TTK4255 Robotic vision - HW2

Håkon Haugann

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Department of Engineering Cybernetics

Part 1 - Theory

Task 1.1

The Hough transform examines lines emerging from a given point (x, y) by voting on all possible lines going through the point. This seems much easier in polar form, as this simply requires a sequential increment of the line angle θ. In the standard form, this operation would require a sequence of cumbersome adjustments to the line parameters a and b in the line equation $y = ax + b$ for the line to pass through (x, y). Hence, polar form is preferred.

Task 1.2

a)

$\theta = 0°$ translates to a vertical line in the image with its position tending to the right with an increasing ρ. Hence, the range of ρ values that produce a visible line in the image is $\underline{\rho \in [0, L]}$.

b)

$\theta = 180°$ is the same case as **a)**, except that the $180°$ rotation gives a tendency to the left in the image with increasing values of ρ. Assuming $\rho \geq 0$, there is <u>no feasible range</u> for ρ that produces a visible line in the image.

c)

$\theta = 45°$ gives a line parallel to the diagonal from the top right to the bottom left corner of the image. The feasible range for ρ is then given by the length of the diagonal. Hence, the range is $\underline{\rho = [0, \sqrt{2}L]}$.

d)

$\theta = -45°$ gives, for $\rho = 0$, the diagonal from the top left to the bottom right corner of the image. Increasing values of ρ pushes the line to the right in the image until it vanishes into the top right corner. The feasible range for ρ is easy to find by observing Figure 1, and comes out as $\rho \in [0, \frac{\cos 45°}{2} L] = [0, \frac{\sqrt{2}}{2} L]$

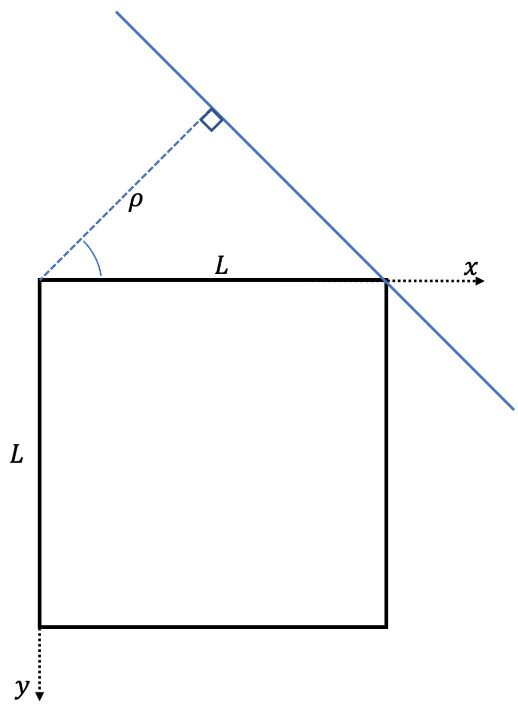

Figure 1: Line segment from task **1.2 d)**

Task 1.3

The derivatives of the image does not change for constant intensity shifts. Also, if w is radially symmetric, the function is independent of pixel position as long as the distance from the center is constant. I would guess these are the main reasons behind that the Harris-Stephens measure is invariant to both rotation and intensity shifts.

If w is *not* radially symmetric I would guess the measure is not invariant the named transformations, at least not to rotations!

Task 1.4

Great question. I'll have to think about that one.

Part 2 - Hough transform

Task 2.1

As stated in the task note, the minimum range for θ is 2π, and an appropriate range for θ is for instance $\underline{\theta \in [-\pi, \pi]}$. The range of the origin ρ depends on the image resolution. The maximum size of ρ will be the diagonal of the image, and is found as $\rho_{max} = \sqrt{H^2 + W^2}$. Letting $\rho_{min} = 0$ will cover all possible *non-directed* lines. To account for all directed lines as described in the *Task 2.1 note*, we must allow negative values of ρ. Hence, an appropriate range for ρ is $\underline{\rho \in [-\sqrt{H^2 + W^2}, \sqrt{H^2 + W^2}]}$.

Task 2.2

Incrementing the accumulator array H at the discretized line coordinates yields the figure shown in Figure 2.

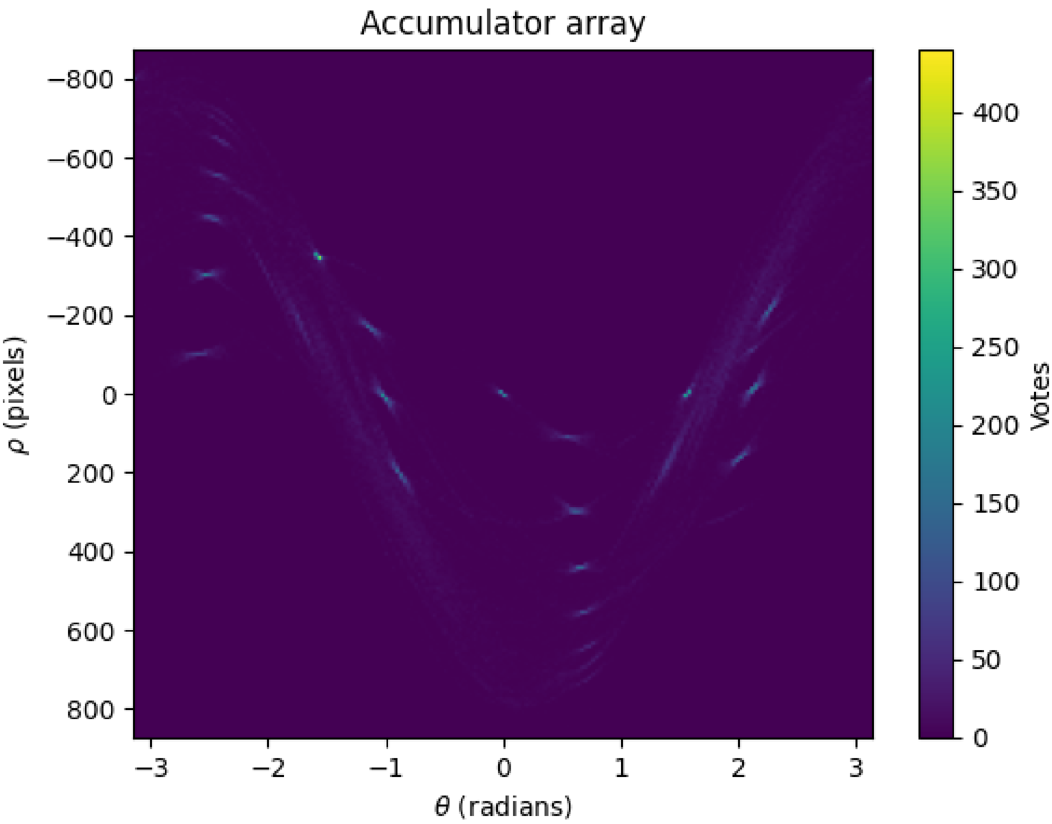

Figure 2: The accumulator array H with several bright spots, indicating a dominant line in those regions.

Task 2.3

The `extract_local_maxima` function pinpoints the dominant lines detected by the Hough transform. These local maximums are highlighted in Figure 3, and drawn in the sample image in Figure 4. These plots were generated using $N_\rho = N_\theta = 1000$ and `line_threshold` $= 0.2$.

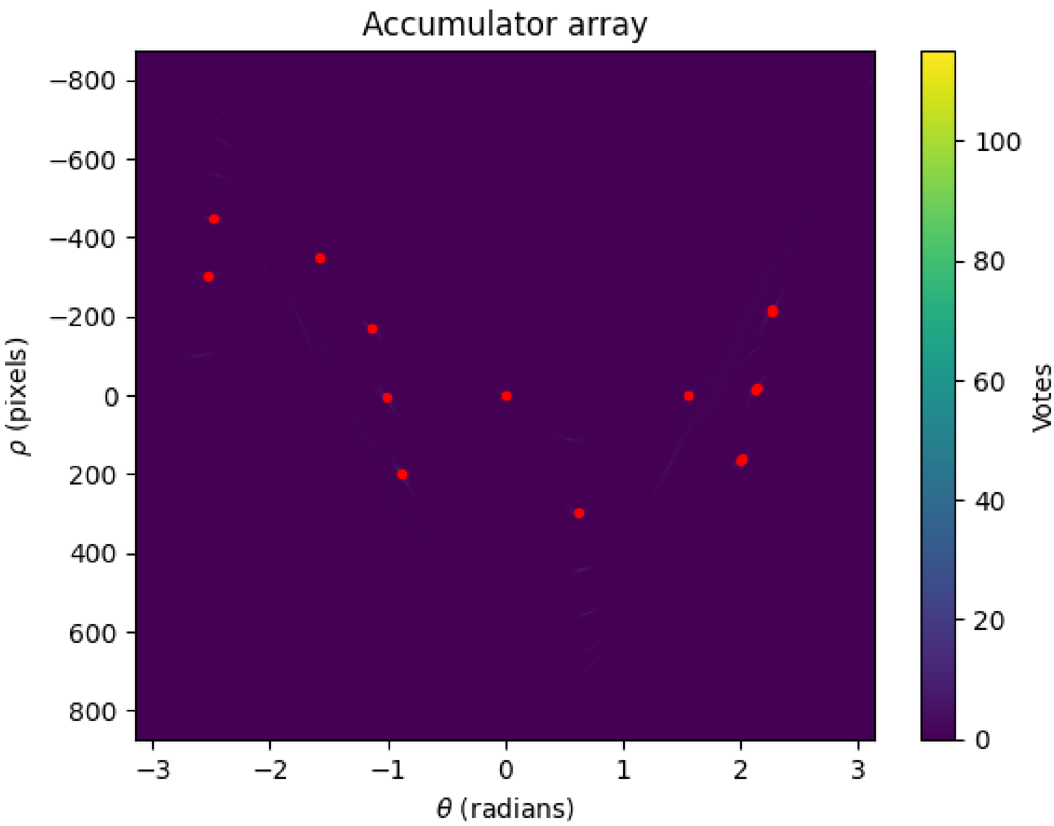

Figure 3: The local maximums in the accumulator array are highlighted as red dots.

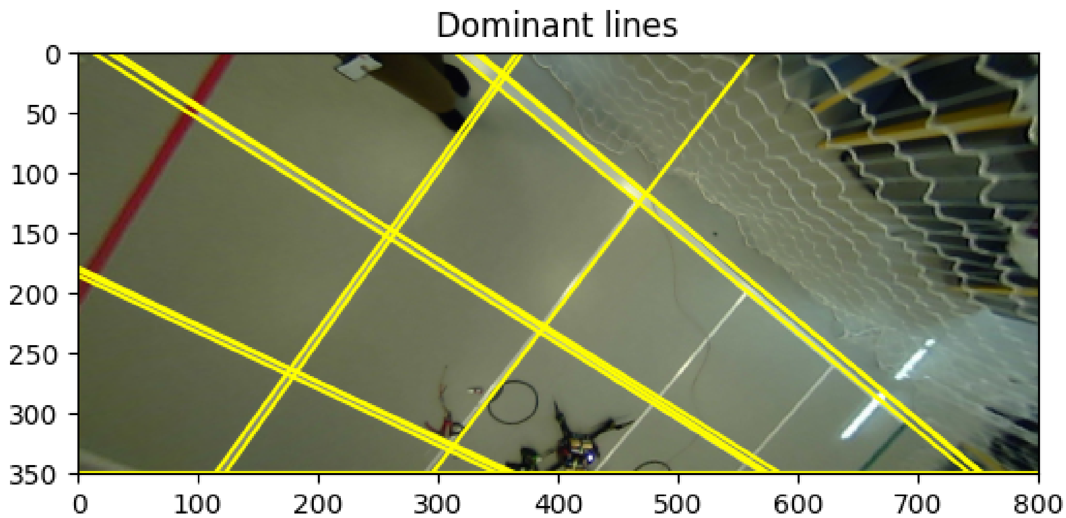

Figure 4: The dominant lines drawn in the sample image

Part 3 - Harris detector

Task 3.1

In computing the Harris-Stephens measure the entries and simple matrix operations were applied. Given a symmetric autocorrelation matrix of the form

$$\boldsymbol{A} = \begin{bmatrix} A_1 & A_2 \\ A_2 & A_3 \end{bmatrix},$$

the Harris-Stephens measure, given as

$$\det(\boldsymbol{A}) - \alpha \operatorname{Tr}(\boldsymbol{A}) = \lambda_0\lambda_1 - \alpha(\lambda_0 + \lambda_1)^2,$$

can be computed using the determinant and trace of $\boldsymbol{A}$ as follows:

$$\det(\boldsymbol{A}) = \boldsymbol{A}_{13} - \boldsymbol{A}_2^2$$
$$\operatorname{Tr}(\boldsymbol{A}) = \boldsymbol{A}_1 + \boldsymbol{A}_3$$

where the matrix products are *element-wise* operations.

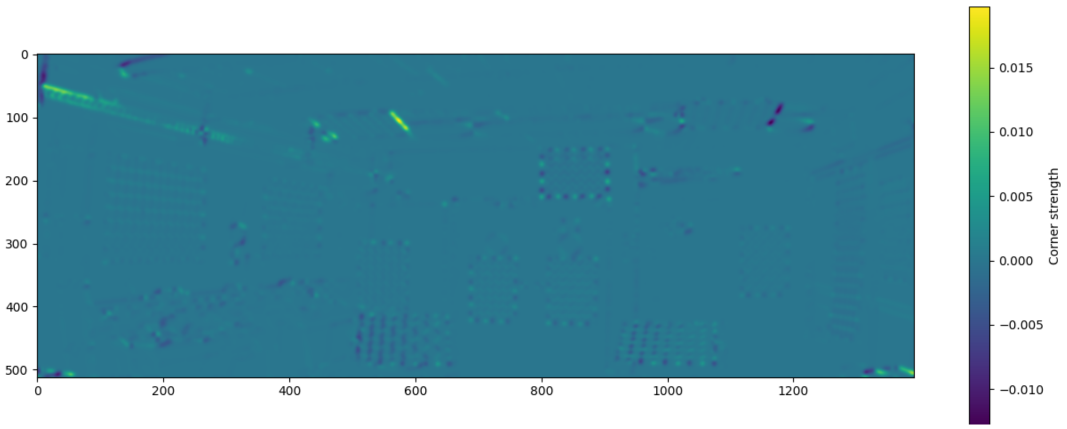

Figure 5: The resulting corner strength from the Harris-Stephens measure.

Task 3.4

Extracting the local maximums from the resulting gray-scale image from the Harris-Stephens measure will yield the domintant corners in the image. These are highlighted in yellow in Figure 6.

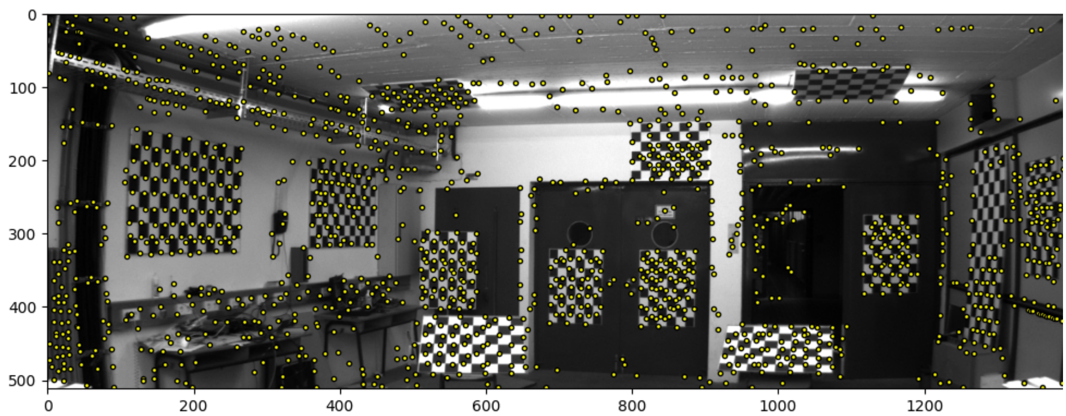

Figure 6: The dominant corners in the sample image.