Certifying C program correctness with respect to CompCert with VeriFast

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Abstract

VeriFast is a powerful tool for verification of various correctness properties of C programs using symbolic execution. However, VeriFast itself has not been verified. We present a proof-of-concept extension which generates a correctness certificate for each successful verification run individually. This certificate takes the form of a Coq script containing two proofs which, when successfully checked by Coq, together remove the need for trusting in the correctness of VeriFast itself.

The first proves a lemma expressing the correctness of the program with respect to a big step operational semantics developed by ourselves, intended to reflect VeriFast's interpretation of C. We have formalized this semantics in Coq as `cbsem`. This lemma is proven by symbolic execution in Coq, which in turn is implemented by transforming the exported AST of the program into a Coq proposition representing the symbolic execution performed by VeriFast itself.

The second proves the correctness of the same C program with respect to CompCert's Clight big step semantics. This proof simply applies our proof of the soundness of `cbsem` with respect to CompCert Clight to the first proof.

Contents

1 Introduction

VeriFast is a general verification tool for programs written in C [4]. If Veri-Fast reports "0 errors found", this is intended to imply that no execution

of the C program has undefined behavior, such as accesses of unallocated memory or data races.

VeriFast performs its verification in a modular fashion by symbolically executing each function individually. Whole program correctness is checked by verifying that each function call satisfies the preconditions for that function, which are provided by user annotations. VeriFast is written in OCaml and relies on an SMT solver for checking the assertions which allow it to conclude that an input program is correct with regards to its specification.

An open problem of course is that desirable qualities in a verification tool such as speed, ease of use and realism in supported language features increase the complexity of the verifier and grow the size of the code base which needs to be trusted. This makes verification of VeriFast itself challenging and prevents us from being sure that it is sound. So the question ultimately is: *what can the user really conclude when VeriFast reports "0 errors found"?*

This simple question involves more than "merely" straightforward implementation bugs within VeriFast. When VeriFast concludes that a program meets its specification, it has also made a large number of assumptions about the semantics of C which are not explicitly provided by the user's annotations.

In effect, the sum of these assumptions results in a version of C specific to VeriFast. We need to trust that this "VeriFast C" relates meaningfully to the version of C implemented by a particular compiler. In other words, VeriFast C should accept a usable but rather strict subset of programs that have defined behavior according to the ISO C standard and especially of the subset of programs which are accepted by this compiler.

More broadly, VeriFast relies on an implicit metatheory which must be assumed correct when trusting its conclusions. To provide a simple example of such a "metalemma": VeriFast concludes that the result of a division of two `int`s n and d is itself a valid `int` and not out-of-bounds ($\texttt{int}_{min} \leq n/d \leq \texttt{int}_{max}$) if: n and d themselves are not out-of-bounds; $d \neq 0$; and $\neg(n = \texttt{int}_{min} \ \wedge \ d = -1)$. From these three conditions it follows that the division result is not out-of-bounds. But the actual proof for this fact is never explicitly checked within VeriFast.

The question then becomes: can we provide an explicit list of all the assumptions that were made in implementing VeriFast? Better yet: can we prove *all of them* correct, preferably in a formal proof checker such as Coq? Finally, can we actually prove them correct *with respect to a third party formal semantics of C?*

In earlier work, Vogels *et al.* [12] already provided a precise definition in Coq for a subset of VeriFast's symbolic execution and proved that subset sound with respect to concrete program execution.

In this technical report, we present a different approach by providing a certification *for each individual run of VeriFast*. We extend VeriFast such that, for each successful verification, it *exports* a machine checkable proof

for the correctness of the program, not only with regards to our own formal semantics (called `cbsem`), but with regards to CompCert's *Clight semantics* as well.

CompCert [7] is a formally verified C compiler which is guaranteed to not introduce new behaviors into a program, ensuring that the guarantees provided by a formal verification tool will be preserved during compilation. Clight [2] is an intermediate C-like language used within CompCert that has pure expressions and implements assignments and function calls as statements. Therefore, proving correctness of the program with regards to Clight allows a VeriFast user to *entirely* shift trust from VeriFast to Coq's computational kernel and to CompCert Clight semantics.

1.1 Overview

The work that is presented in this report provides preliminary validation of our approach for a limited subset of VeriFast and C language features.

We begin our report in Section 2 with a concrete demonstration of our approach for a simple C program. We explain how VeriFast symbolically executes that program and explore the contents of the Coq artefact exported by VeriFast after verification of the program.

Section 3 begins by delimiting the subset of C language features and VeriFast constructs that we chose for our proof of concept. This subset is presented as an intermediate language, called VeriFast Cx, which is substantially similar to C but with some small tweaks to facilitate our work in Coq. The chosen C subset is strong enough to encode terminating and diverging executions and to allow expression of some simple undefined behavior, specifically integer overflows and division by zero.

Section 4 continues with presenting our first main result, which consists of a Coq function $\texttt{exec}_\texttt{f}$ that takes a VeriFast Cx function together with its specification and generates a shallow embedding of VeriFast's symbolic execution for that function into a Coq proposition. This proposition is called the *symbolic execution proposition* or SEP. We then describe our current approach towards automatically proving the SEP in Coq. Proving the SEP remains easy for now, exactly because of the limited scope of VeriFast Cx.

Section 5 presents `cbsem`, which is our formal operational semantics for VeriFast Cx expressions, statements and functions. We chose big step semantics to get started quickly. `cbsem` is actually comprised of two sets of rules, one for terminating programs and one for diverging programs. We present a singular predicate $\texttt{exec}_\texttt{f}^{\Downarrow}$ describing `cbsem`'s notion of correctness of a function. We conclude Section 5 with a theorem proving the soundness of $\texttt{exec}_\texttt{f}$ with regards to $\texttt{exec}_\texttt{f}^{\Downarrow}$, providing us with a method to prove correctness with regards to `cbsem` by symbolic execution.

Section 6 sets up a relation between `cbsem` and Clight's big step semantics for stores, expressions, statements and functions. These relations are set

up for a slightly smaller subset of VeriFast Cx which is still big enough to include the example program from Section 2. We then provide custom-made notions of *program correctness* for both `cbsem` and Clight, and conclude with a soundness theorem stating that correctness of a program in Clight follows from correctness of the equivalent program in `cbsem` (and therefore by symbolic execution in VeriFast).

We end this technical report with a brief discussion of related work (Section 7) and future work (Section 8). Appendix A provides instructions on building and using our development. Appendix B gives an overview of the modules found in our Coq development and links the concise notation used in this report to the names used in the Coq code. Finally, Appendix C shows the entire Coq script that is exported by our extension for the example from Section 2.

1.2 Source code

The source code for our VeriFast extension, effectively a fork of VeriFast, can be downloaded here:

<div align="center">

`https://doi.org/10.5281/zenodo.5585276`

</div>

Our extension generates Coq code and also includes a small Coq library. In this technical report, we will use two methods for rendering such Coq code:

1. Larger chunks of Coq code will be rendered verbatim, similar to how we will render C code.

2. For all other cases, in the interest of readability, we will use a more concise form of notation. VeriFast Cx ASTs embedded within these concisely rendered Coq terms will likewise use a short hand form, marked by use of a different font.

Both forms of notation will be first introduced in Section 2.

2 An example

In this section we illustrate the ideas of our report by applying them to a concrete program. We begin by presenting the program and illustrating its symbolic execution in VeriFast. We then use this example program to showcase the actual workflow of our approach to certified verification.

2.1 Verifying a program with symbolic execution

Listing 1 shows `tests/coq/test_countdown.c`, a C program implementing a simple countdown loop which is included with our VeriFast extension.

```
1   int main()
2   //@ requires true;
3   //@ ensures result == 0;
4   {
5       int x = 32767;
6       while (0 < x)
7       //@ invariant 0 <= x;
8       {
9           x = x - 1;
10      }
11      return x;
12  }
```

Listing 1: `tests/coq/test_countdown.c`

When compiling this program and executing it, the memory location denoted by `x` gets value 32767. The body of the `while` loop will be executed 32767 times, at which point the loop guard detects that `x = 0`, the loop is exited and the program terminates with a return value of 0.

In contrast to this familiar *concrete* mode of execution, VeriFast performs *symbolic* execution. This means that a variable such as `x` does not always have a concrete value, such as 32767. Instead, it may receive a *symbolic* value $\varsigma \in Symbols$, the possible interpretations of which are *constrained* by a *path condition*. This path condition comprises a set of assumptions that further narrows down the possible interpretations for ς, based e.g. on the preconditions (specified at the beginning of the function by the `requires` keyword), branching conditions (such as the loop condition `x < 0`) and user-provided loop invariants (specified using the `invariant` keyword).
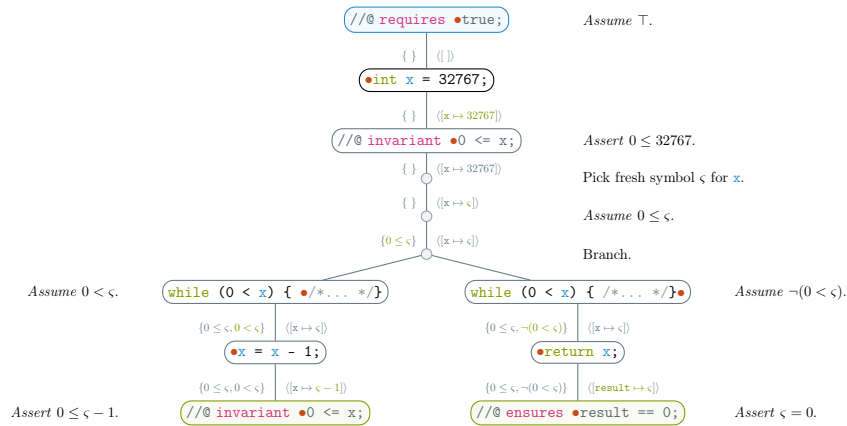

Figure 1: A structural overview of symbolic execution in VeriFast.

Figure 1 shows in detail how VeriFast symbolically executes the above program. Each *node* of this tree can be considered a breakpoint in the symbolic execution performed by VeriFast (marked by the red dot), associated with a *state*. This state is shown above the nodes, with the current path

condition on the left and the current *symbolic store* on the right. The symbolic store links each C variable to its current concrete or symbolic value. Each edge represents a state transition which can change the path condition and symbolic store; these changes are rendered in green.

Since `main` is given the weakest possible precondition (`true` or $\top$), initially nothing can be assumed. In the next line, the variable `x` is initialized with a literal value, so the store now records a concrete value for `x`. Things get more interesting in the `while` loop: first VeriFast will assert that the loop invariant holds, which in this case is easy to see. After this assertion, the value for `x` is replaced by a fresh symbolic value $\varsigma \in Symbols$, for which the loop invariant `0 <= x` is assumed, meaning that we know $0 \leq \varsigma$, whatever the value of ς is otherwise. The symbolic execution now proceeds by splitting in two separate execution branches:

- The left branch executes *all possible iterations of the loop* at once[1]: it tracks *all* executions in which the loop condition `0 < x` holds. This means we assume $0 < \varsigma$. The expression statement `x = x - 1` leads to an update in the symbolic store. If `x` had symbolic value ς before, it now has value $\varsigma - 1$. Having reached the end of the loop body, VeriFast performs another assertion that the loop invariant `0 <= x` holds. Indeed, the SMT solver within VeriFast concludes that $0 \leq \varsigma - 1$, based on the assumptions about ς currently found in the path condition.

- The right branch executes *the code after the loop*. This means that the loop condition does not hold, so our path condition now assumes $\neg(0 < \varsigma)$. It executes the `return x` statement, which leads to the symbolic store receiving a variable `result`, whose symbolic value is also ς. Exiting the function means that we have to assert that the postcondition holds (specified by the `ensures` keyword). This leads to the SMT solver in VeriFast checking whether $\varsigma = 0$, which indeed it can conclude from the assumptions collected in the path condition.

Since both the loop body and the loop continuation satisfy their respective postconditions, VeriFast concludes that the function (and the entire program) is correct in terms of the C language and of its own specification.

2.2 Certified program verification

We will now demonstrate our approach to certified program verification using the program shown in Listing 1. (Appendix A provides further instructions on building and using the extension on your own machine.) Our approach requires three steps:

---

[1]In fact, depending on the size of type `int`, it executes many more iterations than would be possible in concrete execution. In effect, symbolic execution embodies an *over-approximation* of all possible concrete executions.

1. First, we run `clightgen` on the program. This tool, included with the CompCert distribution, generates a Coq script (`test_cc.v`) containing the abstract syntax tree for the program in the Clight intermediate language:

   ```
   $ clightgen tests/coq/test_countdown.c -o test_cc.v
   ```

2. Second, we run VeriFast with the new `-emit_coq_proof` option on the program. This generates a second Coq script containing the VeriFast-generated correctness proof (`test_vf.v`):

   ```
   $ bin/verifast -shared -emit_coq_proof -bindir bin
         tests/coq/test_countdown.c
   ```

3. `test_vf.v` imports the `clightgen`-generated AST from `test_cc.v`. So finally, we can let Coq check our proof by compiling *both* scripts:

   ```
   $ coqc test_cc.v
   $ coqc test_vf.v -Q src/coq verifast
   ```

Let's inspect the contents of both scripts in more detail.

2.3 The Coq script exported by `clightgen`

When we run CompCert's `clightgen` tool on the above example, the resulting Coq script `test_cc.v` contains, among other things, a record `f_main` describing the function body, arguments and return type of function `main`. The function body `f_main.fn_body` is a Clight AST. For the example program, `clightgen` generates the following AST in `test_cc.v`:

```
80  Definition f_main := {|
81    fn_return := tint;
82    fn_callconv := cc_default;
83    fn_params := nil;
84    fn_vars := nil;
85    fn_temps := ((_x, tint) :: nil);
86    fn_body :=
87  (Ssequence
88    (Ssequence
89      (Sset _x (Econst_int (Int.repr 32766) tint))
90      (Ssequence
91        (Swhile
92          (Ebinop Olt (Econst_int (Int.repr 0) tint) (Etempvar _x tint) tint)
93          (Sifthenelse (Ebinop Ole (Etempvar _x tint)
94                        (Econst_int (Int.repr 5) tint) tint)
95            (Sset _x
96              (Ebinop Osub (Etempvar _x tint) (Econst_int (Int.repr 1) tint)
97                tint))
98            (Sset _x
```

```
 99            (Ebinop Osub (Etempvar _x tint) (Econst_int (Int.repr 5) tint)
100              tint))))
101        (Sreturn (Some (Etempvar _x tint)))))
102    (Sreturn (Some (Econst_int (Int.repr 0) tint))))
103  |}.
```

As we will see in the next section, it is this AST that will be imported from within the script exported by VeriFast. The rest of `test_cc.v` consists mostly of declarations of external functions which do not interest us.

2.4 The Coq script exported by VeriFast

A full listing of the Coq script exported by VeriFast (`test_vf.c`) can be found in Listing 2 in Appendix C. Inspecting this script reveals the following major sections:

1. *An import of the Clight script.*

2. *The AST* of the program as parsed by VeriFast, corresponding to what we find in the `clightgen` export, but in the VeriFast Cx language. Notably, it includes the VeriFast annotations, specifically the precondition and postcondition for `main` and the loop invariant.

3. *A lemma* stating that the program represented by this data structure is correct with regards to its own included specification and with regards to VeriFast's own operational semantics `cbsem`. The proof for this lemma reduces the goal to proving success of our Coq encoding of VeriFast's symbolic execution. The rest of the proof consists of tactic applications discharging the proof obligations of the symbolic execution; it is generated by instrumenting VeriFast's symbolic execution engine.

4. *A proof goal* stating that the imported Clight program is correct with regards to Clight's *own big step operational semantics*, proven by application of the above lemma.

Let's consider each of these sections in more detail.

2.4.1 Importing the Clight script

First, we import the Clight script from `test_cc.v` and use it to construct a "local" Clight program `test_cc_prog`:

```
25  Require test_cc.
26
27  Definition test_cc_prog: Clight.program := mkprogram
28    test_cc.composites
29    [(test_cc._main, Gfun(Internal test_cc.f_main))]
30    [test_cc._main]
31    test_cc._main
32    Logic.I.
```

9

This `test_cc_prog` is like the program found in the Clight export, but it does not contain the large set of external global definitions that Clight adds.

2.4.2 Program AST in VeriFast Cx

Next, we look at the description of the program as parsed by VeriFast. Since we currently only support programs with a single main function taking no arguments, the data structure consists *only* of the AST of that function's body. The AST for the example program in `test_vf.v` looks like this:

```
36  Definition main_vf_func :=
37    (StmtLet "x" ((ExprIntLit (32767))) (
38      (
39        (StmtWhile
40          (ExprLt (ExprIntLit (0)) (ExprVar "x"))
41          (ExprLe (ExprIntLit (0)) (ExprVar "x"))
42          (
43            (StmtBlock
44              (
45                (StmtExpr (ExprAssign
46                  (ExprVar "x")
47                  (ExprSub (ExprVar "x") (ExprIntLit (1)))));
48                StmtSkip
49              )
50            );
51            StmtSkip
52          )
53          StmtSkip
54        );
55        (
56          (StmtReturn (ExprVar "x"));
57          StmtSkip
58        )
59      )
60    )).
```

As mentioned in the introduction, in the rest of this text we will often use a more concise notation system for Coq code and, within that concise notation system, a special font for VeriFast Cx ASTs embedded in Coq. This concise notation renders the above Coq definition as follows:

$$
\begin{aligned}
\mathtt{main\_vf\_func} := \;& \mathsf{let}\ x := 32767\ \mathsf{in} \\
& \mathsf{while}\ (0 < x)\ \mathsf{inv}\ (0 \leq x)\ \{ \\
& \quad \{x = x - 1;\ \mathsf{skip}\};\ \mathsf{skip} \\
& \};\ \mathsf{return}\ x;\ \mathsf{skip}.
\end{aligned}
$$

VeriFast Cx itself will be specified in full detail in Section 3.

2.4.3 Correctness with regards to `cbsem`

Below the definition of `main_vf_func` we find the lemma stating correctness of `main_vf_func` in terms of `cbsem`, the operational semantics for VeriFast programs:

```
64   Lemma main_cbsem_by_symexec_tactics:
65     cbsem.exec_func_correct
66       (nil)
67       ExprTrue
68       main_vf_func
69       (ExprEq (ExprVar "result") (ExprIntLit (0))).
```

Translating this into our more concise notation system, the lemma has the following proof goal (see Definition 5.11 for $\text{exec}_f^{\Downarrow}$):

$$\vdash \text{exec}_f^{\Downarrow} \, [\,] \text{ true main\_vf\_func } (\text{result} == 0).$$

This means that we have to provide, for each possible concrete execution of the main function, starting from an initial state satisfying the weak precondition `true`, a derivation showing that the execution either diverges or terminates with a return value satisfying the postcondition.

Instead of constructing these derivations directly, the proof for this lemma begins by applying the soundness result from Theorem 5.13 which states that correctness of a function in the operational semantics `cbsem` can be proven by symbolic execution of that function:

```
64   Lemma main_cbsem_by_symexec_tactics:

        ⋮

70   Proof.
71     apply symexec_cbsem.symexec_func_sound.
72     repeat (autounfold with vf_symexec_clight; simpl).
```

Application of this theorem gives us a new proof goal containing the symbolic execution predicate exec_f (see Definition 4.12 for exec_f):

$$\vdash \text{exec}_f \, [\,] \text{ true main\_vf\_func } (\text{result} == 0).$$

The goal now states that symbolic execution of the function must succeed. As will be seen in 4.1, this symbolic execution consists of constructing a symbolic execution proposition or SEP. Reduction of the current proof goal (performed by the repeated application of the `autounfold` and `simpl` tactics in line 72) effectively computes this SEP for `main_vf_func` with the stated

pre- and postcondition:

$$\vdash \texttt{True} \rightarrow (0 \le 32767) \land ($$
$$\forall\, \texttt{sym} : \texttt{Z}, (\texttt{int}_{\min} \le \texttt{sym} \le \texttt{int}_{\max}) \rightarrow$$
$$(0 \le \texttt{sym}) \rightarrow ($$
$$(0 < \texttt{sym}) \rightarrow$$
$$(\texttt{int}_{\min} \le \texttt{sym} - 1) \land (\texttt{sym} - 1 \le \texttt{int}_{\max}) \land$$
$$(0 \le \texttt{sym} - 1) \land \texttt{True}$$
$$) \land ($$
$$\neg(0 < \texttt{sym}) \rightarrow$$
$$\texttt{sym} = 0 \land \texttt{True}$$
$$)).$$

The structure of this Coq proof goal corresponds to the structure of the graph in Figure 1, with universal quantification taking the place of symbol picking. The rest of the proof is then provided by a tactic script exported by VeriFast, based on recordings of calls to the internal SMT solver:

```
64   Lemma main_cbsem_by_symexec_tactics:

         .
         .
         .

70   Proof.
71     apply symexec_cbsem.symexec_func_sound.
72     repeat (autounfold with vf_symexec_clight; simpl).
73     simpl.
74     intros. (* _: True *)
75     split.
76     (* assert 0 <= 32767 *)
77     lia.
78     intros. (* x: Z *)
79     intros. (* _: 0 <= x *)
80     split.
81     intros. (* _: 0 < x *)
82     split.
83     (* assert INT_MIN <= (x - 1) *)
84     lia.
85     split.

         .
         .
         .

99   Qed.
```

We do not show the entire proof for the sake of brevity (again, see Appendix C for that), but it is clear that we only use three tactics: intros, split and lia. Subsection 4.2 provides more details on the relation between these tactics and the SEP structure.

2.4.4 Correctness with regards to Clight

The fourth and final part of the Coq script consists of a direct proof goal stating that the program `test_cc_prog` is correct, using a predicate for Clight-based program correctness introduced in Definition 6.20. We show at once the proof goal and its actual proof:

```
103  Goal cbsem_clight.compcert_exec_prog_correct test_cc_prog.
104  Proof.
105    apply (cbsem_clight.vf_cl_sound main_vf_func).
106    - repeat econstructor.
107      unfold Globalenvs.Genv.init_mem. simpl.
108      case_eq (Memory.Mem.alloc Memory.Mem.empty 0 1). intros.
109      destruct (Memory.Mem.range_perm_drop_2 m b 0 1 Memtype.Nonempty).
110      + unfold Memory.Mem.range_perm.
111        intros.
112        apply Memory.Mem.perm_alloc_2 with (1:=H) (2:=H0).
113      + rewrite e.
114        reflexivity.
115    - eapply cbsem_clight.cbsem_func_sound.
116      eapply main_cbsem_by_symexec_tactics.
117  Qed.
```

In essence, this expresses that the program either terminates with some return value or diverges according to Clight's own big step semantics.

The proof for this statement is boilerplate, in the sense that it can be automatically constructed *by the same set of Coq tactics for every supported program.* It commences by application of theorem `vf_cl_sound` in line 105, corresponding to our second soundness theorem (Theorem 6.21). As a result, we now need to prove two subgoals:

1. The existence of a correspondence relation between `test_cc_prog` and a slightly transformed version of `main_vf_func` (see Definition 6.17 for more details). The proof for this subgoal involves proving that the Clight and VeriFast ASTs "line up" (`repeat econstructor` on line 106) and the production of an initial execution state for the Clight program (lines 107–114).

2. Through application of an intermediate step (`cbsem_func_sound`, see Lemma 6.19), the second subgoal now becomes (in concise notation):

$$\vdash \mathtt{exec}_{\mathtt{f}}^{\Downarrow} [\,] \text{ true } \mathtt{main\_vf\_func} \ \textcolor{red}{?q}.$$

This subgoal can be proven by `main_cbsem_by_symexec_tactics`, our earlier lemma. Using that our notion of correctness of a Clight program does not say anything about the program's exit code, any postcondition q is acceptable for proving the correctness of the Clight program, including the postcondition result $==$ 0 verified in the above lemma.

Since the Coq type checker accepts the VeriFast Coq and the CompCert libraries and both scripts for the example program, we can be confident in

our conclusion. In fact, in the course of developing the soundness proof of `cbsem` with respect to Clight, we already discovered a bug in VeriFast itself, related to appropriate checks on the result of a division (which was quickly fixed).

3 VeriFast Cx

We begin by describing the C language subset which is currently supported by the export extension to VeriFast[2]. We chose just enough language features to be able to write programs that diverge and that can exhibit some straightforward forms of undefined behavior, specifically out-of-bounds arithmetic and division by zero. This means that our subset has many notable omissions and limitations:

- we currently stick to variables having a single explicit type (`int`);

- we do not support function calls;

- all variables are locally allocated: there is no heap memory.

When VeriFast exports a parsed AST to Coq, the exported AST is actually a program in a slightly different language, which we call *VeriFast Cx*[3], with the x denoting "export". This language differs syntactically from actual C in a few ways which will be quite familiar to many readers:

1. an OCaml/Haskell/... `let`-like variable binding is used for variable declaration;

2. a *list* of statements in C (e.g. `s1`; `s2`; `s3`) is represented in VeriFast Cx by nesting multiple instances of a binary *sequence operator* _; _ terminated by a `skip` statement (for the earlier example we still get s_1; s_2; s_3; `skip`, because the sequence operator notation is right associative);

3. the `while` loop carries its invariant directly within the AST[4].

VeriFast Cx does retain the use of named local variables (as opposed to common use of De Bruijn indices).

The transformations of C into VeriFast Cx make our work in Coq simpler, without altering the meaning of programs substantially. The transformation

---

[2]The notation we use in this paper is more compact than the notation we use in the actual Coq code. In the near future, however, we will adapt our Coq code to use the notation introduced in this paper.

[3]This allows us to retain the name "VeriFast C" to refer to VeriFast's C dialect: the collective set of choices which VeriFast in its implementation makes with regards to the C language.

[4]In the Coq code, the `while` loop actually also has a second statement which is currently not used.

into VeriFast Cx takes place in VeriFast itself. We do not make any direct formal statement about the relationship between the parsed C program and the transformed VeriFast Cx program which is exported to Coq[5]. This means that all our theorems hold for VeriFast Cx rather than "VeriFast C".

This lack of formal relation is indirectly mitigated in section 6, where we will define a relation $\sim_{\mathtt{stmt}}$ between a function body s exported by Veri-Fast and the corresponding function body s_{CL} exported by CompCert's `clightgen` tool. Indeed, a crucial element in proving correctness of a CompCert function with body s_{CL} by symbolically executing s will be our ability to prove that this relation holds between s and s_{CL}.

Definition 3.1. The set of VeriFast Cx *expressions* currently generated by our export code is defined inductively in Coq as:

$$
\begin{aligned}
e \in \mathtt{expr} ::={} & \mathtt{true} \,|\, \mathtt{false} \,|\, \mathtt{z} \,|\, \mathtt{x} \\
& |\, e_1 + e_2 \,|\, e_1 - e_2 \,|\, e_1/e_2 \\
& |\, e_1 < e_2 \,|\, e_1 \le e_2 \,|\, e_1 == e_2 \,|\, e_1 \ne e_2 \\
& |\, e_1 \,\&\&\, e_2 \,|\, e_1 \,||\, e_2 \,|\, !e \\
& |\, e_1 = e_2.
\end{aligned}
$$

The above set includes boolean comparison operators, but we will see how symbolic execution in Coq will limit usage of these operators in the conditions of conditional statements and loops.

Notation 3.2. [z] denotes the actual Coq integer value $z \in \mathtt{Z}$ corresponding to the integer literal z. Likewise, [x] denotes the actual Coq string value $x \in \mathtt{string}$ corresponding to variable reference x.

Definition 3.3. Likewise, the set of VeriFast Cx *statements* currently generated by our export code is defined inductively as:

$$
\begin{aligned}
s \in \mathtt{stmt} ::={} & \mathtt{skip} \\
& |\, s_1;\, s_2 && \text{Sequence statement} \\
& |\, \mathtt{let}\ \mathtt{x} := e\ \mathtt{in}\ s && \texttt{let-style variable declaration} \\
& |\, e && \text{Expression statement} \\
& |\, \mathtt{if}\ e\ \mathtt{then}\ s_1\ \mathtt{else}\ s_2 \\
& |\, \mathtt{return}\ e \\
& |\, \mathtt{while}\ e_c\ \mathtt{inv}\ e_i\ \{s\} && \text{Loop with condition } e_c \text{ and invariant } e_i \\
& |\, \{s\}. && \text{Block statement}
\end{aligned}
$$

---

[5]In Section 6 we will make 2 further transformations to the AST of a function. However, these transformations were specifically geared towards Clight. Since we consider `cbsem` as a result standing on its own, we decided at the time to perform them within Coq rather than within VeriFast.

All VeriFast Cx *functions* currently have return type `int` and take zero or more arguments of type `int`. A VeriFast Cx function in Coq is defined by four things: (1) a list of argument names $\overline{x} \in$ `list string`; (2) an expression e_p representing the function's precondition; (3) the function body s and (4) an expression e_q representing the postcondition. We do not currently define a separate Coq type for a function, but just pass around these four items as separate arguments wherever they are needed.

4   Symbolic execution in Coq

This section discusses our approach to symbolic execution of a function in Coq. This approach consists of two steps. Subsection 4.1 shows how the function's AST, together with its pre- and postcondition, is first transformed into a logical sentence, called the symbolic execution proposition or SEP. This SEP is a Coq term of type `Prop` which corresponds structurally to the tree formed by VeriFast's internal branching and pushing and popping of SMT solver contexts. It is a shallow embedding of the assertions made in the various execution branches, each execution branch having its own set of assumptions representing the path condition of that branch. Subsection 4.2 then describes how the VeriFast extension currently exports a proof for this proposition.

4.1   Constructing the SEP

We will provide our technical description of constructing a SEP in a rather bottom-up fashion, starting from a few conceptual ideas and lower-level definitions and working up towards the main correctness predicate for symbolic execution of a VeriFast Cx function: `exec`$_f$ (see definition 4.12 below). Coq of course works in the opposite direction by gradually unfolding and reducing an instance of `exec`$_f$ into a term of type `Prop`.

4.1.1   Stores, continuations and symbol picking

Definition 4.1. Given that we currently only support variables of type `int`, a *symbolic store* is simply defined as a total function with an empty store $\emptyset$ and a single update operation $\langle [\_ := \_] \rangle :$ `string` $\rightarrow$ `optionZ` $\rightarrow$ `store` $\rightarrow$ `store`, handling both inserts and deletions:

$$\sigma \in \text{store} := \text{string} \rightarrow \text{optionZ}.$$
$$\emptyset := \lambda \ \_, \ \text{None}.$$
$$\langle [x := v] \rangle \sigma := \lambda \ y, \ \begin{cases} v & \text{if } x =_? y, \\ \sigma \ y & \text{otherwise.} \end{cases}$$

Continuations allow us to express construction of the SEP by combining various predicates in a *continuation-passing style*, which corresponds structurally well to the style of programming found in the OCaml source code of VeriFast.

Definition 4.2 (Continuations taking *only* a store)**.**

$$C \in \mathtt{cont} := \mathtt{store} \to \mathtt{Prop}.$$

$\mathtt{cont}$ has been given its own type because it is so common throughout the development. Other, less common continuations will take additional parameters on top of a store. For instance, a *return continuation* will have type $\mathtt{Z} \to \mathtt{cont}$. In addition to a store, a return continuation also takes a return value z to generate a SEP. Important instances of these continuation passing-style SEP-producing functions are the functions to pick fresh symbols.

Definition 4.3. The function $\mathtt{for_Z} : \mathtt{string} \to \mathtt{cont} \to \mathtt{cont}$ is defined as:

$$\mathtt{int_{min}} : \mathtt{Z} := -2147483648.$$
$$\mathtt{int_{max}} : \mathtt{Z} := 2147483647.$$
$$\mathtt{is\_int} \ z := \mathtt{int_{min}} \leq z \leq \mathtt{int_{max}}.$$
$$\mathtt{for_Z} \ x \ C \ \sigma := \forall \ z, \ \mathtt{is\_int} \ z \to C \ (\langle [x := \mathtt{Some} z] \rangle \ \sigma).$$

$\mathtt{for_Z}$ takes a variable identifier x and, through a continuation, generates a proposition quantifying over $z \in \mathtt{Z}$, such that z is within hardcoded integer bounds[6].

The fixpoint function $\mathtt{for_{Zs}}$ lifts $\mathtt{for_Z}$ to pick fresh symbols for a whole list of identifiers $\overline{x} \in \mathtt{list} \ \mathtt{string}$ at once. Likewise, the fixpoint function $\mathtt{havoc_{Zs}}$ picks fresh symbols for a list of identifiers $\overline{x}$, *with the added condition that the identifiers were already bound in the store which is passed to* $\mathtt{havoc_{Zs}}$. This condition does not exist for $\mathtt{for_{Zs}}$. We omit definitions for both functions.

4.1.2 Evaluation and translation of expressions

An expression $e \in \mathtt{expr}$ can be shallowly embedded in a SEP in two distinct ways. When *evaluating* an expression in the context of actual symbolic execution, we require extra conditions to be embedded, for instance to check integer bounds or to check for division by zero. These extra verification conditions correspond to VeriFast making calls to the SMT solver during execution of some C function. However, when *translating* an expression in the context of embedding pre- and postconditions and invariants in the SEP,

---

[6]In the future, we will allow architectural parametrization in our exported Coq script, similar to that found in CompCert.

there is no need to add any extra checks in the SEP. Rather, the expression *is itself* the assumption made by the SMT solver or the assertion verified by the SMT solver.

Since our concern with this work was to get a simple proof of concept working, the total set of expressions which is handled properly by our Coq code is a limited subset of what is allowed in C. To this end, we make a second, rather artificial distinction between *arithmetic expressions* (z, x, $e_1 + e_2$, $e_1 - e_2$, e_1/e_2) and *"Boolean" expressions* ($e_1 < e_2$, $e_1 \leq e_2$, $e_1 == e_2$, $e_1 \neq e_2$, e_1 && e_2, $e_1 \mathbin{||} e_2$, !e, true and false). The assignment expression $e_1 = e_2$ forms a third category of its own. To enable further simplifications, only arithmetic and Boolean expression are evaluated. Both are evaluated as r-expressions[7].

Definition 4.4. Fixpoint function $\mathtt{eval}^{\mathtt{c}}_{\mathtt{Z}} : \mathtt{expr} \to (\mathtt{Z} \to \mathtt{cont}) \to \mathtt{cont}$ attempts to evaluate an expression e to a Coq value z. The superscript $\mathtt{c}$ denotes that it is a CPS function: if successful, a continuation C is called with value z, producing a sub-SEP which is conjoined with any side conditions arising from the evaluation. That is, $\mathtt{eval}^{\mathtt{c}}_{\mathtt{Z}}\ e\ C\ \sigma$ is true if and only if in store σ, e has no undefined behavior and evaluates to a value z such that $C\ z\ \sigma$ is true.

For integer literals and variable references, this evaluation is rather simple:

$$\mathtt{eval}^{\mathtt{c}}_{\mathtt{Z}}\ \mathtt{z}\ C\ \sigma := \begin{cases} C\ [\mathtt{z}]\ \sigma & \text{if } \mathtt{int_{min}} \leq_? [\mathtt{z}] \leq_? \mathtt{int_{max}}, \\ \mathtt{False} & \text{otherwise.} \end{cases}$$

$$\mathtt{eval}^{\mathtt{c}}_{\mathtt{Z}}\ \mathtt{x}\ C\ \sigma := \begin{cases} C\ z\ \sigma & \text{if } \sigma\ [\mathtt{x}] =_? \mathtt{Some}z, \\ \mathtt{False} & \text{otherwise.} \end{cases}$$

Evaluation of the arithmetic expression $\_ + \_$ is defined as:

$$\begin{aligned}
\mathtt{eval}^{\mathtt{c}}_{\mathtt{Z}}\ (e_1 + e_2)\ C\ \sigma := {} & \mathtt{eval}^{\mathtt{c}}_{\mathtt{Z}}\ e_1\ (\lambda\ z_1\ \sigma, \\
& \quad \mathtt{eval}^{\mathtt{c}}_{\mathtt{Z}}\ e_2\ (\lambda\ z_2\ \sigma, \\
& \qquad \mathtt{let}\ z := z_1 + z_2\ \mathtt{in} \\
& \qquad (\mathtt{int_{min}} \leq z) \wedge ((z \leq \mathtt{int_{max}}) \wedge (C\ z\ \sigma)) \\
& \quad)\ \sigma \\
&)\ \sigma.
\end{aligned}$$

We omit the definition for $\_ - \_$, which is very similar to that for addition.

---

[7]As will be seen in section 4.1.3, we don't need evaluation of l-expressions for now.

For division we have:

$$\mathtt{eval^c_Z}\ (e_1/e_2)\ C\ \sigma := \mathtt{eval^c_Z}\ e_1\ (\lambda\ z_1\ \sigma,$$
$$\mathtt{eval^c_Z}\ e_2\ (\lambda\ z_2\ \sigma,$$
$$\mathtt{let}\ z := z_1 \div z_2\ \mathtt{in}$$
$$z_2 \neq 0 \wedge ((z_1 \neq \mathtt{int_{min}}) \vee (z_2 \neq -1)) \wedge (C\ z\ \sigma)$$
$$)\ \sigma$$
$$)\ \sigma.$$

Finally, for all other cases (Boolean expressions and the assignment expression), $\mathtt{eval^c_Z}$ generates `False`:

$$\mathtt{eval^c_Z}\ \_\ C\ \sigma := \mathtt{False}.$$

In principle, we could have simplified $\mathtt{eval^c_Z}$, based on the observation that the current set of supported expressions does not change σ during evaluation. This will however not remain the case in the future.

The important idea is that $\mathtt{eval^c_Z}$ tightly mirrors what happens in VeriFast itself. It embeds verification conditions whenever VeriFast performs SMT calls. Even the *order* of the embedded verification conditions corresponds to the order in which VeriFast calls the SMT solver: this can be seen in the case for $+$, where first a comparison against $\mathtt{int_{min}}$ is asserted, followed by a comparison against $\mathtt{int_{max}}$, followed by the continuation. Likewise, for the direct computational checks in the cases of z and x, VeriFast correspondingly performs the same checks directly in the OCaml code (thereby *avoiding* calls to the SMT solver).

Definition 4.5. Fixpoint function $\mathtt{eval^c_{Prop}} : \mathtt{expr} \to (\mathtt{Prop} \to \mathtt{cont}) \to \mathtt{cont}$ transforms the Boolean expressions into a SEP. Let us begin with the simple definitions for `true` and `false`:

$$\mathtt{eval^c_{Prop}}\ \mathtt{true}\ C\ \sigma := C\ \mathtt{True}\ \sigma.$$
$$\mathtt{eval^c_{Prop}}\ \mathtt{false}\ C\ \sigma := C\ \mathtt{False}\ \sigma.$$

The cases for $\_ == \_$, $\_ < \_$, $\_ \leq \_$ and $\_ \neq \_$ are very similar and rest on calls to $\mathtt{eval^c_Z}$ (Definition 4.4), meaning the resulting proposition may contain side conditions. We only give the case for equality testing:

$$\mathtt{eval^c_{Prop}}\ (e_1 == e_2)\ C\ \sigma := \mathtt{eval^c_Z}\ e_1\ (\lambda\ z_1\ \sigma,$$
$$\mathtt{eval^c_Z}\ e_2\ (\lambda\ z_2\ \sigma,$$
$$C\ (z_1 = z_2)\ \sigma$$
$$)\ \sigma$$
$$)\ \sigma.$$

The cases for the logical binary operations are likewise very similar. We only provide a definition for logical conjunction:

$$\texttt{eval}^{\texttt{c}}_{\texttt{Prop}} \ (e_1 \wedge e_2) \ C \ \sigma := \texttt{eval}^{\texttt{c}}_{\texttt{Prop}} \ e_1 \ (\lambda \ E_1 \ \sigma,$$
$$\texttt{eval}^{\texttt{c}}_{\texttt{Prop}} \ e_2 \ (\lambda \ E_2 \ \sigma,$$
$$C \ (E_1 \wedge E_2) \ \sigma$$
$$) \ \sigma$$
$$) \ \sigma.$$

The definition for logical negation is given as:

$$\texttt{eval}^{\texttt{c}}_{\texttt{Prop}} \ (!e) \ C \ \sigma := \texttt{eval}^{\texttt{c}}_{\texttt{Prop}} \ e \ (\lambda \ E \ \sigma, \ C \ (\neg E) \ \sigma) \ \sigma.$$

All other cases (assignment expressions and arithmetic expressions) currently evaluate to `False`.

Definition 4.6. Fixpoint function $\texttt{translate}^{\texttt{c}}_{\texttt{Prop}} : \texttt{expr} \rightarrow (\texttt{Prop} \rightarrow \texttt{Prop}) \rightarrow \texttt{cont}$ *translates* expression e in store σ into $E \in \texttt{Prop}$, such that $\texttt{translate}^{\texttt{c}}_{\texttt{Prop}} \ e \ C \ \sigma$ is true if $C \ E \ \sigma$ is true.

It differs from $\texttt{eval}^{\texttt{c}}_{\texttt{Prop}}$ in that no side conditions for division by zero and integer boundedness are generated for the cases involving arithmetic. The exact definition for $\texttt{translate}^{\texttt{c}}_{\texttt{Prop}}$ is not provided here.

Definition 4.7. The functions $\texttt{produce} : \texttt{expr} \rightarrow \texttt{cont} \rightarrow \texttt{cont}$ and $\texttt{consume} : \texttt{expr} \rightarrow \texttt{cont} \rightarrow \texttt{cont}$ are defined as:

$$\texttt{produce} \ e \ C \ \sigma := \texttt{translate}^{\texttt{c}}_{\texttt{Prop}} \ e \ (\lambda \ E, \ E \rightarrow C \ \sigma) \ \sigma.$$
$$\texttt{consume} \ e \ C \ \sigma := \texttt{translate}^{\texttt{c}}_{\texttt{Prop}} \ e \ (\lambda \ E, \ E \ \wedge \ C \ \sigma) \ \sigma.$$

These functions leverage $\texttt{translate}^{\texttt{c}}_{\texttt{Prop}}$ for a common use case: the actual *production* (assumption) of pre-conditions and *consumption* (assertion) of post-conditions.

4.1.3 Symbolic execution of statements

Before we can get to constructing a SEP which embeds the symbolic execution of statements, we need to introduce two auxiliary functions.

Definition 4.8. Fixpoint function $\texttt{targets} : \texttt{stmt} \rightarrow \texttt{list string}$ collects the names of all free variables that are *potentially* modified by some statement s. We omit definition of this function.

Definition 4.9. Function $\texttt{leak\_check} : \texttt{cont}$ is defined as:

$$\texttt{leak\_check} := \lambda \ \_, \ \texttt{True}.$$

It is a placeholder SEP generating function which corresponds to the checks performed by VeriFast upon exiting a function or at the end of a loop body. Since we do not deal with heap memory in this proof of concept, its implementation remains trivial for now.

Definition 4.10. Symbolic execution of a *statement s* is implemented by the fixpoint function $\texttt{exec}_{\texttt{stmt}} : \texttt{stmt} \rightarrow \texttt{cont} \rightarrow (\texttt{Z} \rightarrow \texttt{cont}) \rightarrow \texttt{cont}$. This function takes two continuations, which correspond to the two possible outcomes for executing a statement: *normal continuation* ($C_N \in \texttt{cont}$) and *returning from the function* ($C_R \in \texttt{Z} \rightarrow \texttt{cont}$):

$$\texttt{exec}_{\texttt{stmt}} \; \texttt{skip} \; C_N \; C_R \; \sigma := C_N \; \sigma.$$
$$\texttt{exec}_{\texttt{stmt}} \; (s_1; \, s_2) \; C_N \; C_R \; \sigma := \texttt{exec}_{\texttt{stmt}} \; s_1 \; (\texttt{exec}_{\texttt{stmt}} \; s_2 \; C_N \; C_R) \; C_R \; \sigma.$$
$$\texttt{exec}_{\texttt{stmt}} \; (\texttt{return} \; e) \; C_N \; C_R \; \sigma := \texttt{eval}_{\texttt{Z}}^{\texttt{c}} \; e \; C_R \; \sigma.$$
$$\texttt{exec}_{\texttt{stmt}} \; \{s\} \; C_N \; C_R \; \sigma := \texttt{exec}_{\texttt{stmt}} \; s \; C_N \; C_R \; \sigma.$$

For local variable declarations, we have:

$$\texttt{exec}_{\texttt{stmt}} \; (\texttt{let} \; x := e \; \texttt{in} \; s) \; C_N \; C_R \; \sigma :=$$
$$\texttt{if} \; (\sigma \; x) \; \texttt{then False else}$$
$$\texttt{eval}_{\texttt{Z}}^{\texttt{c}} \; e \; (\lambda \; z \; \sigma,$$
$$\texttt{let} \; C_N' := (\lambda \; \sigma, \; C_N \; \langle[x := \texttt{None}]\rangle\sigma) \; \texttt{in}$$
$$\texttt{let} \; C_R' := (\lambda \; z \; \sigma, \; C_R \; z \; \langle[x := \texttt{None}]\rangle\sigma) \; \texttt{in}$$
$$\texttt{exec}_{\texttt{stmt}} \; s \; C_N' \; C_R' \; \langle[x := \texttt{Some} \; z]\rangle\sigma$$
$$) \; \sigma.$$

In terms of expression statements, our symbolic execution currently only supports direct assignments of an expression e to a variable identifier expression x, which is why in practice we only need to care about evaluation of r-expressions:

$$\texttt{exec}_{\texttt{stmt}} \; (\texttt{x} = e) \; C_N \; C_R \; \sigma := \texttt{eval}_{\texttt{Z}}^{\texttt{c}} \; e \; (\lambda \; z \; \sigma, \; C_N \; \langle[[\texttt{x}] := \texttt{Some} \; z]\rangle\sigma) \; \sigma.$$

Conditional statements branch into two path conditions, one in which the condition $e \in \texttt{expr}$ is true and one in which it is not true:

$$\texttt{exec}_{\texttt{stmt}} \; (\texttt{if} \; e \; \texttt{then} \; s_1 \; \texttt{else} \; s_2) \; C_N \; C_R \; \sigma := \texttt{eval}_{\texttt{Prop}}^{\texttt{c}} \; e \; (\lambda \; E \; \sigma,$$
$$(E \rightarrow (\texttt{exec}_{\texttt{stmt}} \; s_1 \; C_N \; C_R \; \sigma))$$
$$\wedge$$
$$(\neg E \rightarrow (\texttt{exec}_{\texttt{stmt}} \; s_2 \; C_N \; C_R \; \sigma))$$
$$) \; \sigma.$$

Finally, `while` loops are a little bit more involved:

$$\texttt{exec}_{\texttt{stmt}} \ (\textsf{while } e_c \textsf{ inv } e_i \ \{s\}) \ C_N \ C_R \ \sigma := \texttt{consume } e_i \ ($$

$$\texttt{havoc}_{\texttt{Zs}} \ (\texttt{targets } s) \ ($$

$$\texttt{produce } e_i \ ($$

$$\texttt{eval}_{\texttt{Prop}}^{\texttt{c}} \ e_c \ (\lambda \ E_c \ \sigma,$$

$$(E_C \rightarrow (\texttt{exec}_{\texttt{stmt}} \ s \ (\texttt{consume } e_i \ \texttt{leak\_check}) \ C_R \ \sigma))$$

$$\wedge$$

$$(\neg E_C \rightarrow C_N \ \sigma)$$

$$)$$

$$)$$

$$)) \ \sigma.$$

This chaining of continuations will unfold to reproduce the verification steps shown graphically in Figure 1.

4.1.4 Symbolic execution of a function

Before we can finally define symbolic execution of a function, we need to examine one last predicate, `ret`, which is used to generate the symbolic store in which the postcondition can be asserted upon return.

Definition 4.11. Predicate `ret : store → cont → Z → cont` *retains* an "old" symbolic store σ_0 and ultimately passes on this store, updated with a return value z, to its own continuation C (which will embed the actual postcondition, as shown below in definition 4.12):

$$\texttt{ret } \sigma_0 \ C \ z := \lambda \ \_, \ C \ (\langle[\text{"return"} := \texttt{Some } z]\rangle\sigma_0).$$

The continuation produced by `ret` simply discards the "new" store with which it is called. Indeed, a postcondition will be evaluated in terms of the original arguments provided to the function, together with the return value.

Definition 4.12. The function correctness predicate `exec`$_{\texttt{f}}$ `: list string →` `expr → stmt → expr → Prop` for a function with arguments $\overline{x}$, pre- and postconditions e_p and e_q and body s is defined as:

$$\texttt{exec}_{\texttt{f}} \ \overline{x} \ e_p \ s \ e_q := \texttt{for}_{\texttt{Zs}} \ \overline{x} \ (\texttt{produce } e_p \ (\lambda \ \sigma,$$

$$\texttt{exec}_{\texttt{stmt}} \ s$$

$$(\lambda \ \_, \ \texttt{False})$$

$$(\texttt{ret } \sigma \ (\texttt{consume } e_q \ \texttt{leak\_check}))$$

$$\sigma$$

$$)$$

$$)) \ \emptyset.$$

Intuitively, definition 4.12 states that nothing is known about the function arguments, except that they are integers and that the precondition must hold. Definition 4.12 makes it necessary for a function to either *diverge* or, if it does terminate, to *return* an integer result z, the value of which is limited by a path condition which satisfies the function's postcondition:

- A diverging symbolic execution path will accumulate contradictory assumptions, the presence of which instantly terminates symbolic execution.

- If some symbolic execution path of s does not somehow end with a return statement, the normalization of the subterm with $exec_{stmt}$ would end up calling the normal continuation, which is λ _, `False`. Proving the resulting SEP would require proving `False`, which is impossible.

4.2 Proving the SEP

After we have constructed and normalized a SEP for the function, we need to provide a proof for it. Proof steps for the SEP in Coq are closely related to changes in the VeriFast SMT solver state. We currently identify four types of proof steps:

- *Assumptions*, for instance introduced by predicates for_{Zs} or `produce`, require us to move the variable or hypothesis into the proof context. In VeriFast this is accomplished by making `#assume` calls to the SMT solver API.

- If the proof goal is a *conjunction*, for instance produced by branching in some cases of $exec_{stmt}$ or by `consume`, we need to split it into two subgoals. In VeriFast, this is implemented by a function `#branch`, which pushes a new SMT solver context for each branch, popping the context after the branch finishes.

- *Assertions* are generated by e.g. `consume` or some continuations. They are handled in VeriFast by `#assert` calls to the SMT solver.

- As mentioned earlier, a path condition may accumulate *contradictory assumptions*, for instance in the context of a diverging execution. At that point we need to point out these contradictions discharging the proof goal. In VeriFast, the SMT solver's internal mechanism detects this.

In Table 1 we summarize our current approach to implementing these four types of proof steps in Coq. Since we may benefit from the limited, integer arithmetic nature of the current set of expressions which has to be

	Tactic
Assumptions	`intro`
Conjunctions	`split`
Assertions	`lia`
Contradictions	`lia`

Table 1: Four types of proof steps.

supported, a second option would be to make a single tactic which repeatedly applies `intros`, `split` and `lia` from within a syntax-driven goal match.

For now, we choose to make VeriFast export a proof. We record all calls to the SMT API and the `#branch` function and use these recordings to generate a tactics proof in the exported proof. The main difficulty in recording so far was that contradictions "break out" of the SMT solver, meaning they are not explicitly recorded. Contradictions can be seen in the recordings as the *absence* of an assertion or equivalently, as a "dangling" assumption. Since contradictions in our case stem from assumptions about integer expressions, we also use `lia` to deal with these.

Exporting the proof is at this point not only more cumbersome than a simple syntax-driven tactic in Coq, it is also quite *fragile*. Ironically, this fragility can be seen as an advantage. It can point out bugs, in VeriFast or in the Coq code, when both do different things. If the SEP generated in Coq misses an assertion or assumption, or if it has too many assertions or assumptions, the exported proof may not line up with the SEP.

5 Big step semantics for VeriFast Cx

Before we prove the soundness of VeriFast's symbolic execution with respect to Clight, let us first look at an intermediate step. In this section, we develop `cbsem`, a formal semantics for VeriFast Cx.

We chose to express ourselves using big step semantics (see Subsection 5.1), because this approach was somewhat easier to get started with. The downside is that our inductive big step semantics, seen by itself, does not allow us to distinguish program executions that have undefined behavior from those that diverge. Neither type of execution outcome can be derived in this big step semantics.

At first we solved this problem by adding a timeout counter to each semantic rule. This worked well in itself and would have allowed us to stick to a single set of derivation rules. However, in order to prove soundness with regards to Clight, we decided it would be easier to reflect Clight's approach to big step semantics [2, 9] by *adding* a separate, *coinductive* set of semantic rules for diverging programs (see Subsection 5.2). `cbsem` therefore refers to the *combined* inductive and coinductive sets of semantic rules.

Throughout this section, we will relate symbolic execution as specified in Section 4 to the inductive and the coinductive semantics that are being developed, again building up from expressions to functions. Finally, in Subsection 5.3 we provide a *soundness theorem* proving that symbolic execution of a function is sound with regards to our formal big step semantics `cbsem`.

5.1 Terminating statements

Definition 5.1. The set `outcome` specifies the *possible outcomes* of executing some VeriFast Cx statement. It is defined inductively as:

$$o \in \texttt{outcome} ::= \texttt{N} \mid \texttt{R}\ z.$$

In the same way that predicate $\texttt{exec}_{\texttt{stmt}}$ for symbolic execution accepts two continuations, one for a normal continuation and one for a return situation, `outcome` is defined by two constructors.

Before we can get to the actual inference rules for our big step semantics, we need to mention variations of $\texttt{eval}_{\texttt{Z}}^{\texttt{c}}$ and $\texttt{eval}_{\texttt{Prop}}^{\texttt{c}}$ which do *not* use the continuation passing style.

Definition 5.2. Fixpoint function $\texttt{eval}_{\texttt{Z}} : \texttt{expr} \rightarrow \texttt{store} \rightarrow \texttt{option Z}$ attempts to evaluate an expression e to a value z. This method will evaluate in exactly the same way as the method $\texttt{eval}_{\texttt{Z}}^{\texttt{c}}$ from Definition 4.4. The lack of the superscript $\texttt{c}$ denotes that it directly returns z (as an `option` value), instead of passing it to some continuation.

Likewise, fixpoint function $\texttt{eval}_{\texttt{bool}} : \texttt{expr} \rightarrow \texttt{store} \rightarrow \texttt{option bool}$ evaluates expression e in a way comparable to $\texttt{eval}_{\texttt{Prop}}^{\texttt{c}}$ from Definition 4.5; but it computes an actual boolean value, instead of a logical proposition and does not pass it on to a continuation.

We omit precise definitions for both functions.

Definition 5.3. The big step relation $\sigma, s \Downarrow \sigma', o$ describes the *finite execution* of statement s starting from store σ, resulting in store σ' with outcome o. It is inductively defined as:

$$\frac{}{\sigma, \texttt{skip} \Downarrow \sigma, \texttt{N}} \qquad \frac{\sigma, s_1 \Downarrow \sigma', \texttt{N} \qquad \sigma', s_2 \Downarrow \sigma'', o}{\sigma, (s_1;\ s_2) \Downarrow \sigma'', o} \qquad \frac{\sigma, s_1 \Downarrow \sigma', o \qquad o \neq \texttt{N}}{\sigma, (s_1;\ s_2) \Downarrow \sigma', o}$$

$$\frac{\sigma\ x = \texttt{None} \qquad \texttt{eval}_{\texttt{Z}}\ e\ \sigma = \texttt{Some}\ z \qquad \langle[x := \texttt{Some}\ z]\rangle\sigma, s \Downarrow \sigma', o}{\sigma, (\texttt{let}\ x := e\ \texttt{in}\ s) \Downarrow \langle[x := \texttt{None}]\rangle\sigma', o}$$

$$\frac{\texttt{eval}_{\texttt{Z}}\ e\ \sigma = \texttt{Some}\ z}{\sigma, (\texttt{x} = e) \Downarrow \langle[[\texttt{x}] := \texttt{Some}\ z]\rangle\sigma, \texttt{N}}$$

$$\frac{\texttt{eval}_\texttt{bool}\ e\ \sigma = \texttt{Some true} \qquad \sigma, s_1 \Downarrow \sigma', o}{\sigma, (\texttt{if } e \texttt{ then } s_1 \texttt{ else } s_2) \Downarrow \sigma', o}$$

$$\frac{\texttt{eval}_\texttt{bool}\ e\ \sigma = \texttt{Some false} \qquad \sigma, s_2 \Downarrow \sigma', o}{\sigma, (\texttt{if } e \texttt{ then } s_1 \texttt{ else } s_2) \Downarrow \sigma', o} \qquad \frac{\texttt{eval}_\texttt{z}\ e\ \sigma = \texttt{Some } z}{\sigma, (\texttt{return } e) \Downarrow \sigma, \texttt{R } z}$$

$$\frac{\texttt{eval}_\texttt{bool}\ e_c\ \sigma = \texttt{Some true} \qquad \sigma, s \Downarrow \sigma', o \qquad o \neq \texttt{N}}{\sigma, (\texttt{while } e_c \texttt{ inv } e_i\ \{s\}) \Downarrow \sigma', o}$$

$$\frac{\begin{array}{c}\texttt{eval}_\texttt{bool}\ e_c\ \sigma = \texttt{Some true} \\ \sigma, s \Downarrow \sigma', \texttt{N} \qquad \sigma', (\texttt{while } e_c \texttt{ inv } e_i\ \{s\}) \Downarrow \sigma'', o\end{array}}{\sigma, (\texttt{while } e_c \texttt{ inv } e_i\ \{s\}) \Downarrow \sigma'', o}$$

$$\frac{\texttt{eval}_\texttt{bool}\ e_i\ \sigma = \texttt{Some false}}{\sigma, (\texttt{while } e_c \texttt{ inv } e_i\ \{s\}) \Downarrow \sigma, \texttt{N}} \qquad \frac{\sigma, s \Downarrow \sigma', o}{\sigma, \{s\} \Downarrow \sigma', o}$$

We can now proceed to explore the relation between symbolic execution and our semantics for terminating statements.

Definition 5.4. The *well-boundedness* predicate $\texttt{wb} : \texttt{store} \to \texttt{Prop}$ is defined as:

$$\texttt{wb } \sigma := \forall\ x\ z,\ \sigma\ x = \texttt{Some } z \to \texttt{is\_int } z.$$

It asserts that all values in some store σ have a value z within integer bounds $\texttt{int}_\texttt{min} \leq z \leq \texttt{int}_\texttt{max}$ (see also Definition 4.3).

Lemma 5.5. *Given a store σ such that $\texttt{wb } \sigma$, if symbolic execution of some statement s succeeds ($\texttt{exec}_\texttt{stmt}\ s\ C_N\ C_R\ \sigma$) and a terminating execution can be derived in our semantics ($\sigma, s \Downarrow \sigma', o$), then:*

$$\begin{array}{ll} C_N\ \sigma' & \textit{if } o = \texttt{N}, \\ C_R\ z\ \sigma' & \textit{if } o = \texttt{R } z. \end{array}$$

This lemma makes it possible to use the outcome o from an execution in $\Downarrow$ as a witness for the validity of either the normal or returning continuation in the equivalent symbolic execution.

Proof. The while loop requires us to proceed by induction on the derivation of $\sigma, s \Downarrow \sigma', o$. $\qquad\square$

5.2 Diverging statements

Definition 5.6. The big step relation $\sigma, s \Downarrow_\infty$ describes the *diverging execution* of statement s starting from store σ. Since it does not terminate, it is not associated with a final state or an outcome. It is coinductively defined as:

$$\frac{\sigma, s_1 \Downarrow_\infty}{\sigma, (s_1;\, s_2) \Downarrow_\infty} \qquad \frac{\sigma, s_1 \Downarrow \sigma', \mathtt{N} \qquad \sigma', s_2 \Downarrow_\infty}{\sigma, (s_1;\, s_2) \Downarrow_\infty}$$

$$\frac{\sigma\, x = \mathtt{None} \qquad \mathtt{eval_Z}\ e\ \sigma = \mathtt{Some}\ z \qquad \langle[x := \mathtt{Some}\ z]\rangle\sigma, s \Downarrow_\infty}{\sigma, (\mathsf{let}\ x := e\ \mathsf{in}\ s) \Downarrow_\infty}$$

$$\frac{\mathtt{eval_{bool}}\ e\ \sigma = \mathtt{Some\ true} \qquad \sigma, s_1 \Downarrow_\infty}{\sigma, (\mathsf{if}\ e\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2) \Downarrow_\infty}$$

$$\frac{\mathtt{eval_{bool}}\ e\ \sigma = \mathtt{Some\ false} \qquad \sigma, s_2 \Downarrow_\infty}{\sigma, (\mathsf{if}\ e\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2) \Downarrow_\infty}$$

$$\frac{\mathtt{eval_{bool}}\ e_c\ \sigma = \mathtt{Some\ true} \qquad \sigma, s \Downarrow_\infty}{\sigma, (\mathsf{while}\ e_c\ \mathsf{inv}\ e_i\ \{s\}) \Downarrow_\infty}$$

$$\frac{\mathtt{eval_{bool}}\ e_c\ \sigma = \mathtt{Some\ true} \qquad \sigma, s \Downarrow \sigma', \mathtt{N} \qquad \sigma', (\mathsf{while}\ e_c\ \mathsf{inv}\ e_i\ \{s\}) \Downarrow_\infty}{\sigma, (\mathsf{while}\ e_c\ \mathsf{inv}\ e_i\ \{s\}) \Downarrow_\infty}$$

$$\frac{\sigma, s \Downarrow_\infty}{\sigma, \{s\} \Downarrow_\infty}$$

We can now relate the coinductive relation $\Downarrow_\infty$ to symbolic execution.

Definition 5.7. Predicate $\sigma, s \Downarrow$ expresses the *ability* to derive a terminating execution for some statement s starting from σ:

$$\sigma, s \Downarrow := \exists\ \sigma'\ o,\ \sigma, s \Downarrow \sigma', o.$$

The *inability* to derive a terminating execution, expressed $\neg(\sigma, s \Downarrow)$, may stem from either stuckness (for instance, in the case of division-by-zero) or from divergence.

Lemma 5.8. *Given a store σ such that* $\mathtt{wb}\sigma$, *if symbolic execution of some statement s succeeds (*$\mathtt{exec_{stmt}}\ s\ C_N\ C_R\ \sigma$*) but a terminating execution cannot be derived using* cbsem *(in other words, $\neg(\sigma, s \Downarrow)$), then we can derive a diverging execution $\sigma, s \Downarrow_\infty$.*

Proof. The proof is coinductive and proceeds by case analysis on s. Many cases can be discharged *ex falso* by construction of a terminating derivation, contradicting the assumption $\neg(\sigma, s \Downarrow)$.

For the case of a while loop where the loop body s' is executed, we need to *decide* whether the loop body terminates from σ: $(\sigma, s' \Downarrow) \vee \neg(\sigma, s' \Downarrow)$. Because of the halting problem, we are forced at this point to employ an axiomatic instance of the excluded middle, leading to two subcases corresponding to the two rules for diverging while loops in Definition 5.6. $\square$

Lemma 5.8 states that if we cannot make a terminating derivation for some s starting from σ, then the evidence provided by successful symbolic execution of s allows us to narrow down stuckness and divergence as causes for $\neg(\sigma, s \Downarrow)$ to just divergence.

5.3 Soundness of symbolic execution wrt `cbsem`

Definition 5.9. Two stores σ and σ' are *equivalent modulo a list of identifiers* $\overline{x}$, written $\sigma \equiv_{\overline{x}} \sigma'$, if they satisfy $\forall\, x, (x \in \overline{x}) \vee (\sigma\ x = \sigma'\ x)$.

Definition 5.10. Store σ *binds* variables $\overline{x}$, notation $\overline{x} \subseteq \texttt{bindings}\ \sigma$, if $\sigma\ x \neq \texttt{None}$ for $x \in \overline{x}$.

Since we have implemented stores as functions, a list of variables bound by a store must be passed around explicitly in various places. This is a bit of a nuisance and will be fixed in future versions of this work, by representing stores as finite maps rather than functions.

Definition 5.11. The predicate $\texttt{exec}_{\texttt{f}}^{\Downarrow} : \texttt{list string} \to \texttt{expr} \to \texttt{stmt} \to \texttt{expr} \to \texttt{Prop}$ for a function with arguments $\overline{x}$, pre- and postconditions e_p and e_q and function body s expresses correctness of that function with regards to `cbsem`. Formally, this predicate asserts that:

$\texttt{translate}_{\texttt{Prop}}^{\texttt{c}}\ e_p\ (\lambda\ P,$

$\quad P \to ($

$\qquad \exists\ \sigma,\ z, \texttt{translate}_{\texttt{Prop}}^{\texttt{c}}\ e_q\ (\lambda\ Q,\ (\sigma, s \Downarrow \sigma', \texttt{R}\ z) \wedge Q)\ \langle[\text{"result"} := z]\rangle\sigma$

$\quad) \vee (\sigma, s \Downarrow_{\infty})$

$)\ \sigma,$

for any store σ such that $\texttt{wb}\ \sigma$, $\emptyset \equiv_{\overline{x}} \sigma$ and $\overline{x} \subseteq \texttt{bindings}\ \sigma$.

$\texttt{exec}_{\texttt{f}}^{\Downarrow}$ expresses that a function is correct with regards to `cbsem` if, given that the precondition holds for the call arguments, it either terminates with a return outcome which satisfies the postcondition, or diverges. It excludes any undefined behavior and it does not allow termination with a normal outcome.

We will now conclude this section by proving soundness of the earlier notion of function correctness from symbolic execution, $\mathtt{exec_f}$, to $\mathtt{exec_f^{\Downarrow}}$. The proof for this requires a similar soundness lemma for statements.

Lemma 5.12 (Soundness for statements). *If* $\mathtt{exec_{stmt}}\ s\ C_N\ C_R\ \sigma$ *holds with* $\mathtt{wb}\ \sigma$, *then either:*

1. *s terminates normally; that is, there exists some σ' such that we can derive $\sigma, s \Downarrow \sigma', \mathtt{N}$ and $C_N\ \sigma'$ holds;*

2. *s terminates by returning a value; that is, there exists some σ' and z such that we can derive $\sigma, s \Downarrow \sigma', \mathtt{R}\ z$ and $C_R\ z\ \sigma'$ holds;*

3. *s diverges; that is, $\sigma, s \Downarrow_\infty$.*

Proof. Similar to what we encountered in the proof of Lemma 5.8 for the case of the while loop, the halting problem forces us to commence this proof with a case analysis on the axiomatic tautology $(\sigma, s' \Downarrow)\ \vee\ \neg(\sigma, s' \Downarrow)$. The terminating case proceeds by case analysis of the outcome of terminating execution, followed by application of Lemma 5.5. The diverging case depends on Lemma 5.8. $\qquad\square$

Theorem 5.13 (Soundness for functions). *Given a function with arguments $\overline{x}$, pre- and postconditions e_p and e_q and function body s, then $\mathtt{exec_f}\ \overline{x}\ e_p\ s\ e_q$ implies $\mathtt{exec_f^{\Downarrow}}\ \overline{x}\ e_p\ s\ e_q$.*

Proof. The proof proceeds by examining the three cases provided by Lemma 5.12 for symbolic execution of the function body s. The case for normal *termination* of the function body can be easily dispelled, because, as Definition 4.12 shows, the normal *continuation* for s in $\mathtt{exec_f}$ is $\lambda\ \_,\ \mathtt{False}$. The other two cases are trivial. $\qquad\square$

6 Proving correctness in Clight big step semantics

Theorem 5.13 proves that symbolic execution of a function allows us to conclude semantic correctness of that function with regards to its pre- and postcondition within $\mathtt{cbsem}$. But can we be confident that $\mathtt{cbsem}$ itself is a sound semantics for function execution?

In this section we will prove soundness of $\mathtt{cbsem}$ with regards to Clight for a subset of VeriFast Cx. This further narrowing of VeriFast Cx is mainly restricted in terms of some operations for boolean expressions (we drop $e_1 == e_2$, $e_1 \neq e_2$, $e_1\ \&\&\ e_2$, $e_1\ ||\ e_2$ and $!e$) and block statements $\{s\}$. The resulting subset of VeriFast Cx remains expressive enough for meaningful functions with loops, conditionals and the capacity for displaying undefined behavior, including the example from Section 2.

First we will draw relations between VeriFast stores (6.1), expressions (6.2) and statements (6.3) and their counterparts in Clight. Next, we will introduce two small transformations which are necessary to line up the ASTs exported from VeriFast with those exported by `clightgen` (6.4). Finally, because we support only a limited subset of C, we define custom notions of program correctness in both VeriFast and Clight and provide our second main result, proving that proving correctness of program execution in `cbsem` implies correctness of program execution in Clight (6.5).

6.1 Stores and temporary environments

In VeriFast Cx, local variable values are stored in a store σ. But the local variable x introduced by $\mathsf{let}\ x := e\ \mathsf{in}\ s$ exist only for the duration of statement s. Once s terminates, the variable goes out of scope, meaning that the value for x in the resulting store is `None`.

CompCert has two mechanisms for local variables, but we use the so-called *temporary variables*, which are stored in a *temporary environment* `tenv` $\in$ `temp_env`. A major difference with local variables in VeriFast Cx is that temporaries in CompCert exist from the very beginning of a function. This means there is no "unsetting" of temporary variables, e.g. when exiting some block.

Because of this important difference, we need to track a *partial* correspondence between a VeriFast store σ and a Clight temporary environment `tenv` throughout an execution, limited to the variables which are currently in scope *in the VeriFast program.*

Definition 6.1. The relation $\lesssim$: `store` $\to$ `temp_env` $\to$ `Prop` is inductively defined as:

$$\frac{}{\emptyset \lesssim \mathtt{tenv}} \qquad \frac{i_{\mathrm{CL}} = \mathtt{Int.repr}\ z \qquad \sigma \lesssim \mathtt{tenv}}{\langle[x := \mathtt{Some}\ z]\rangle\sigma \lesssim (\mathtt{PTree.set}\ \$x\ (\mathtt{Vint}\ i_{\mathrm{CL}})\ \mathtt{tenv})}$$

$$\frac{\sigma \lesssim \mathtt{tenv}}{\langle[x := \mathtt{None}]\rangle\sigma \lesssim \mathtt{tenv}}$$

In the above definition, CompCert's `PTree.set` function updates a temporary variable identified by $\$x$ (mapping string x to some internal identifier representation) in `tenv` with a CompCert `Int` value built from a value z.

6.2 Expressions

For expressions, the correspondence between VeriFast and Clight for the supported subset of VeriFast Cx is described by two straightforward inductive relations, one for integer evaluation and another one for boolean evaluation, in line with the definitions for $\mathtt{eval_Z}$ and $\mathtt{eval_{bool}}$ from Definition 5.2.

Definition 6.2. The relation $\sim_{\texttt{int}}$: $\texttt{expr} \to \texttt{Clight.expr} \to \texttt{Prop}$ is inductively defined as:

$$\frac{i_{\mathrm{CL}} = \texttt{Int.repr }[\texttt{z}]}{\texttt{z} \sim_{\texttt{int}} \texttt{Econst\_int } i_{\mathrm{CL}} \texttt{ tint}} \qquad \frac{}{\texttt{x} \sim_{\texttt{int}} \texttt{Etempvar \$[x] tint}}$$

$$\frac{e_1 \sim_{\texttt{int}} e_{\mathrm{CL},1} \qquad e_2 \sim_{\texttt{int}} e_{\mathrm{CL},2}}{(e_1 + e_2) \sim_{\texttt{int}} (\texttt{Ebinop Oadd } e_{\mathrm{CL},1} \, e_{\mathrm{CL},2} \texttt{ tint})}$$

$$\frac{e_1 \sim_{\texttt{int}} e_{\mathrm{CL},1} \qquad e_2 \sim_{\texttt{int}} e_{\mathrm{CL},2}}{(e_1 - e_2) \sim_{\texttt{int}} (\texttt{Ebinop Osub } e_{\mathrm{CL},1} \, e_{\mathrm{CL},2} \texttt{ tint})}$$

$$\frac{e_1 \sim_{\texttt{int}} e_{\mathrm{CL},1} \qquad e_2 \sim_{\texttt{int}} e_{\mathrm{CL},2}}{(e_1/e_2) \sim_{\texttt{int}} (\texttt{Ebinop Odiv } e_{\mathrm{CL},1} \, e_{\mathrm{CL},2} \texttt{ tint})}$$

Lemma 6.3 (Soundness of integer evaluation). *Assume a store σ with $\texttt{wb } \sigma$ and a temporary environment $\texttt{tenv}$, such that $\sigma \lesssim \texttt{tenv}$. Let e and e_{CL} be expressions such that $e \sim_{\texttt{int}} e_{CL}$. Then $\texttt{eval}_{\texttt{z}} \, e \, \sigma = \texttt{Some } z$ implies that e_{CL} evaluates to $\texttt{Vint } (\texttt{Int.repr } z)$ by Clight's $\texttt{eval\_expr}$ semantics, starting from $\texttt{tenv}$.*

Proof. By induction on $e \sim_{\texttt{int}} e_{\mathrm{CL}}$. The main issues are the unpacking of CompCert's definitions for evaluation and integer values and proving that VeriFast's $\texttt{eval}_{\texttt{z}}$ implies CompCert's boundedness of the results. $\square$

Definition 6.4. The relation $\sim_{\texttt{bool}}$: $\texttt{expr} \to \texttt{Clight.expr} \to \texttt{Prop}$ is inductively defined as:

$$\frac{}{\texttt{true} \sim_{\texttt{bool}} (\texttt{Econst\_int Int.one tint})}$$

$$\frac{}{\texttt{false} \sim_{\texttt{bool}} (\texttt{Econst\_int Int.zero tint})}$$

$$\frac{e_1 \sim_{\texttt{int}} e_{\mathrm{CL},1} \qquad e_2 \sim_{\texttt{int}} e_{\mathrm{CL},2}}{(e_1 < e_2) \sim_{\texttt{bool}} (\texttt{Ebinop Olt } e_{\mathrm{CL},1} \, e_{\mathrm{CL},2} \texttt{ tint})}$$

$$\frac{e_1 \sim_{\texttt{int}} e_{\mathrm{CL},1} \qquad e_2 \sim_{\texttt{int}} e_{\mathrm{CL},2}}{(e_1 \le e_2) \sim_{\texttt{bool}} (\texttt{Ebinop Ole } e_{\mathrm{CL},1} \, e_{\mathrm{CL},2} \texttt{ tint})}$$

Lemma 6.5 (Soundness of boolean evaluation). *Assume a store σ with $\texttt{wb } \sigma$ and a temporary environment $\texttt{tenv}$, such that $\sigma \lesssim \texttt{tenv}$. Let e and e_{CL} be expressions such that $e \sim_{\texttt{bool}} e_{CL}$. Then $\texttt{eval}_{\texttt{bool}} \, e \, \sigma = \texttt{Some } b$, with b a Coq boolean, implies that there is some Clight value b_{CL}, such that:*

1. *Clight's* `eval_expr` *evaluates expression e_{CL} to value b_{CL};*

2. *and Clight's* `bool_val` *function evaluates value b_{CL} to* `Some b`.

In other words, if VeriFast Cx evaluates a VeriFast expression e to a boolean value b, then Clight evaluates the corresponding expression e_{CL} to a corresponding value b_{CL}.

Proof. By induction on $e \sim_{\mathsf{bool}} e_{\mathrm{CL}}$. As with Lemma 6.3, most of the proof revolves around unpacking CompCert's structures. $\qquad\square$

6.3   Statements

As with expressions, the correspondence between VeriFast Cx and Clight statements is described by an inductively defined relation. We then describe two lemmas describing soundness of statement execution, one for the terminating and one for the diverging case.

Definition 6.6. The relation $\sim_{\mathsf{stmt}}$: `stmt` $\to$ `Clight.statement` $\to$ `Prop` is inductively defined as:

$$\frac{}{\mathsf{skip} \sim_{\mathsf{stmt}} \mathtt{Sskip}} \qquad \frac{s_1 \sim_{\mathsf{stmt}} s_{\mathrm{CL},1} \qquad s_2 \sim_{\mathsf{stmt}} s_{\mathrm{CL},2}}{(s_1;\, s_2) \sim_{\mathsf{stmt}} (\mathtt{Ssequence}\ s_{\mathrm{CL},1}\ s_{\mathrm{CL},2})}$$

$$\frac{e \sim_{\mathsf{int}} e_{\mathrm{CL}} \qquad s \sim_{\mathsf{stmt}} s_{\mathrm{CL}}}{(\mathsf{let}\ x := e\ \mathsf{in}\ s) \sim_{\mathsf{stmt}} (\mathtt{Ssequence}\ (\mathtt{Sset}\ \$x\ e_{\mathrm{CL}})\ s_{\mathrm{CL}})}$$

$$\frac{e \sim_{\mathsf{int}} e_{\mathrm{CL}}}{(\mathsf{x} = e) \sim_{\mathsf{stmt}} (\mathtt{Sset}\ \$[\mathsf{x}]\ e_{\mathrm{CL}})} \qquad \frac{e \sim_{\mathsf{int}} e_{\mathrm{CL}}}{(\mathsf{return}\ e) \sim_{\mathsf{stmt}} (\mathtt{Sreturn}\ (\mathtt{Some}\ e_{\mathrm{CL}}))}$$

$$\frac{e \sim_{\mathsf{bool}} e_{\mathrm{CL}} \qquad s_1 \sim_{\mathsf{stmt}} s_{\mathrm{CL},1} \qquad s_2 \sim_{\mathsf{stmt}} s_{\mathrm{CL},2}}{(\mathsf{if}\ e\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2) \sim_{\mathsf{stmt}} (\mathtt{Sifthenelse}\ e\ s_{\mathrm{CL},1}\ s_{\mathrm{CL},2})}$$

$$\frac{e \sim_{\mathsf{bool}} e_{\mathrm{CL}} \qquad s \sim_{\mathsf{stmt}} s_{\mathrm{CL}}}{(\mathsf{while}\ e\ \mathsf{inv}\ i\ \{s\}) \sim_{\mathsf{stmt}} (\mathtt{Swhile}\ e_{\mathrm{CL}}\ s_{\mathrm{CL}})}$$

Note that we use a combination of statements to model our own scoped variable declaration $\mathsf{let}\ x := e\ \mathsf{in}\ s$.

Next, we provide two lemmas proving the soundness of VeriFast Cx's big step semantics for *statement execution*, $\Downarrow$ and $\Downarrow_\infty$, to their counterparts in Clight's big step semantics, `exec_stmt` and `execinf_stmt`.

For both the terminating and diverging case, we need to point out again the fact that our bigstep semantics `cbsem` does not deal with heap memories. This means that the soundness lemmas for statement execution are universally quantified over Clight memory states *without further side conditions*;

and in the terminating case, the post-execution memory state is identical with the pre-execution memory state.

In addition, `cbsem` does not handle function calls or I/O, whereas Clight keeps track of such observable events using a inductive type `trace` for terminating executions and a coinductive type `traceinf` for diverging executions. So terminating Clight executions in Lemma 6.7 will always be derived with the empty Clight `trace` E0. Somewhat surprisingly perhaps, the lemma for soundness of infinite statement execution (Lemma 6.8) universally quantifies *over every possible Clight* `traceinf`.

Lemma 6.7 (Soundness of a terminating statement execution)*. Assume a store σ with* `wb` σ *and a temporary environment* `tenv`*, such that* $\sigma \lesssim$ `tenv`*. Let s and s_{CL} be statements such that $s \sim_{\texttt{stmt}} s_{CL}$. If $\sigma, s \Downarrow \sigma', o$, then:*

1. *for $o = $ N, there exists a* `tenv`$'$ *such that Clight's* `exec_stmt` *relates execution of s_{CL}, starting from* `tenv`*, with* `tenv`$'$ *and with Clight outcome* `Out_normal`*.*

2. *likewise, for $o = $ R z, there exists a* `tenv`$'$ *such that* `exec_stmt` *relates execution of s_{CL}, starting from* `tenv`*, with* `tenv`$'$ *and outcome* `Out_return`*, together with the proper Clight representation of z as return value.*

In both cases, $\sigma' \lesssim$ `tenv`$'$ and the Clight execution is derived with the empty Clight event trace E0*.*

Proof. As in earlier proofs, the presence of the while loop requires us to proceed by induction on the derivation of $\sigma, s \Downarrow \sigma', o$. □

Lemma 6.8 (Soundness of a diverging statement execution)*. Assume a store σ with* `wb` σ *and a temporary environment* `tenv`*, such that* $\sigma \lesssim$ `tenv`*. Let s and s_{CL} be statements such that $s \sim_{\texttt{stmt}} s_{CL}$. If $\sigma, s \Downarrow_\infty$, then there is diverging execution using Clight's* `execinf_stmt`*, starting from* `tenv`*.*

Proof. The proof is coinductive and proceeds by case analysis on the statement s. □

6.4   Lining up ASTs

We have now constructed a notion $\sim_{\texttt{stmt}}$ of statement equivalence and proven that, if $s \sim_{\texttt{stmt}} s_{CL}$, execution of s in `cbsem` is sound wrt execution of s_{CL} in Clight. However, imagine some C code which falls within the subset of supported expressions and statements, such as the example from Section 2. If VeriFast parses this code and produces a statement s and if CompCert parses this code and produces a statement s_{CL}: will the relation $s \sim_{\texttt{stmt}} s_{CL}$ hold? For our simple test cases, the answer is *yes, except for the following two differences*:

- Our VeriFast export code currently exports a lot of $(s; \mathtt{skip})$ instances, which in Clight are all represented as *just* the Clight equivalent of s.

- For the statement representing a `main` function body, the Clight compiler `clightgen` always wraps a main function with a `return` 0 statement.

So before moving to program correctness, we introduce two AST transformation functions which deal with these differences. For each transformation, we prove that they preserve relevant execution properties within `cbsem`.

Definition 6.9. Fixpoint function $\mathtt{simpl} : \mathtt{stmt} \to \mathtt{stmt}$ recursively removes instances of $(\_; \mathtt{skip})$ from a statement s:

$$\mathtt{simpl}\ (s; \mathtt{skip}) := s.$$
$$\mathtt{simpl}\ (s_1; s_2) := (\mathtt{simpl}\ s_1); (\mathtt{simpl}\ s_2).$$
$$\mathtt{simpl}\ (\mathsf{let}\ x := e\ \mathsf{in}\ s) := \mathsf{let}\ x := e\ \mathsf{in}\ (\mathtt{simpl}\ s).$$
$$\mathtt{simpl}\ (\mathsf{if}\ e\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2) := \mathsf{if}\ e\ \mathsf{then}\ (\mathtt{simpl}\ s_1)\ \mathsf{else}\ (\mathtt{simpl}\ s_2).$$
$$\mathtt{simpl}\ (\mathsf{while}\ e\ \mathsf{inv}\ i\ \{s\}) := \mathsf{while}\ e\ \mathsf{inv}\ i\ \{(\mathtt{simpl}\ s)\}.$$
$$\mathtt{simpl}\ \{s\} := \{\mathtt{simpl}\ s\}.$$
$$\mathtt{simpl}\ s := s, \text{ for all other cases.}$$

Definition 6.10. $\sim_{\mathtt{simpl}} : \mathtt{stmt} \to \mathtt{stmt} \to \mathtt{Prop}$ is an inductively defined propositional relation which reflects the computational function $\mathtt{simpl}$. The only purpose of this extra construct is that it makes the proofs for preservation of termination and divergence (see Lemmas 6.12 and 6.13) easier. We omit the definition for this relation.

Lemma 6.11. *For each statement s, $s \sim_{\mathtt{simpl}} (\mathtt{simpl}\ s)$.*

Proof. By straightforward induction on s. $\qquad\square$

Lemma 6.12 ($\sim_{\mathtt{simpl}}$ preserves termination)**.** *Given statements s_1 and s_2 such that $s_1 \sim_{\mathtt{simpl}} s_2$, then $\sigma, s_1 \Downarrow \sigma', o$ implies $\sigma, s_2 \Downarrow \sigma', o$.*

Proof. By induction on $\sigma, s_1 \Downarrow \sigma', o$. $\qquad\square$

Lemma 6.13 ($\sim_{\mathtt{simpl}}$ preserves divergence)**.** *Given statements s_1 and s_2 such that $s_1 \sim_{\mathtt{simpl}} s_2$, then $\sigma, s_1 \Downarrow_\infty$ implies $\sigma, s_2 \Downarrow_\infty$.*

Proof. Perhaps surprisingly, this proof was slightly more difficult than expected. Starting the proof with coinduction fails for the case in which $s_1 = (s_1'; \mathtt{skip})$, due to the difficulty in fulfilling the guardedness condition. So we start by induction on $s_1 \sim_{\mathtt{simpl}} s_2$, which is a finite derivation, allowing the application of the proper coinductive constructors to produce $\sigma, s_2 \Downarrow_\infty$. We then only need to make a co-recursive call within the case for the while loop, which can be properly guarded. $\qquad\square$

Definition 6.14. Fixpoint function $\texttt{program} : \texttt{stmt} \to \texttt{stmt}$ appends a return statement to some s:

$$\texttt{program}s := s; \texttt{return } 0.$$

This transformation will be necessary because CompCert does it automatically to each main function.

Lemma 6.15 ($\texttt{program}$ preserves return outcomes)**.** *Given* $\sigma, s \Downarrow \sigma', \texttt{R } z$ *then* $\sigma, (\texttt{program}s) \Downarrow \sigma', \texttt{R } z$*; that is,* $\texttt{program}$ *does not change the outcome of an execution which returns a value.*

Proof. By case analysis on the derivation of $\sigma, s \Downarrow \sigma', \texttt{R } z$. $\qquad\square$

Lemma 6.16 ($\texttt{program}$ preserves divergence)**.** *Given that* $\sigma, s \Downarrow_\infty$ *holds, then* $\sigma, (\texttt{program}s) \Downarrow_\infty$*.*

Proof. By application of the $\Downarrow_\infty$ constructor for divergence of the first statement of a sequence. $\qquad\square$

6.5 Correctness of programs

We want to build confidence in our $\texttt{cbsem}$ semantics by proving soundness with respect to Clight. To this end, Lemmas 6.7 and 6.8 already provide good results. But ultimately, our primary goal remains to work towards *full end-to-end soundness on the level of programs*, giving us confidence that a program which is proven correct by symbolic execution in VeriFast, when afterwards compiled with CompCert and executed, will not crash.

The subset of C that we chose for this proof-of-concept does not feature *function calls*, meaning that any statements about program correctness must consider only the $\texttt{main}$ function in a program. With this shortcut in mind, we still need to provide precise notions for the following things:

- program correspondence between a $\texttt{cbsem}$ program and a Clight program (see Definition 6.17);

- program correctness in $\texttt{cbsem}$[8] (see Definition 6.18), together with a connection to $\texttt{cbsem}$'s function correctness $\texttt{exec}_\texttt{f}^\Downarrow$, which is required to make the connection to symbolic execution;

- program correctness for Clight (Definition 6.20).

With all these definitions in place we will then state and prove the second main soundness theorem of this work (Theorem 6.21).

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[8]The notion of $\texttt{cbsem}$ program correctness introduced here will be tailored for usage with Clight, which is why we define it here and not in Section 5.

Definition 6.17. The relation $\sim_{\texttt{prog}}: \texttt{stmt} \to \texttt{Clight.program} \to \texttt{Prop}$ is inductively defined with a single constructor, relating a VeriFast Cx statement (representing the body of the `main` function as parsed with VeriFast) with an entire Clight program p_{CL} with main function `cl_main`:

$$\frac{(\dots) \qquad \texttt{Clight.fn\_body cl\_main} = s_{\text{CL}} \qquad s \sim_{\texttt{stmt}} s_{\text{CL}}}{s \sim_{\texttt{prog}} p_{\text{CL}}}$$

Although not shown explicitly in the above inference rule, this constructor also requires evidence about the initial execution state and the memory layout of the CompCert program. As our C subset does not support programs using heap memory and main functions taking arguments, this can be trivially provided by the VeriFast-exported Coq script, after it has imported the Coq script generated by CompCert.

Definition 6.18. A VeriFast Cx program consists of a `main` function with body s which takes no arguments. This program is *correct*, written $\texttt{exec}_{\texttt{prog}}^{\Downarrow} s$, if either:

- there exist σ' and z such that $\emptyset, s \Downarrow \sigma', \texttt{R } z$;

- or $\sigma, s \Downarrow_{\infty}$.

Predicate $\texttt{exec}_{\texttt{prog}}^{\Downarrow}$ will be proven sound with regards to Clight. But in order to apply the full end-to-end chain of soundness proofs, we need to link it to symbolic execution. In order to use Theorem 5.13, which links the correctness of symbolic execution $\texttt{exec}_{\texttt{f}}$ to semantic correctness $\texttt{exec}_{\texttt{f}}^{\Downarrow}$, we need another small lemma which links $\texttt{exec}_{\texttt{f}}^{\Downarrow}$ to $\texttt{exec}_{\texttt{prog}}^{\Downarrow}$.

Lemma 6.19. *Given* $\texttt{exec}_{\texttt{f}}^{\Downarrow}$ *[ ]* `true` s q*, that is the correct execution in* `cbsem` *of a function without arguments and with body* s*, precondition* `true` *and postcondition* q*, then we may conclude* $\texttt{exec}_{\texttt{prog}}^{\Downarrow}$ (`program` (`simpl` s))*.*

Proof. Hypothesis $\texttt{exec}_{\texttt{f}}^{\Downarrow}$ [ ] `true` s q is a disjunction which allows us to consider two cases:

1. For the case of termination of s with a return outcome, we proceed by Lemmas 6.12, 6.13 and 6.11.

2. For the case of divergence of s, we proceed by Lemmas 6.15 and 6.16.

$\square$

Definition 6.20. Predicate `exec_prog` : `Clight.program` $\to$ `Prop` expresses correctness of a program p with regards to Clight's own big step semantics (`ClightBigstep` [2]) if:

- either there exists a return value v such that, for the empty trace `E0`, `bigstep_program_terminates` p `E0` (`Int.repr` v);

- or `bigstep_program_diverges` p $\mathtt{trc}_\infty$ holds for all infinite event traces $\mathtt{trc}_\infty$.

`bigstep_program_terminates` and `bigstep_program_diverges` describe CompCert's notion of program correctness for terminating and diverging programs.

The universal quantifier in the diverging case results from the fact that `traceinf` is defined coinductively akin to a stream, so there is no empty `traceinf` constructor. Since there is no concept in `cbsem` which is equivalent to Clight execution traces, any infinite event trace is accepted.

Theorem 6.21 (Soundness of program execution). *Consider a main function with body s. Let s' be shorthand notation for `program (simpl s)`. If $s' \sim_{\mathtt{prog}} p$ for some Clight program p, then program correctness $\mathtt{exec}_{\mathtt{prog}}^{\Downarrow} s'$ in VeriFast implies program correctness `exec_prog` p in Clight.*

Proof. Hypothesis $\mathtt{exec}_{\mathtt{prog}}^{\Downarrow} s'$ provides us with two cases, both of which provide all the evidence needed to conclude `bigstep_program_terminates` p for the returning case or `bigstep_program_diverges` p for the diverging case. $\qquad\square$

7 Related work

Due to the limited scale of this TR, it is early to provide a deep comparison of advantages and disadvantages with respect to approaches taken by other projects. We do however want to list the following related projects.

Certificate-generating program verification tools

Boogie verifies a program, written in its intermediate language, by transforming it through a number of steps into a Verification Condition (VC). This VC can then be validated by an SMT solver. Similar to our approach, a recent extension to the Boogie verifier [10] allows the automatic generation of machine checkable certificates for a verification run, thereby avoiding direct verification of Boogie itself. These certificates import the input- and output programs for three of the most complicated steps in Boogie's verification pipeline into Isabelle. For each step, it is proven that correctness of the output program (or VC) implies correctness of the input program. The certificates however do not currently prove the VC itself and therefore do not constitute a full machine checkable correctness proof of the verified program.

Certified verification of C programs

Verified Software Toolchain (VST) [1] allows semi-automatic production of a correctness proof for C programs, stated in a program logic called Verifiable

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C. It is developed entirely within Coq and tightly integrated with CompCert, making it possible to conclude that the correctness properties of a verified program are preserved during compilation with CompCert. Verifiable C contains a very complete set of C language features and allows reasoning about memory states through separation logic. The actual proving in VST, while assisted by a library of powerful tactics (VST-Floyd), still requires manual guidance. Our own aim however is for our certificates to be standalone Coq scripts, meaning that we want to avoid interactive proof guidance at the level of Coq. We plan to achieve this by translating into Coq the pre-existing VeriFast constructs such as loop invariants, lemmas and heap predicates; and by replicating the reasoning performed by VeriFast's SMT solver.

RefinedC [11] takes an approach to foundational verification not unlike that taken by VST, in the sense that it allows stating the correctness of C programs featuring heap memory and concurrency and that proving these properties takes place entirely within Coq. Its language semantics also describe a language with a low-level memory model similar to (but not identical with) that of CompCert [8]. However, RefinedC differs significantly from VST in that it promises fully automatic proving, meaning that somebody who is certifying a C program with RefinedC should not have to delve into Coq. To this end, RefinedC annotations describing pre- and postconditions and invariants leverage a sophisticated typing system, which is translated into a subset of the Iris framework for separation logic [5] that facilitates efficient, syntax-directed proof search. Automation therefore may require an expert extending the typing system for new use cases. Likewise, by relying on the translation of existing VeriFast constructs and by inspection of the SMT solver, we aim to offer a similar level of automation. We hope that, to some degree, our approach will make it possible to handle new use cases *merely* through these existing VeriFast constructs, rather than by having an expert make changes to the Coq library or to the exporting code.

8 Future work

The most obvious need for expansion of course exists in the subset of C that we support. Our ultimate goal is to support the full set of C supported by VeriFast. This will require symbolic execution in Coq and `cbsem` to support function calls, pointer and `struct` types, heap memory and separation logic. We will also need to translate supporting VeriFast constructs such as heap predicates, lemmas and fixpoints into Coq and make these translated versions useable for our automatically generated SEP proofs. Keeping these proofs automated for the general case will require us to move beyond the simplistic syntax-driven tactics presented in Section 4.2.

In terms of setting up a *soundness chain* with respect to a third party operational semantics for C, we aim to prove `cbsem` sound with respect to

CH2O [6], which provides a very strict interpretation of the C11 standard [3]. If VeriFast can prove a program correct in CH2O, this should mean that the program will be correct with respect to any common compiler which respects the C standard – which, while in no way a *formal* statement, is a very useful result.

We can then still further extend the soundness chain by proving CH2O operational semantics sound with respect to one of CompCert's operational semantics. This would allow us to keep the benefit of being able to make a formal statement about program correctness down to the level of assembly code, a benefit that we currently possess by virtue of Theorem 6.21, proving the soundness of `cbsem` with respect to CompCert.

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A   Building and using the extension

The extension described in this TR is not yet included in the VeriFast trunk. But as mentioned in the introduction, it can be downloaded here:

$$\texttt{https://doi.org/10.5281/zenodo.5585276}$$

In this appendix we provide instructions on how to build this extension to VeriFast and how to use it for verification of simple programs which fall within the limited subset supported by VeriFast Cx.

A.1   Building the extension

First we need to build VeriFast itself. First, download the code from the above address and check out the branch which contains the extension:

```
$ git clone verifast-coq-export-tr-2021.bundle verifast
$ cd verifast
$ git checkout coq-export-tr-2021
```

Next, we follow the standard build instructions for VeriFast. For macOS, we can build VeriFast like this:

```
$ ./setup-build.sh
$ cd src
$ export PATH=/usr/local/vfdeps-509f16f/bin:$PATH
$ export DYLD_LIBRARY_PATH=/usr/local/vfdeps-509f16f/lib:
    $DYLD_LIBRARY_PATH
$ make
```

For other platforms such as Windows and Linux, consult the VeriFast documentation itself (these can be found through the main `README.md` included with the VeriFast project).

Next we need to install CompCert and compile our own Coq code. We have developed this project using Coq 8.13.2 and CompCert 3.9. Our current suggestion is to do all of this using OCaml's package manager Opam. Opam can be easily installed using your system's package manager, such as

Homebrew on macOS. Opam allows us to build a local switch (which is a local, self-contained installation of OCaml and Coq).

Once you have installed Opam, execute the following commands from the main VeriFast directory to install the Coq and CompCert requirements:

```
$ cd ..
$ opam switch create . 4.12.0
$ eval $(opam env)
$ opam install coq.8.13.2
$ opam repo add coq-released https://coq.inria.fr/opam/released
$ opam install coq-compcert.3.9
```

Now we can compile the VeriFast Coq code itself:

```
$ coq_makefile -f "_CoqProject" -o "Makefile.coq"
$ make -f Makefile.coq
```

A.2   Using the extension

A number of test files are included in folder `tests/coq`. These have not yet been added to the main test suite, because our setup will change in the future[9]. To run one of these included examples (such as `test_countdown.c`, which is the example from Section 2), use the `-emit_coq_proof` option:

```
$ bin/verifast -shared -emit_coq_proof -bindir bin
    tests/coq/test_countdown.c
```

Upon successful verification, this will generate a Coq artefact `test_vf.v` in your present working directory. This artefact will need to be successfully typechecked by Coq to prove the correctness of VeriFast's verification. But because this artefact imports another Coq file, generated by CompCert's `clightgen`, we first need to generate this second script and compile it using Coq:

```
$ clightgen tests/coq/test_countdown.c -o test_cc.v
$ coqc test_cc.v
```

With the CompCert export compiled, we can now finally typecheck the artefact exported by VeriFast:

```
$ coqc test_vf.v -Q src/coq verifast
```

---

[9]Specifically, we may decide to no longer automatically load the CompCert-generated Coq script from within our own artefact. More fundamentally, we will likely opt to replace CompCert with another third-party C semantics.

Note that some examples from `tests/coq` require the `-allow_dead_code` option to be used with VeriFast, since they explicitly test how our export code deals with unreachable code.

Finally, a simple shell script `test_coq.sh` has been included to perform the above 4 steps automatically:

```
$ ./test_coq.sh tests/coq/test_countdown.c
```

This script leaves the resulting scripts around for further inspection.

B   Overview of the extension code

In the course of our work, we developed a Coq library which must be loaded by each proof artefact. In this appendix, we briefly discuss the Coq modules which comprise this library. We also provide lookup tables to locate the implementations of the definitions, lemmas and theorems in this report. The files for this library can be found under `src/coq/*.v`. Finally, we also briefly describe how we instrumented the VeriFast OCaml code to export the certificates themselves.

base.v

`cx.v` contains basic Coq notation settings shared by our entire library.

cx.v

`cx.v` contains the main type definitions for embedding VeriFast Cx ASTs in Coq. Its contents are mainly introduced in Section 3.

Name in Coq module	Concise	Introduced
expr	$e \in \mathtt{expr}$	Definition 3.1
stmt	$s \in \mathtt{stmt}$	Definition 3.3
stmt_to_free_targets	$\mathtt{targets}\ s$	Definition 4.8

shared.v

Module `shared.v` currently collects a variety of basic data structures and lemmas related to stores, store equivalence, variable binding and generating propositions that quantify over variables.

Name in Coq module	Concise	Introduced
store	$\sigma \in \texttt{store}$	Definition 4.1
empty_store	$\emptyset$	Definition 4.1
store_update	$\langle[x := v]\rangle\sigma$	Definition 4.1
cont	$C \in \texttt{cont}$	Definition 4.2
min_signed	$\texttt{int}_{\texttt{min}}$	Definition 4.3
max_signed	$\texttt{int}_{\texttt{max}}$	Definition 4.3
is_int	$\texttt{is\_int}\ z$	Definition 4.3
for_Z	$\texttt{for}_{\texttt{Z}}\ x\ C\ \sigma$	Definition 4.3
for_Zs	$\texttt{for}_{\texttt{Zs}}\ \overline{x}\ C\ \sigma$	Definition 4.3
havoc_Zs	$\texttt{havoc}_{\texttt{Zs}}\ \overline{x}\ C\ \sigma$	Definition 4.3
well_bound	$\texttt{wb}\ \sigma$	Definition 5.4
store_eq_mod_xs	$\sigma \equiv_{\overline{x}} \sigma'$	Definition 5.9
binds	$\overline{x} \subseteq \texttt{bindings}\ \sigma$	Definition 5.10

`translate_expr.v`

This module implements a number of functions to translate VeriFast Cx expressions into Coq data types such as `Prop`. Translation differs from evaluation in that no side conditions (such as checks for division by zero) are generated.

Name in Coq module	Concise	Introduced
translate_expr_to_Prop_cps	$\texttt{translate}_{\texttt{Prop}}^{\texttt{C}}\ e\ C\ \sigma$	Definition 4.6

`eval_expr.v`

Module `eval_expr.v` implements functions for the evaluation of VeriFast Cx expressions into Coq data types such as `Prop`. Side conditions (such as checks for division by zero) are generated whenever needed. (However, CPS versions of these functions are found in `symexec.v` because they are uniquely related to symbolic execution.)

Name in Coq module	Concise	Introduced
eval_expr_to_Z	$\texttt{eval}_{\texttt{Z}}\ e\ C\ \sigma$	Definition 5.2
eval_expr_to_bool	$\texttt{eval}_{\texttt{bool}}\ e\ C\ \sigma$	Definition 5.2

`symexec.v`

Module `symexec.v` implements everything related to symbolic execution in Coq: (CPS-based) evaluation of expressions and functions constructing the SEP for execution of a statement and a function. It also provides a number of useful lemmas for working with SEPs in proofs.

Name in Coq module	Concise	Introduced
eval_expr_to_Z_cps	$\text{eval}_Z^{\text{c}}\ e\ C\ \sigma$	Definition 4.4
eval_expr_Prop_cps	$\text{eval}_{\text{Prop}}^{\text{c}}\ e\ C\ \sigma$	Definition 4.5
produce	$\text{produce}\ e\ C\ \sigma$	Definition 4.7
consume	$\text{consume}\ e\ C\ \sigma$	Definition 4.7
leak_check	$\text{leak\_check}\ \sigma$	Definition 4.9
exec_stmt	$\text{exec}_{\text{stmt}}\ s\ C_N\ C_R\ \sigma$	Definition 4.10
ret	$\text{ret}\ \sigma_0\ C\ z\ \sigma$	Definition 4.11
exec_func_correct	$\text{exec}_{\text{f}}\ \overline{x}\ e_p\ s\ e_q$	Definition 4.12

cbsem.v

Module `cbsem.v` implements the `cbsem` operational semantics itself, together with some useful lemmas.

Name in Coq module	Concise	Introduced
outcome	$o \in \texttt{outcome}$	Definition 5.1
exec_stmt	$\sigma, s \Downarrow \sigma', o$	Definition 5.3
execinf_stmt	$\sigma, s \Downarrow_\infty$	Definition 5.6
exec_stmt_derivable	$\sigma, s \Downarrow$	Definition 5.7
exec_func_correct	$\text{exec}_{\text{f}}^{\Downarrow}\ \overline{x}\ e_p\ s\ e_q$	Definition 5.11

symexec_cbsem.v

Module `symexec_cbsem.v` implements the first soundness result which proves the soundness of symbolic execution with respect to `cbsem`.

Name in Coq module	Concise	Introduced
symexec_stmt_sound_termination		Lemma 5.5
symexec_stmt_sound_divergence		Lemma 5.8
symexec_stmt_sound		Lemma 5.12
symexec_func_sound		Theorem 5.13

cbsem_clight.v

Module `cbsem_clight.v` provides the structures needed to express the correspondence between a VeriFast Cx program and a Clight program. Using these structures, it then provides a number of lemmas, ending with the second soundness theorem which proves soundness of `cbsem` with respect to CompCert's Clight for a subset of VeriFast Cx.

Name in Coq module	Concise	Introduced
`st_tenv_rel`	$\sigma \lesssim \mathtt{tenv}$	Definition 6.1
`expr_equiv_int`	$e \sim_{\mathtt{int}} e_{\mathrm{CL}}$	Definition 6.2
`vf_cl_expr_to_int_sound`		Lemma 6.3
`expr_equiv_b`	$e \sim_{\mathtt{bool}} e_{\mathrm{CL}}$	Definition 6.4
`vf_cl_expr_to_bool_sound`		Lemma 6.5
`stmt_equiv`	$s \sim_{\mathtt{stmt}} s_{\mathrm{CL}}$	Definition 6.6
`vf_cl_exec_stmt_sound`		Lemma 6.7
`vf_cl_execinf_stmt_sound`		Lemma 6.8
`simplify_vf_stmt`	$\mathtt{simpl}\ s$	Definition 6.9
`simplify_vf_stmt_rel`	$s \sim_{\mathtt{simpl}} s'$	Definition 6.10
`simplify_vf_stmt_rel_intro`		Lemma 6.11
`simplify_vf_stmt__preserves__termination`		Lemma 6.12
`simplify_vf_stmt__preserves__divergence`		Lemma 6.13
`programify_vf_stmt`	$\mathtt{program}\ s$	Definition 6.14
`programify_vf_stmt__preserves__return`		Lemma 6.15
`programify_vf_stmt__preserves__divergence`		Lemma 6.16
`prog_equiv`	$s \sim_{\mathtt{prog}} p_{\mathrm{CL}}$	Definition 6.17
`cbsem_exec_prog_correct`	$\mathtt{exec}_{\mathtt{prog}}^{\Downarrow}\ s$	Definition 6.18
`cbsem_func_sound`		Lemma 6.19
`compcert_exec_prog_correct`	$\mathtt{exec\_prog}\ p_{\mathrm{CL}}$	Definition 6.20
`vf_cl_sound`		Theorem 6.21

Instrumenting VeriFast to generate certificates

The main export code can be found in the OCaml module `src/coq.ml`. Our exporting code depends on constructing a `recorder` object when setting up a verification run (see function `verify_program` in `src/verifast.ml`). For the duration of the actual verification, this recording object is available everywhere in the code.

We then added recorder calls at various strategic points in the VeriFast code base. These calls correspond to high level events such as the SMT solver making assumptions and checking assertions, fresh symbol picking, pushing and popping SMT solver contexts and branching.

This intermediate recording tree is folded to produce a tree with nodes corresponding to the four basic types of proof steps discussed in Subsection 4.2: *assumptions* and *assertions* (for calls to the SMT solver); *conjunctions* (for branching the execution) and a node expressing that an execution branch was terminated due to the presence of a *contradictory path condition*. This final tree is then used, together with the program's AST, to export the certificate (see function `verify` in `src/vfconsole.ml`).

C   Full listing for `test_vf.v`

```
1   From Coq Require Import Bool.
2   From Coq Require Import List.
3   From Coq Require Import Psatz.
4   From Coq Require Import String.
5   From Coq Require Import ZArith.
6   From compcert Require Import Coqlib.
7   From compcert Require Import Integers.
8   From compcert Require Import Floats.
9   From compcert Require Import AST.
10  From compcert Require Import Ctypes.
11  From compcert Require Import Cop.
12  From compcert Require Import Clight.
13  From compcert Require Import Clightdefs.
14  From verifast Require Import cbsem.
15  From verifast Require Import symexec_clight.
16
17  Import Clightdefs.ClightNotations.
18  Import ListNotations.
19
20  Open Scope string_scope.
21  Open Scope Z_scope.
22
23  (* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *)
24
25  Require test_cc.
26
27  Definition test_cc_prog: Clight.program := mkprogram
28    test_cc.composites
29    [(test_cc._main, Gfun(Internal test_cc.f_main))]
30    [test_cc._main]
31    test_cc._main
32    Logic.I.
33
34  (* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *)
35
36  Definition main_vf_func :=
37    (StmtLet "x" ((ExprIntLit (32767))) (
38      (
39        (StmtWhile
40          (ExprLt (ExprIntLit (0)) (ExprVar "x"))
41          (ExprLe (ExprIntLit (0)) (ExprVar "x"))
42          (
43            (StmtBlock
44              (
45                (StmtExpr (ExprAssign
46                  (ExprVar "x")
47                  (ExprSub (ExprVar "x") (ExprIntLit (1)))));
48                StmtSkip
49              )
50            );
51            StmtSkip
```

```
52         )
53           StmtSkip
54         );
55         (
56           (StmtReturn (ExprVar "x"));
57           StmtSkip
58         )
59       )
60    )).
61
62  (* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *)
63
64  Lemma main_cbsem_by_symexec_tactics:
65    cbsem.exec_func_correct
66      (nil)
67      ExprTrue
68      main_vf_func
69      (ExprEq (ExprVar "result") (ExprIntLit (0))).
70  Proof.
71    apply symexec_cbsem.symexec_func_sound.
72    repeat (autounfold with vf_symexec_clight; simpl).
73    simpl.
74    intros. (* _: True *)
75    split.
76    (* assert 0 <= 32767 *)
77    lia.
78    intros. (* x: Z *)
79    intros. (* _: 0 <= x *)
80    split.
81    intros. (* _: 0 < x *)
82    split.
83    (* assert INT_MIN <= (x - 1) *)
84    lia.
85    split.
86    (* assert (x - 1) <= INT_MAX *)
87    lia.
88    split.
89    (* assert 0 <= (x - 1) *)
90    lia.
91    (* assert True *)
92    lia.
93    intros. (* _:  (0 < x) *)
94    split.
95    (* assert x = 0 *)
96    lia.
97    (* assert True *)
98    lia.
99  Qed.
100
101 (* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *)
102
103 Goal cbsem_clight.compcert_exec_prog_correct test_cc_prog.
104 Proof.
105   apply (cbsem_clight.vf_cl_sound main_vf_func).
```

```
106    - repeat econstructor.
107      unfold Globalenvs.Genv.init_mem. simpl.
108      case_eq (Memory.Mem.alloc Memory.Mem.empty 0 1). intros.
109      destruct (Memory.Mem.range_perm_drop_2 m b 0 1 Memtype.Nonempty).
110    + unfold Memory.Mem.range_perm.
111      intros.
112      apply Memory.Mem.perm_alloc_2 with (1:=H) (2:=H0).
113    + rewrite e.
114      reflexivity.
115   - eapply cbsem_clight.cbsem_func_sound.
116     eapply main_cbsem_by_symexec_tactics.
117  Qed.
```

Listing 2: The full listing for `test_vf.v`, the script exported by VeriFast for the example in Section 2.