Lab 1: Ian + Toby

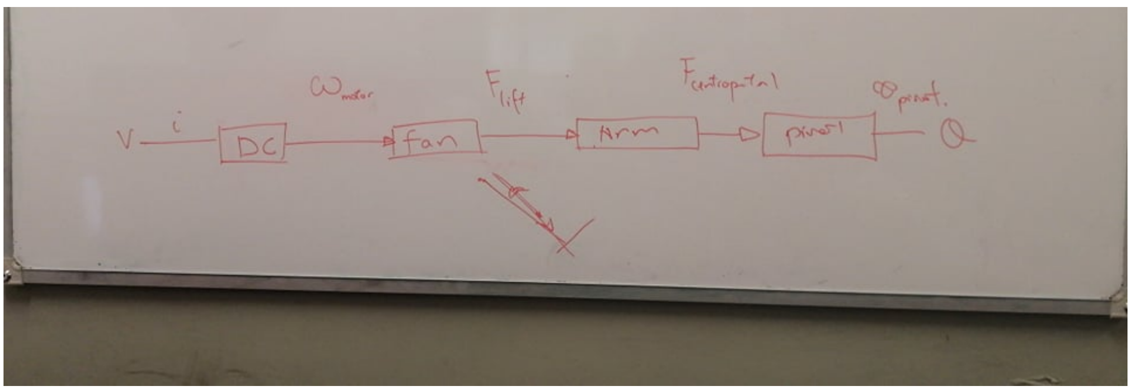
Introduction

The purpose of this lab is to design a complex system; motorised propeller driven rigid pendulum. This lab is meant to be relatively open-ended in solving the various problems. In the first lab my lab partner (Toby) and I was tasked to derive a mathematical model for the system and measure the open loop response. However, the ultimate goal for this lab is to design a closed loop control system.

Part 1:

The purpose of the first lab is to create a block diagram for the system. Furthermore a health and safety briefing was covered in order to prevent lab accidents. (see appendix)

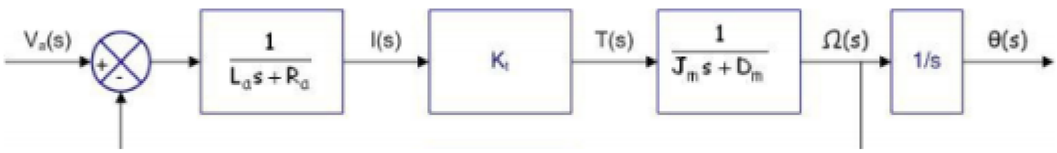
Block diagram of the pendulum



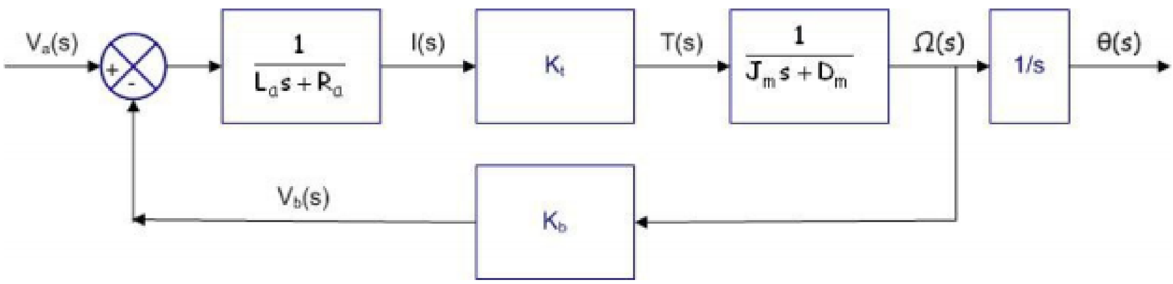
A simplified version of the open loop system which Toby and I attempted.

Part 2:

Input-output characteristic of block diagrams



A model laplace block diagram for the open loop pendulum system.



A model laplace block diagram for the close loop pendulum system.

Transfer function of armature system

$$\frac{\Omega_m(s)}{V_a(s)} = \frac{K_t/(J_m L_a)}{s^2 + \left(\frac{J_m R_a + D_m L_a}{J_m L_a}\right)s + \left(\frac{R_a D_m + K_t K_b}{J_m L_a}\right)}$$

R_a = Armature resistance
L_a = Armature inductance
K_t = Torque constant
K_b = Back emf constant
J_m = Inertia of the motor
D_m = Damping of the motor.

Ω is given as the angular velocity and Va is given as the input voltage.

Derivation process, (variables see appendix)

3. $B \cdot L \cdot I(t) = f(t)$ → arm force

$V_b(t) = K_b \cdot \frac{d\theta(t)}{dt} = K_b \cdot \omega(t)$

feed back

back emf

voltage - current relationship in arm

$V_a(t) = I_a(t) + L_a \cdot \frac{dI(t)}{dt} + V_b(t)$

feed back voltage could be negative

Laplace Transform

$V_a(s) = RI(s) + L_a s I(s) + V_b(s)$

$\tau_{motor} \propto I_a(t)$

$\therefore \tau_m = K_{Torque} \cdot I_a(s)$ $\qquad I_a(s) = \frac{\tau_m(s)}{K_{Torque}}$

sub into Laplace (Electrical System) of Arm

$V_a(s) = R\frac{\tau_{motor}(s)}{K_{Torque}} + L_a \cdot s \cdot \frac{\tau_m(s)}{K_{Torque}} + K_b \omega_{motor}(s)$

$$Va(s) = \frac{T(s)(R_a + L_a s)}{Kt} + K_{fb} s * \Theta$$

Mechanical system

$$T_m(t) = J_m \cdot \frac{\partial^2 \theta(t)}{\partial t^2} + D_m \frac{\partial \theta(t)}{\partial t}$$

Laplace in Terms of $\Omega(s)$ angular velocity

$$\cancel{T(s) = J_m}$$

$$T(s) = J_m \cdot s \cdot \Omega(s) + D_m \Omega(s) = (J_m \cdot s + D_m)\,\Omega(s)$$

Sub in

$$\frac{T(s)}{K_t}(R_a + L_a \cdot s) + K_{fb} \cdot s \cdot \theta_m(s) = V_a$$

$$(J_m \cdot s + D_m)\frac{1}{K_t}(R_a + L_a \cdot s) + K_{fb} \cdot s \cdot \theta_m = V_a(s)$$

Transfer function in terms of θ_m displacement

$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_t/(J_m L_a)}{s^3 + \left(\frac{J_m R_a + D_m \cdot L_a}{J_m \cdot L_a}\right) + \left(\frac{R_a D_m + K_t \cdot K_b}{J_m \cdot L_a}\right)s}$$

divide to get in terms of angular velocity

$$\frac{\Omega_m(s)}{V_a(s)} = \frac{K_t/(J_m L_a)}{s^2 + \left(\frac{J_m \cdot R_a + D_m \cdot L_a}{J_m \cdot L_a}\right)s + \left(\frac{R_a D_m + K_t \cdot K_b}{J_m \cdot L_a}\right)}$$

a

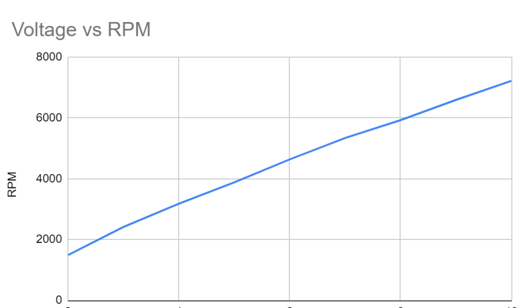
Ra (values)
10.40
14.8
19.32
27.99
12.2
13.2
12.8
18.744
17.8
15
11.8
15.8ohm (mean)

b

La = time constant = 159ms
V = 10V
L = (Ra+220)/T = 235/.159 = 1477.98742138

c

$$K_b = \frac{v_b - i_b R_a}{\omega}$$



It shows that it's linear

d

Therefore our Kb

voltage	current	rpm	W	kb(abs)
1	0.14	524	54.873	0.0200462887
2	0.19	1491	156.1374	0.005443923109
3	0.29	2414.833333	252.8807	0.005338485697
4	0.41	3179	332.9041	0.006458316374
5	0.54	3880.333333	406.31	0.007629642391
6	0.7	4633.333333	485.1666	0.009275164449
7	0.88	5334.5	588.5752	0.01053391308
8	1.05	5922.333333	620.1853	0.01249626523
9	1.25	6586.833333	689.68431	0.01413690272
10	1.45	7214.666667	755.518119	0.01555224118

To obtain the kb a formula was used which requires V,I, RPM and w (omega). RPM was obtained using a laser that detects movement. This laser device is not very accurate when it comes to the measurement of the RPM of the fan. To increase the precision Toby and I attached a small bandaid to one of the blades of the motor which will aid in the laser's ability to detect movement. 10 tests were made to increase the accuracy of the RPM of the fan.

All previous variables and their average.

Jm = τKt

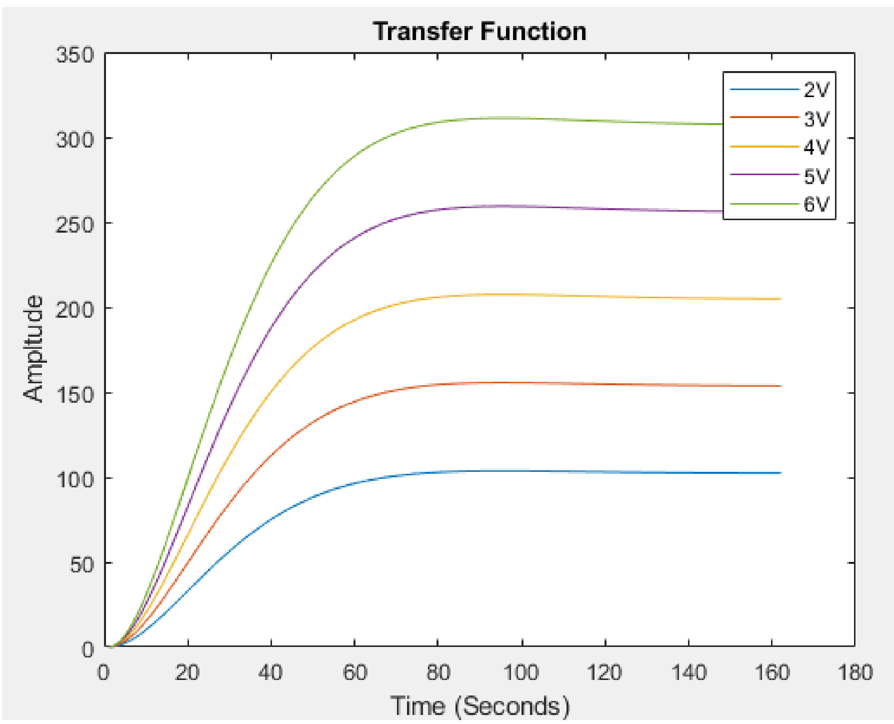
V	I	10.4	La	Kb=Kt	Dm	Jm
1	0.14	14.8	1477.987421	0.0200462887	0.000051145015	0.003187359904
2	0.19	19.32	1477.987421	0.005443923109	0.000006624584	0.000865583774
3	0.29	27.99	1477.987421	0.005338485697	0.000006122099	0.000848819225
4	0.41	12.2	1477.987421	0.006458316374	0.000007953971	0.001026872303
5	0.54	13.2	1477.987421	0.007629642391	0.000010140057	0.00121311314
6	0.7	12.8	1477.987421	0.009275164449	0.000013382238	0.001474751147
7	0.88	18.744	1477.987421	0.01053391308	0.000015749633	0.00167489218
8	1.05	17.8	1477.987421	0.01249626523	0.000021156706	0.001986906171
9	1.25	15	1477.987421	0.01413690272	0.000025622053	0.002247767533
10	1.45	11.8	1477.987421	0.01555224118	0.000029848059	0.002472806347
		15.82309091	1477.987421	0.006829106404	0.000008844590	0.001085827918

4) simulation of the motor response

Motor parameters	Average values
Ra	15.8231
La	1477.987421
Kb = Kt	0.00682911
Dm	8.84*10^-6
Jm	1.1*10^-3
Kt/(Jm*La)	4.2*10^-3
(Jm*Ra + Dm*La)/Jm*La	0.01872
(Ra*Dm+Kb*Kt)/Jm*La	0.00011

$$\frac{\Omega_m(s)}{V_a(s)} = \frac{K_t/(J_m L_a)}{s^2 + \left(\frac{J_m R_a + D_m L_a}{J_m L_a}\right)s + \left(\frac{R_a D_m + K_t K_b}{J_m L_a}\right)}$$

The transfer function was plotted with the inputs of 2 to 6V



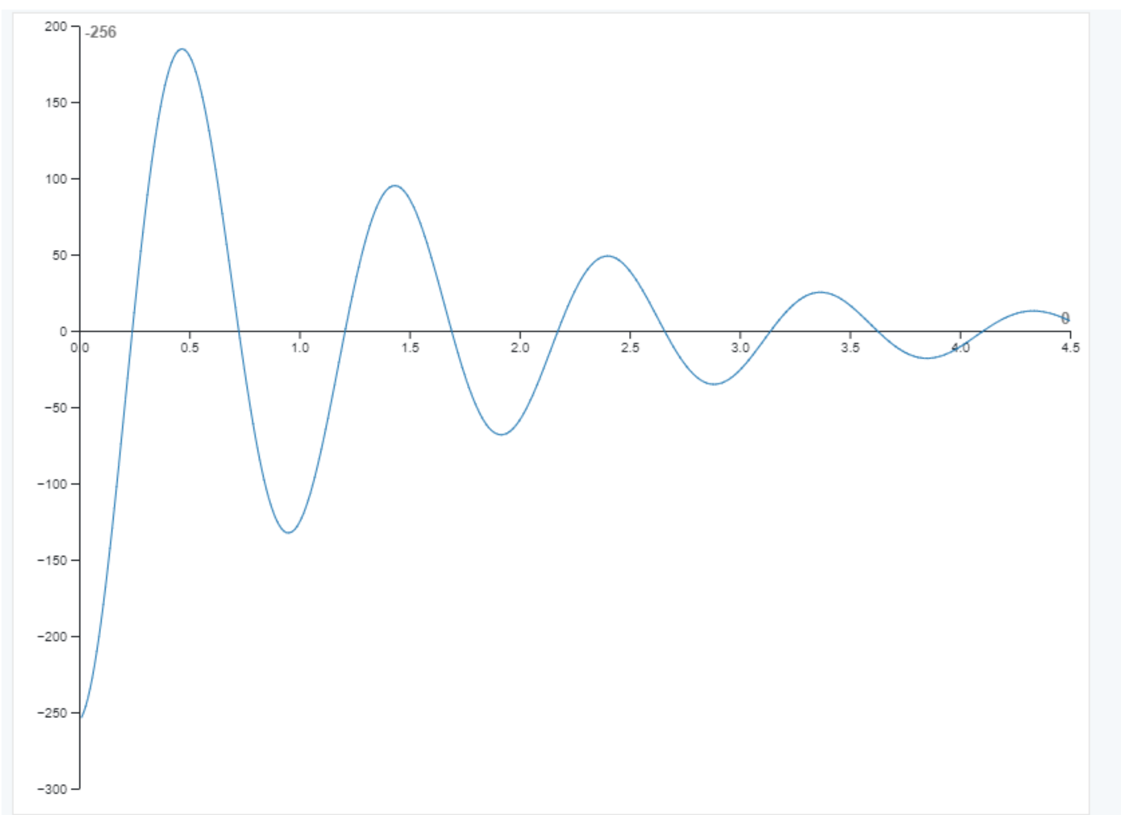
Steady states

voltage	1	2	3	4	5	6
Steady state	-	102.4	153.6	205.8	256.5	310
ω	54.873	156.1374	252.8807	332.9041	406.31	485.1666

There was an enormous difference between experimental results and modelled response. It is safe to assume that the way Kb/Kt was measured was inaccurate due to the errors in the measuring device (laser tracker). Thus resulting in RPM being incorrect. We have altered the Kb to match previous years Kb's result in order to improve our findings.

Part 3:

Measurement of system parameter for the pendulum



$$y = A e^{-\beta \cdot t} \cos(\omega t + \varphi)$$

$$\varphi = 0$$

$$A = -256 \quad @\ t = 0$$

$$\omega = 2\pi / T \qquad T = 0.95 \qquad \varphi = 0$$

$$\cancel{-256 = -256 \cdot e^{-\beta \cdot t} \cos(6.6138\ldots t)}$$

$$\cancel{184 = -256 \cdot e^{-\beta \cdot 0.47} \cdot \cos(6.614 \cdot 0.47)}$$

$$184 = -256 \cdot e^{-\beta \cdot 0.47} \cdot \cos(6.614 \cdot 0.47)$$

$$\frac{184}{-256 \cdot \cos(6.614 \times 0.47)} = e^{-\beta \cdot 0.47}$$

$$0.719 1418 = e^{-\beta \cdot 0.47}$$

$$\ln(0.7191418) = \ln(e^{-\beta \cdot 0.47})$$

$$-0.32969 = -\beta \cdot 0.47$$

$$\frac{-0.32969}{-0.47} = \beta = 0.7$$

$$\omega = 6.614 = \sqrt{\frac{m \cdot g \cdot d}{J_p} - \beta^2} = \sqrt{\frac{0.168 \cdot 9.81 \cdot 0.14}{J_p} - 0.7^2}$$

$$6.614^2 = \frac{0.168 \cdot 9.81 \cdot 0.14}{J_p} - 0.49$$

$$44.234996 = \frac{0.2307312}{J_p} \qquad J_p = \frac{0.2307312}{44.234996} = 0.00521$$

Therefore c = B*2*Jp ⇒ c = 0.7*2*0.00521 = 0.007294

Previos formula

$$\frac{\Omega_m(s)}{V(s)} = \frac{K_t/(J_m L_a)}{s^2 + \left(\frac{J_m \cdot R_a + D_m L_a}{J_m \cdot L_a}\right)s + \left(\frac{R_a D_m + K_t \cdot K_b}{J_m \cdot L_a}\right)}$$

angular velocity / voltage.

integrate to get displacement.

$$G(s) = \frac{\theta(s)}{V(s)} = \frac{1}{s} \cdot \frac{\Omega(s)}{V(s)}.$$

$$G(s) = \frac{K_t/(J_m \cdot L_a)}{s^3 + \left(\frac{J_m \cdot R_a + D_m \cdot L_a}{J_m \cdot L_a}\right)s^2 + \left(\frac{R_a \cdot D_m + K_t \cdot K_b}{J_m \cdot L_a}\right)s}$$

$$G(s) = \frac{4.2*10^{-3}}{s^3+0.01872s^2+0.00011s}$$

It is worth mentioning that Toby and I did not have enough time to tinker with the real system to give a solid comparison between theoretical calculations and real calculations.

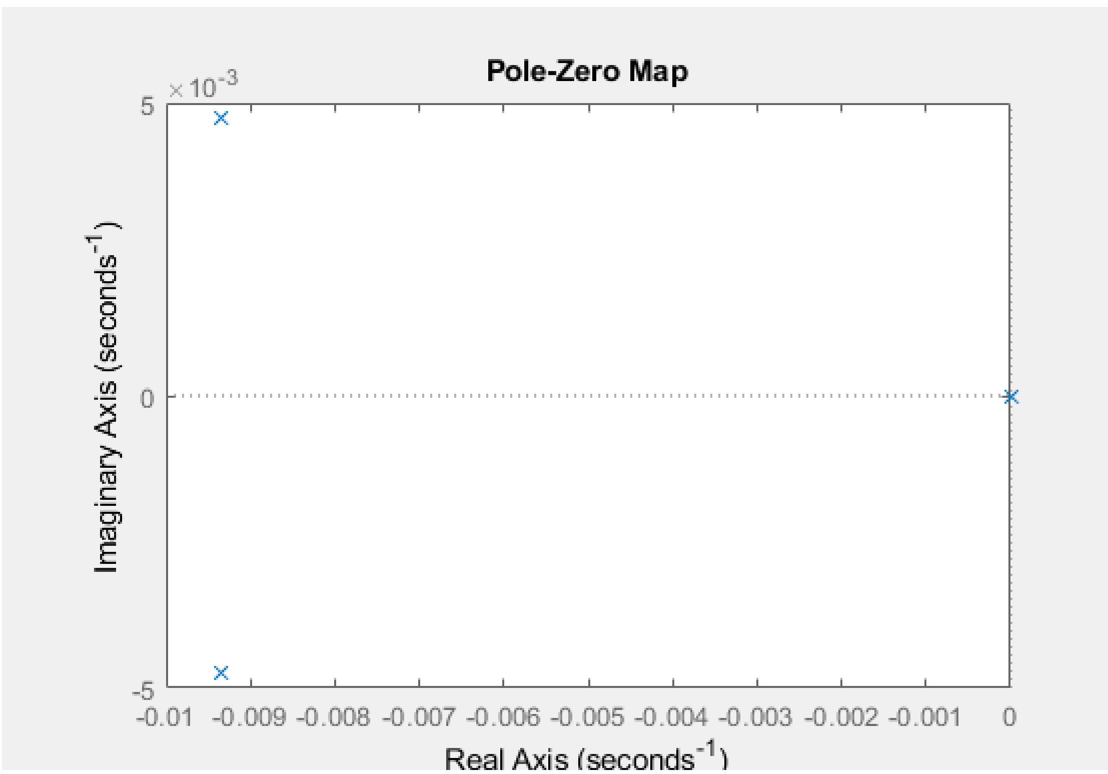
Matlab inputs:

```
>> sys = tf([0.0042] [1 0.01872 0.00011 0]);
 sys = tf([0.0042] [1 0.01872 0.00011 0]);
                   ↑
Error: Invalid expression. When calling a function or indexing a
variable, use parentheses. Otherwise, check for mismatched delimiters.

>> sys = tf([0.0042], [1 0.01872 0.00011 0]);
>> pretty(sys)
Undefined function 'pretty' for input arguments of type 'tf'.

>> pzmap(sys)
>>
```

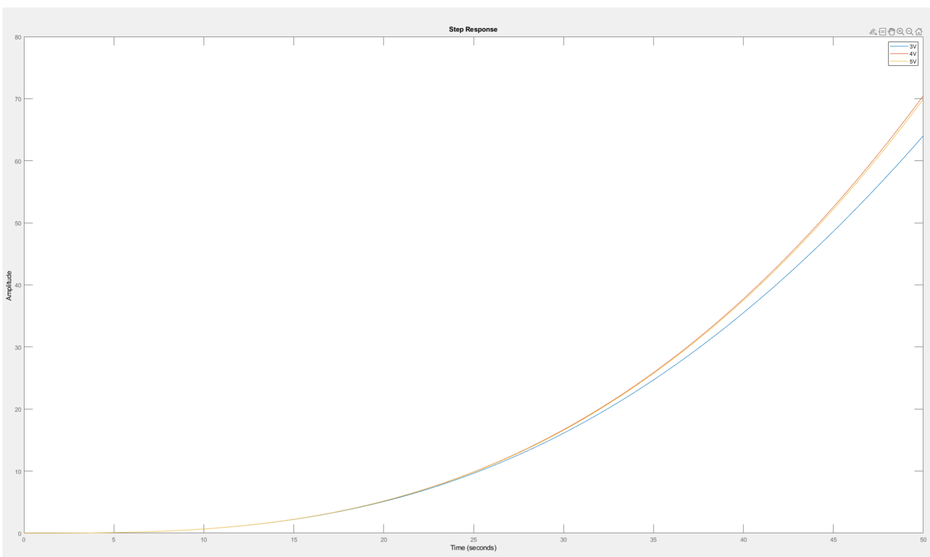
Poles output:



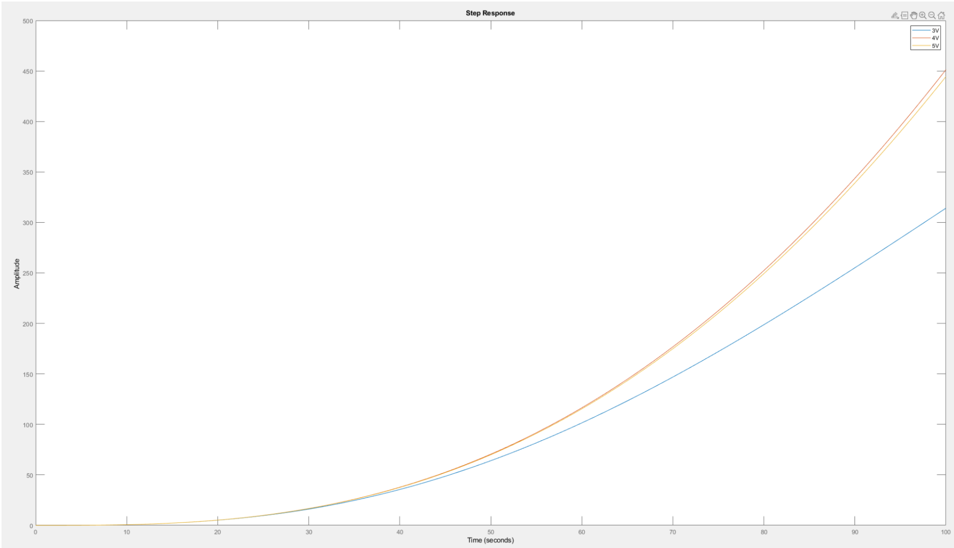
Simulation at 3V and 4V and 5V

	Kt/(JmLa)	(JmRa+DmLa)/(JmLa)	(RaDm+KtKb)/(JmLa)
3V	0.00425	0.001790	0.00099
4V	0.004255	0.01845165933	0.0001104077117
5V	0.00425	0.01905	0.00012

Plotted onto step function of a 3rd order system displace/Voltage:



A further observation shows that 3V does not go up as much as 4V and 5V



Part 1: Safety **Part of appendix of first report**

Hazard	Cause	Probability	Servierty	Mitigation
Fan	Finger	9	3	Safety gloves
Electrocution	Touching exposed parts	5	6	Rubber gloves
System overheat	Touching exposed part	4	5	Monitor temp
Integral windup	Going beyond saturation limits	2	2	Monitor PID
Explosion	overheating	1	8	Risk management
Device falling apart	Bad assembly	2	6	Quality control
Finger jam	Putting finger in pendulum/tight area	7	3	Watch where your hand goes
Static shock	Positive/Negative neutron unbalanced	2	1	Rubber gloves
Puncture	Exposed sharp parts	5	5	Cover up sharp objects
Inhaling smoke	Device fries itself, smokes	2	7	Monitor temperature

Part 2 variables

Variables

B = Fixed magnetic field (Tesla)
$i_a(t)$ = armature current, function of time (A)
$i_a(\infty)$ = steady state armature current (A)
J_L = Inertia of the load
D_L = Viscous damping coefficient of the load
J_a = Inertia of the armature and rotor
J_L = Inertia of the load.
J_M = Total inertia presented to the motor
D_a = Viscous damping coefficient of the armature and rotor
D_L = Viscous damping coefficient of the load.
D_M = Viscous damping coefficient of the armature plus load.
K_b = back emf constant
K_m = Motor constant
K_t = torque constant (N.m/A)
l = length of the conductor (m)
L_a = armature inductance (Henries)
N_1 = Gear on motor side
N_2 = Gear on load side
P_E = Electrical power (Watt)
P_M = Mechanical power (Watt)
R_a = Armature resistance (ohm)
Rm = Speed regulation constant
R_s = Series resistance (ohm)
$T(t)$ = Torque (newton.meter)
T_{stall} = Stall torque of motor (newton.meter)
$v_a(t)$ = applied armature voltage (volts)
$v_b(t)$ = induced back emf (volts)
$\theta_m(t)$ = angular displacement of the motor (radians)
$\omega_m(t)$ = angular velocity of motor (radians/second)
ω_0 = no-load speed of motor (radians/second)