

Question 1a

$$\frac{50}{s^2 + 5s} = \frac{50}{s(s+5)} = 50 \cdot \frac{1}{s(s+5)} = 50 \cdot \frac{1}{s} \cdot \frac{1}{s+5} = 50 \cdot \frac{1}{s} \cdot \frac{1}{5} \cdot \frac{1}{\frac{s}{5} + 1}$$

$$G(s) = 10 \cdot \frac{1}{s} \cdot \frac{1}{\frac{s}{5} + 1}$$

constant gain of 10 or 20dB

red: gain

blue: pole

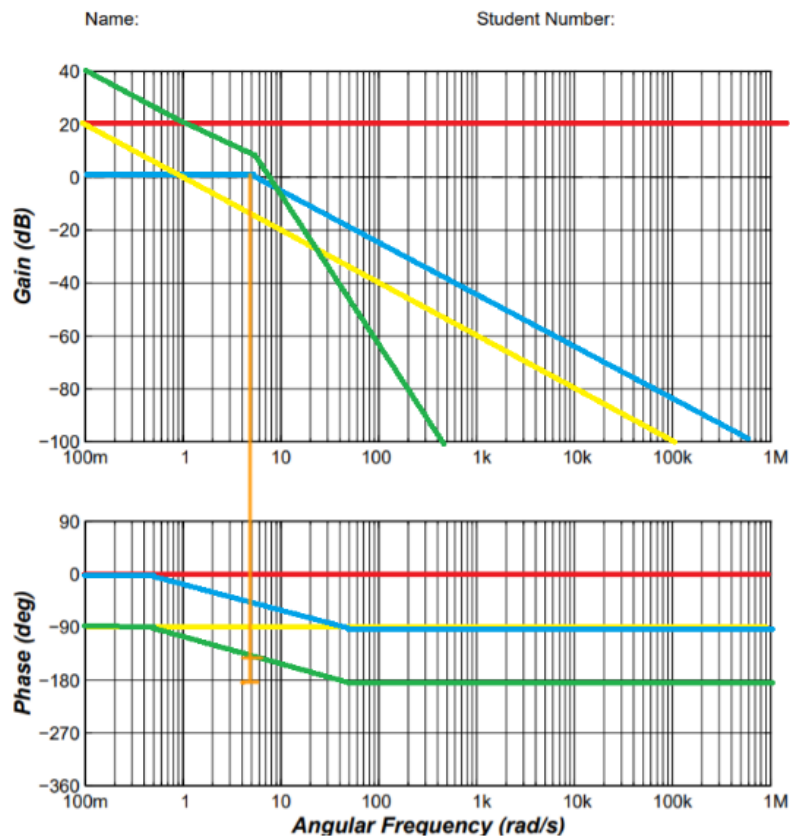
yellow: integrator

green: total

orange = phase margin of ~40

gain margin is inf

system is stable due to it's positive cross over angle (~120 degrees)

**Question 1b**

$$\frac{(s+25)}{s^2+4s} = (s+25) \cdot \frac{1}{s(s+4)} = (s+25) \cdot \frac{1}{s} \cdot \frac{1}{(s+4)} = (s+25) \cdot \frac{1}{s} \cdot \frac{1}{4} \cdot \frac{1}{\frac{s}{4}+1}$$

$$G(s) = (s+25) \cdot \frac{1}{4} \cdot \frac{1}{s} \cdot \frac{1}{\frac{s}{4}+1}$$

constant gain of 0.25 or -12.0411998266dB

red = gain

blue = pole

yellow = integrator

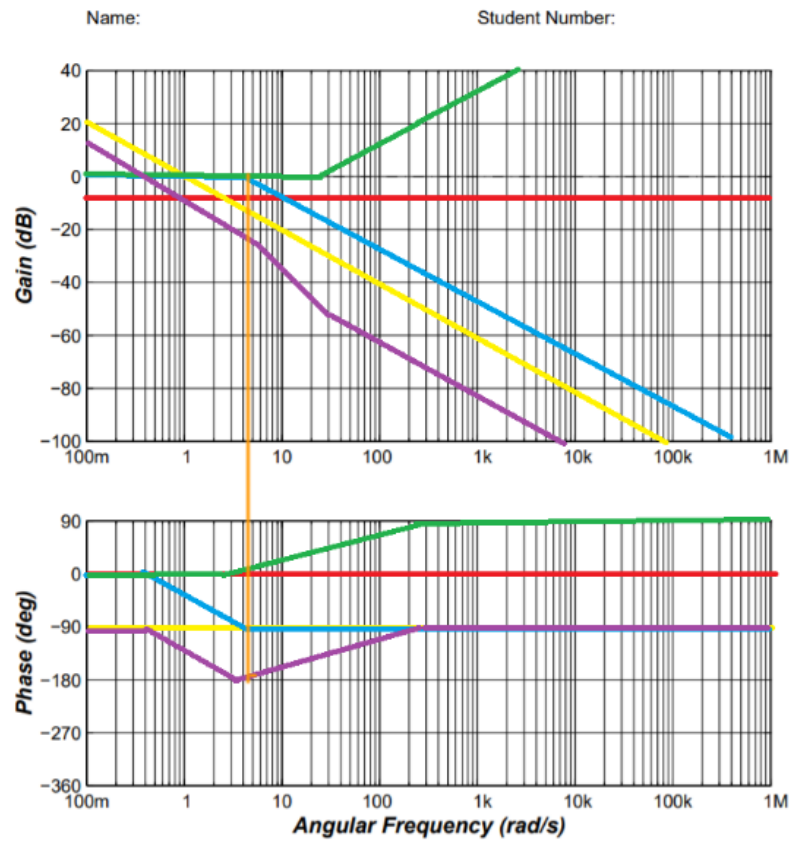
green = zeros

purple = total

orange = phase margin of ~10 degrees

gain margin of inf

system is stable as it has a positive cross over frequency (~170 degrees)



Question 1c

$$\frac{640}{s^3 + 16s^2 + 64} = 10 \cdot \frac{1}{s} \cdot \frac{64}{s^2 + 16s + 64}$$

given $\frac{w^2}{s^2 + 2\zeta ws + w^2}$ format is $10 \cdot \frac{1}{s} \cdot \frac{64}{s^2 + 16s + 64}$

$$\zeta = 1$$

$$G(s) = 10 \cdot \frac{1}{s} \cdot \frac{64}{(s+8)^2} = \frac{10}{8^2} \cdot \frac{1}{s} \cdot \frac{64}{\left(\frac{s}{8} + 1\right)^2}$$

$$G(s) = 10 \cdot \frac{1}{s} \cdot \frac{1}{\left(\frac{s}{8} + 1\right)^2}$$

constant gain of 10 or -16.1235994797dB (needs correction)

red = gain

yellow = integrator

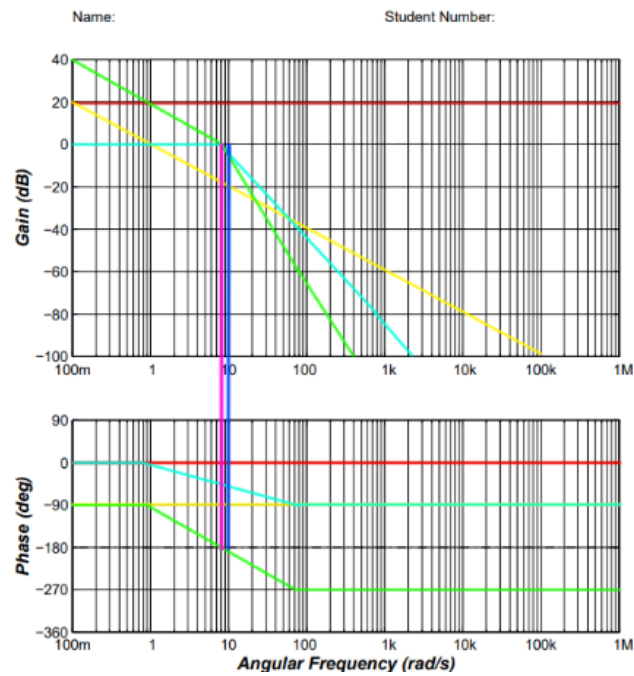
blue = repeated poles

green = total

magenta = phase margin: ~10

blue = gain margin: ~4

system is unstable since there's a gain margin after the cross over frequency.



Question 2

$$\begin{aligned} \frac{5400}{s^2 + 60s + 900} &= 5400 \cdot \frac{1}{s^2 + 60s + 900} \\ &= 5400 \cdot \frac{1}{(s + 30)^2} = \frac{5400}{30^2} \cdot \frac{1}{\left(\frac{s}{30} + 1\right)^2} \\ G &= 6 \cdot \frac{1}{\left(\frac{s}{30} + 1\right)^2} \end{aligned}$$

blue = gain

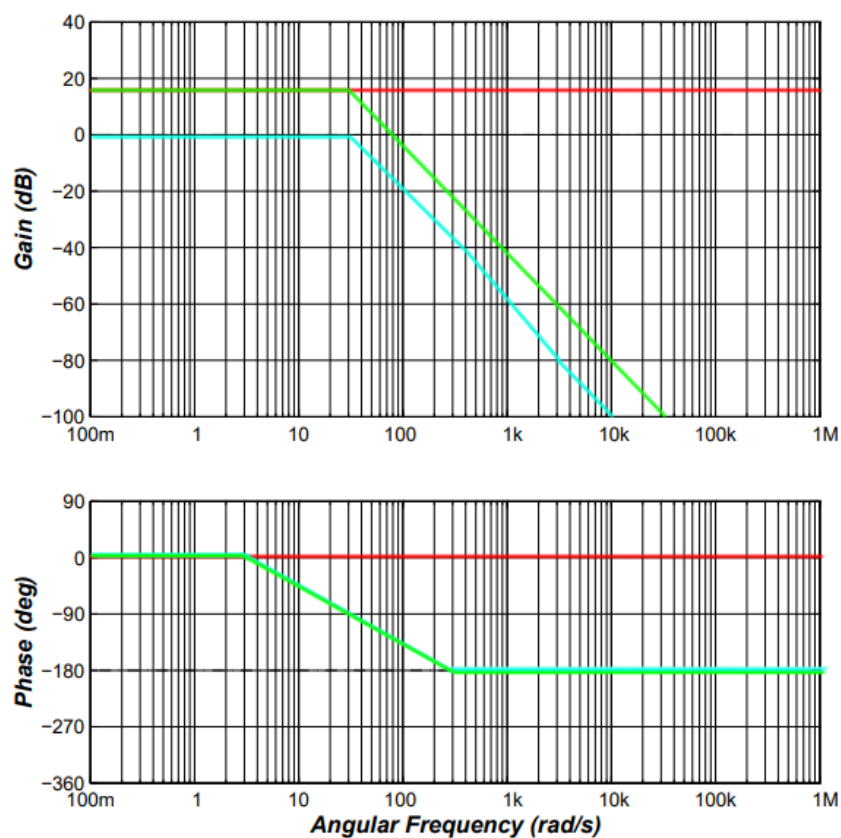
red = pole at 30 x2

green = total

(uncompensated plant)

Name:

Student Number:



insert a $G < 1$, to shift the gain down so that it may have a phase margin greater than 50 degrees. let $G = 0.5$. thus an additional line will be added at -6.0206. as a result, the phase margin is now shifted to be greater than 50 degrees.

green = uncompensated system

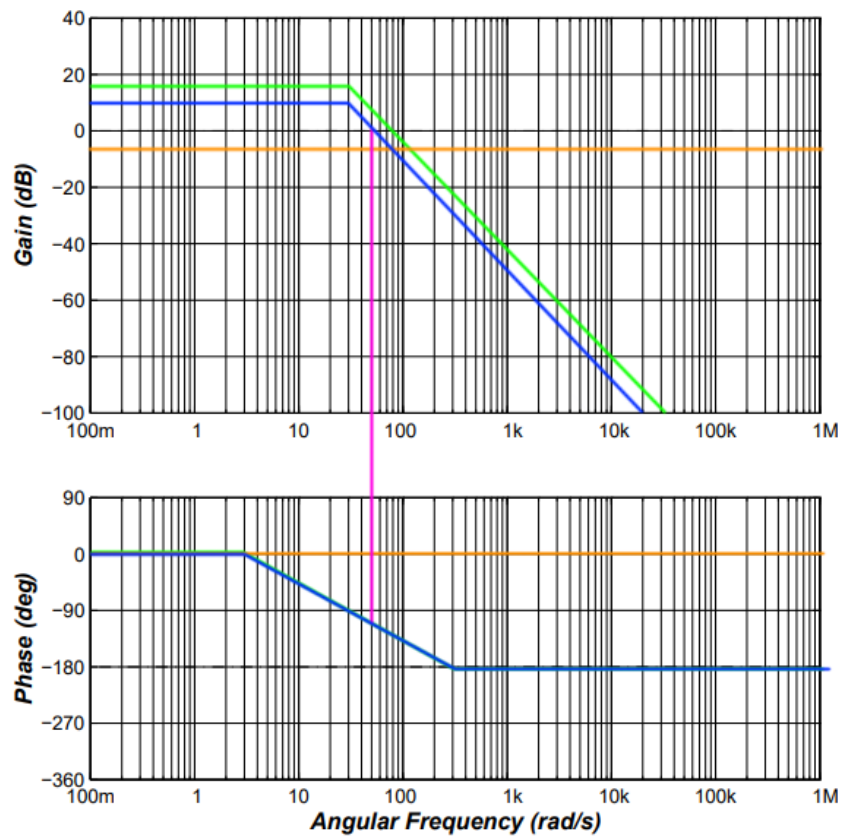
orange = Kp at 0.5

magenta = phase margin ~80

blue = uncompensated * 0.5

Name:

Student Number:



new transfer function is now

$$G = 3 \cdot \frac{1}{\left(\frac{s}{30} + 1\right)^2}$$

it would be ideal to add a pole at the origin and a zero to cancel out the pole. However, if the zero is too far away from the pole the system will become unstable. General rule of thumb is to have a zero a decade before the phase margin in this case -5. This will remove the steady state error as it reduces the system to a type 1 system where a step input will have 0 steady state error. thus a PI will be introduced

$$PI = \left(\frac{s + 5}{s}\right)$$

blue = K_p * uncompensated

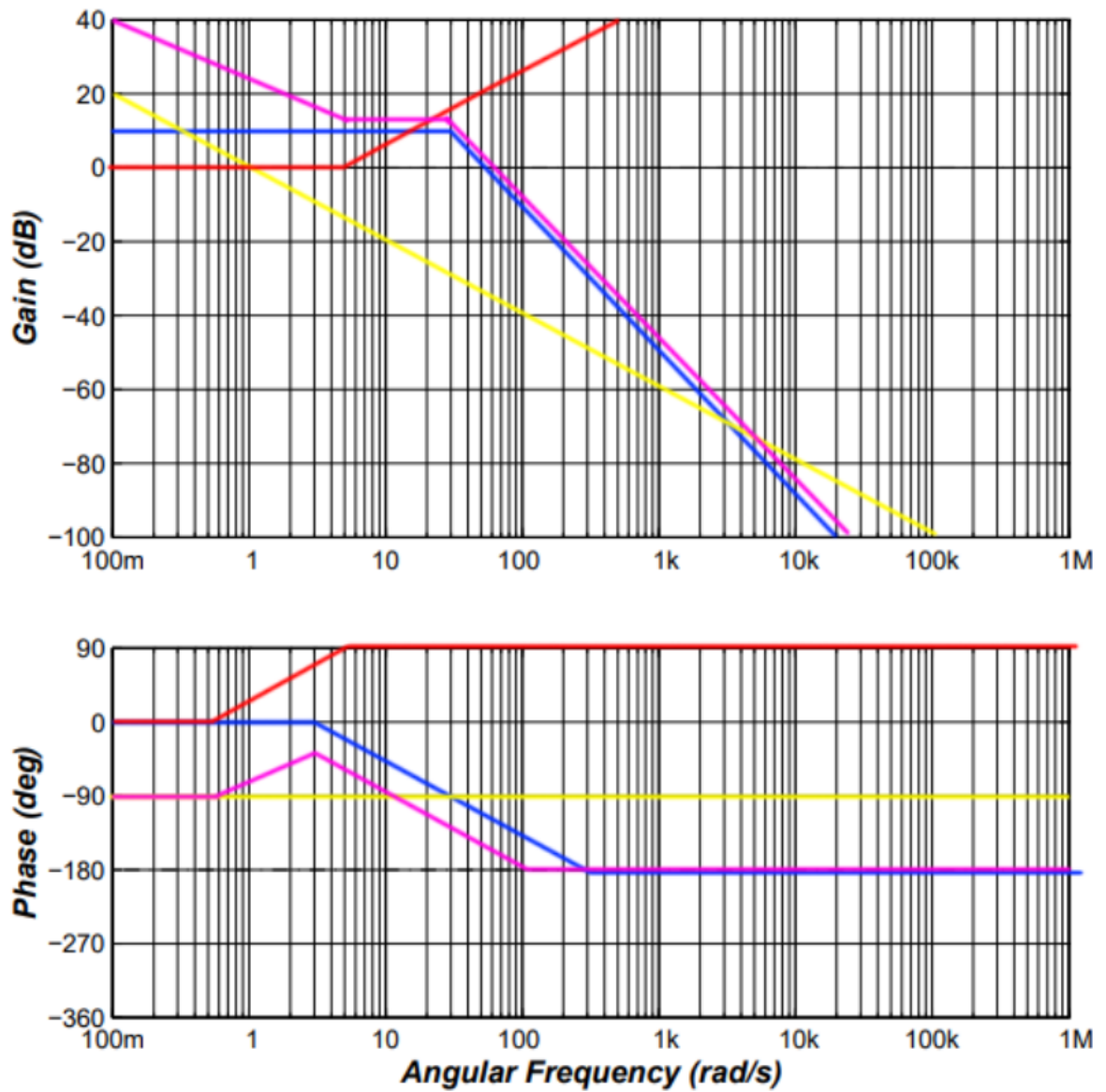
red = zero at -5

yellow = integrator

magenta = fully compensated system with phase margin of 63.6 degrees

Name:

Student Number:



therefore our actual compensator can be modelled as

$$compensator = 0.5 \left(\frac{s+5}{s} \right)$$

red = zero at -5

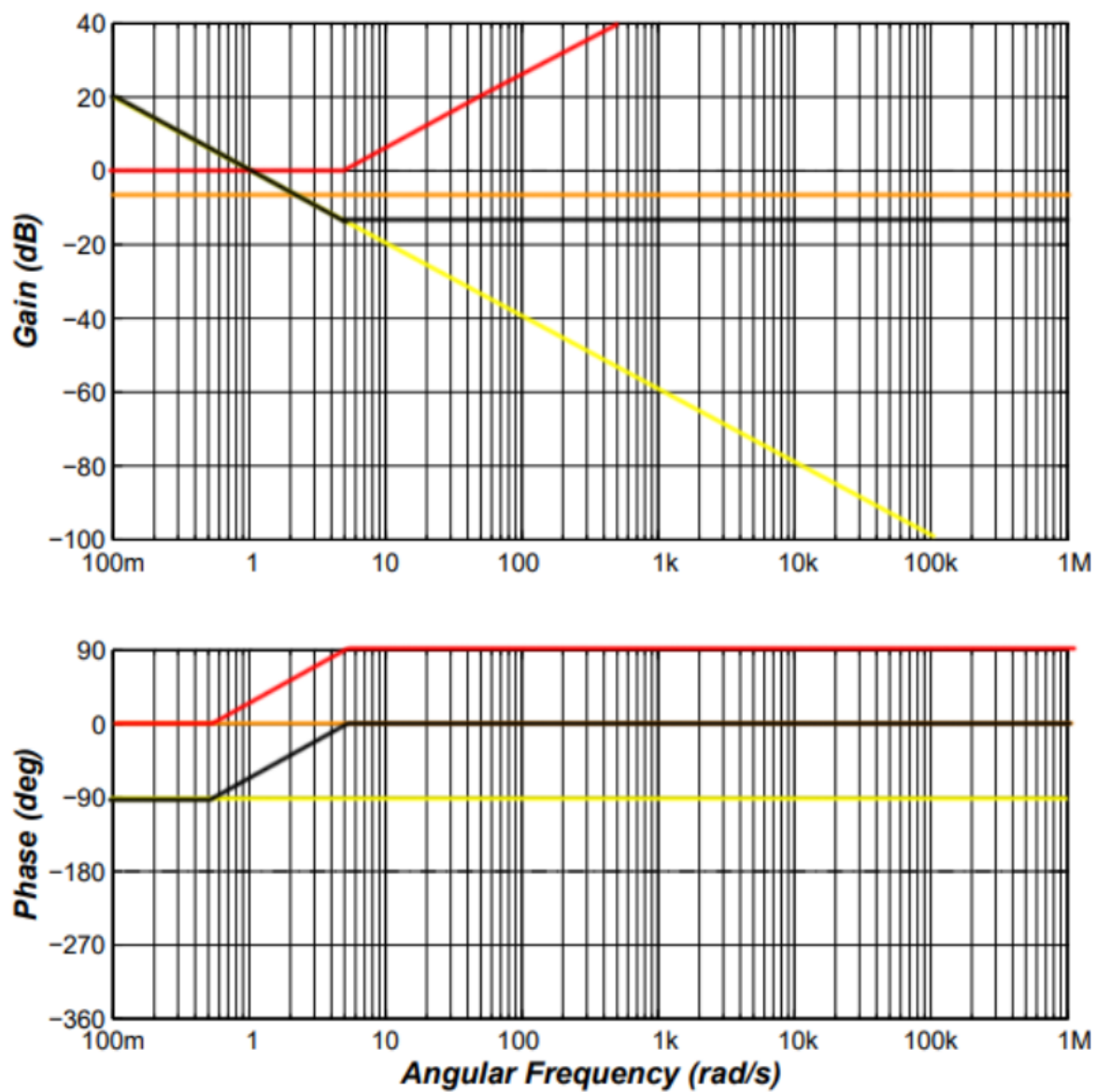
yellow = integrator

orange = Kp (gain)

black = full compensator

Name:

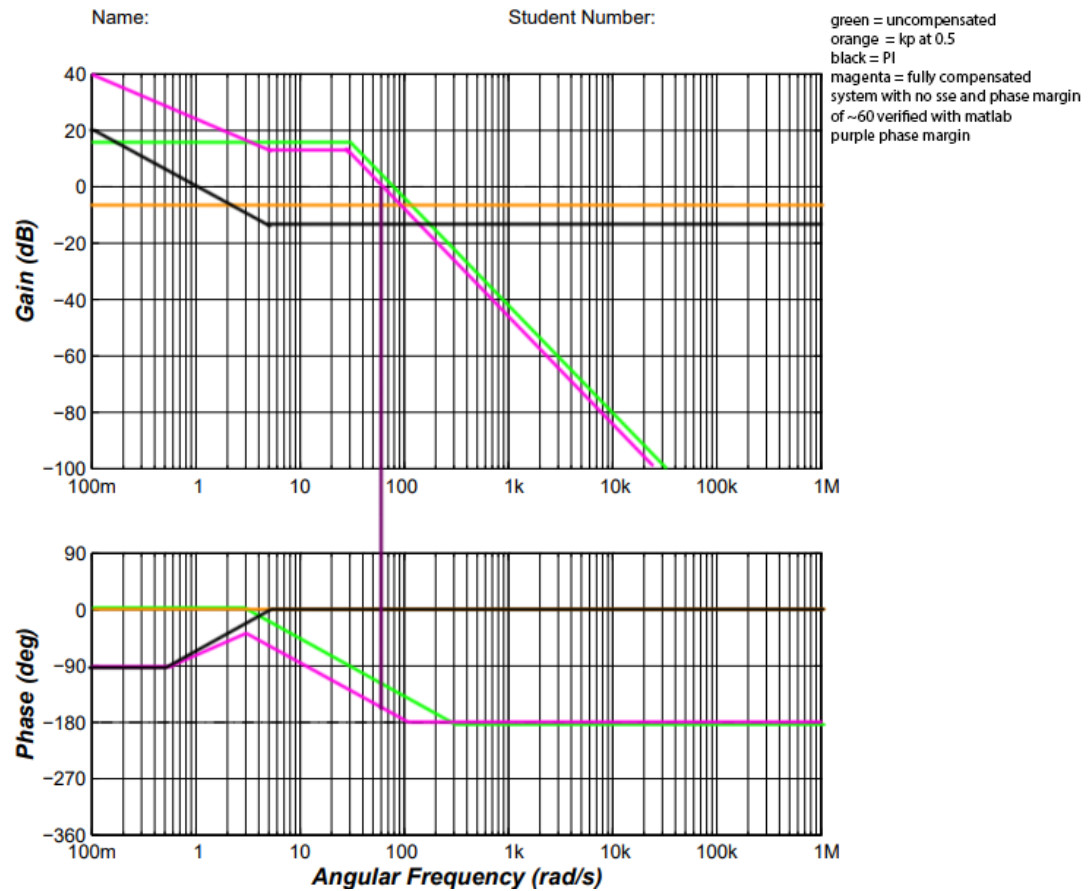
Student Number:



Finally the transfer function of the compensated system would be

$$\frac{2700s + 13500}{s^3 + 60s^2 + 900s}$$

macro look of the compensator



Question 3a

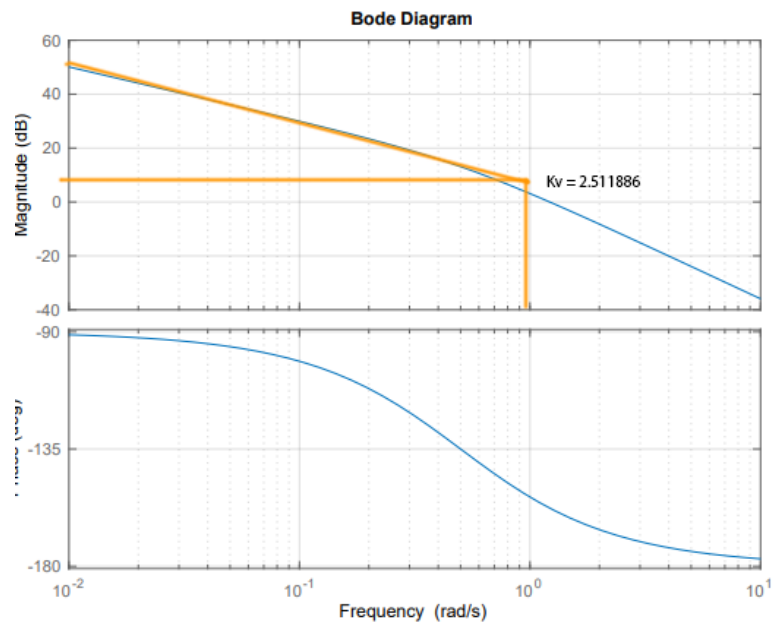
a) type 1 system

reason: phase went from -90 to -180 degrees, meaning there's 2 poles and 1 zero. A drop off point was observed to be at approximately 8dB which gives us a gain value of 2.51186.

step input sse = 0

ramp input sse = $1/2.51186 = 0.39811135971$

parabolic input sse = inf



Question 3b

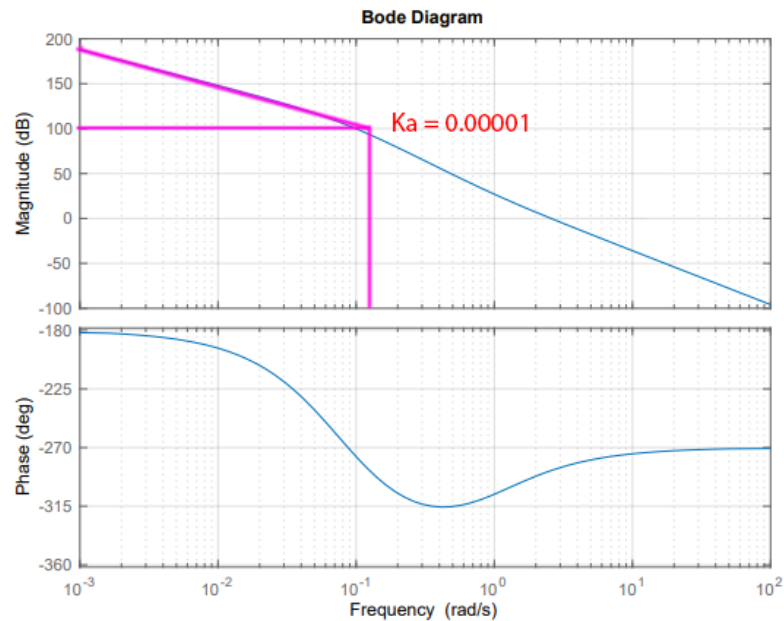
this is a type 2 system.

reason: it has a zero which allowed the phase to recover from -315 degrees and back to -270 degrees. A drop off point was observed to be approximately 100dB. This results to a gain of 100000

step input sse: 0

ramp input sse: 0

parabolic input sse: $1/100000 = 0.00001$

**Question 3c**

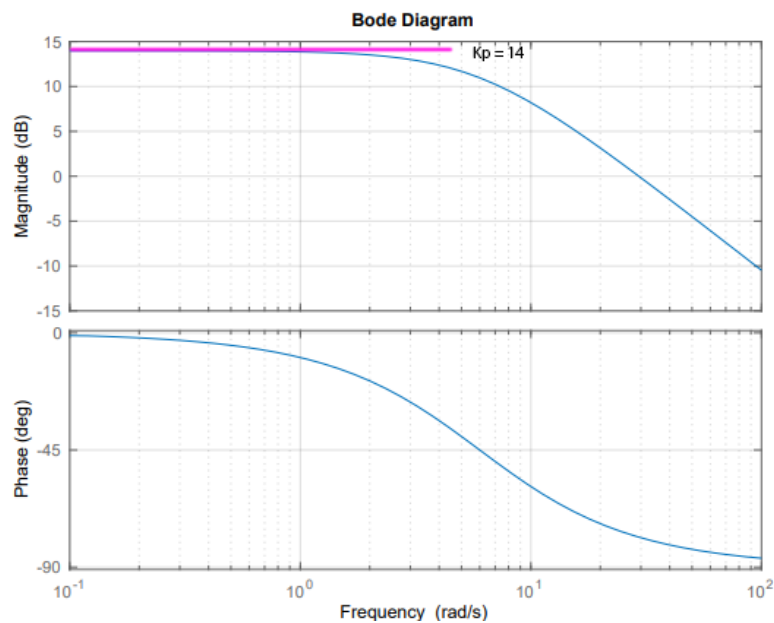
type 0

reason: there's a zero to cancel out a pole. A drop off point was observed to be at approximately 14dB which means that there's a gain 5.01187233627

step input sse: $1/(1+5.01187233627) = 0.16633753081$

ramp input sse: inf

parabolic input sse: inf



In []: