Discrete Mathematics 1 Supervisior: Marton Havasi

Lectures 7-9 19/11/2017

Practice questions (Think about the solution even if you do not end up writing it down.)

1. Exercise sheet 3.1.1

   Calculate the set CD(666, 330) of common divisors of 666 and 330.

2. Exercise sheet 3.2.6

   Prove that for all positive integers a and b,

$$gcd(13a + 8b, 5a + 3b) = gcd(a, b)$$

3. Exercise sheet 3.2.2

   Prove that for all positive integers a, b, c,

$$gcd(a, c) = 1 \implies gcd(ab, c) = gcd(b, c)$$

Core questions

1. Exercise sheet 3.3.1.a

   Let a and b be natural numbers such that $a^2|b(b + a)$. Prove that $a|b$.

   Hint: For positive a and b, consider $a_0 = \frac{a}{gcd(a,b)}$ and $b_0 = \frac{b}{gcd(a,b)}$ so that $gcd(a_0, b_0) = 1$, and show that $a^2|b(b + a) \implies a_0 = 1$.

2. 1959 IMO Problem 1

   Prove that the fraction $\frac{21n+4}{14n+3}$ is irreducible for every natural number n.

3. 2007 Paper 2 Question 3 link

4. 2015 Paper 2 Question 9, Part A link

Tryhard questions (recommended)

1. For $n \in \mathrm{N}$, find $\min(a, b)$ $(a, b \in \mathrm{N})$ such that the Euclidean algorithm applied to the pair (a, b) takes n steps to finish. I.e. what is the worst case for the Euclidean algorithm?

2. Let n be a positive integer relatively prime to 6. We paint the vertices of a regular n-gon with three colours so that there is an odd number of vertices of each colour. Show that there exists an isosceles triangle whose three vertices are of different colours.