

### Practice questions

1. Exercise sheet 1.2.1 . Either prove or disprove that, for all sets A and B,

- (a)  $A \subseteq B \Rightarrow P(A) \subseteq P(B)$ ,
- (b)  $P(A \cup B) \subseteq P(A) \cup P(B)$ ,
- (c)  $P(A) \cup P(B) \subseteq P(A \cup B)$ ,
- (d)  $P(A \cap B) \subseteq P(A) \cap P(B)$ ,
- (e)  $P(A) \cap P(B) \subseteq P(A \cap B)$

2. Exercise sheet 1.2.4. Either prove or disprove that, for all sets A, B, C and D,

- (a)  $(A \subseteq B \wedge C \subseteq D) \Rightarrow A \uplus C \subseteq B \uplus D$ ,
- (b)  $(A \cup B) \uplus C \subseteq (A \uplus C) \cup (B \uplus C)$ ,
- (c)  $(A \uplus C) \cup (B \uplus C) \subseteq (A \cup B) \uplus C$ ,
- (d)  $(A \cap B) \uplus C \subseteq (A \uplus C) \cap (B \uplus C)$ ,
- (e)  $(A \uplus C) \cap (B \uplus C) \subseteq (A \cap B) \uplus C$ .

3. Exercise sheet 1.2.5 For  $\mathcal{F} \subseteq P(A)$ , let  $\mathcal{U} = \{X \subseteq A \mid \forall S \in \mathcal{F}. S \subseteq X\} \subseteq P(A)$ .

Prove that  $\bigcup \mathcal{F} = \bigcap \mathcal{U}$ .

Analogously, define  $\mathcal{L} \subseteq P(A)$  such that  $\bigcap \mathcal{F} = \bigcup \mathcal{L}$ . Also prove this statement.

4. Exercise sheet 2.2.1 Let  $\mathcal{F} \subseteq P(A \times B)$  be a collection of relations from A to B. Prove that,

- (a) for all  $R : X \rightarrow A$ ,

$$(\bigcup \mathcal{F}) \circ R = \bigcup \{S \circ R \mid S \in \mathcal{F}\} : X \rightarrow B$$

and that,

- (b) for all  $R : B \rightarrow Y$ ,

$$R \circ (\bigcup \mathcal{F}) = \bigcup \{R \circ S \mid S \in \mathcal{F}\} : A \rightarrow Y$$

5. Exercise sheet 2.2.2 For a relation R on a set A, let

$$\mathcal{T}_R = \{Q \subseteq A \times A \mid R \subseteq Q \wedge Q \text{ is transitive} \}$$

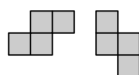
For  $R^{\circ+} = R \circ R^{\circ*}$ , prove that (i)  $R^{\circ+} \in \mathcal{T}_R$  and (ii)  $R^{\circ+} \subseteq \bigcap \mathcal{T}_R$ . Hence,  $R^{\circ+} = \bigcap \mathcal{T}_R$

6. 2007 Paper 2 Question 5 Link

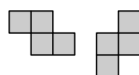
### Tryhard questions

1. 2008 Paper 2 Question 3 Link

2. (Not related to the exam material) Construct a tetromino by attaching two  $2 \times 1$  dominoes along their longer sides such that the midpoint of the longer side of one domino is a corner of the other domino. This construction yields two kinds of tetrominoes with opposite orientations. Let us call them S- and Z-tetrominoes, respectively. Assume that a lattice polygon P can be tiled with S-tetrominoes. Prove that no matter how we tile P using only S- and Z-tetrominoes, we always use an even number of Z-tetrominoes.



S-tetrominoes



Z-tetrominoes