

Core questions (everyone is expected to solve these exercises)

1. Exercise sheet 1.2.1 . Either prove or disprove that, for all sets A and B,

- (a) $A \subseteq B \Rightarrow P(A) \subseteq P(B)$,
- (b) $P(A \cup B) \subseteq P(A) \cup P(B)$,
- (c) $P(A) \cup P(B) \subseteq P(A \cup B)$,
- (d) $P(A \cap B) \subseteq P(A) \cap P(B)$,
- (e) $P(A) \cap P(B) \subseteq P(A \cap B)$

2. Exercise sheet 1.2.4. Either prove or disprove that, for all sets A, B, C and D,

- (a) $(A \subseteq B \wedge C \subseteq D) \Rightarrow A \uplus C \subseteq B \uplus D$,
- (b) $(A \cup B) \uplus C \subseteq (A \uplus C) \cup (B \uplus C)$,
- (c) $(A \uplus C) \cup (B \uplus C) \subseteq (A \cup B) \uplus C$,
- (d) $(A \cap B) \uplus C \subseteq (A \uplus C) \cap (B \uplus C)$,
- (e) $(A \uplus C) \cap (B \uplus C) \subseteq (A \cap B) \uplus C$.

3. Exercise sheet 1.2.5 For $\mathcal{F} \subseteq P(A)$, let $\mathcal{U} = \{X \subseteq A \mid \forall S \in \mathcal{F}. S \subseteq X\} \subseteq P(A)$.

Prove that $\bigcup \mathcal{F} = \bigcap \mathcal{U}$.

Analogously, define $\mathcal{L} \subseteq P(A)$ such that $\bigcup \mathcal{F} = \bigcap \mathcal{L}$. Also prove this statement.

4. Exercise sheet 2.2.1 Let $\mathcal{F} \subseteq P(A \times B)$ be a collection of relations from A to B. Prove that,

- (a) for all $R : X \rightarrowtail A$,

$$(\bigcup \mathcal{F}) \circ R = \bigcup \{S \circ R \mid S \in \mathcal{F}\} : X \rightarrowtail B$$

and that,

- (b) for all $R : B \rightarrowtail Y$,

$$R \circ (\bigcup \mathcal{F}) = \bigcup \{R \circ S \mid S \in \mathcal{F}\} : A \rightarrowtail Y$$

5. Exercise sheet 2.2.2 For a relation R on a set A, let

$$\mathcal{T}_R = \{Q \subseteq A \times A \mid R \subseteq Q \wedge Q \text{ is transitive} \}$$

For $R^{\circ+} = R \circ R^{\circ*}$, prove that (i) $R^{\circ+} \in \mathcal{T}_R$ and (ii) $R^{\circ+} \subseteq \bigcap \mathcal{T}_R$. Hence, $R^{\circ+} = \bigcap \mathcal{T}_R$

6. 2007 Paper 2 Question 5 Link

Tryhard questions (recommended)

1. 2008 Paper 2 Question 3 Link