21. a) $\bar{A}B + \bar{A}D + B\bar{C}D = (\bar{A}+D)(\bar{A}+\bar{C})(B+\bar{D})$

$VP = (\bar{A}+D)(\bar{A}B + \bar{C}B + \bar{A}\bar{D} + \bar{C}\bar{D})$

$= \bar{A}B + \bar{A}\bar{C}B + \bar{A}\bar{D} + \bar{A}\bar{C}\bar{D} + D\bar{A}B + D\bar{C}B + D\bar{A}\bar{D} + D\bar{C}\bar{D}$

$= \bar{A}B + \bar{A}\bar{C}B + \bar{A}\bar{D} + \bar{A}\bar{C}\bar{D} + D\bar{A}B + D\bar{C}B$

$= \bar{A}B(1+\bar{C}) + \bar{A}\bar{D}(1+\bar{C}) + D\bar{A}B + B\bar{C}D$

$= \bar{A}B + \bar{A}\bar{D} + B\bar{C}D + D\bar{A}B = (\bar{A}DB + \bar{A}B) + \bar{A}\bar{D} + B\bar{C}D$

$= \bar{A}B(D+1) + \bar{A}\bar{D} + B\bar{C}D = \bar{A}B + \bar{A}\bar{D}B\bar{C}D = VT$

c) $z + xy + \bar{x}z = (x+z)(y+z)$

$VP = xy + xz + zy + z = xy + zy + z(x+1)$

$= xy + zy + z = xy + zy(\bar{x}+1)z = xy + \bar{x}zy + zy + z$

$= xy + \bar{x}zy + z(y+1) = xy + \bar{x}zy + z$

~~$= xy + \bar{x}zy + z(y+1) = xy$~~ +

$= xy + \bar{x}zy + z(\bar{x}+1) = xy + \bar{x}zy + z\bar{x} + z$

$= xy + \bar{x}z(y+1) + z = xy + \bar{x}z + z = VT$

d) $\overline{A \oplus B} = \bar{A} \oplus B$

$VT = \overline{(\bar{A}+B)(A+\bar{B})} = \overline{(\bar{A}+B)} + \overline{(A+\bar{B})}$

$= A\bar{B} + \bar{A}B$

$VP = \bar{A} \oplus B = (\bar{\bar{A}}+B)(\bar{B}+\bar{A}) = A\bar{B} + \bar{A}B$

e) $AB(A \oplus B \oplus C) = ABC$

$VT = AB(\bar{A}+B+C)(A+\bar{B}+C)(A+B+\bar{C})$

$= AB(\bar{A}\bar{B} + AC + AB + BC + AC + \bar{B}C + C)(A+B+\bar{C})$

$= AB(\bar{A}\bar{B} + AC + AB + BC + AC + C)(A+B+\bar{C})$

$= AB(AC + BC + C)(A+B+\bar{C})$

$= AB(AC + C)(A+B+\bar{C})$

$= AB(C)(A+B+\bar{C})$

$= AB(AC + BC + 0) = ABC = VP$

2-1/

a) $F_1 = \bar{C}B\bar{A} + A\bar{B}C + ABC$

$= \Sigma(2,5,7)$

$F_1 = (C+B+A)(C+B+\bar{A})(C+\bar{B}+\bar{A})(\bar{C}+B+A)(\bar{C}+\bar{B}+A)$

$= \Pi(0,1,3,4,6)$

$F_2 = \bar{A}B\bar{C} + AB\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC$

$= \Sigma(0,3,4,5,6)$

$F_2 = (\bar{A}+B+C)(A+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$

$= \Pi(1,2,7)$

2-3/ $F_1 = \bar{A}\bar{B}\bar{C} + A\bar{B}C = \Sigma(0,5) + d(2,6)$

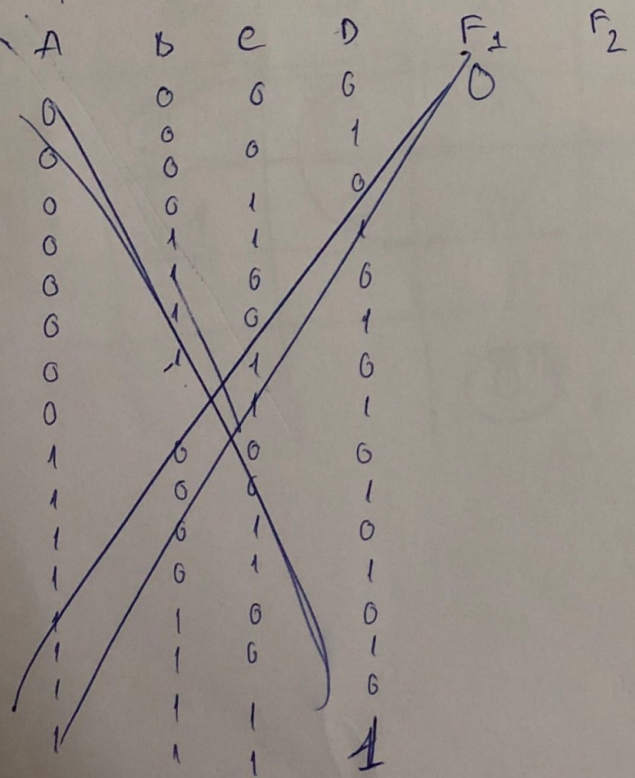
$F_1 = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$

$= \Pi(1,3,4,7) \cdot d(2,6)$

$F_2 = \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} = \Sigma(0,3,4) + d(1,5,6)$

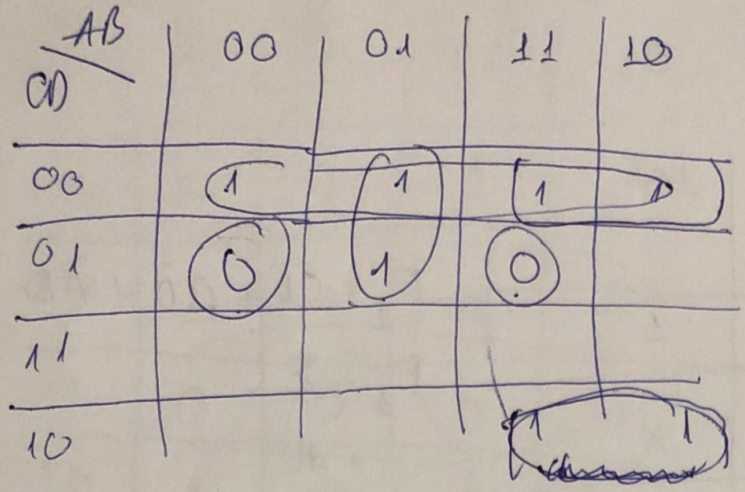
$F_2 = (A+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C}) = \Pi(2,7) \cdot d(1,5,6)$

2-4/



A	b	C	D	F_1	F_2
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	0	1
0	0	1	1	0	1
0	1	0	0	1	0
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	0	1
1	0	0	0	0	0
1	0	0	1	1	0
1	0	1	0	0	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	0	1
1	1	1	1	1	1

A	b	C	D	F_1	F_2
0	0	0	0	1	x
0	0	0	1	1	0
0	0	1	0	1	1
0	0	1	1	x	0
0	1	0	0	1	0
0	1	0	1	0	0
0	1	1	0	1	x
0	1	1	1	0	x
1	0	0	0	1	x
1	0	0	1	0	1
1	0	1	0	0	1
1	0	1	1	0	0
1	1	0	0	1	0
1	1	0	1	x	1
1	1	1	0	x	0
1	1	1	1	0	0

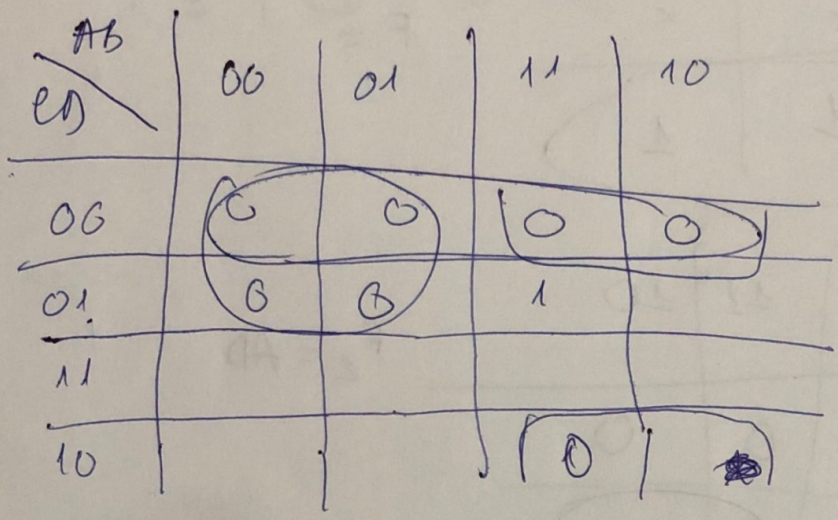


$f_1 = \bar{C}\bar{D} + CA + \bar{A}B\bar{C}$

$F_1 = \Sigma(0, 4, 5, 12, 8, 14, 10)$

$f_1 = (A+B+C+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})$

$= \Pi(1, 13)$

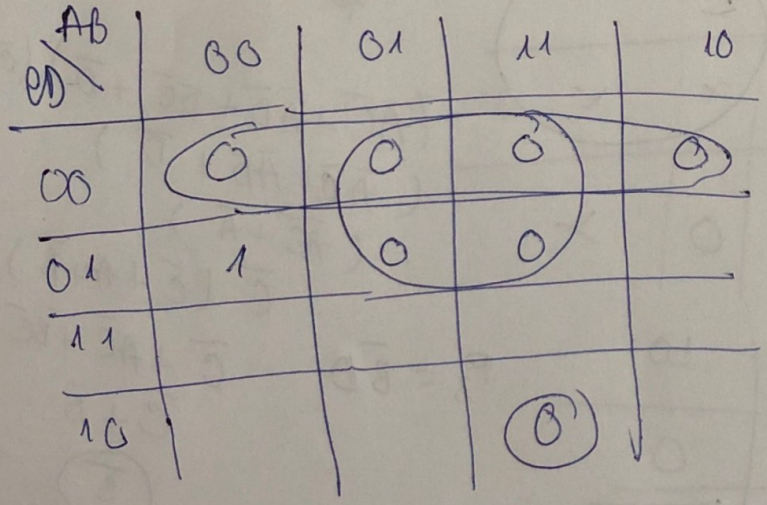


$F_2 = AB\bar{C}D$

$f_2 = \Sigma(15)$

$f_2 = (\bar{A}+C)(\bar{A}+D)$

$F_2 = \Pi(0, 4, 12, 8, 1, 5, 14, 10)$



$F_3 = \bar{A}\bar{D}\bar{C}D$

$F_3 = \Sigma(1)$

$F_3 = (C+D)(\bar{B}+C)(\bar{A}+\bar{B}+\bar{C}+D)$

$F_3 = \Pi(0, 4, 12, 8, 5, 15, 14)$

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	1	0	0
11	0	0	0	0
10	1	0	1	1

$F = (\bar{A} + \bar{D})(\bar{C} + \bar{D})(\bar{C} + A + \bar{B})$

$= (\overline{AD})(\overline{CD})(\overline{CAB})$