Stubborn consensus for multi robots' control

Master's Degree in Artificial Intelligence and Robotics

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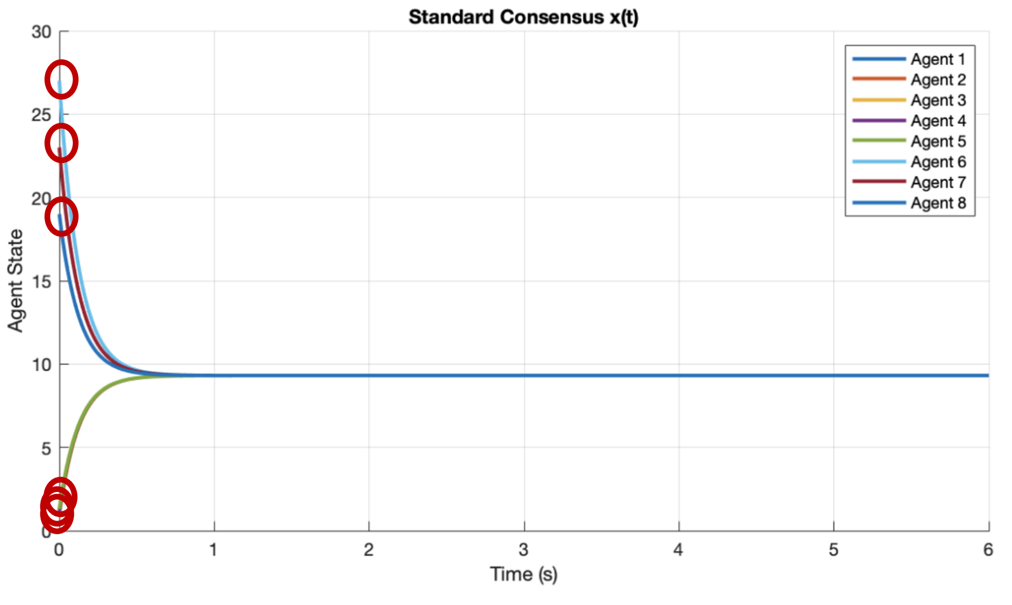
Problem Introduction

Introduction

- In standard consensus, agents eventually agree on a constant state, which for **rendez-vous** problem correspond to a common position
- For undirected graphs it correspond to the **arithmetic mean** of the agent's initial conditions
- The arithmetic mean gives the same weight to all agents, so each initial state influences the final agreement equally
- But what if there are **large outliers** (e.g. located much farther from the origin than the majority)?.....
- … the final agreement is **pulled away** from the majority because of the outliers!



How to neglect large outliers?

Introduction

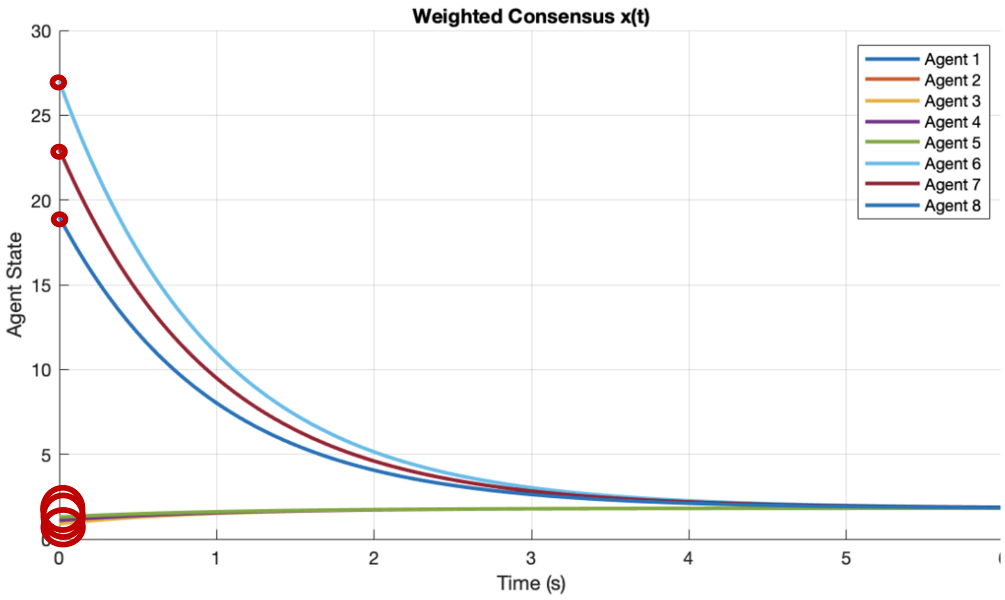
- Two different approaches have been explored to reduce the influence of agents that behave as large outliers

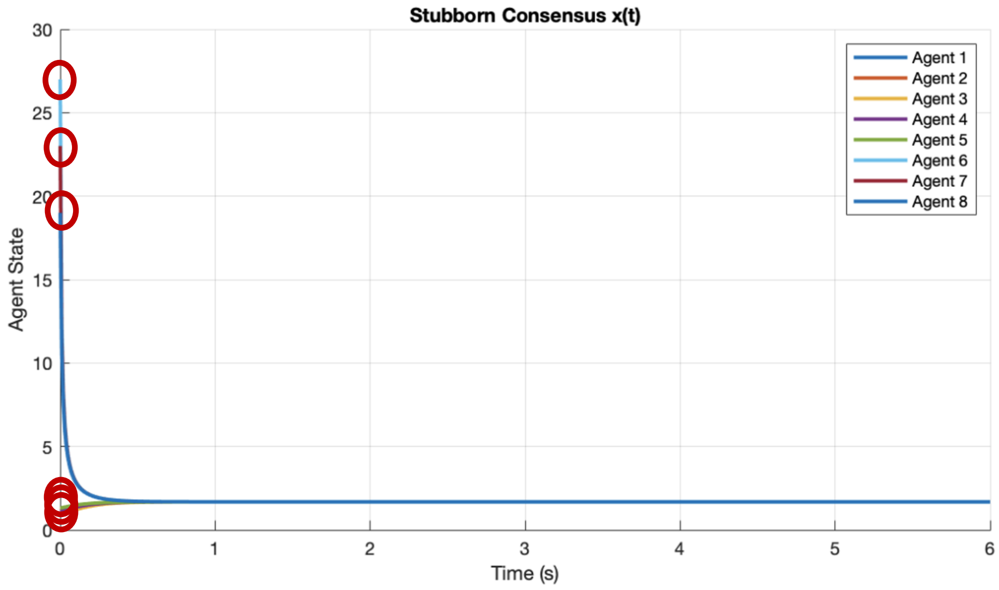
WEIGHTED CONSENSUS

- assign a weight to each agent making them converge to a **weighted average** of their initial conditions

STUBBORN CONSENSUS

- make the agents converse to the **harmonic mean** of the initial conditions thus neglecting the large outliers





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Proposed Approach

Stubborn consensus
Proposed Approach

- **Objective**: design a consensus protocol so that all agents converge to the **harmonic mean** of their initial states

- introducing a **state transformation** for each agent :

$$\begin{cases} y_i(t) = \dfrac{1}{x_i(t)} \\ \dot{y}_i = -\dfrac{\dot{x}_i}{x_i^2} \end{cases}$$

- then we can apply the classic consensus protocol to the transformed states with the Laplacian $L = D - A$

- the explicit solution is:

$$y(t) = e^{-Lt} y(0)$$

- therefore, in the original variables we obtain:

$$x_i(t) = \frac{1}{\left[e^{-Lt} \frac{1}{x(0)}\right]_i}$$

- our goal is to make the denominator of x to converge to the arithmetic mean, thus x will converge to the harmonic mean

Stubborn consensus

Proposed Approach

- **FOR UNDIRECTED GRAPHS**
  - L is symmetric, positive semidefinite
  - if the graph is **connected**:

$$\lambda_1 = 0 \; and \; 0 < \lambda_2 \leq \cdots \leq \lambda_N$$

  - by diagonalizing L by an orthogonal matrix U

$$e^{-Lt}\frac{1}{x(0)} = u_1 \, u_1^T \frac{1}{x(0)} + \sum_{i=2}^{N} u_i \, u_i^T e^{-\lambda_i t}\frac{1}{x(0)}$$

  - considering the limit for $t \to \infty$:

$$x_i(t) = \frac{1}{\left[e^{-Lt}\frac{1}{x(0)}\right]_i} \longrightarrow \frac{1}{\frac{1}{N}\sum_{k=1}^{N}\frac{1}{x_k(0)}}$$

$$\forall i$$

- **FOR DIRECTED GRAPHS**
  - L is no longer symmetric, but $L\,\mathbf{1} = 0$
  - if the graph is **balanced** $\mathbf{1}^T L = 0$
  - assuming that $\boldsymbol{rank(L) = N - 1}$, with N the number of agents:

$$\lambda_1 = 0 \; and \; 0 < Re(\lambda_2) \leq \cdots \leq Re(\lambda_N)$$

  - by applying the **Jordan decomposition** of L and analysing the limit as $t \to \infty$:

$$x_i(t) = \frac{1}{\left[e^{-Lt}\frac{1}{x(0)}\right]_i} \longrightarrow \frac{1}{\frac{1}{N}\sum_{k=1}^{N}\frac{1}{x_k(0)}}$$

$$\forall i$$

Weighted consensus

Proposed Approach

- **Objective**: consensus protocol that enables all agents to converge to a **weighted average** of their initial states

- By defining the **non-symmetric weighted adjacency matrix W** as

$$W(i,j) = A(i,j) \cdot w(j)$$

and the weights as the **absolute value of the inverse of the agents' initial conditions**, the corresponding graph Laplacian becomes:

$$L = D - W$$

- If $rank(L) = N - 1$, where N is the number of agents:

$$\lambda_1 = 0 \; and \; 0 < Re(\lambda_2) \leq \cdots \leq Re(\lambda_N)$$

- By using **Jordan decomposition** and imposing as $t \rightarrow \infty$, one finds that:

$$\lim_{t \rightarrow \infty} x(t) = (p_1 \cdot q_1^T) \cdot x(0)$$

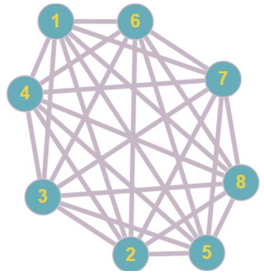
With p_1 and q_1 being right and left eigenvectors of L associated to $\lambda_1 = 0$

- Imposing normalization condition $q_1^T \cdot p_1 = 1$ and π (normalized version of q_1):

$$\lim_{t \rightarrow \infty} x(t) = (\pi \cdot x(0)) \cdot \mathbf{1}$$

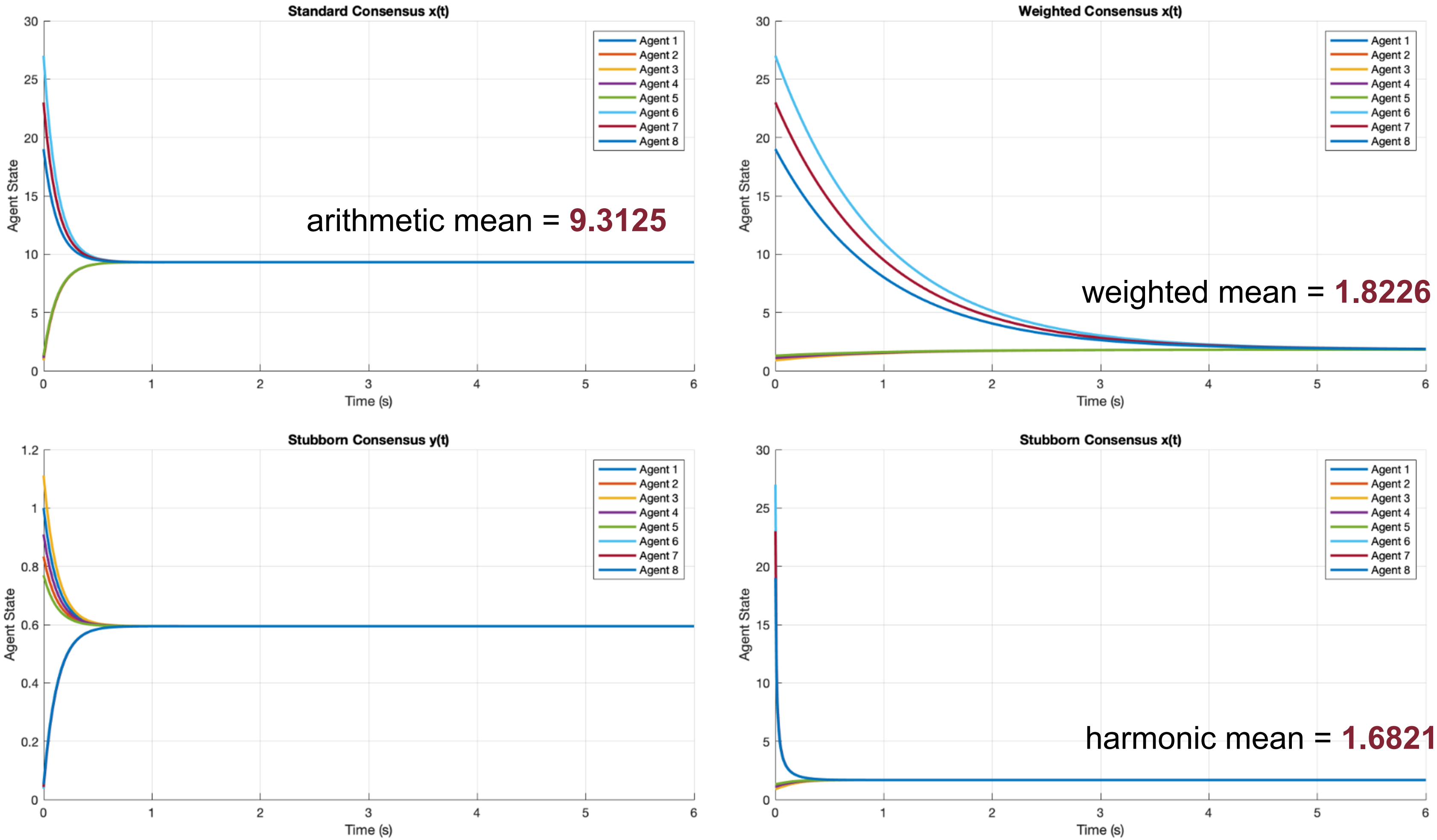
Experiment I

Experiments



Setup

- **Agent dimension:** 1D
  $x_0 = [1\ 1.2\ 0.9\ 1.1\ 1.3\ \mathbf{27}\ \mathbf{23}\ \mathbf{19}\,]$
- **Dynamics:** simple integrator
- **Topology:** fully connected, undirected graph

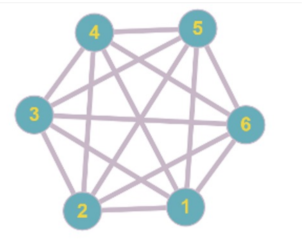


$\lambda_2^{std} = 8.00$

$\lambda_2^{wgt} = 1.0083$

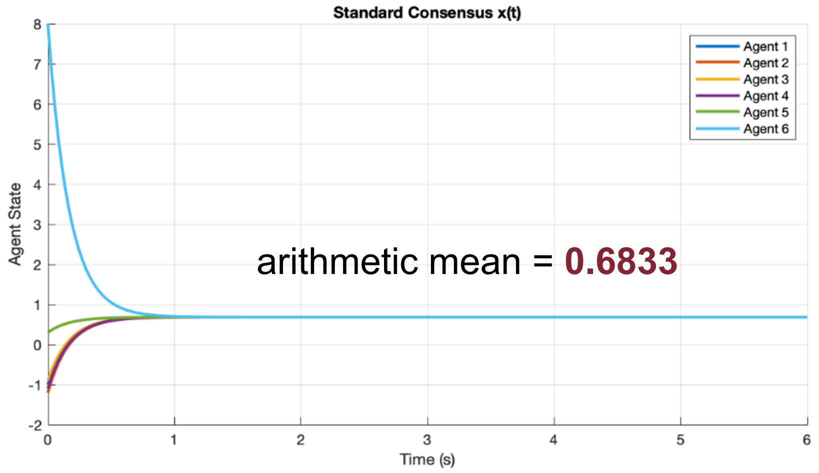
Experiment II

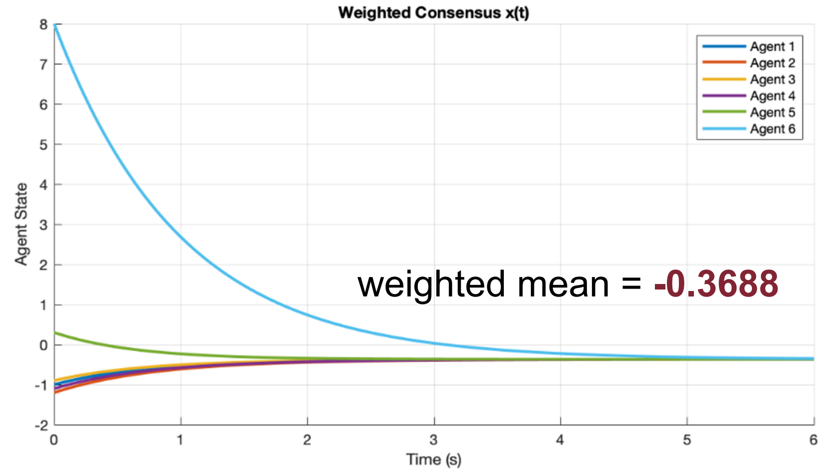
Experiments

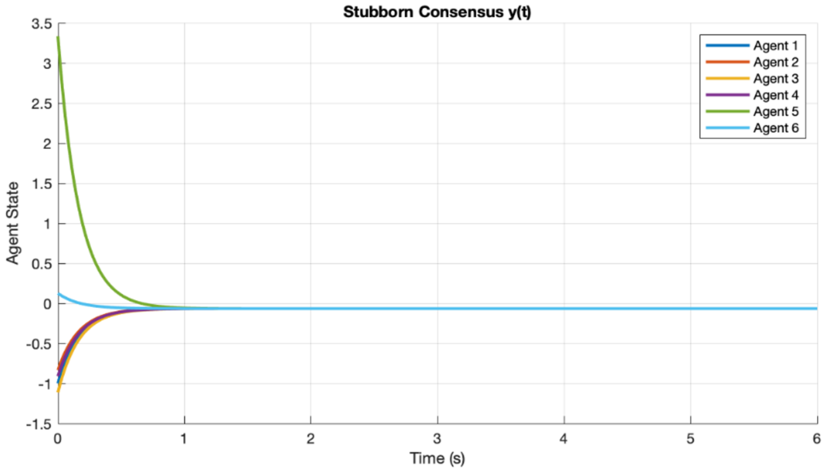


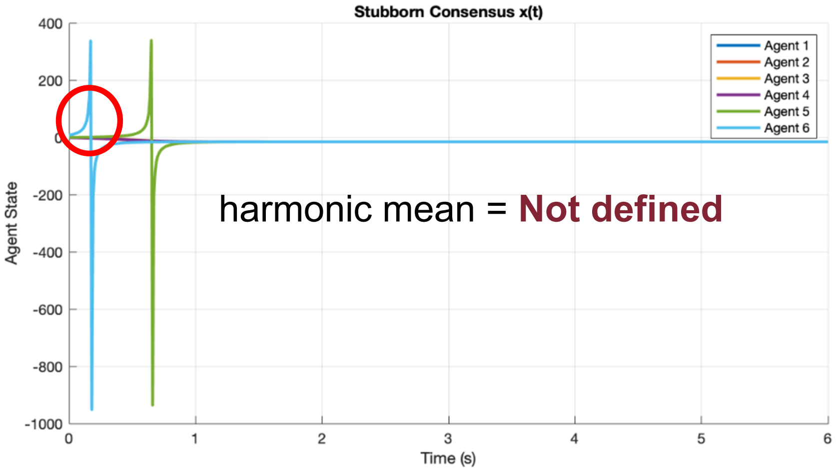
Setup

- **Agent dimension:** 1D
  $\mathbf{x}_0 = [\,-1 \quad -1.2 \quad -0.9 \quad -1.1 \quad 0.3 \quad 8]$
- **Dynamics:** simple integrator
- **Topology:** fully connected, undirected graph



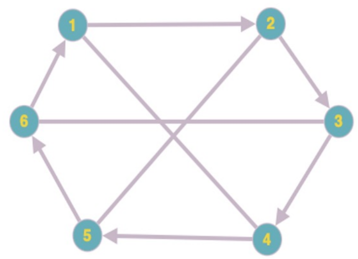






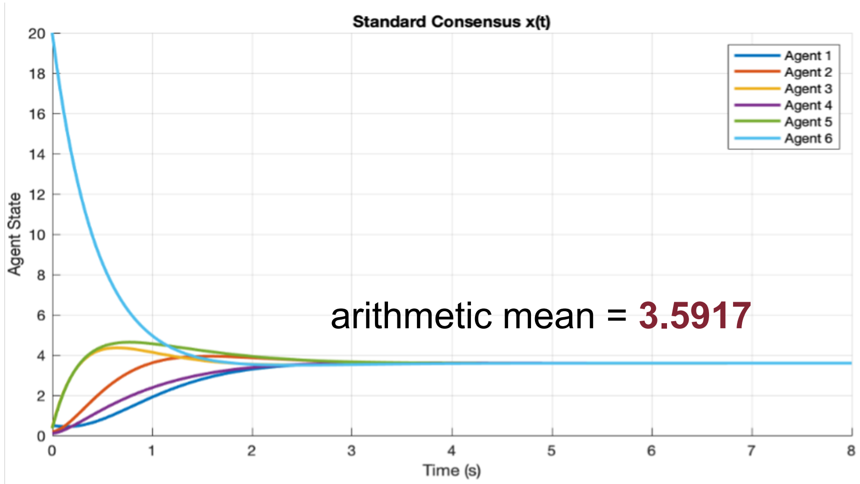
Experiment III

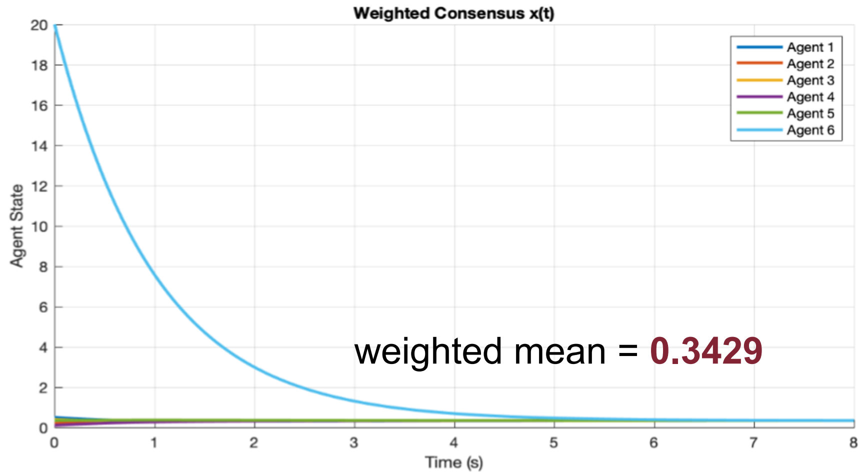
Experiments

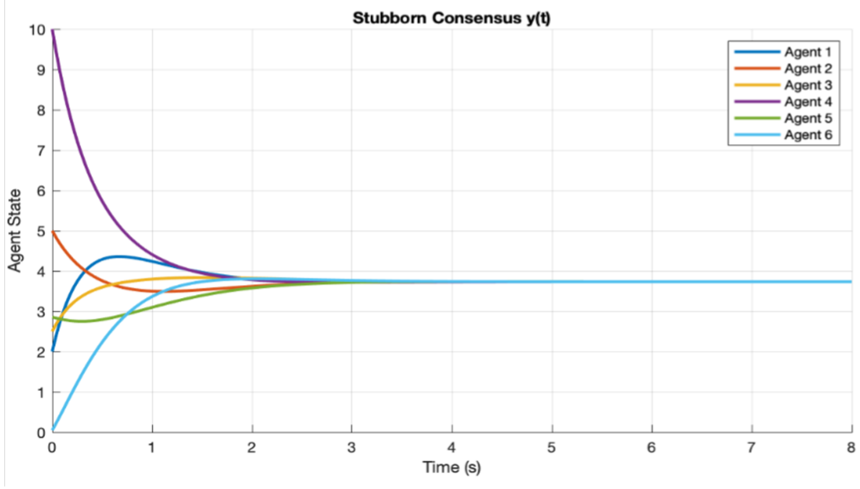


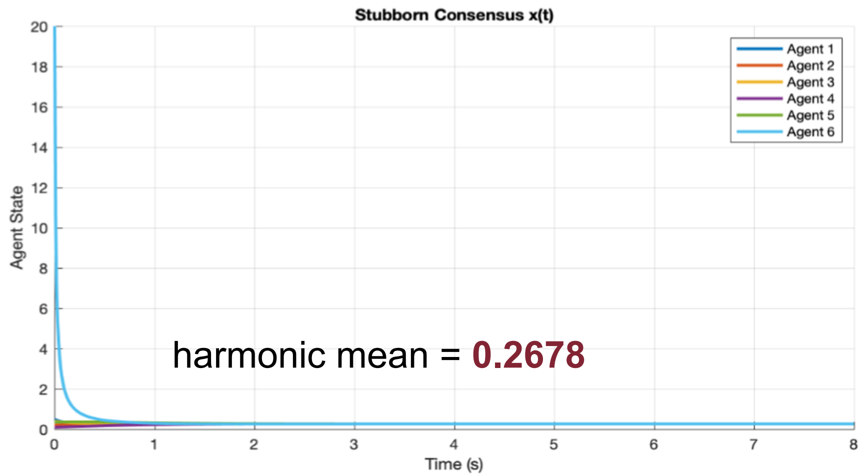
Setup

- **Agent dimension:** 1D
  $\mathbf{x}_0 = [\,0.5 \quad 0.2 \quad 0.4 \quad 0.1 \quad 0.35 \quad 20]$
- **Dynamics:** simple integrator
- **Topology: Balanced, directed graph** (rooted-out branching)





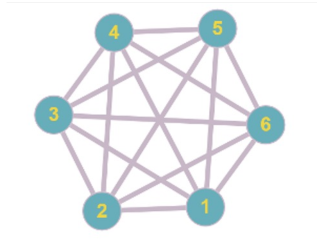




$\lambda_2^{std} = 1.5000 - 0.8660i$

$\lambda_2^{wgt} = 1.0000 - 0.0928i$

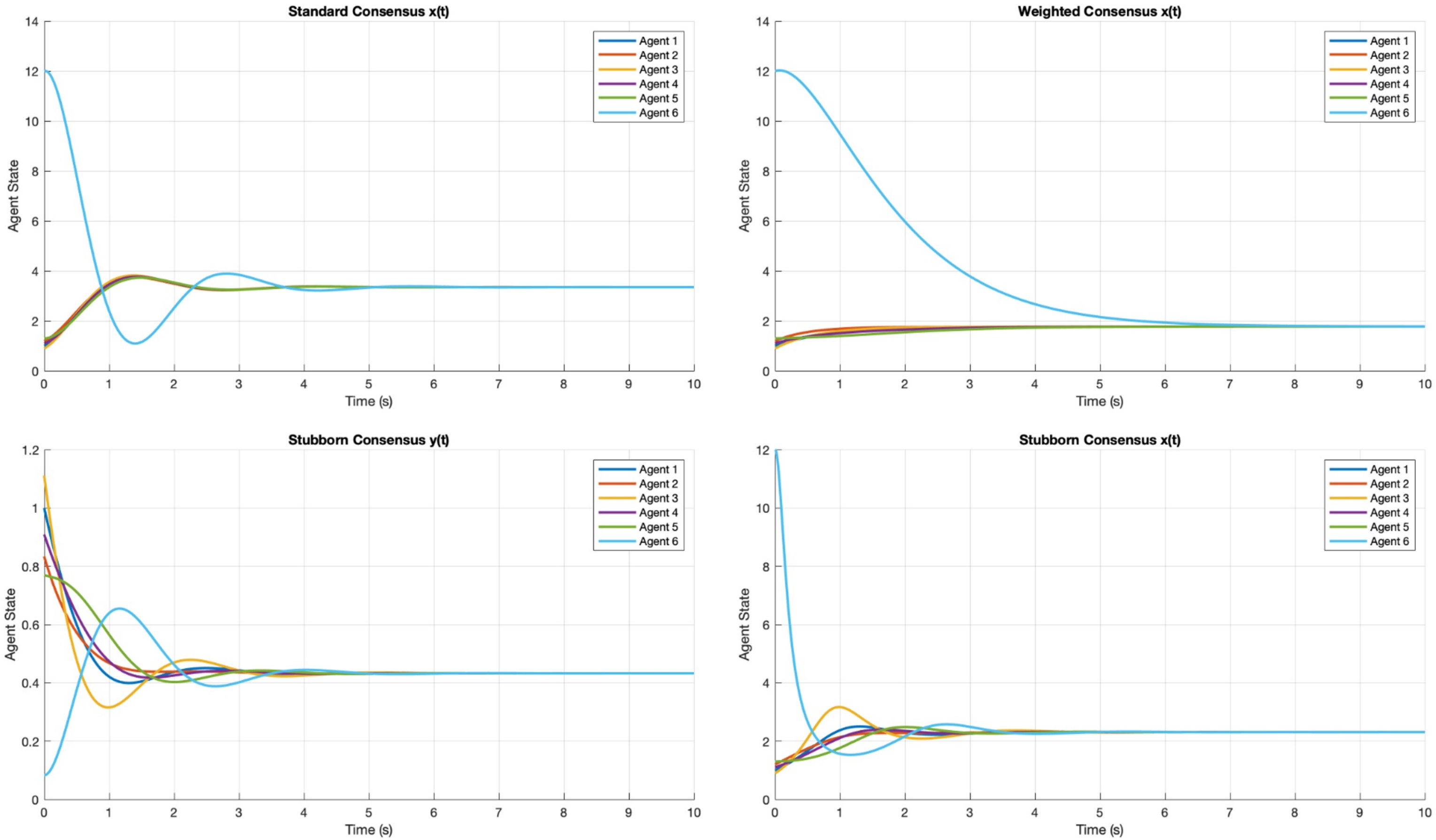
Experiment IV
Experiments



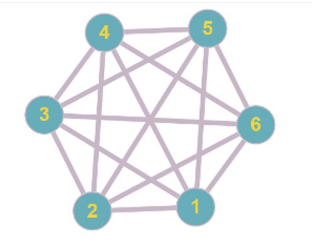
Setup

- **Agent dimension:** 1D
  x_0=[1 1.2 0.9 1.1 1.3 12]
  v_0=[1 1.1 1.4 0.8 0.1 0.8]
- **Dynamics:** double integrator
- **Topology:** fully connected, undirected graph

- ## Behaviour 1: Position convergence



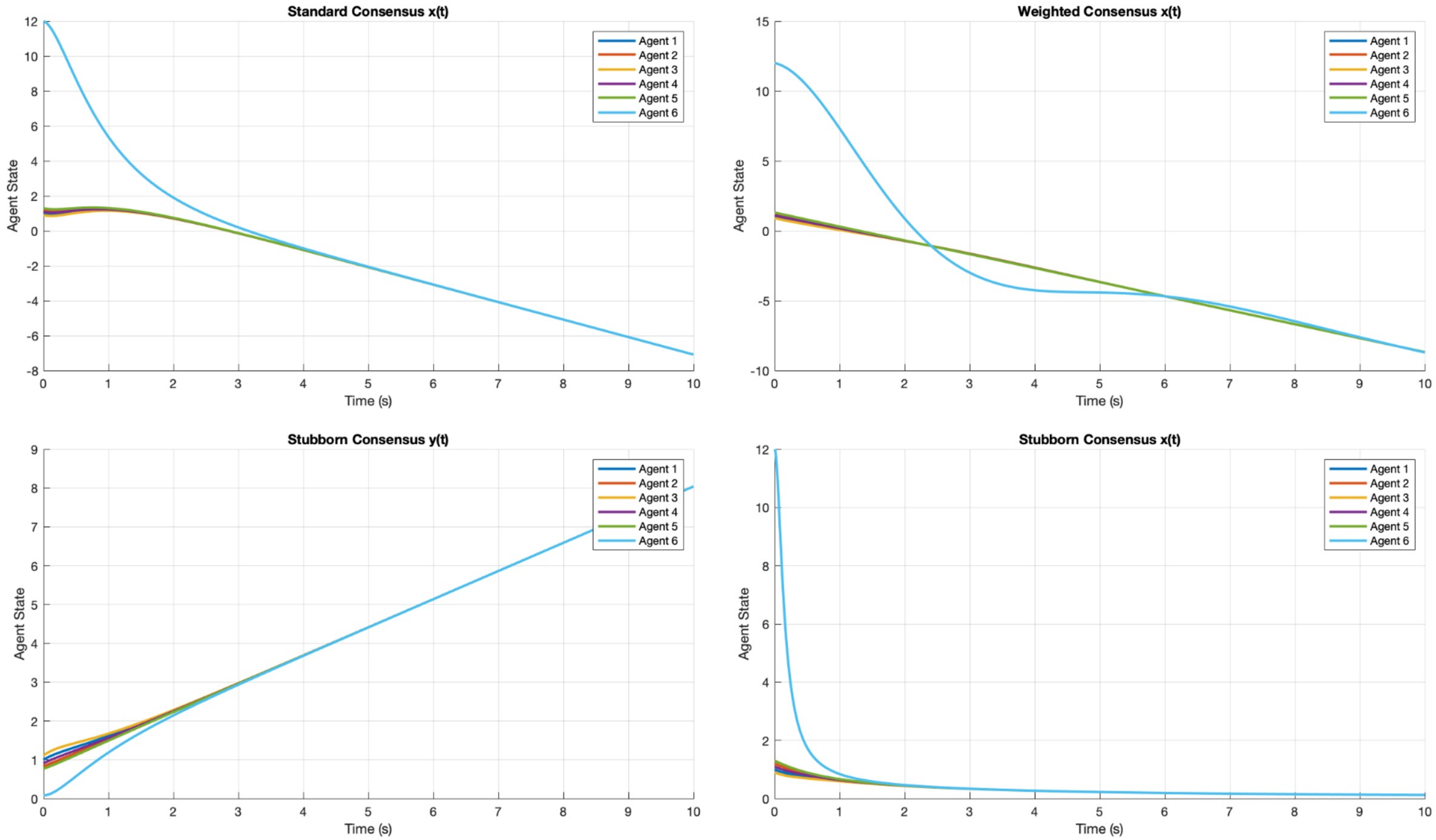
Experiment IV
Experiments



Setup

- **Agent dimension:** 1D
  x_0=[1 1.2 0.9 1.1 1.3 12]
  v_0=[-1 -1 -1 -1 -1 -1]
- **Dynamics:** double integrator
- **Topology:** fully connected, undirected graph

- **Behaviour 2: Travel together at same velocity**



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Conclusions

Conclusions

From the previous analysis we can conclude that:

- **Stubborn Consensus**:
  - Converges to **harmonic mean**
  - **Faster convergence**
  - For directed graphs **requires balance**
  - Harmonic mean not defined for **mixed values** (positive/negative initial state)

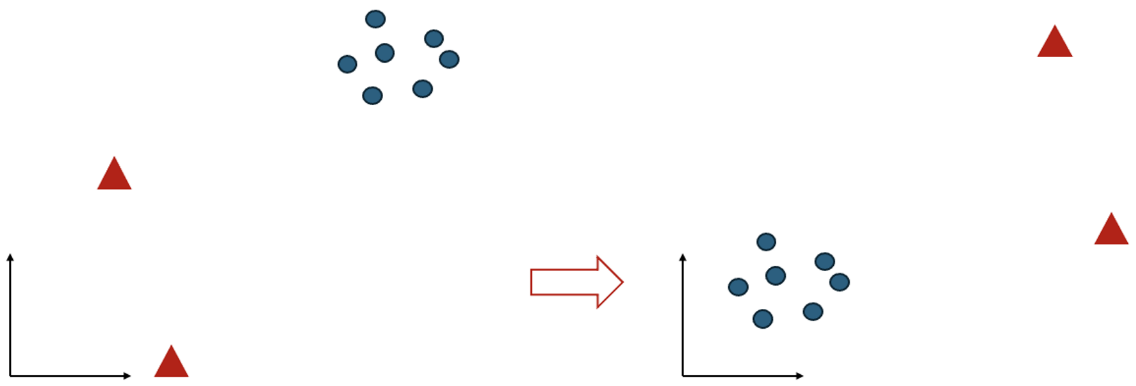
- **Weighted Consensus**:
  - Allows **mixed values**
  - **Slower convergence** (due to smaller connectivity eigenvalue)

Both this approaches have a **more robust behaviour** in the presence of outliers wrt the Standard approach.

Future Works

Conclusions

- Extend to **higher-dimensional states**.
- Consider **non-holonomic** constraints.
- Improve **outlier definition** (RANSAC strategy).
- Apply homogeneous transformations for **robustness**.



Thanks for your attention!

Reference

[1] R. Olfati-Saber , R. M. Murray, “Consensus Problems in Networks of Agents With Switching Topology and Time-Delays”, IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 49, NO. 9, SEPTEM-BER 2004.