Discrete Mathematics 2 Supervisior: Marton Havasi
Lectures 4-6 11/12/2017

Topics unique existence; disjunction; Fermat's Little Theorem; negation; contrapositive; proof by contradiction; natural numbers; monoids; commutativity; semirings

Core questions

1. Exercise sheet 2.2.4

   What are $rem(55^2, 79)$, $rem(23^2, 79)$, $rem(23 \cdot 55, 79)$, and $rem(55^{78}, 79)$?

2. Consider the statement

   $$\forall \text{ natural number } x.\ x^{100} - 1 \equiv \prod_{1 \leq i \leq k} (x - a_i) \mod 101$$

   Where $\{a_1, \ldots, a_k\}$ is a finite set of natural numbers.

   Find the minimum of $\sum_{1 \leq i \leq k} a_i$ such that satisfies the statement above.

3. Exercise sheet 2.1.4

   Let m be a positive integer.

   1. Prove the associativity of the addition and multiplication operations in $\mathcal{Z}_m$; that is, that for all i, j, k in $\mathcal{Z}_m$,

      $$(i +_m j) +_m k = i +_m (j +_m k) \text{ and } (i \cdot_m j) \cdot_m k = i \cdot_m (j \cdot_m k)$$

   2. Prove that the additive inverse of k in $\mathcal{Z}_m$ is $[-k]_m$.

4. 2014, Paper 2, Question 7 Link

Tryhard questions (entirely optional, can be difficult)

1. One of Euler's conjectures was disproved in the 1960s by three American mathematicians when they showed there was a positive integer such that $133^5 + 110^5 + 84^5 + 27^5 = n^5$. Find the value of n without using a calculator.

2. (This is difficult. Do not prioritize this over other supervision work.)

   Is it true that if p is a prime number, and k is an integer $2 \leq k \leq p$, then the sum of the products of each k-element subset of $\{1, 2, \ldots, p\}$ will be divisible by p?

Survey Questions

1. How long did it take to complete the core questions?

2. How do you rate your understanding of the topics of this week's supervision? (select one or more)

   - I have little clue
   - I understand some of the topics
   - I understand most of the topics
   - Take me to the exam hall