

**Core questions (everyone is expected to solve these exercises)**

1. Without using the handout on enumerability, show that  $\mathbb{N} \times \mathbb{N}$  is enumerable by exhibiting a surjection  $e : \mathbb{N} \rightarrow \mathbb{N} \times \mathbb{N}$ .
2. Exercise sheet 6.2.2 For an equivalence relation  $E$  on a set  $A$ , show that if  $a_1 E a_2$  then  $[a_1]_E = [a_2]_E$  where  $[a]_E = \{x \in A \mid x E a\}$ .
3. Exercise sheet 10.2.4 For a set  $X$ , prove that there is no injection  $P(X) \rightarrow X$ .
4. Exercise sheet 9.2.1 For  $X \subseteq A$ , prove that the direct image  $\vec{f}(X) \subseteq B$  under an injective function  $f : A \rightarrow B$  is in bijection with  $X$ ; that is,  $X \cong \vec{f}(X)$ .
5. Prove that if  $B$  is a countable set then  $\mathbb{R} \cup B \cong \mathbb{R}$ .
6. 2006 Paper 2 Question 5 [Link](#)

**Tryhard questions (recommended)**

1. Prove that if  $A$  and  $B$  are sets such that  $A \cong \mathbb{R}$  and  $B \cong \mathbb{R}$  then  $A \cup B \cong \mathbb{R}$ .
2. If you did not do it for last supervision: 2008 Paper 2 Question 3 [Link](#)