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ECEN315 LABORATORY REPORT ONE

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ECEN315 Lab script 2

1. Introduction

The aim of this lab is to demonstrate the effect of a Proportional Integral Derivative (PID) controller on a complex closed loop pendulum system. The complex closed loop system is derived from the previous lab after the implementation of the open loop transfer function of the pendulum system.

2. Background

In the previous lab, the lab produced a false transfer function. To resolve this issue, the open loop transfer function of the pendulum system was provided by a classmate. This transfer function would match the open loop transfer function of the system.

$$G(s) = \frac{4.2*10^{-3}}{s^3+0.01872s^2+0.00011s}$$

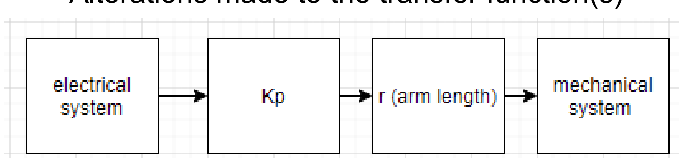
Previously false open loop transfer function

$$\frac{0.0042}{s^4+0.01872s^3+0.00011s^2+0.0042}$$

Previously false closed loop transfer function

An open loop pendulum system does not provide a feedback loop to the input for comparison. For the purpose of the previous lab there was no current difference to alter the power supplied to the fan. However, for the goal of this lab a closed loop variation of the pendulum system was derived to demonstrate the effects of a PID controller. Additionally, a PID examines the proportional, integral and derivative factors of the feedback input to produce a near critically damped output. Furthermore, the Ziegler-Nichols method is employed to fine tune a PID system. [1]

Alterations made to the transfer function(s)



$$\frac{V_o}{V_i} = \frac{\Omega_m(s)}{V_a(s)} = \frac{K_t/(J_m L_a)}{s^2 + \left(\frac{J_m R_a + D_m L_a}{J_m L_a}\right)s + \left(\frac{R_a D_m + K_t K_b}{J_m L_a}\right)} * Kp * r * \frac{\frac{1}{J_p}}{s^2 + \frac{C}{J_p}s + \frac{DMG}{J_p}}$$

motor			**pendulum**	
Ra	15.8231		jp	0.00521
La	1477.987421		C	0.007294
Kb = Kt	0.00682911		DMG (mass * g)	0.2307312
Dm	0.00000884		B	0.7
Jm	0.0011			
			1/jp	191.9385797
Kt/(Jm*La)	0.0042		c/jp	1.4
(Jm*Ra + Dm*La)/Jm*La	0.01872		dmg/jp	44.28621881
(Ra*Dm + Kb*Kt)/Jm*La	0.00011			

$$\frac{\theta}{Vi} = \frac{0.0042}{s^2 + 0.01872s + 0.00011} \cdot Kp \cdot 0.165 \cdot \frac{191.9385797}{s^2 + 1.4s + 44.28621881}$$

$$\frac{\theta}{Vi} = Kp \frac{0.133013}{s^4 + 1.41872\, s^3 + 44.3125\, s^2 + 0.829192\, s + 0.00487148}$$

*

The correct open loop transfer function of the pendulum system

Therefore, the correct transfer function for the close loop pendulum system is given as (without the Kp): (from voltage input to angle output)

$$\frac{\theta}{Vi} = \frac{0.133\,s^4 + 0.1887\,s^3 + 5.894\,s^2 + 0.1103\,s + 0.000648}{s^8 + 2.837\,s^7 + 90.64\,s^6 + 127.4\,s^5 + 1966\,s^4 + 73.69\,s^3 + 7.013\,s^2 + 0.1184\,s + 0.0006717}$$

However, only the mechanical part of this pendulum system will have a feedback system implemented as the close loop system takes the error of the angle. As a result, the open loop transfer function for the pendulum system is provided below.

$$\frac{\theta o}{\theta i} = \frac{191.9385797}{s^2 + 1.4s + 44.28621881}$$

3.1 motor

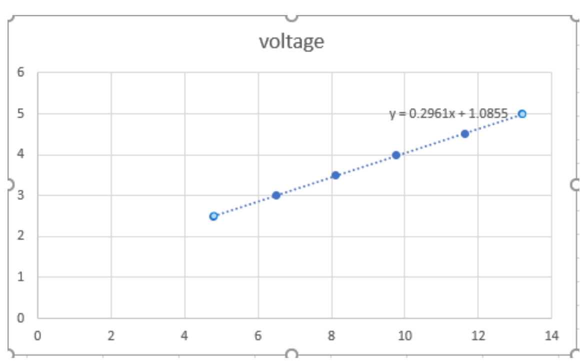
To get the pulse width modulation (PWM), an input voltage from 0 to 5V was used to obtain the respective angle output which produces a linear equation for the angle in terms of voltage.

To calibrate the open loop system, a linear equation that models the angle input to the PWM output was created.

The first part of the procedure is to build on the previously made open loop response of the system. In the previous lab, the pendulum's position was dictated by the power output of the power supply. To improve this open loop system a PWM was implemented to provide the system with automatic motor speed control.

To calibrate the PWM unit, a constant supply of 10V is provided to the open loop system which can then be observed in Simulink for the output angle. An input voltage from 0 to 5V was used to find the angle output. This table produced a linear equation as shown below which shows a voltage output in terms of an angle input.

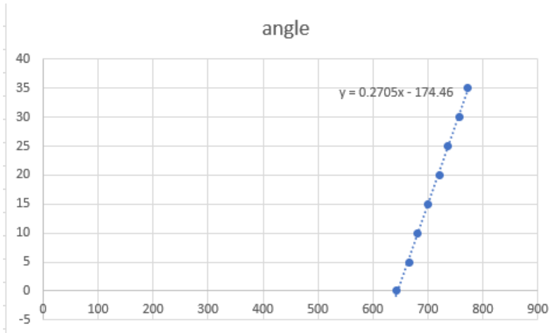
angle	voltage
4.8	2.5
6.5	3
8.1	3.5
9.75	4
11.64	4.5
13.2	5



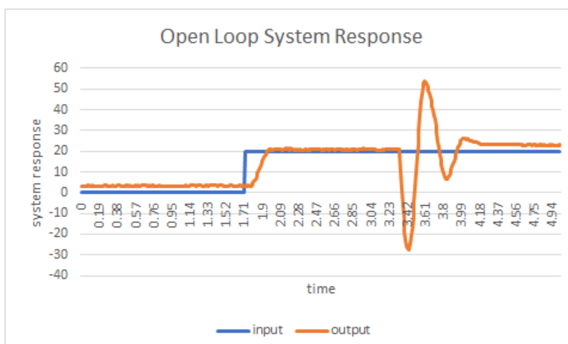
3.2 calibration of the open loop system

The linear equation was extrapolated so that an input angle will produce a PWM output as shown below. This was obtained by increasing the voltage to an input angle before observing the output PWM.

output	angle
642	0
666	5
682	10
701	15
721	20
737	25
757	30
772	35



The Simulink schematic was modified to fit the parameters of the ideal linear model. Below is the input vs output of the open loop system response with some disturbance added after the step input. Furthermore, this graph suggests that this is the ideal open loop system for the pendulum due to the low steady state error. Thus, the open loop control system for the pendulum is completed.



This dataset cannot be compared with the previous lab's dataset as it was incorrectly done. However, it is safe to assume the data set would be similar. As observed in the table below, the open loop system has high overshoot and settling time which needs to be modified in a closed loop system for improvements.

```
    RiseTime: 0.1701
SettlingTime: 5.3264
 SettlingMin: 2.0953
 SettlingMax: 7.4059
   Overshoot: 71.7162
  Undershoot: 0
        Peak: 7.4059
    PeakTime: 0.4721
```

The disadvantage of an open loop system is that it cannot act against disturbance. To overcome this issue, a close system was created based on the current open loop system of the pendulum. In addition, this system will compare the input angular position of the pendulum and compare it to the current position of the pendulum with the employment of a rotary potentiometer. This will produce an error signal which can be used in a compensator to improve the system's response.

3.3 designing a closed loop system

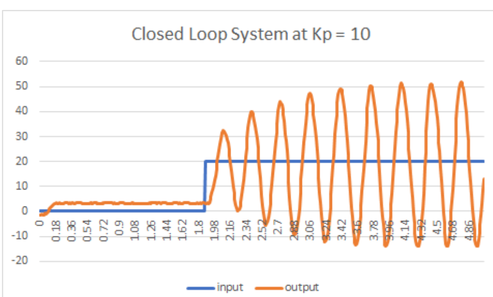
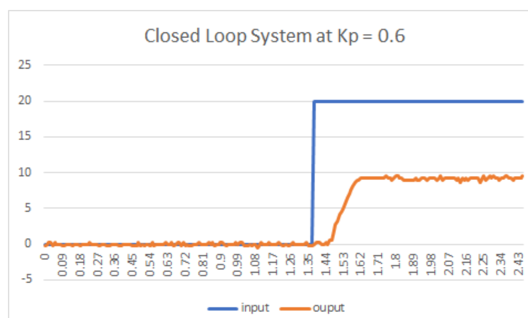
To design a closed loop system for the pendulum, the open loop transfer function must be analysed to determine what value of proportional gain will make it unstable.

$$negative\ feedback\ =\ \frac{forward}{1+foward}$$

Thus, the pendulum close loop system can be modelled by this transfer function by employing the negative feedback equation above bringing it into a 4th order system.

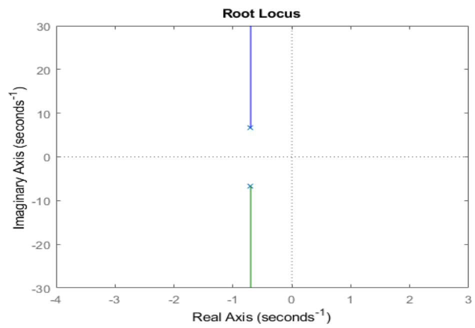
```
systemfeedback =

             192 s^2 + 268.8 s + 8502
  ---------------------------------------------
  s^4 + 2.8 s^3 + 282.5 s^2 + 392.8 s + 1.046e04
```

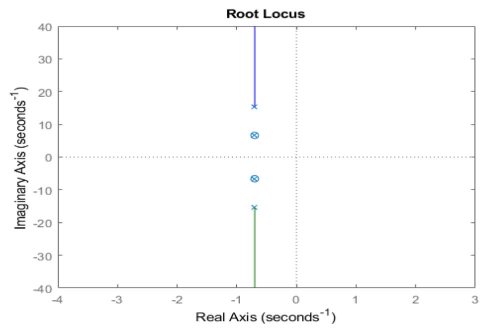
It was observed that, for Kp (proportional gain) values greater than 10 it will cause the close loop system to be unstable. This value of Kp appears to be higher than expected (ideally Kp should be less than 1) which leads the system to be unstable after a step input as observed in the figure below. As a result, manual tuning will be required. Finally, the optimal value of Kp has been obtained and was observed to be at Kp = 0.6 after extensive amounts of trial-and-error. All trial-and-errors were carried with a step input from 0 to 20.

unstable system	stable system
	

As observed in the root locus of the open loop system it has 2 poles. These poles are dominant poles as they are located quite close to the imaginary axis. Furthermore, these poles tend to +-inf on the imaginary axis which can explain why there was an error in terms of calculating the theoretical value of Kp. When arranged into a closed loop system the pendulum behaves like a 2nd order system as there's zeros to cancel out 2 additional poles. Furthermore, the steady state error of the close loop system at Kp = 0.6 is quite high at around a 10 degrees difference. This can prove to be problematic in a real-world scenario. E.g. helicopter at the right altitude. Thus, a PID controller will be introduced to further improve on the close loop pendulum system.

Root locus of open loop pendulum system	Root locus of close loop pendulum system

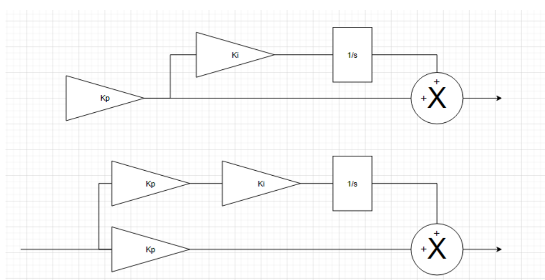




3.4 Adjusting PID (trial-and-error)

An integrator (Ki) is first introduced to allow a smoother oscillation. It was observed that increasing the Ki value causes the PWM to adjust faster. The disadvantage to the fast correction is that it will cause the system to become unstable. Moreover, the higher the value of Ki, the further the zeros travel from the origin.

The position of the new zero that has been created is determined by the ratio of Ki/Kp. This only holds true when Kp is parallel to Ki. However, in the circuit the Kp and Ki were series.

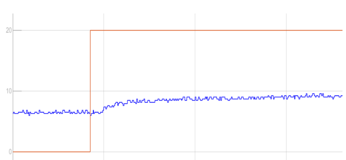


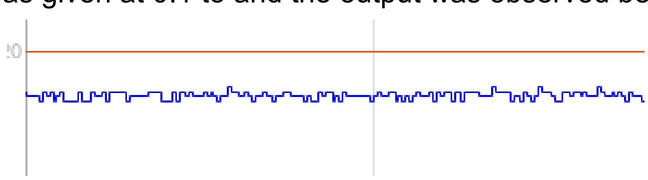
$$Kp + \frac{Kp \cdot Ki}{s} = \frac{Kp \cdot s + Kp \cdot Ki}{s} = \frac{Kp\,(s + Ki)}{s}$$

As a result, Ki can be directly adjusted to move the zero from the origin independent to Kp. e.g. let Kp = 5 and Ki = 10, using the formula 5(s+10) the zero would still be at -10 even if Kp was increased to 7.

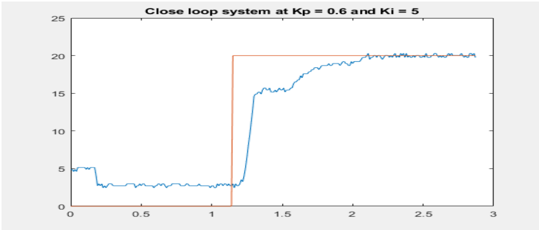
3.5 introducing integrator value

An estimated value of Ki was given at 0.1 to and the output was observed below.

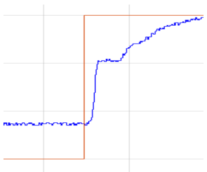
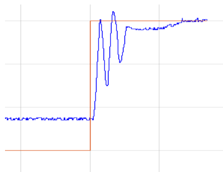




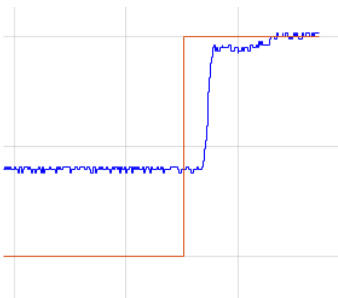
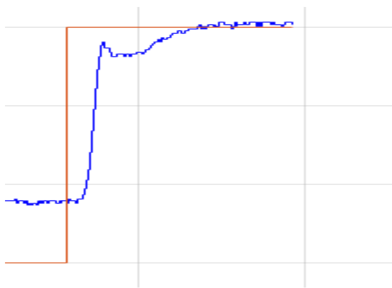
The system proved to be stable, though the Ki value was too small as the PWM adjusted at a slow pace. In other words, this shows that the pendulum corrects at a slow rate. Additionally, the observed steady state error is marginal compared to the input (orange line) angle. Ki is now increased to 5, this results in achieving steady state in a faster time frame. In addition, there was also a decrease in the steady state error as observed in the Matlab figure below.



An input angle of 30 was experimented with the current Ki and Kp configuration. It was noticed that it took approximately 6 seconds to achieve steady state as observed below. In an attempt to increase the settling time, Kp was adjusted to 1. However, this configuration resulted in greater oscillation thus was abandoned.

Input angle of 30	Input angle of 30 and Kp = 1
	

Different input angles were tested and observed to be able to reach steady state at a decent rate. However, when disturbance was added (dropping the fan from additional height) the system became underdamped to the point of instability. Thus, Kp was adjusted to 0.4 and Ki was adjusted to be 15 which resulted in the best possible damping output.

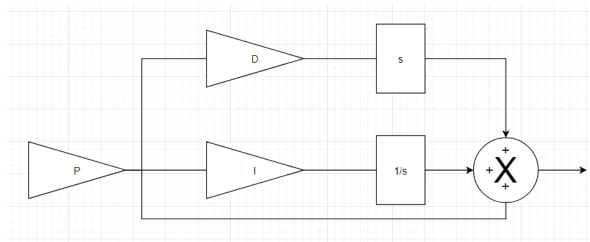
Disturbance at angle of 20	**input angle of 20**	**input angle of 30:**
		

3.6 Introducing derivative value

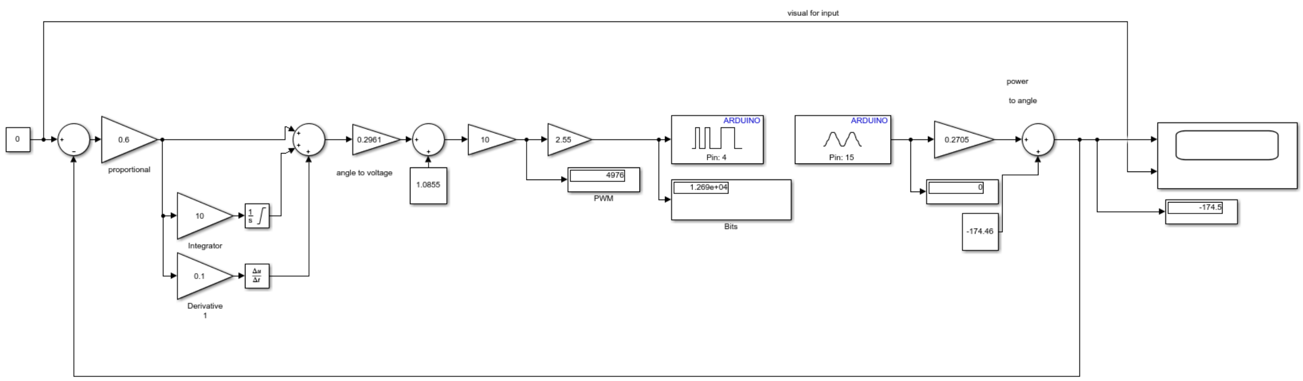
The same process for Ki was repeated to find the value of Kd. First deriving an equation for the PID system now with a derivative factor involved and how it could be modelled in a flow diagram.

$$G(c) = P\left(Ds + \frac{I}{s} + 1\right)$$

$$G(c) = \frac{P}{s}\left(Ds^2 + I + s\right)$$



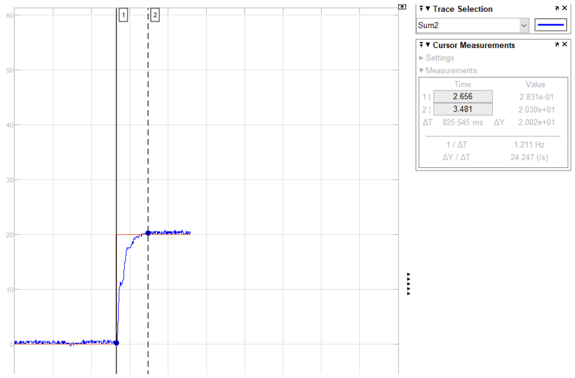
The formula derived will be different from the formula provided in the lab script as our model was based on a series variation. As observed below the Simulink figure has a full PID system that is implemented in the series variation.

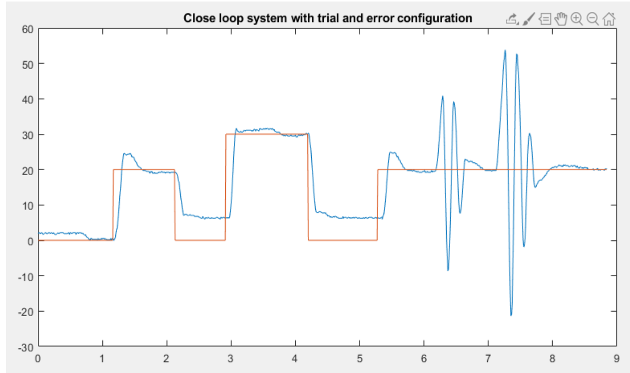


A hypothesis was made that increasing the Ki value to be greater than 15 will cause a slight overshoot in the damping. This hypothesis has been tested between the values of 10 to 15, and shown the best result when Ki is 15.

It was discovered that the ideal Kd value for this system should be less than 1 as it is a very noisy system. This is because a huge derivative would make the system noisier. Additionally, a disturbance was implemented to test the Kd value to be less than 0.1. The result of a controlled disturbance caused the system output to be noisier.

As stated in the lab report, the target settling time is 6 seconds given a step input of 20. As a result, this would require a compensator system that would settle with threefold improvement in the settling time and have 0 steady state error. At the configuration of Kp = 0.4, Ki = 15 and Kd = 0.1. A low steady state error was achieved including a settling point of approximately 1s.



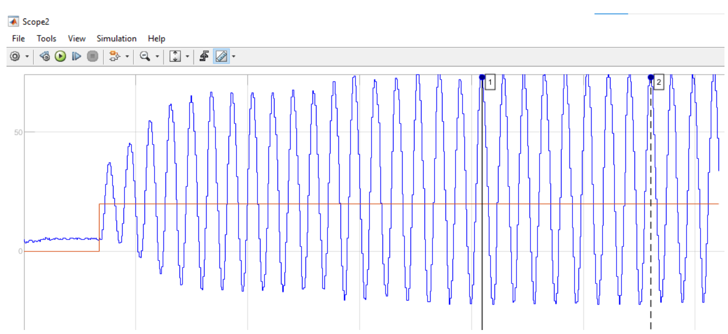


Improving this threefold has proved to be difficult. However, the advantage to this configuration was achieving a settling time of approximately 1s.

3.7 Fine tuning PID controller

Using the Ziegler-Nichols table:

Rule Name	Tuning Parameters
Classic Ziegler-Nichols	Kp = 0.6 Ku Ti = 0.5 Tu Td = 0.125 Tu
Pessen Integral Rule	Kp = 0.7 Ku Ti = 0.4 Tu Td = 0.15 Tu
Some Overshoot	Kp = 0.33 Ku Ti = 0.5 Tu Td = 0.33 Tu
No Overshoot	Kp = 0.2 Ku Ti = 0.5 Tu Td = 0.33 Tu



A more accurate reading of the frequency could be made using repeated measurements. Thus, we obtained 8 oscillations over 1.508s which gives us a frequency of 5.31Hz. The 5.31Hz obtained does not seem accurate. This is believed to be the fault of the internal clock. The Ku was given from trial-and-error. In addition, the parameters for finding Ku was the lowest value of the proportional controller (Kp) which gives us a stable oscillation. The determined value Ku is 2.25 and the determined value of Tu is the inverse of 5.31Hz which is 0.188.

3.8 PID configuration (ziegler-Nichols variations)

Ziegler-Nichols Table and values:

Given that Ku = 2.25 and Tu = 0.188 and using the Ziegler Nichols table (Shown above)

Rule Name	Kp	Ti	Td	Ki	kd
Classic Ziegler-Nichols	0.6*2.25=**1.35**	0.5*0.188= **0.094**	0.125*0.188= **0.0235**	1.2 * 2.25/ 0.188 = **14.36**	0.075*2.25*0.188 = **0.032**
Pessen Integral Rule	0.7*2.25=**1.575**	0.4*0.188= **0.0752**	0.15*0.188=**0.0282**	1.75 * 2.25/ 0.188 = **20.94**	0.105*2.25*0.188 = **0.044**
Some Overshoot	0.33*2.25=**0.7425**	0.5*0.188= **0.094**	0.33*0.188=**0.06204**	0.66 * 2.25/ 0.188 = **7.90**	0.11*2.25*0.188 = **0.047**
No Overshoot	0.2*2.25=**0.45**	0.5*0.188= **0.094**	0.33*0.188=**0.06204**	0.4 * 2.25/ 0.188 = **4.79**	0.066*2.25*0.188 = **0.028**

Method:

The experimental method was to set the input angle (IA) to 0, then change it to 20 and observe the damping. If the system is stable, the same process will be repeated from angle 0 to 20. In addition disturbance will be added to the system at angle 20. Furthermore, after each angle is resetted to 0 (on Simulink). Then a physical manual reset of the armature will be made to make each test fair.

Classic Ziegler-Nichols:

As observed in figure 6.6.1 the IA starts off at 0 and increases to 20 creating a small overshoot. However, when the method was repeated for angle 30 the system was made unstable. The assumption made was that the Kp value. This was because in the trial-and-error phase an observation was made that any value of Kp greater than 1 would result in an unstable system.

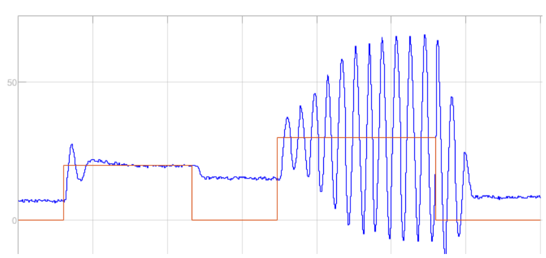


Fig. 6.6.1

Pessen Integral Rule:

As observed in Fig. 6.6.2. Pessen Integral rule configuration gave a very similar output as the Classic Ziegler-Nichols but had greater underdamping. This is the result of a high Kp value and

Ki value as observed in the trial-and-error phase, Ki values greater than 15 led to instability in the system which can be noticed when the IA was changed to 30.

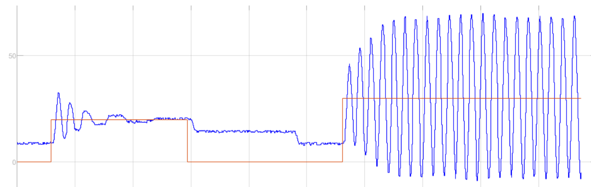


Fig. 6.6.2

Some Overshoot:

It can be observed that the Some Overshoot configuration resulted in almost critically dampening the system when the IA was increased to 20 and to 30. Thus the next phase of the method was carried on. The additional disturbance was generated that resulted in an underdamped oscillation as observed in Fig 6.6.3. However, if too much disturbance was added the system will become unstable.

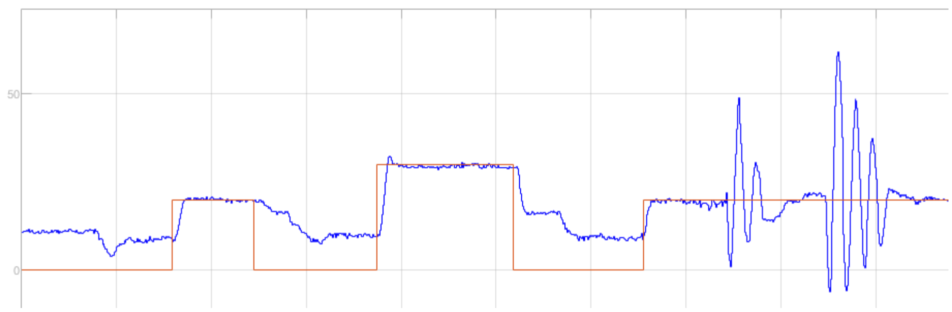


Fig 6.6.3

No overshoot:

As observed in Fig 6.6.4, A No Overshoot configuration resulted in a slight overdamped system. Furthermore, it also resulted in a slight underdamped output when disturbance was added at a 20 IA. However, this is the recommended configuration as it's safer for users since it has a reduced oscillation and when greater disturbance was added, the system did not become unstable like the Some Overshoot configuration.

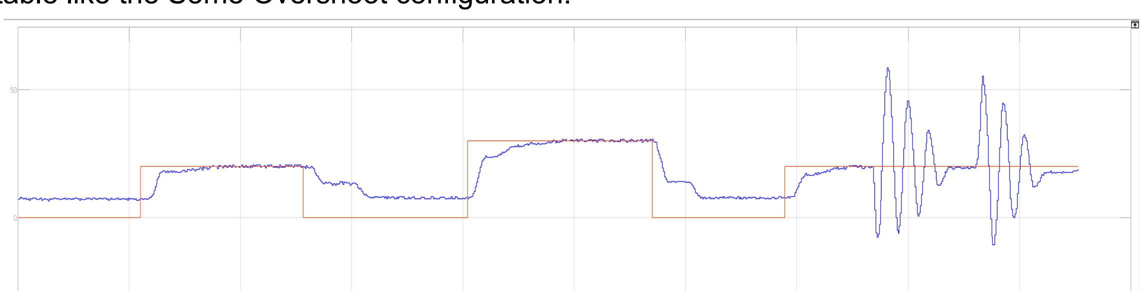
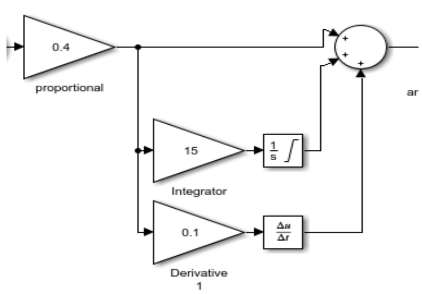
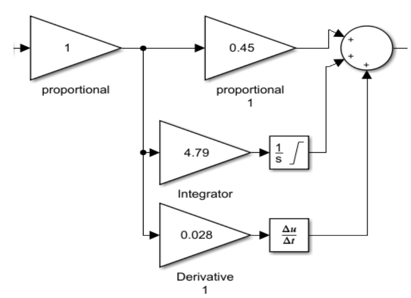
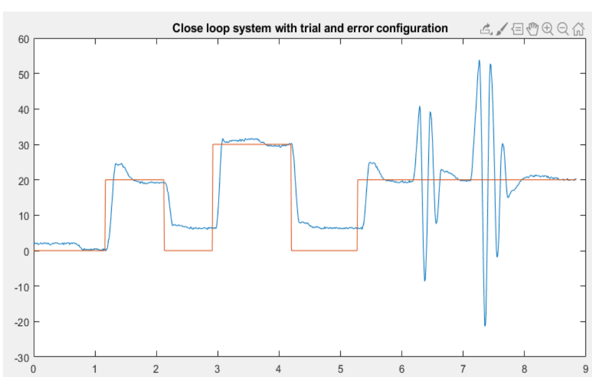
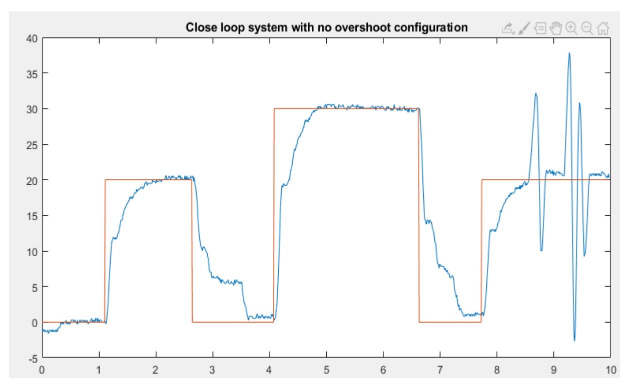


Fig 6.6.4

Some Overshoot vs No Overshoot:

It can be argued that the Some Overshoot's output reached the IA more quickly compared to No Overshoot. However, in control systems it is in the health and safety's best interest to have a more stable system. No Overshoot may reach steady state at a slower rate compared to Some Overshoot, but it is safer.

trial-and-error model vs Zeigler-Nichols model

trial-and-error controller and output	**Zeigler-Nichols model (no overshoot) and output**
	
	

As observed, the trial-and-error output and the No Overshoot output looks very similar. However, the time it took to obtain the values for the PID system was done much faster with the Zeigler-Nichols method (No Overshoot configuration). Thus, it is safe to conclude that the Zeigler-Nichols is a more efficient method in tuning PID systems.

4. Discussion/conclusion

A major disadvantage to the open loop system is that it cannot auto adjust itself. In a real-world scenario a closed loop system can save a lot of power which is a huge benefit to the environment. E.g. air conditioning units or even assist pilots in landing planes. However, a close loop system can sometimes be disadvantages if not used correctly as they can become unstable. As a result, PID tuning will need to be implemented to fully utilize a closed loop system. Furthermore, Ziegler-Nichols variations can be employed to tune a PID controller system in a short amount of time.

5. References:

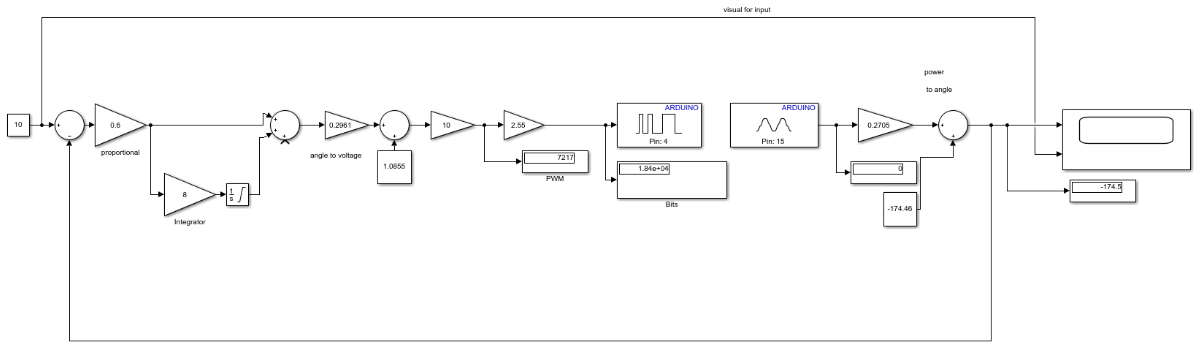
[1] Ziegler, J.G & Nichols, N. B. (1942). "Optimum settings for automatic controllers" (PDF). Transactions of the ASME. 64: 759–768.
[2] G. Gouws, “Understanding Small, Permanent Magnet Brushed DC Motors.”, March 2008, Last Updated 21st April 2015, Unpublished

Appendices

Health and safety

Hazard	Cause	Probability	Severity	Mitigation
Fan	Finger	9	3	Safety gloves
Electrocution	Touching exposed parts	5	6	Rubber gloves
System overheat	Touching exposed part	4	5	Monitor temp
Integral windup	Going beyond saturation limits	2	2	Monitor PID
Explosion	overheating	1	8	Risk management
Device falling apart	Bad assembly	2	6	Quality control
Finger jam	Putting finger in pendulum/tight area	7	3	Watch where your hand goes
Static shock	Positive/Negative neutron unbalanced	2	1	Rubber gloves
Puncture	Exposed sharp parts	5	5	Cover up sharp objects
Inhaling smoke	Device fries itself, smokes	2	7	Monitor temperature

Full Simulink diagram



Variables used to calculate transfer functions

Variables

B = Fixed magnetic field (Tesla)
$i_a(t)$ = armature current, function of time (A)
$i_a(\infty)$ = steady state armature current (A)
J_L = Inertia of the load
D_L = Viscous damping coefficient of the load
J_a = Inertia of the armature and rotor
J_L = Inertia of the load.
J_M = Total inertia presented to the motor
D_a = Viscous damping coefficient of the armature and rotor
D_L = Viscous damping coefficient of the load.
D_M = Viscous damping coefficient of the armature plus load.
K_b = back emf constant
K_m = Motor constant
K_t = torque constant (N.m/A)
l = length of the conductor (m)
L_a = armature inductance (Henries)
N_1 = Gear on motor side
N_2 = Gear on load side
P_E = Electrical power (Watt)
P_M = Mechanical power (Watt)
R_a = Armature resistance (ohm)
Rm = Speed regulation constant
R_s = Series resistance (ohm)
T(t) = Torque (newton.meter)
T_{stall} = Stall torque of motor (newton.meter)
$v_a(t)$ = applied armature voltage (volts)
$v_b(t)$ = induced back emf (volts)
$\theta_m(t)$ = angular displacement of the motor (radians)
$\omega_m(t)$ = angular velocity of motor (radians/second)
ω_0 = no-load speed of motor (radians/second)