Discrete Mathematics 5 Supervisior: Marton Havasi

Lectures 13-15 09/02/2019

Practice questions

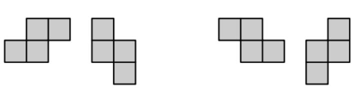
1. Exercise sheet 1.2.1 . Either prove or disprove that, for all sets A and B,
   (a) $A \subseteq B \Rightarrow P(A) \subseteq P(B)$,
   (b) $P(A \cup B) \subseteq P(A) \cup P(B)$,
   (c) $P(A) \cup P(B) \subseteq P(A \cup B)$,
   (d) $P(A \cap B) \subseteq P(A) \cap P(B)$,
   (e) $P(A) \cap P(B) \subseteq P(A \cap B)$
2. Exercise sheet 1.2.4. Either prove or disprove that, for all sets A, B, C and D,
   (a) $(A \subseteq B \wedge C \subseteq D) \Rightarrow A \uplus C \subseteq B \uplus D$,
   (b) $(A \cup B) \uplus C \subseteq (A \uplus C) \cup (B \uplus C)$,
   (c) $(A \uplus C) \cup (B \uplus C) \subseteq (A \cup B) \uplus C$,
   (d) $(A \cap B) \uplus C \subseteq (A \uplus C) \cap (B \uplus C)$,
   (e) $(A \uplus C) \cap (B \uplus C) \subseteq (A \cap B) \uplus C$.

Core questions

1. Exercise sheet 1.2.5 For $\mathcal{F} \subseteq P(A)$, let $\mathcal{U} = \{X \subseteq A \mid \forall S \in \mathcal{F}.S \subseteq X\} \subseteq P(A)$.
   Prove that $\bigcup \mathcal{F} = \bigcap \mathcal{U}$.
   Analogously, define $\mathcal{L} \subseteq P(A)$ such that $\bigcap \mathcal{F} = \bigcup \mathcal{L}$. Also prove this statement.
2. Exercise sheet 2.2.1 Let $\mathcal{F} \subseteq P(A \times B)$ be a collection of relations from A to B. Prove that,
   (a) for all $R : X \nrightarrow A$,
   $$(\bigcup \mathcal{F}) \circ R = \bigcup \{S \circ R \mid S \in \mathcal{F}\} : X \nrightarrow B$$
   and that,
   (b) for all $R : B \nrightarrow Y$,
   $$R \circ (\bigcup \mathcal{F}) = \bigcup \{R \circ S \mid S \in \mathcal{F}\} : A \nrightarrow Y$$
3. Exercise sheet 2.2.2 For a relation R on a set A, let
   $$\mathcal{T}_R = \{Q \subseteq A \times A \mid R \subseteq Q \wedge Q \text{ is transitive }\}$$
   For $R^{\circ +} = R \circ R^{\circ *}$, prove that (i) $R^{\circ +} \in \mathcal{T}_R$ and (ii) $R^{\circ +} \subseteq \bigcap \mathcal{T}_R$. Hence, $R^{\circ +} = \bigcap \mathcal{T}_R$
4. 2007 Paper 2 Question 5 Link

Tryhard questions

1. 2008 Paper 2 Question 3 Link
2. (Not related to the exam material) Construct a tetromino by attaching two 2×1 dominoes along their longer sides such that the midpoint of the longer side of one domino is a corner of the other domino. This construction yields two kinds of tetrominoes with opposite orientations. Let us call them Sand Z-tetrominoes, respectively. Assume that a lattice polygon P can be tiled with S-tetrominoes. Prove than no matter how we tile P using only S- and Z-tetrominoes, we always use an even number of Z-tetrominoes.



S-tetrominoes Z-tetrominoes