EF	Description	Suggestion
x	Specific Share	i
G	Index/Benchmark Portfolio	M
B, S	Side of Market	S
T	Time Interval	T
P	Price Range	R
	% around last execution price	
	I suggest price range	
	around last execution price	
λ	Liquidity	${}^{S}\lambda_t^i\,(T,R)$
Vol	Volume	
	of what?	
$n(T)$	Number of trades	$N(T)$
	within Time Interval	
Trades	traded volume	${}^{x}q_t^i$
ΔCLOB^P	Increase of CLOB volume	
	within price range	
		${}^{S}q_t^{R,i}$
(t_i, t_{i+1})	Time segment between two trades	
FF_x	Free Float of share	F_t^i
	Weight of Stock i in Index	ω_t^i
i	running index of time	τ
	within an interval	
i	running index of trades	n

1 EF Formulas

$$
\begin{aligned}
Liquidity &= f(P,T,Vol) && (1)\\
&\sum_{i=0}^{n(T)} \text{Trades}_x && (2)\\
&\sum_{i=0}^{n(T)} \Delta\text{CLOB}^P_{x(t_i,t_{i+1})} && (3)\\
&\frac{\sum_{i=0}^{n(T)} \text{Trades}_x + \sum_{i=0}^{n(T)} \Delta\text{CLOB}^P_{x(t_i,t_{i+1})}}{FF_x} && (4)\\
&\frac{\sum_{i=0}^{n(T)} \text{Trades}_G + \sum_{i=0}^{n(T)} \Delta\text{CLOB}^P_{G(t_i,t_{i+1})}}{FF_G} && (5)\\
\text{Sell Side:}\lambda^S_x(T,p) &= \frac{\left(\sum_{i=0}^{n(T)} \text{Trades}_x + \sum_{i=0}^{n(T)} \Delta\text{CLOB}^P_{x(t_i,t_{i+1})}\right)\cdot FF_G}{\left(\sum_{i=0}^{n(T)} \text{Trades}_G + \sum_{i=0}^{n(T)} \Delta\text{CLOB}^P_{G(t_i,t_{i+1})}\right)\cdot FF_x} && (6)\\
\text{Buy Side:}\lambda^B_x(T,p) &= \frac{\left(\sum_{i=0}^{n(T)} \text{Trades}_x + \sum_{i=0}^{n(T)} \Delta\text{CLOB}^P_{x(t_i,t_{i+1})}\right)\cdot FF_G}{\left(\sum_{i=0}^{n(T)} \text{Trades}_G + \sum_{i=0}^{n(T)} \Delta\text{CLOB}^P_{G(t_i,t_{i+1})}\right)\cdot FF_x} && (7)\\
\lambda_T &= \frac{\lambda_S + \lambda_B}{2} && (8)
\end{aligned}
$$

2 Revised Formulas

Based on Formula 5 of Francioni and Egloff we define for each side of the market S, stock i, and market index M:

$$
\begin{aligned}
{}^S\ell^i_t(T,R) &= \frac{\max\{{}^Sq^{R,i}_{t+\tau}\}^T_{\tau=0} + \sum_{\tau=0}^{T} {}^xq^i_{t+\tau}}{F^i_t}, && (5)\\
{}^S\ell^M_t(T,R) &= \sum_{i=1}^{I} \omega^i_t \cdot {}^S\ell^i_t(T,R), && (6)
\end{aligned}
$$

where ${}^Sq^{R,i^i}$ denotes the sum of all bid respectively asked quantities for share i at price ${}^Sp^i \in \left[(1-R)\cdot {}^ep^i; (1+R)\cdot {}^ep^i\right]$. T denotes the length of an interval, for which the statistic is computed, R denotes the range around the most recent trade price. Francioni and Egloff state that *only increases* in the order book should be considered in computing the sum of $\Delta\,{}^Sq^{R,i}_{(t_\tau,t_{\tau+1})}$ in equations (5) and(6). To operationalize this idea, we propose to use the maximal length of the order book during time interval T.

We propose to investigate to ways of computing the liquidity measure λ. First, as suggested by Francioni and Egloff simply as ratio of the stock and market ℓ (see equation 7a). Alternatively, analogous to the CAPM, where the β is the regression coefficient when regressing excess stock returns on the market portfolio, we can define $\hat{\lambda}$ as in equation 7b:

$$
\begin{aligned}
{}^S\lambda^i_t(T,R) &= \frac{{}^S\ell^i_t(T,R)}{{}^S\ell^M_t(T,R)} && (7a)\\
{}^S\ell^i_t(T,R) &= {}^S\hat{\lambda}^i_t(T,R)\cdot {}^S\,\ell^M_t(T,R) + \epsilon_t. && (7b)
\end{aligned}
$$

Open Questions

1. End of Day Auction: wird auf die Volumina aus dem Limit Order Buch zugegriffen?
2. Warum gibt es die Standard drei Auktionen pro Tag? Warum manchmal mehr?
3. run ℓ regression in logs, then then intercept should be the ℓ-ratio and the slope have an elasticity interpretation
4. by construction, the λ is relative to the market, overall liquidity fluctuations might not be captured in λ but only in ℓ.
5. What are useful 'case/event' studies to establish meaningfulness of the measure?
   - US-Thanksgiving (or generally bank holidays in US)
   - Neue nicht mehr handelsfreie Tage: e.g. 3. Oktober 2023 und 2024
   - CoViD: Maerz Marktturbulenz
   - Stock specific news releases: quarterly/annual results
   - Januar und/oder Dezember
6. Compare 'our' liquidity measure to other measures, which ones?
7. Looking at $\frac{VolAuction}{VolBuy+VolSell+VolAuction}$ we see for first quarter 2020 that median share of auction volume is one third in daily trading, very rarely above 60%.
8. Looks like there is a positive association between auction volume and ℓ for each stock. We had a hypothesis that less liquid stocks have higher importance of auctions. Need to look at this with precise graph/stats rather than the proxies we are currently using.
9. Alternative Liquidty Measures:
   - ask1$_price$: $bid1_pricespreadTradingVolume$

- Read Engle/Lange, use their references

10. Estimate AR on daily (latent) volume,