

Core questions (everyone is expected to solve these exercises)

1. Without using the handout on enumerability, show that $\mathbb{N} \times \mathbb{N}$ is enumerable by exhibiting a surjection $e : \mathbb{N} \rightarrow \mathbb{N} \times \mathbb{N}$.
2. Exercise sheet 6.2.2 For an equivalence relation E on a set A , show that if $a_1 E a_2$ then $[a_1]_E = [a_2]_E$ where $[a]_E = \{x \in A \mid x E a\}$.
3. Exercise sheet 10.2.4 For a set X , prove that there is no injection $P(X) \rightarrow X$.
4. Exercise sheet 9.2.1 For $X \subseteq A$, prove that the direct image $\vec{f}(X) \subseteq B$ under an injective function $f : A \rightarrow B$ is in bijection with X ; that is, $X \cong \vec{f}(X)$.
5. Prove that if B is a countable set then $\mathbb{R} \cup B \cong \mathbb{R}$.
6. 2006 Paper 2 Question 5 [Link](#)

Tryhard questions (recommended)

1. Prove that if A and B are sets such that $A \cong \mathbb{R}$ and $B \cong \mathbb{R}$ then $A \cup B \cong \mathbb{R}$.
2. If you did not do it for last supervision: 2008 Paper 2 Question 3 [Link](#)