Discrete Mathematics 1 Supervisior: Marton Havasi

Lectures 1-3 08/11/2018

Core questions (everyone is expected to solve these)

1. Prove or disprove the following statements. Clear, step-by-step proofs are expected.
   - Suppose n is a natural number larger than 2, and n is not a prime number. Then $2n + 13$ is not a prime number.
   - If $x^2 + y = 13$ and $y \neq 4$ then $x \neq 3$.
   - For an integer n, n^2 is even if and only if n is even.
   - For all real numbers x and y there is a real number z such that $x + z = y - z$.
   - For all integers x and y there is an integer z such that $x + z = y - z$.
   - For all integers m and n, if mn is even, then either m is even or n is even.
   - $10|1526^{19} + 2^{58}$

2. Find all p prime numbers, such that $\frac{p^2-1}{p-1}$ is also prime.

3. Find all natural numbers n, such that $n^3 - 27$ is a prime.

4. Let $P(m)$ be a statement for m ranging over the natural numbers, and consider the derived statement

   $$P^{\#}(m) = (\forall \text{ natural number } k \;.\; 0 \leq k \leq m \implies P(k))$$

   again for m ranging over the natural numbers.
   - Show that for all natural numbers l, $P^{\#}(l) \implies P(l)$
   - Prove by exhibiting a counter-example that $P(n) \implies P^{\#}(n)$ does not hold.
   - Prove or disprove:
     - $P^{\#}(0) \iff P(0)$
     - $\forall$ natural number $n.(P^{\#}(n) \implies P^{\#}(n+1)) \iff (P^{\#}(n) \implies P(n+1))$
     - $(\forall$ natural number $m.P^{\#}(m)) \iff (\forall$ natural number $m.P(m))$

5. Prove that for all integers d, k, l, m, n,
   - $d|m \wedge d|n \implies d|(m+n)$
   - $d|m \implies d|km$
   - $d|m \wedge d|n \implies d|(km + ln)$

Tryhard questions (do them if you can)

1. Prove that there are infinitely many natural numbers n, such that $4n + 3$ is prime.

2. (Hard) Is it true that if p is a prime number, and k is an integer $2 \leq k \leq p$, then the sum of the products of each k-element subset of $\{1, 2, \ldots, p\}$ will be divisible by p?