

Discrete Mathematics 1

Lectures 1-3

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Topics proof, implication, contrapositive, modus ponens, bi-implication, divisibility, congruence, universal quantification, equality, conjunction, existential quantification, unique existence

Core questions

1. Prove or disprove the following statements. Clear, step-by-step proofs are expected.
 - Suppose n is a natural number larger than 2, and n is not a prime number. Then $2n + 13$ is not a prime number.
 - If $x^2 + y = 13$ and $y \neq 4$ then $x \neq 3$.
 - For an integer n , n^2 is even if and only if n is even.
 - For all real numbers x and y there is a real number z such that $x + z = y - z$.
 - For all integers x and y there is an integer z such that $x + z = y - z$.
 - For all integers m and n , if mn is even, then either m is even or n is even.
 - $10 \mid 1526^{19} + 2^{58}$

2. Find all p prime numbers, such that $\frac{p^2-1}{p-1}$ is also prime.

3. Let $P(m)$ be a statement for m ranging over the natural numbers, and consider the derived statement

$$P^\#(m) = (\forall \text{ natural number } k . 0 \leq k \leq m \implies P(k))$$

again for m ranging over the natural numbers.

- Show that for all natural numbers l , $P^\#(l) \implies P(l)$
 - Prove by exhibiting a counter-example that $P(n) \implies P^\#(n)$ does not hold.
 - Prove or disprove:
 - $P^\#(0) \iff P(0)$
 - $\forall \text{ natural number } n. (P^\#(n) \implies P^\#(n+1)) \iff (P^\#(n) \implies P(n+1))$
 - $(\forall \text{ natural number } m. P^\#(m)) \iff (\forall \text{ natural number } m. P(m))$
4. Prove that for all integers d, k, l, m, n ,
 - $d \mid m \wedge d \mid n \implies d \mid (m+n)$
 - $d \mid m \implies d \mid km$
 - $d \mid m \wedge d \mid n \implies d \mid (km + ln)$

Tryhard questions (entirely optional, can be difficult)

1. Find all natural numbers n , such that $n^3 - 27$ is a prime.
2. Prove that there are infinitely many natural numbers n , such that $4n + 3$ is prime.

Survey Questions

1. How long did it take to complete the core questions?
2. How do you rate your understanding of the topics of this week's supervision? (select one or more)
 - I have little clue
 - I understand some of the topics
 - I understand most of the topics
 - Take me to the exam hall