Discrete Mathematics 2 Supervisior: Marton Havasi

Lectures 4-6 11/15/2018

Instructions: Everyone has to do the Core questions. You may skip practice questions under two conditions: 1. You are **absolutely** certain that you will gain nothing from solving it. That means you went through the solution mentally and you are 100% sure you know how to solve it for full marks in the exam. 2. You are spending the time you saved to solve the tryhard questions. I trust everyone to make responsible decisions.

Practice questions

1. Exercise sheet 2.2.4

   What are $rem(55^2, 79)$, $rem(23^2, 79)$, $rem(23 \cdot 55, 79)$, and $rem(55^{78}, 79)$?

2. Exercise sheet 2.1.4

   Let m be a positive integer.

   1. Prove the associativity of the addition and multiplication operations in $\mathcal{Z}_m$; that is, that for all i, j, k in $\mathcal{Z}_m$,

   $$(i +_m j) +_m k = i +_m (j +_m k) \text{ and } (i \cdot_m j) \cdot_m k = i \cdot_m (j \cdot_m k)$$

   2. Prove that the additive inverse of k in $\mathcal{Z}_m$ is $[-k]_m$.

3. Exercise sheet 2.3.2

   A decimal (respectively binary) repunit is a natural number whose decimal (respectively binary) representation consists solely of 1's.

   1. What are the first three decimal repunits? And the first three binary ones?
   2. Show that no decimal repunit strictly greater than 1 is square, and that the same holds for binary repunits. Is this the case for every base? Hint: Use Lemma 26 of the notes.

Core questions

1. 2014, Paper 2, Question 7 Link

2. Consider the statement

   $$\forall \text{ natural number } x.\ x^{100} - 1 \equiv \prod_{1 \leq i \leq k} (x - a_i) \mod 101$$

   Where $\{a_1, \ldots, a_k\}$ is a finite set of natural numbers.

   Find the minimum of $\sum_{1 \leq i \leq k} a_i$ such that satisfies the statement above.

   Did you consider all the edge cases?

Tryhard questions (please don't cheat by looking them up online)

1. One of Euler's conjectures was disproved in the 1960s by three American mathematicians when they showed there was a positive integer such that $133^5 + 110^5 + 84^5 + 27^5 = n^5$. Find the value of n without using a calculator.

2. Show that there exists an infinite set S of positive integers such that for any two distinct elements m and n of S, the integers $2^m - 3$ and $2^n - 3$ are coprime.