CS1101
Discrete Mathematics

Chapter 01
The Foundations: Logic and Proofs

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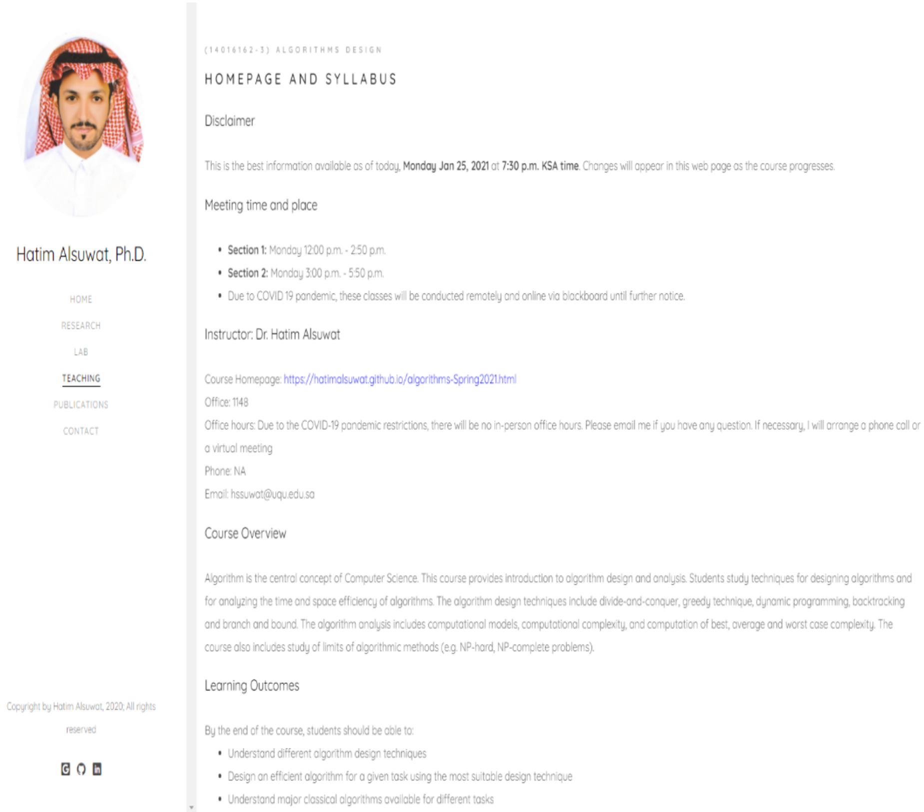
Faculty of Computers and Information Systems

UMM ALQURA UNIVERSITY

Fall 2022

Course Information

https://hatimalsuwat.github.io/DM1-Fall2022.html



Communication:

- Announcements on webpage/ emails/ blackboard
- Questions? Email me.
- Staff email: hssuwat@uqu.edu.sa

Course technology:

- Website
- UQU Blackboard
- Regular homework
- Help us make it awesome!

Course Information

- Course Website https://hatimalsuwat.github.io/DM1-Fall2022.html
- Discussion:
  - Please ask any question during the lecture (don’t be shy)
  - There is no such thing as a stupid question.
  - Answer others’ questions - if you know the answer ;-)
  - Learn from others’ questions and answers

Course Information

- **Grading:**
  - **Midterm Exam: 20%**
  - **Homework Assignments: 20%**
  - **Quizzes: 20%**
  - **Final Exam: 40%**

- **Total score that can be achieved: 100**

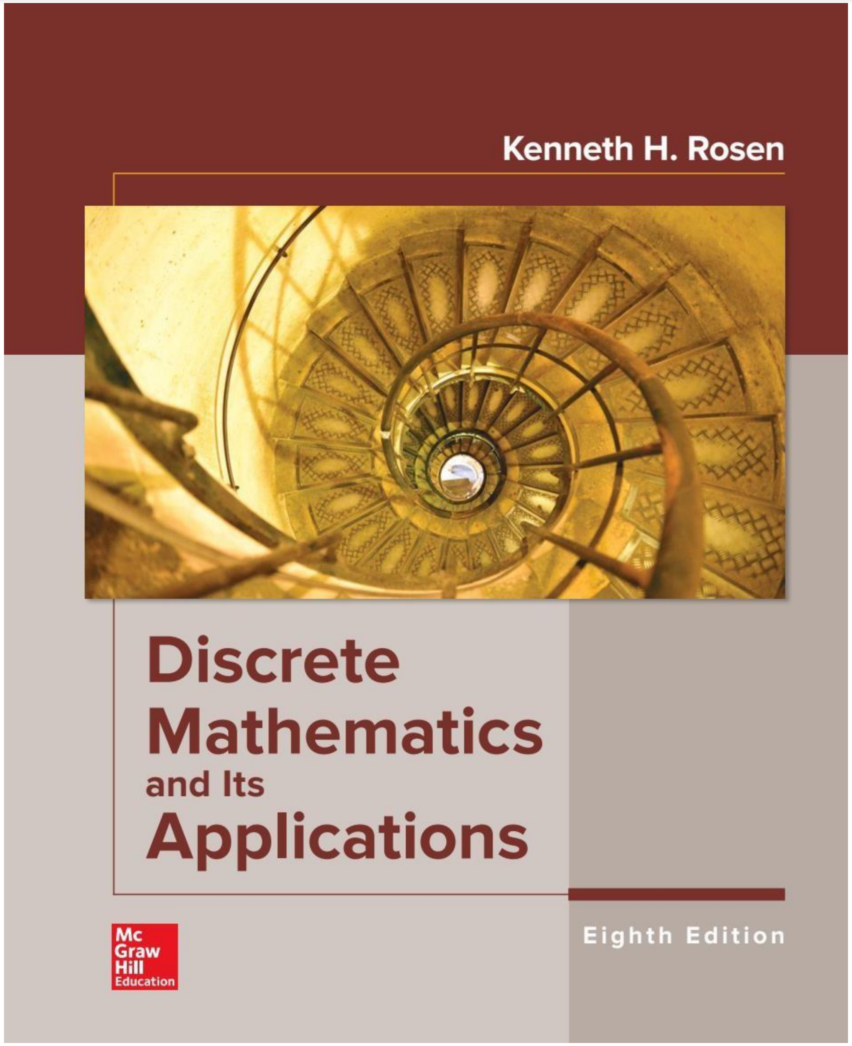
Course Information

- **Meeting time and place:**
  - **Office:** Department of Computer Science (office #1148)
  - **Office hours:** Please email me if you have any question. If necessary, I will arrange a phone call or in-person meeting
  - **Email:** Hssuwat@uqu.edu.sa

Course Information: Feedback

- Please give feedback positive or negative as early as you can via email.

Lectures Reference



Textbook 2018

Course Objectives

- Learn how to think mathematically.
- Grasp the basic logical and reasoning mechanisms of mathematical thought.
- Acquire logic and proof as the basics for abstract thinking.
- Improve problem-solving skills.
- Grasp the basic elements of induction, recursion, combination and discrete structures.

DM is a Gateway Course

Topics in discrete mathematics will be important in many courses that you will take in the future:

- **Computer Science:** Computer Architecture, Data Structures, Algorithms, Programming Languages, Compilers, Computer Security, Databases, Artificial Intelligence, Networking, Graphics, Game Design, Theory of Computation, ……
- **Mathematics:** Logic, Set Theory, Probability, Number Theory, Abstract Algebra, Combinatorics, Graph Theory, Game Theory, Network Optimization, …
- **Other Disciplines:** You may find concepts learned here useful in courses in philosophy, economics, linguistics, and other departments.

Course Syllabus

- The Foundations: Logic and Proofs.
- Basic Structures: Sets, Functions, Sequences, and Sums.
- Algorithms.
- Induction and Recursion.

Chapter 1: Logic

- Introduction to Propositional Logic.
- Compound Propositions.
- Applications of Propositional Logic.
- Propositional Equivalences.

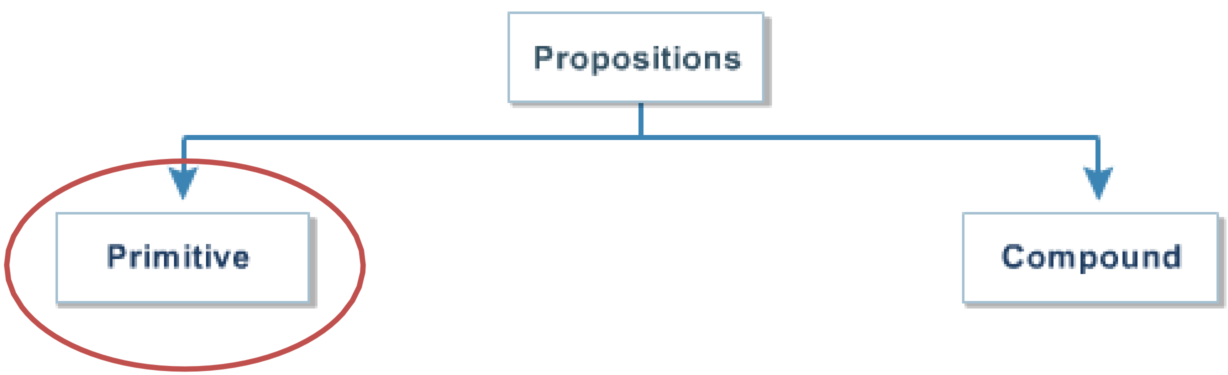
Introduction to Propositional Logic (1/4)

What is Logic?

- Logic is the discipline that deals with the methods of reasoning.
- On an elementary level, logic provides rules and techniques for determining whether a given argument is valid.
- Logical reasoning is used in mathematics to prove theorems.

Introduction to Propositional Logic (2/4)

- The basic building blocks of logic is **Proposition**
- A proposition (or statement) is a **declarative sentence** that is either **true** or **false**, but **not both**.
- The area of logic that deals with propositions is called **propositional logics**.



Introduction to Propositional Logic (3/4)

Examples:

Propositions	Truth value
$2 + 3 = 5$	True
$5 - 2 = 1$	False
Today is Friday	False
$x + 3 = 7$, for $x = 4$	True
Cairo is the capital of Egypt	True

Sentences	Is a Proposition
What time is it?	**Not** propositions
Read this carefully.	**Not** propositions
$x + 3 = 7$	**Not** propositions

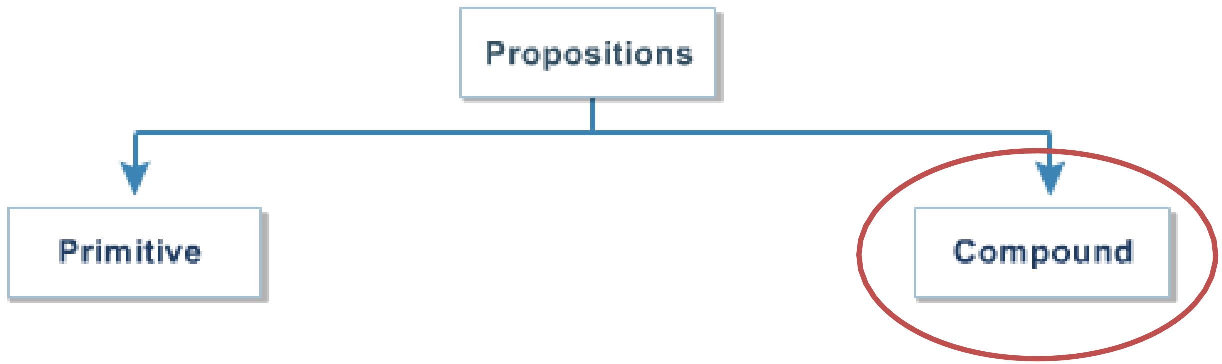
Introduction to Propositional Logic (4/4)

- We use letters to denote propositional variables $\boldsymbol{p, q, r, s, \ldots}$
- The truth value of a proposition is true, denoted by **T**, if it is a true proposition and false, denoted by **F**, if it is a false proposition.

Compound Propositions (1/23)

Compound Proposition

- Compound Propositions are formed from existing propositions using **logical operators**.



Compound Propositions (2/23)

Negation

DEFINITION 1

Let p be a proposition. The *negation of* p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

Other notations you might see are $\sim p$, $-p$, p', Np, and $!p$.

Compound Propositions (3/23)

Example

Find the negation of the proposition

p: "Cairo is the capital of Egypt"

Compound Propositions (4/23)

Example: Solution

Find the negation of the proposition

p: "Cairo is the capital of Egypt"

The negation is

$\neg p$: "It is not the case that Cairo is the capital of Egypt"

This negation can be more simply expressed as

$\neg p$: "Cairo is not the capital of Egypt"

Truth Table

- Truth Table: is a table that gives the truth values of a compound statement.

The Truth Table for the Negation of a Proposition

Proposition

Truth Values

p	$\neg p$
T	
F	

Truth Table

- Truth Table: is a table that gives the truth values of a compound statement.

The Truth Table for the Negation of a Proposition

Proposition

Truth Values

p	$\neg p$
T	F
F	T

Compound Propositions (6/23)

Negation

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Compound Propositions (7/23)

Logical Connectives

DEFINITION 2

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Example

p: Today is Friday.

q: It is raining today.

$p \wedge q$: Today is Friday and it is raining today.

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Compound Propositions (8/23)

Logical Connectives

DEFINITION 3

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \lor q$, is the proposition "p or q." The disjunction $p \lor q$ is false when both p and q are false and is true otherwise.

Example

p: Today is Friday.

q: It is raining today.

$p \lor q$: Today is Friday or it is raining today.

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

Compound Propositions (9/23)

Logical Connectives

DEFINITION 4

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$ (or p XOR q), is the proposition that is true when exactly one of p and q is true and is false otherwise.

Example

p : They are parents.

q : They are children.

$p \oplus q$: They are parents or children but not both.

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Compound Propositions (10/23)

Logical Connectives

DEFINITION 5

Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"

TABLE 5 **The Truth Table for the Conditional Statement $p \rightarrow q$.**

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

Compound Propositions (10/23)

Logical Connectives

DEFINITION 5

Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"

TABLE 5 **The Truth Table for the Conditional Statement $p \rightarrow q$.**

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

Compound Propositions (11/23)

Logical Connectives

EXAMPLE 1

"If you get 100% on the final, then you will get an A."

If you manage to get a 100% on the final, then you would expect to receive an A. If you do not get 100% you may or may not receive an A depending on other factors. However, if you do get 100%, but the professor does not give you an A, you will feel cheated.

Compound Propositions (12/23)

Logical Connectives

EXAMPLE 2

Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

Compound Propositions (12/23)

Logical Connectives

EXAMPLE 2

Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \to q$ as a statement in English.

"If Maria learns discrete mathematics, then she will find a good job."

"Maria will find a good job when she learns discrete mathematics."

Compound Propositions (13/23)

Logical Connectives

EXAMPLE 3

"If today is Friday, then $2 + 3 = 6$."

Compound Propositions (13/23)

Logical Connectives

EXAMPLE 3

"If today is Friday, then $2 + 3 = 6$."

is true every day except Friday, even though $2 + 3 = 6$ is false.

Compound Propositions (14/23)

Logical Connectives

DEFINITION 6

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

TABLE 6 **The Truth Table for the Biconditional $p \leftrightarrow q$.**

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

"p is necessary and sufficient for q"
"if p then q, and conversely"
"p iff q." "p exactly when q."

"You can take the flight if and only if you buy a ticket."

Compound Propositions (15/23)

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

Compound Propositions (16/23)

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

TABLE 7 **The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.**

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T				
T	F				
F	T				
F	F				

Compound Propositions (16/23)

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \lor \neg q) \rightarrow (p \land q)$.

TABLE 7 The Truth Table of $(p \lor \neg q) \rightarrow (p \land q)$.

p	q	$\neg q$	$p \lor \neg q$	$p \land q$	$(p \lor \neg q) \rightarrow (p \land q)$
T	T	F			
T	F	T			
F	T	F			
F	F	T			

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T		
T	F	T	T		
F	T	F	F		
F	F	T	T		

Compound Propositions (16/23)

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	
T	F	T	T	F	
F	T	F	F	F	
F	F	T	T	F	

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Compound Propositions (17/23)

Precedence of Logical Operators

TABLE 8
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Compound Propositions (18/23)

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

Compound Propositions (19/23)

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

p	q	r	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \rightarrow r$

Compound Propositions (19/23)

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

p	q	r	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \rightarrow r$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Compound Propositions (19/23)

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

p	q	r	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \rightarrow r$
T	T	T	F		
T	T	F	F		
T	F	T	T		
T	F	F	T		
F	T	T	F		
F	T	F	F		
F	F	T	T		
F	F	F	T		

Compound Propositions (19/23)

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \to r$

p	q	r	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \to r$
T	T	T	F	F	
T	T	F	F	F	
T	F	T	T	T	
T	F	F	T	T	
F	T	T	F	F	
F	T	F	F	F	
F	F	T	T	F	
F	F	F	T	F	

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

p	q	r	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

Compound Propositions (20/23)

Logic and Bit Operations

- Computers represent information using **bits**. A bit is a symbol with two possible values, namely, 0 (zero) and 1 (one).

Truth Value	***Bit***
T	1
F	0

Compound Propositions (21/23)

Computer Bit Operations

- We will also use the notation OR, AND, and XOR for the operators $\vee$, $\wedge$, and $\oplus$, as is done in various programming languages.

TABLE 9 **Table for the Bit Operators *OR*, *AND*, and *XOR*.**

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Compound Propositions (22/23)

Bit Strings

- Information is often represented using bit strings, which are lists of zeros and ones. When this is done, operations on the bit strings can be used to manipulate this information.

A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

$\boxed{\text{101010011 is a bit string of length nine.}}$

Compound Propositions (23/23)

Example

- Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings 01 1011 0110 and 11 0001 1101

01 1011 0110	
11 0001 1101	
11 1011 1111	bitwise *OR*
01 0001 0100	bitwise *AND*
10 1010 1011	bitwise *XOR*

Applications of Propositional Logic (1/13)

1- Translating English Sentences.

2- System Specifications.

3- Boolean Searches.

4- Logic Puzzles.

5- Logic Circuits.

Applications of Propositional Logic (1/13)

1- Translating English Sentences.

2- System Specifications.

3- Boolean Searches.

4- Logic Puzzles.

5- Logic Circuits.

Applications of Propositional Logic (1/13)

Translating English Sentences

- There are many reasons to translate English sentences into expressions involving propositional variables and logical connectives. In particular, English (and every other human language) is often ambiguous. Translating sentences into compound statements (and other types of logical expressions, which we will introduce later in this chapter) removes the ambiguity.

Applications of Propositional Logic (3/13)

Example 1

You can access the Internet from campus only if you are a computer science major or you are not a student.

Applications of Propositional Logic (4/13)

Example 1

You can access the Internet from campus only if you are a computer science major or you are not a student.

Solution:

Let p, q and r be the propositions:

p: You can access the Internet from campus.
q: You are a computer science major.
r: You are a student.

Applications of Propositional Logic (4/13)

Example 1

(You can access the Internet from campus) only if (you are a computer science major or you are not a student).

Solution:

Let p, q and r be the propositions:

$p \rightarrow q$
"p only if q"

p: You can access the Internet from campus.
q: You are a computer science major.
r: You are a student.

Applications of Propositional Logic (5/13)

Example 1

(You can access the Internet from campus) only if (you are a computer science major or you are not a student).

Solution:

Let p, q and r be the propositions:

$p \rightarrow q$
"p only if q"

p: You can access the Internet from campus.
q: You are a computer science major.
r: You are a student.

The sentence can be represented by logic as

$p \rightarrow (q \vee \neg r)$

Applications of Propositional Logic (6/13)

Example 2

The automated reply cannot be sent when the file system is full.

Example 2

The automated reply cannot be sent when the file system is full.

Solution:

Let p and q be the propositions:

$p \rightarrow q$

"q when p"

p: The automated reply can be sent .

q: The file system is full.

Applications of Propositional Logic (8/13)

Example 2

(The automated reply cannot be sent) when (the file system is full.)

Solution:

Let p and q be the propositions:

$p \rightarrow q$
"q when p"

p: The automated reply can be sent .
q: The file system is full.

The sentence can be represented by logic as

$\boxed{q \rightarrow \neg p}$

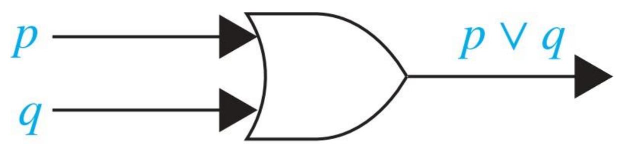
Applications of Propositional Logic (9/13)

Logic Circuits

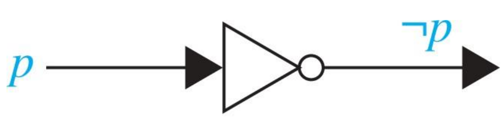
- A logic circuit (or digital circuit) receives input signals $p_1, p_2, \ldots, p_n$, each a bit [either 0 (off) or 1 (on)], and produces output signals $s_1, s_2, \ldots, s_n$, each a bit.
- In this course, we will restrict our attention to logic circuits with a single output signal; in general, digital circuits may have multiple outputs.

Logic Circuits

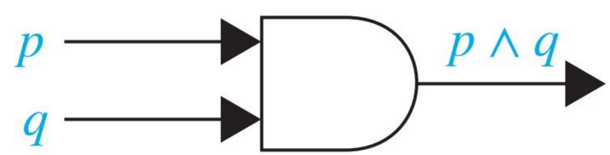
- Complicated digital circuits can be constructed from three basic circuits, called gates.



OR gate



Inverter

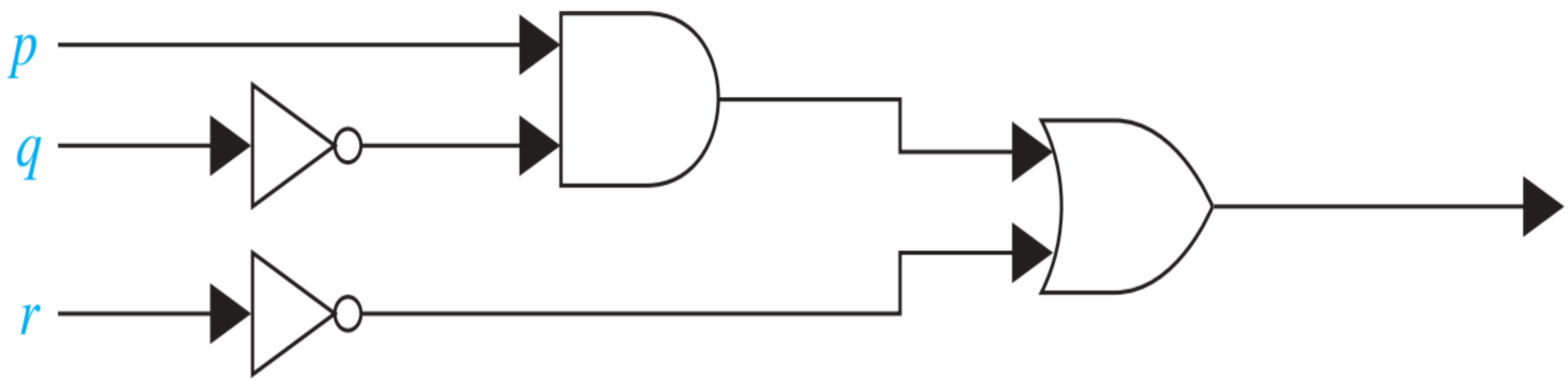


AND gate

Applications of Propositional Logic (11/13)

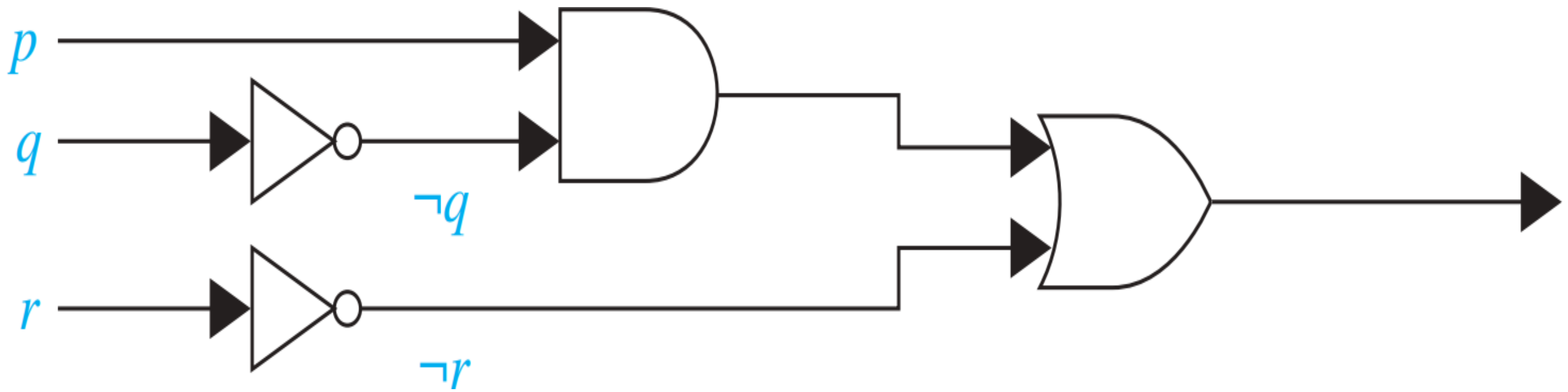
Example 1

- Determine the output for the combinatorial circuit in the following figure.



Example 1

- Determine the output for the combinatorial circuit in the following figure.



Example 1

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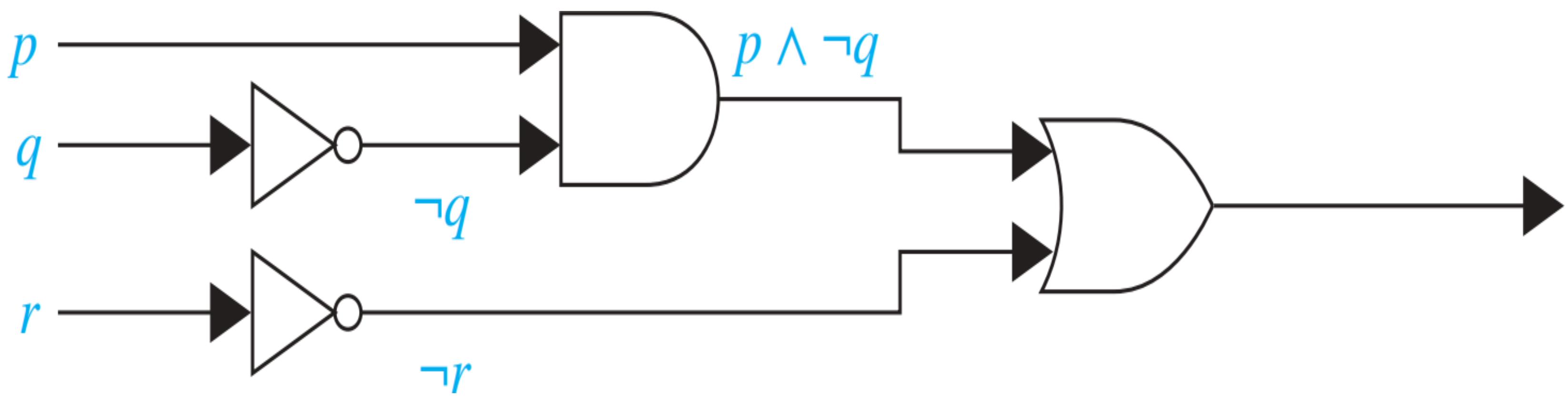
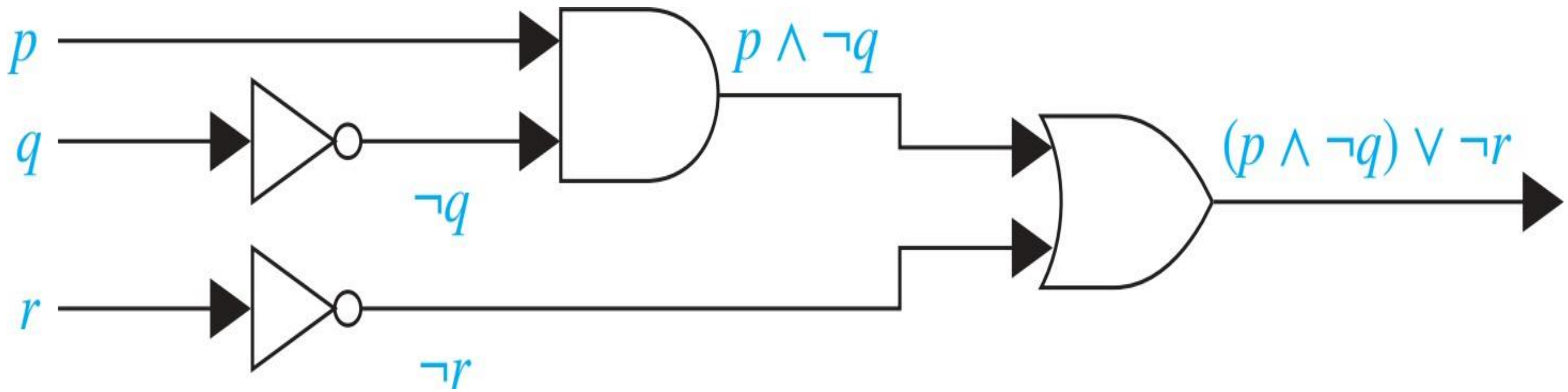


Applications of Propositional Logic (11/13)

Example 1

- Determine the output for the combinatorial circuit in the following figure.



Applications of Propositional Logic (12/13)

Example 2

- Build a digital circuit that produces the output

$$(p \lor \neg r) \quad \land \ (\neg p \quad \lor \ (q \ \lor \ \neg r))$$

when given input bits p, q, and r.

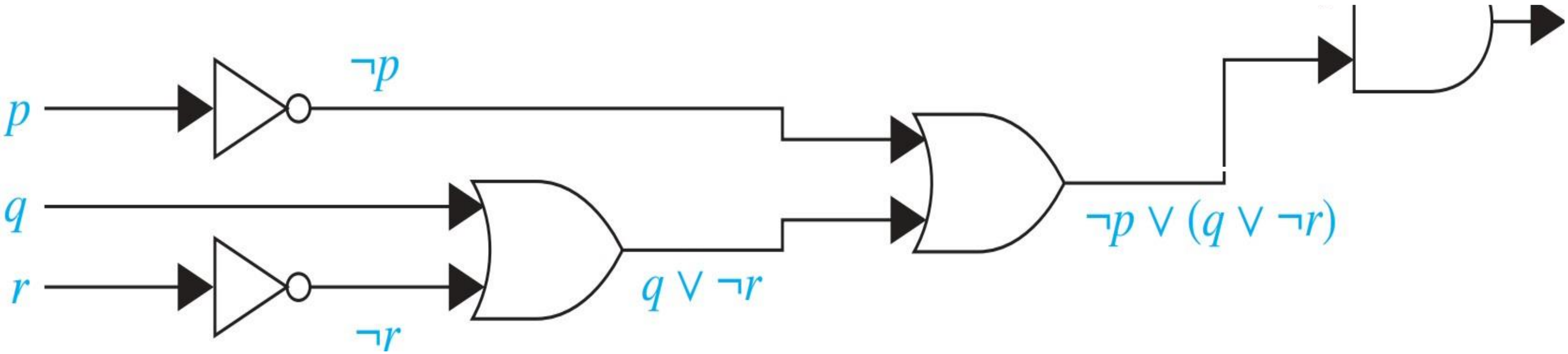
Applications of Propositional Logic (13/13)

Example 2

$$\boxed{(p \vee \neg r)} \wedge (\neg p \vee (q \vee \neg r))$$

Example 2

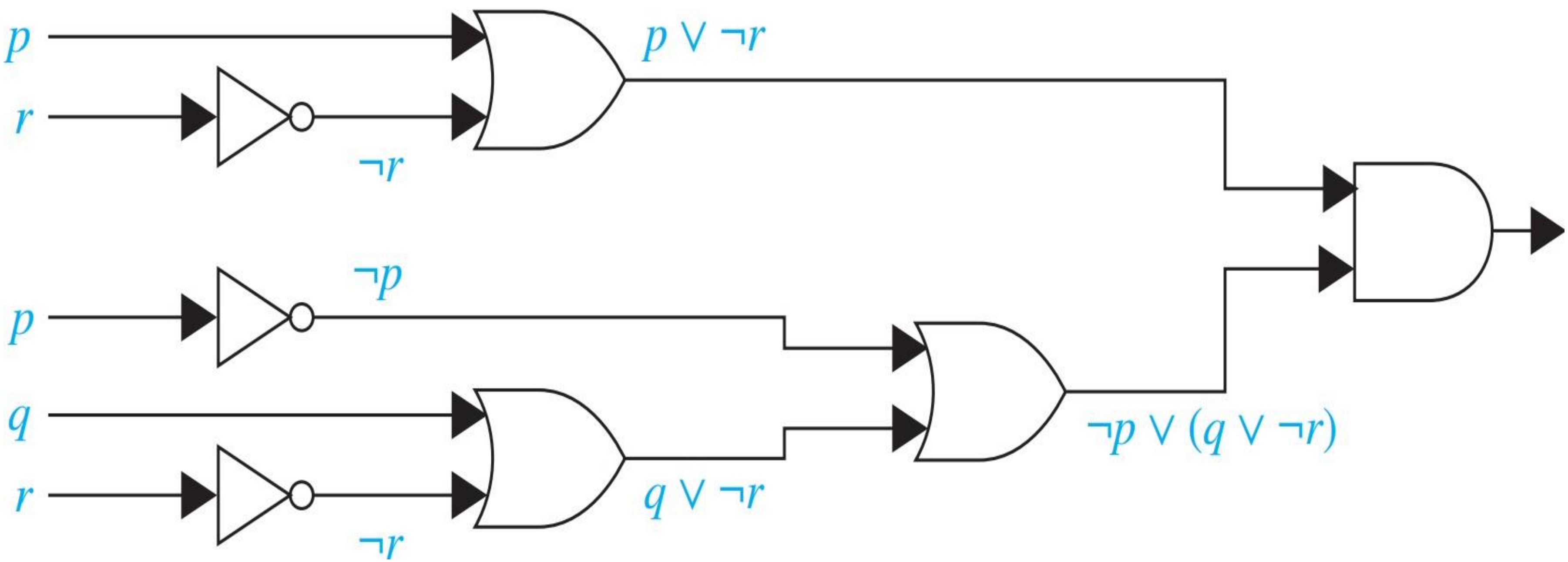
$$(p \vee \neg r) \quad \wedge \boxed{(\neg p \quad \vee \; (q \;\; \vee \;\; \neg r))}$$



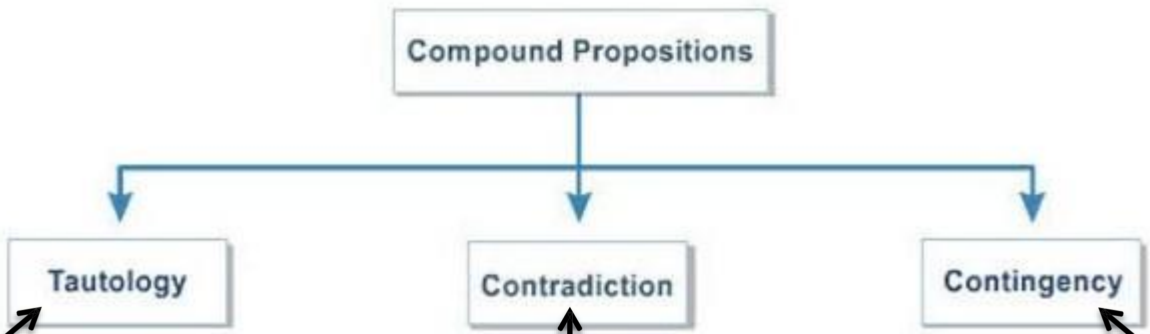
Applications of Propositional Logic (13/13)

Example 2

$$\boxed{(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))}$$



Compound Propositions Classification (1/2)



A compound proposition that is always true

A compound proposition that is always false

A compound proposition that is sometimes true and sometimes false

Compound Propositions Classification (2/2)

Example:

- Show that following conditional statement is a **tautology** by using truth table

$$(p \wedge q) \rightarrow p$$

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$

Compound Propositions Classification (2/2)

Example:

- Show that following conditional statement is a **tautology** by using truth table

$$(p \wedge q) \rightarrow p$$

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	**T**	**T**	**T**
T	**F**	**F**	**T**
F	**T**	**F**	**T**
F	**F**	**F**	**T**

Logical Equivalences (1/6)

Logically equivalent:

The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

Compound propositions that have the **same truth values** in **all** possible cases are called **logically equivalent**.

$\equiv$

$\Leftrightarrow$

Logical Equivalences (2/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T					
T	F					
F	T					
F	F					

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T				
T	F	T				
F	T	T				
F	F	F				

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F			
T	F	T	F			
F	T	T	F			
F	F	F	T			

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	
T	F	T	F	F	T	
F	T	T	F	T	F	
F	F	F	T	T	T	

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Logical Equivalences (3/6)

Example1:

Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

Truth Tables for $\neg(p \vee q)$ and $\neg p \wedge \neg q$.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Logical Equivalences (4/6)

Logical Equivalences (1/3)

Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws

Logical Equivalences (4/6)

Logical Equivalences (2/3)

Logical Equivalences.	
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Logical Equivalences (4/6)

Logical Equivalences (3/3)

Logical Equivalences Involving Conditional Statements.
$p \to q \equiv \neg p \lor q$
$p \to q \equiv \neg q \to \neg p$
$p \lor q \equiv \neg p \to q$
$p \land q \equiv \neg(p \to \neg q)$

Logical Equivalences Involving Biconditional Statements.
$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Logical Equivalences (5/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$ by the second De Morgan law

$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$$\begin{aligned} \neg(p \vee (\neg p \wedge q)) &\equiv \neg p \wedge \neg(\neg p \wedge q) && \text{by the second De Morgan law} \\ &\equiv \neg p \wedge [\neg(\neg p) \vee \neg q] && \text{by the first De Morgan law} \end{aligned}$$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law

$\neg(\neg p) \equiv p$	Double negation law

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law
$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q))$	$\equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
	$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
	$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law
	$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
	$\equiv \mathbf{F} \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv \mathbf{F}$

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee(\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q))$	$\equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
	$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
	$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law
	$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
	$\equiv \mathbf{F} \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv \mathbf{F}$
	$\equiv (\neg p \wedge \neg q) \vee \mathbf{F}$	by the commutative law for disjunction
	$\equiv \neg p \wedge \neg q$	by the identity law for $\mathbf{F}$

$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\neg(p \vee (\neg p \wedge q))$	$\equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
	$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
	$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law
	$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
	$\equiv \mathbf{F} \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv \mathbf{F}$
	$\equiv (\neg p \wedge \neg q) \vee \mathbf{F}$	by the commutative law for disjunction
	$\equiv \neg p \wedge \neg q$	by the identity law for $\mathbf{F}$

$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws

Logical Equivalences (6/6)

Example 1:

Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$\boxed{\neg(p \vee (\neg p \wedge q))}$	$\equiv \neg p \wedge \neg(\neg p \wedge q)$	by the second De Morgan law
	$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$	by the first De Morgan law
	$\equiv \neg p \wedge (p \vee \neg q)$	by the double negation law
	$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	by the second distributive law
	$\equiv \mathbf{F} \vee (\neg p \wedge \neg q)$	because $\neg p \wedge p \equiv \mathbf{F}$
	$\equiv (\neg p \wedge \neg q) \vee \mathbf{F}$	by the commutative law for disjunction
	$\boxed{\equiv \neg p \wedge \neg q}$	by the identity law for $\mathbf{F}$

Next class

- 1.4 Predicates and Quantifiers
- 1.5 Nested Quantifiers
- 1.6 Rules of Inference