TTK4255 Robotic vision - HW3

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Part 1 - Choosing a sensor and lens

Task 1.1

The task translates to a geometry problem involving finding f such that

$$X' = -f\frac{X}{Z}$$

where $X' = -10\mu m, X = 1cm$ and $Z = 50m$. Inserting the numerical values we find that $\underline{f = 50mm.}$

Task 1.2

The half-width of the sensor will be $\frac{1024}{2} \cdot 10\mu m = 512mm$. Using a focal length of $f = 50mm$ we find that this corresponds to a ground half-distance of $X = \frac{0.00512}{0.05} \cdot 50 = 5.12m$, giving a total ground distance of $10.24m$. Taking 5 images per second and traveling at $50m/s$ implies a travel distance of $10m$ per image, giving $0.24m$ overlap of each image. Assuming movement in only one of the camera axes, this corresponds to a $\underline{2.34\%\text{ coverage of the total image area.}}$

Part 2 - Implementing the pinhole camera model

Task 2.1

Given the linear relationship

$$\begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} s_x f & 0 & c_x \\ 0 & s_y f & c_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

we find the dehomogenized coordinates, that is dividing the vector $\tilde{\boldsymbol{u}}$ by its last element, as

$$\begin{bmatrix} \tilde{u}/\tilde{w} \\ \tilde{v}/\tilde{w} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{X}{Z} s_x f + c_x \\ \frac{Y}{Z} s_y f + c_y \\ 1 \end{bmatrix}, \quad \forall Z \neq 0,$$

which is equal to the pixel coordinate equations (4) in the task set.

Task 2.2

Figure 1 illustrates the results from the `project`-function.

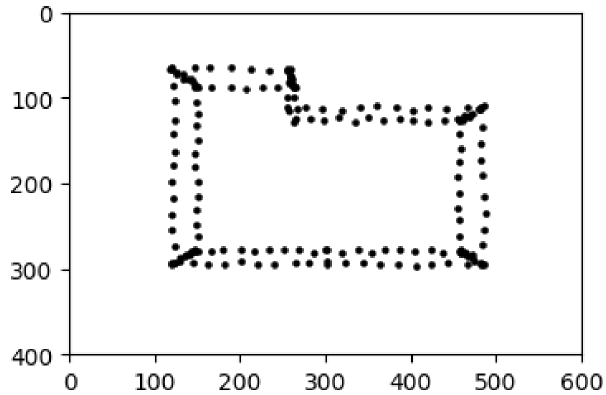


Figure 1: The points from *task2points.txt* manipulated using the pinhole model.

Part 3 - Homogeneous coordinates and transformations

Task 3.1

Given the homogeneous coordinates

$$\tilde{\boldsymbol{X}} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \\ \tilde{W} \end{bmatrix}$$

we find the associated 3D-coordinates by dividing by the last element, that is

$$\boldsymbol{X} = \frac{1}{\tilde{W}} \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix}.$$

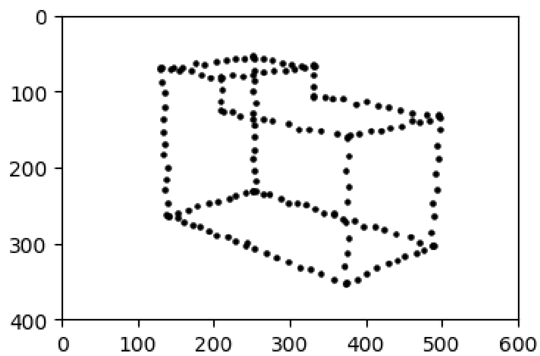
These coordinates can be inserted into the equations for (u, v), giving

$$u = c_x + s_x f \frac{\tilde{X}/\tilde{W}}{\tilde{Z}/\tilde{W}} = c_x + s_x f + \frac{\tilde{X}}{\tilde{Z}}$$
$$v = c_y + s_y f \frac{\tilde{Y}/\tilde{W}}{\tilde{Z}/\tilde{W}} = c_x + s_x f + \frac{\tilde{Y}}{\tilde{Z}}$$

which illustrates that the division by $\tilde{W}$ is unnecessary as the quantity cancels out. Hence, the last component of the homogeneous coordinate $\tilde{\boldsymbol{X}}$ can be dropped when computing the projection (u, v).

Task 3.2

The composite transformation that yields the wanted orientation of the points is found as a combination of a translation of 6 units, a rotation of 15° about the x-axis and a rotation of 45° about the y-axis.



Part 4 - Image formation model for the Quanser helicopter

Task 4.1

The coordinates of the four screws on the platform, given in the coordinate frame shown in fig. 7a in the task set, is

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 11.45 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 11.45 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 11.45 \\ 11.45 \\ 0 \end{bmatrix},$$

given that the distance between adjacent skrews is 11.45 cm.

Task 4.2

These vectors seem to coincide nicely with the screws after transforming into the camera frame as described in the task, as illustrated in Figure 2

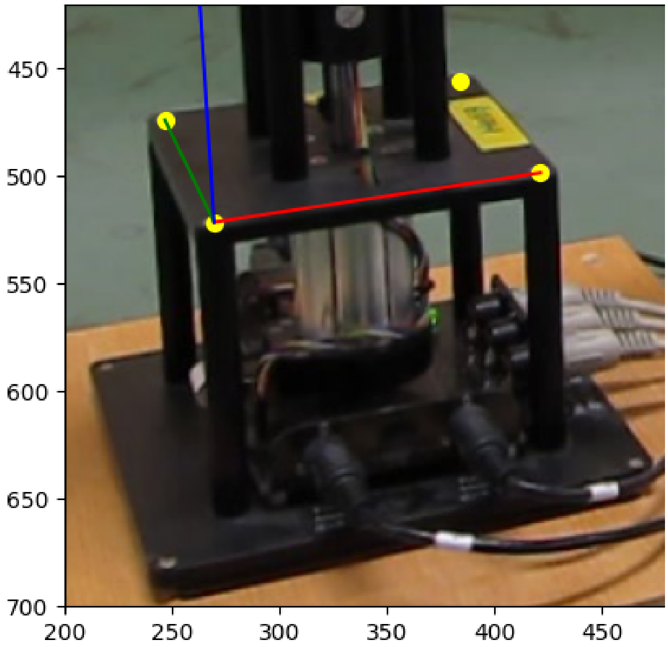


Figure 2: The coordinates transformed into the camera frame.

Task 4.3

The transformation matrix $\boldsymbol{T}_{\text{base}}^{\text{platform}}(\psi)$ is found by translating in x and y by $\frac{11.45}{2}$ cm with respect to the platform frame, and then doing a rotation ψ about the z-axis:

$$\boldsymbol{T}_{\text{base}}^{\text{platform}}(\psi) = \boldsymbol{T}_{xyz}(0.05725, 0.05725, 0) \cdot \boldsymbol{R}_z(\psi)$$

Task 4.4

The transformation matrix $\boldsymbol{T}_{\text{hinge}}^{\text{base}}(\psi)$ is found by first translating in z by 32.5 cm with respect to the base frame, and then doing a rotation θ about the y-axis:

$$\boldsymbol{T}_{\text{hinge}}^{\text{base}}(\psi) = \boldsymbol{T}_{xyz}(0, 0, 0.325) \cdot \boldsymbol{R}_y(\theta)$$

Task 4.5

The transformation matrix $\boldsymbol{T}_{\text{arm}}^{\text{hinge}}$ is found by simply translating -5 cm in the z-axis with respect to the hinge frame.

$$\boldsymbol{T}_{\text{hinge}}^{\text{base}}(\psi) = \boldsymbol{T}_{xyz}(0, 0, -0.05)$$

Task 4.6

The transformation matrix $\boldsymbol{T}^{\text{arm}}_{\text{rotors}}(\phi)$ is found by first translating in z and x by -3 cm and 65 cm respectively with respect to the arm frame, and then doing a rotation ϕ about the x-axis:

$$\boldsymbol{T}^{\text{base}}_{\text{hinge}}(\psi) = \boldsymbol{T}_{xyz}(65, 0, -0.03) \cdot \boldsymbol{R}_x(\phi)$$

Task 4.7

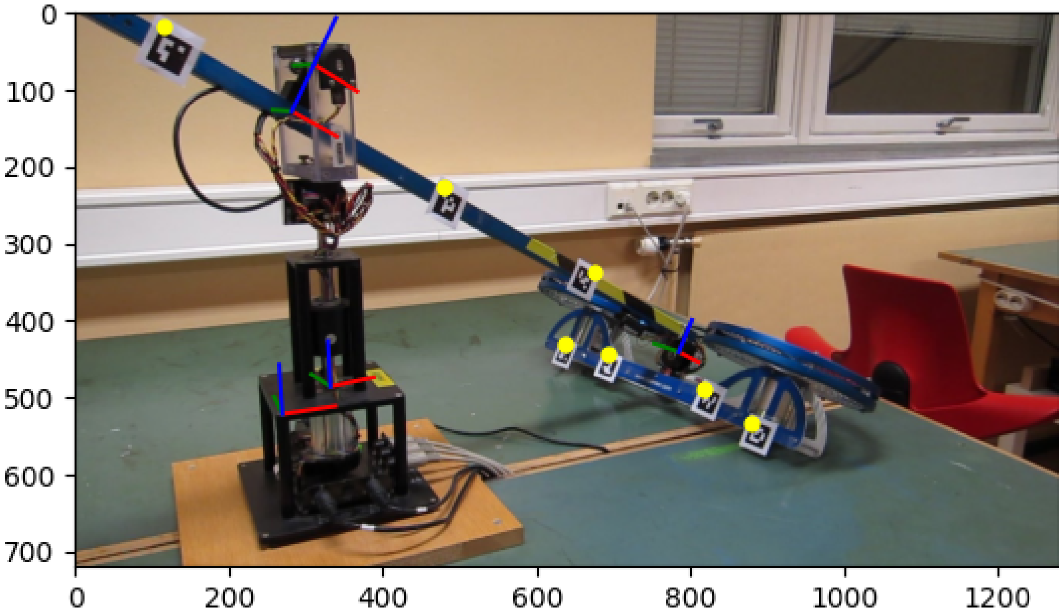


Figure 3: Results after plotting each frame and heli points to the quanser image.