AcPL

2.4.) $\rho^j_k = \rho^j(x_k) \propto e^{-\frac{i}{2}(\lambda - i\varepsilon)x_k^2} \prod_{\ell \in \partial k \setminus j} \int e^{i x_\ell x_k M_{k\ell}} \rho^k(x_\ell) dx_\ell$

and $\rho^k(x_\ell) \propto e^{-\frac{1}{2}\omega_\ell^k x_\ell^2}$; $\rho(x_j) \propto e^{-\frac{1}{2}\omega_j x_j^2}$

=> $\rho^k(x_\ell)$ einsetzen

$$\Rightarrow \rho^j_k \propto e^{-\frac{i}{2}(\lambda - i\varepsilon)x_k^2} \prod \underbrace{\int e^{i x_\ell x_k M_{k\ell}} e^{-\frac{1}{2}\omega_\ell^k x_\ell^2} dx_\ell}_{I}$$

I: Gaussintegral: $\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)} dx = \sqrt{\frac{\pi}{a}}\, e^{\frac{b^2}{4a}-c}$

$\Rightarrow a = \frac{1}{2}\omega_\ell^k$; $b = -i x_k M_{k\ell}$; $c = 0$

$\Rightarrow I = \sqrt{\frac{2\pi}{\omega_\ell^k}}\, e^{\frac{-x_k^2 M_{k\ell}^2}{2\omega_\ell^k}}$

$\Rightarrow \rho^j_k \propto \prod_\ell \sqrt{\frac{2\pi}{\omega_\ell^k}}\, e^{\frac{-x_k^2 M_{k\ell}^2}{2\omega_\ell^k} - \frac{i}{2}(\lambda - i\varepsilon)x_k^2} = \prod_\ell \sqrt{\frac{2\pi}{\omega_\ell^k}}\, e^{-\frac{x_k^2}{2}\left(\frac{M_{k\ell}^2}{\omega_\ell^k} + i(\lambda - i\varepsilon)\right)}$

$\Rightarrow e^{-\frac{1}{2}\omega_k^j x_k^2} \propto \prod_\ell \sqrt{\frac{2\pi}{\omega_\ell^k}}\, e^{-\frac{x_k^2}{2}\left(\frac{M_{k\ell}^2}{\omega_\ell^k} + i(\lambda - i\varepsilon)\right)} \Leftrightarrow e^{-\frac{x_k^2}{2}\omega_k^j} \propto \prod_\ell \sqrt{\frac{2\pi}{\omega_\ell^k}}\, e^{-\frac{x_k^2}{2}\left(\frac{M_{k\ell}^2}{\omega_\ell^k} + i\lambda + \varepsilon\right)}$

$\Leftrightarrow e^{-\frac{x_k^2}{2}\omega_k^j} \propto e^{-\frac{x_k^2}{2}\sum_\ell \frac{M_{k\ell}^2}{\omega_\ell^k} + i\lambda + \varepsilon} \prod_\ell \sqrt{\frac{2\pi}{\omega_\ell^k}}$

=> for function in x_k $\prod_\ell \sqrt{\frac{2\pi}{\omega_\ell}}$ is const => coefficient evaluating

$\Rightarrow e^{-\frac{x_k^2}{2}\underbrace{\omega_k^j}_{u}} \propto e^{-\frac{x_k^2}{2}\underbrace{\sum_\ell \frac{M_{k\ell}^2}{\omega_\ell^k} + i\lambda + \varepsilon}_{u}} \Rightarrow \omega_k^j = \sum_\ell \frac{M_{k\ell}^2}{\omega_\ell^k} + i\lambda + \varepsilon$

$\Leftrightarrow \omega_k^j = i\lambda + \varepsilon + \sum_\ell \frac{M_{k\ell}^2}{\omega_\ell^k}$

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for ω_j use eq. 18: $\rho_j \propto e^{-\frac{i}{2}(\lambda - i\varepsilon)x_j^2} \underbrace{\int e^{i x_j \sum_{k \in \partial j} x_k M_{kj}} \underbrace{\rho^i_{(x_{\partial j})}}_{e^{-\frac{x_k^2}{2}\omega_k^i}} dx_{\partial j}}_{I}$

I: Gauss => $\sqrt{\frac{2\pi}{\omega_k^j}}\, e^{\frac{-x_k^2 M_{kj}^2}{2\omega_k^j}}$

$\Rightarrow e^{\frac{x_j^2}{2}\left(\underbrace{-\omega_j + i\lambda + \varepsilon + \sum_{k \in \partial j} \frac{M_{kj}^2}{\omega_k^j}}_{=0}\right)} \propto$ const

$\Rightarrow \omega_j = i\lambda + \varepsilon + \sum_{\partial j} \frac{M_{kj}^2}{\omega_k^j}$