiv \neg p \vee q}$

$p \to q \equiv \neg q \to \neg p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(p \to \neg q)$

Rules of Inference (7/9)

Example4 – Same as Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

$\neg q \rightarrow s$

1.	$\cancel{p \rightarrow q}$	Premise 1
2.	$\neg p \vee q$	Logical Equivalence
3.	$\cancel{\neg p \rightarrow r}$	Premise 2
4.	$p \vee r$	Logical Equivalence
5.	$r \rightarrow s$	Premise 3
6.	$\neg r \vee s$	Logical Equivalence

Logical Equivalences Involving Conditional Statements.

$\boxed{p \rightarrow q \equiv \neg p \vee q}$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

Rules of Inference (7/9)

Example4 – Same as Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

$\neg q \rightarrow s$

1.	$\cancel{p \rightarrow q}$	Premise 1
2.	$\neg p \vee q$	Logical Equivalence
3.	$\cancel{\neg p \rightarrow r}$	Premise 2
4.	$p \vee r$	Logical Equivalence
5.	$\cancel{r \rightarrow s}$	Premise 3
6.	$\neg r \vee s$	Logical Equivalence

Logical Equivalences Involving Conditional Statements.

$\boxed{p \rightarrow q \equiv \neg p \vee q}$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \vee q \equiv \neg p \rightarrow q$

$p \wedge q \equiv \neg(p \rightarrow \neg q)$

Rules of Inference (7/9)

Example4 – Same as Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

$\neg q \rightarrow s$

1. $\cancel{p \rightarrow q}$ Premise 1
2. $\neg p \vee q$ Logical Equivalence
3. $\cancel{\neg p \rightarrow r}$ Premise 2
4. $p \vee r$ Logical Equivalence

$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Rules of Inference (7/9)

Example4 – Same as Example3

$p \to q$

$\neg p \to r$

$r \to s$

$\neg q \to s$

1. $\cancel{p \to q}$ Premise 1
2. $\neg p \lor q$ Logical Equivalence
3. $\cancel{\neg p \to r}$ Premise 2
4. $p \lor r$ Logical Equivalence
5. $q \lor r$ Resolution

$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

Rules of Inference (7/9)

Example4 – Same as Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

$\neg q \rightarrow s$

1. $\cancel{p \rightarrow q}$ Premise 1
2. $\cancel{\neg p \vee q}$ Logical Equivalence
3. $\cancel{\neg p \rightarrow r}$ Premise 2
4. $\cancel{p \vee r}$ Logical Equivalence
5. $q \vee r$ Resolution

$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Rules of Inference (7/9)

Example4 – Same as Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

$\neg q \rightarrow s$

1. $\cancel{p \rightarrow q}$ Premise 1
2. $\cancel{\neg p \lor q}$ Logical Equivalence
3. $\cancel{\neg p \rightarrow r}$ Premise 2
4. $\cancel{p \lor r}$ Logical Equivalence
5. $q \lor r$ Resolution
6. $\cancel{r \rightarrow s}$ Premise 3
7. $s \lor \neg r$ Logical Equivalence

Logical Equivalences Involving Conditional Statements.

$\boxed{p \rightarrow q \equiv \neg p \lor q}$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$p \lor q \equiv \neg p \rightarrow q$

$p \land q \equiv \neg(p \rightarrow \neg q)$

Rules of Inference (7/9)

Example4 – Same as Example3

$p \to q$

$\neg p \to r$

$r \to s$

$\neg q \to s$

1. $\cancel{p \to q}$ Premise 1
2. $\cancel{\neg p \vee q}$ Logical Equivalence
3. $\cancel{\neg p \to r}$ Premise 2
4. $\cancel{p \vee r}$ Logical Equivalence
5. $q \vee r$ Resolution
6. $\cancel{r \to s}$ Premise 3
7. $s \vee \neg r$ Logical Equivalence

$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$	Resolution

Rules of Inference (7/9)

Example4 – Same as Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

$\neg q \rightarrow s$

1.	$\cancel{p \rightarrow q}$	Premise 1
2.	$\cancel{\neg p \vee q}$	Logical Equivalence
3.	$\cancel{\neg p \rightarrow r}$	Premise 2
4.	$\cancel{p \vee r}$	Logical Equivalence
5.	$q \vee r$	Resolution
6.	$\cancel{r \rightarrow s}$	Premise 3
7.	$s \vee \neg r$	Logical Equivalence
8.	$q \vee s$	Resolution

$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Rules of Inference (7/9)

Example4 – Same as Example3

$p \rightarrow q$

$\neg p \rightarrow r$

$r \rightarrow s$

$\neg q \rightarrow s$

1.	$\cancel{p \rightarrow q}$	Premise 1
2.	$\cancel{\neg p \vee q}$	Logical Equivalence
3.	$\cancel{\neg p \rightarrow r}$	Premise 2
4.	$\cancel{p \vee r}$	Logical Equivalence
5.	$\cancel{q \vee r}$	Resolution
6.	$\cancel{r \rightarrow s}$	Premise 3
7.	$\cancel{s \vee \neg r}$	Logical Equivalence
8.	$q \vee s$	Resolution

$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

Rules of Inference (7/9)

Example4 – Same as Example3

$p \to q$

$\neg p \to r$

$r \to s$

$\neg q \to s$

1. $\cancel{p \to q}$ Premise 1
2. $\cancel{\neg p \vee q}$ Logical Equivalence
3. $\cancel{\neg p \to r}$ Premise 2
4. $\cancel{p \vee r}$ Logical Equivalence
5. $\cancel{q \vee r}$ Resolution
6. $\cancel{r \to s}$ Premise 3
7. $\cancel{s \vee \neg r}$ Logical Equivalence
8. $\cancel{q \vee s}$ Resolution
9. $\neg q \to s$ Logical Equivalence

Logical Equivalences Involving Conditional Statements.

$\boxed{p \to q \equiv \neg p \vee q}$

$p \to q \equiv \neg q \to \neg p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(p \to \neg q)$

Example4 – Same as Example3

$p \to q$

$\neg p \to r$

$r \to s$

$\boxed{\neg q \to s}$

1. $\cancel{p \to q}$ Premise 1
2. $\cancel{\neg p \vee q}$ Logical Equivalence
3. $\cancel{\neg p \to r}$ Premise 2
4. $\cancel{p \vee r}$ Logical Equivalence
5. $\cancel{q \vee r}$ Resolution
6. $\cancel{r \to s}$ Premise 3
7. $\cancel{s \vee \neg r}$ Logical Equivalence
8. $\cancel{q \vee s}$ Resolution
9. $\boxed{\neg q \to s}$ Logical Equivalence

Logical Equivalences Involving Conditional Statements.

$\boxed{p \to q \equiv \neg p \vee q}$

$p \to q \equiv \neg q \to \neg p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(p \to \neg q)$

Rules of Inference (8/9)

Rules of Inference for Quantified Statements

TABLE 2 **Rules of Inference for Quantified Statements.**

Rule of Inference	*Name*
$\dfrac{\forall x P(x)}{\therefore\ P(c)}$	Universal instantiation
$\dfrac{P(c) \text{ for an arbitrary } c}{\therefore\ \forall x P(x)}$	Universal generalization
$\dfrac{\exists x P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation
$\dfrac{P(c) \text{ for some element } c}{\therefore\ \exists x P(x)}$	Existential generalization

Rules of Inference (9/9)

Example1

Show that the premises “A student in this class has not read the book”, and “Everyone in this class passed the first exam” imply the conclusion that “Someone who passed the first exam has not read the book”.

Rules of Inference (9/9)

Example1

Show that the premises “A student in this class has not read the book”, and “Everyone in this class passed the first exam” imply the conclusion that “Someone who passed the first exam has not read the book”.

$P(x)$: x in this class

$Q(x)$: x has read the book

$S(x)$: x passed the first exam

Rules of Inference (9/9)

Example1

Show that the premises "A student in this class has not read the book", and "Everyone in this class passed the first exam" imply the conclusion that "Someone who passed the first exam has not read the book".

$P(x)$: x in this class

$Q(x)$: x has read the book

$S(x)$: x passed the first exam

Premises:

$\exists x\big(P(x) \wedge \neg Q(x)\big)$

$\forall x\big(P(x) \rightarrow S(x)\big)$

Conclusion:

$\exists x\big(S(x) \wedge \neg Q(x)\big)$

Rules of Inference (9/9)

Example1

1. $\exists x(P(x) \wedge \neg Q(x))$ Premise 1

$P(x)$: x in this class

$Q(x)$: x has read the book

$S(x)$: x passed the first exam

Premises:

$\exists x(P(x) \wedge \neg Q(x))$

$\forall x(P(x) \rightarrow S(x))$

Conclusion:

$\exists x(S(x) \wedge \neg Q(x))$

$\dfrac{\exists x P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation

Rules of Inference (9/9)

Example1

$P(x)$: x in this class

$Q(x)$: x has read the book

$S(x)$: x passed the first exam

Premises:

$\exists x\left(P(x) \wedge \neg Q(x)\right)$

$\forall x\left(P(x) \rightarrow S(x)\right)$

Conclusion:

$\exists x\left(S(x) \wedge \neg Q(x)\right)$

1. $\exists x\left(P(x) \wedge \neg Q(x)\right)$ Premise 1
2. $P(a) \wedge \neg Q(a)$ Existential instantiation

$\dfrac{\exists x P(x)}{\therefore\ P(c) \text{ for some element } c}$	Existential instantiation

Rules of Inference (9/9)

Example1

$P(x)$: x in this class

$Q(x)$: x has read the book

$S(x)$: x passed the first exam

Premises:

$\exists x(P(x) \land \neg Q(x))$

$\forall x(P(x) \to S(x))$

Conclusion:

$\exists x(S(x) \land \neg Q(x))$

1. $\exists x(P(x) \land \neg Q(x))$ Premise 1
2. $P(a) \land \neg Q(a)$ Existential instantiation

$\begin{array}{l} p \land q \\ \hline \therefore p \end{array}$	$(p \land q) \to p$	Simplification

Rules of Inference (9/9)

Example1

$P(x)$: x in this class

$Q(x)$: x has read the book

$S(x)$: x passed the first exam

Premises:

$\exists x(P(x) \wedge \neg Q(x))$

$\forall x(P(x) \rightarrow S(x))$

Conclusion:

$\exists x(S(x) \wedge \neg Q(x))$

1.	$\exists x(P(x) \wedge \neg Q(x))$	Premise 1
2.	$P(a) \wedge \neg Q(a)$	Existential instantiation
3.	$P(a)$	Simplification
4.	$\neg Q(a)$	Simplification

$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array}$	$(p \wedge q) \rightarrow p$	Simplification

Rules of Inference (9/9)

Example1

$P(x)$: x in this class

$Q(x)$: x has read the book

$S(x)$: x passed the first exam

Premises:

$\exists x\left(P(x) \wedge \neg Q(x)\right)$

$\forall x\left(P(x) \rightarrow S(x)\right)$

Conclusion:

$\exists x\left(S(x) \wedge \neg Q(x)\right)$

1.	$\exists x\left(P(x) \wedge \neg Q(x)\right)$	Premise 1
2.	$P(a) \wedge \neg Q(a)$	Existential instantiation
3.	$P(a)$	Simplification
4.	$\neg Q(a)$	Simplification
5.	$\forall x\left(P(x) \rightarrow S(x)\right)$	Premise 2

Rules of Inference (9/9)

Example1

$P(x)$: x in this class

$Q(x)$: x has read the book

$S(x)$: x passed the first exam

Premises:

$\exists x(P(x) \land \neg Q(x))$

$\forall x(P(x) \rightarrow S(x))$

Conclusion:

$\exists x(S(x) \land \neg Q(x))$

1.	$\exists x(P(x) \land \neg Q(x))$	Premise 1
2.	$P(a) \land \neg Q(a)$	Existential instantiation
3.	$\boxed{P(a)}$	Simplification
4.	$\neg Q(a)$	Simplification
5.	$\boxed{\forall x(P(x) \rightarrow S(x))}$	Premise 2

p $p \rightarrow q$ $\therefore q$	$(p \land (p \rightarrow q)) \rightarrow q$	Modus ponens

Rules of Inference (9/9)

Example1

$P(x)$: x in this class

$Q(x)$: x has read the book

$S(x)$: x passed the first exam

Premises:

$\exists x(P(x) \wedge \neg Q(x))$

$\forall x(P(x) \rightarrow S(x))$

Conclusion:

$\exists x(S(x) \wedge \neg Q(x))$

1.	$\exists x(P(x) \wedge \neg Q(x))$	Premise 1
2.	$P(a) \wedge \neg Q(a)$	Existential instantiation
3.	$\boxed{P(a)}$	Simplification
4.	$\neg Q(a)$	Simplification
5.	$\boxed{\forall x(P(x) \rightarrow S(x))}$	Premise 2
6.	$S(a)$	Modus ponens

p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens

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7.	$S(a) \land \neg Q(a)$	Conjunction

p q $\therefore p \land q$	$((p) \land (q)) \rightarrow (p \land q)$	Conjunction

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$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

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7.	$S(a) \wedge \neg Q(a)$	Conjunction
8.	$\exists x\left(S(x) \wedge \neg Q(x)\right)$	Existential generalization

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