

Practice questions

1. Exercise sheet 6.2.2 For an equivalence relation E on a set A , show that if $a_1 E a_2$ then $[a_1]_E = [a_2]_E$ where $[a]_E = \{x \in A \mid x E a\}$.

Core questions

1. Without using the handout on enumerability, show that $\mathbb{N} \times \mathbb{N}$ is enumerable by exhibiting a surjection $e : \mathbb{N} \rightarrow \mathbb{N} \times \mathbb{N}$.
2. Exercise sheet 10.2.4 For a set X , prove that there is no injection $P(X) \rightarrow X$.
3. Exercise sheet 9.2.1 For $X \subseteq A$, prove that the direct image $\vec{f}(X) \subseteq B$ under an injective function $f : A \rightarrow B$ is in bijection with X ; that is, $X \cong \vec{f}(X)$.
4. Prove that if B is a countable set then $\mathbb{R} \cup B \cong \mathbb{R}$.
5. 2006 Paper 2 Question 5 [Link](#)
6. 2009 Paper 1 Question 4 [Link](#)

Tryhard questions

1. Prove that if A and B are sets such that $A \cong \mathbb{R}$ and $B \cong \mathbb{R}$ then $A \cup B \cong \mathbb{R}$.