



CSI3105: Software Testing

Module 8a: Combinatorial Testing Introduction

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Lecture Summary



The learning goal for this week is to familiarize you with Combinatorial Testing

Do you know what combinatorial testing is?

Why its used?

How to use mutually orthogonal Latin squares to derive test cases?

Can you complete the workshop?

Introduction

The behavior of software applications may be affected by many factors such as the operating environment, network connection, hardware platform and input parameters.



Windows 10

X Z

Y



Introduction

Software applications may be affected by many factors such input parameters

Black box techniques help identify possible inputs

- equivalence partitioning (EP) and,
- boundary value analysis (BVA) can be applied for identifying possible values for each factor.

The goal here was to try generate a set of tests that adequately test the input domains for our software under test

Introduction

Shortcomings of EP:

- Possibility of a large number of sub-domains in the partition
- Lacks guidelines on how to select inputs from the sub-domains
- **What about combinations of input?????**
- **Does not account for faults due to specific interactions amongst values of different input variables**

Shortcomings of BVA:

- Other interactions in the input domain remain untested.
- Doesn't account for interaction of variables

Combinatorial Testing

We are going to look at techniques that generate small test configurations OR test sets even when the possible configurations or the input domain and the sub domains are large and complex.



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Module 8b: Combinatorial Testing Terms

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More Terminology – Input Space

Your **input space** is your input domain.

The variables being assigned to your software under test.

The **input space** of a program P consists of k-tuples of values that could be input to P during execution.

Example:

- Consider program P that takes two integers $x > 0$ and $y > 0$ as inputs.
 - The **input space** of P is the set of all pairs of positive non-zero integers.

Input Space

- Another example..
- Date function that outputs the next date
- Day, month, year – Input space (days, months and years)

| Day | Month | Year | Expected Output |
|-----|-------|------|-----------------|
| 15 | 6 | 1912 | 16/6/1912 |

Modeling: Input and configuration space (from slides, Mathur (2014))

The **configuration space** of P consists of all possible settings of the environment variables under which P could be used.

Example:

- This program is also intended to be executed under the Windows and the MacOS operating system, through the Chrome or Safari browsers, and must be able to print to a local or a networked printer.
 - The **configuration space** of P consists of triples (X, Y, Z) where X represents an **operating system**, Y a **browser**, and Z a **local or a networked** printer.

Many Combinations

Input or Configurations space leads to many combinations

- Will firefox on mac handle the same as firefox on windows?
 - Better test this!

Should I test the day and month input to a function in isolation or is there a relationship between these variables I should account for?

New Terminology



Levels,
Factors and,
Factor Combinations!

Factors

Consider a program P that takes n inputs corresponding to variables X_1, X_2, \dots, X_n . We refer to the inputs as **factors**. The inputs are also referred to as **test parameters** or as **values**.

You can think about factors as the input parameters to a program.

If an addition program required two inputs, x and y, these would be the factors

- This program would have 2 factors, being the x and y inputs.

Levels

Let us assume that each factor may be set at any one from a total of c_i , $1 \leq i \leq n$ values.....

The levels of your factors are essentially, your equivalence classes.

Each value assignable to a factor is known as a **level**. The notation $|F|$ refers to the number of levels for factor F.

Pizza ordering
program
inputs



| Factor | Levels |
|---------|------------------|
| Size | Large Small |
| Topping | Custom Pre-set |
| Address | Valid Invalid |

Factor Combination

A set of values, one for each factor, is known as a **factor combination**.

Let's for a second assume that we have a pizza ordering application that takes in three inputs...

$$Levels^{Factors} = 2^3 = 8 \text{ factor combinations}$$

$$\text{Or } |Size| \times |Toppings| \times |Address| = 2 \times 2 \times 2$$

(L, C, V), (L, P, V), (L, C, I), (L, P, I),
(S, C, V), (S, P, V), (S, C, I), (S, P, I),

| Factor | Levels |
|---------|------------------|
| Size | Large Small |
| Topping | Custom Pre-set |
| Address | Valid Invalid |

Factors, Levels and Factor Combination

Example the second!

- Program P
 - two input variables X and Y
 - X from the set {a, b, c}
 - Y from the set {d, e, f}
- Thus we have **2 factors** (X AND Y)
- **3 levels** for each factor.
- This leads to a total of $3^2=9$ **factor combinations** (a, d), (a, e), (a, f), (b, d), (b, e), (b, f), (c, d), (c, e), and (c, f).

If each factor combination yields one test case, for many programs, the number of tests generated for exhaustive testing could be **exorbitantly large**.

If a program has 15 factors with 4 levels each, the total number of tests is 4^{15} OR 1,073,741,824 test cases.

Executing a billion tests might be impractical for many software applications



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Module 8c: Combinatorial Testing and Exhaustive Testing

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Common Problem



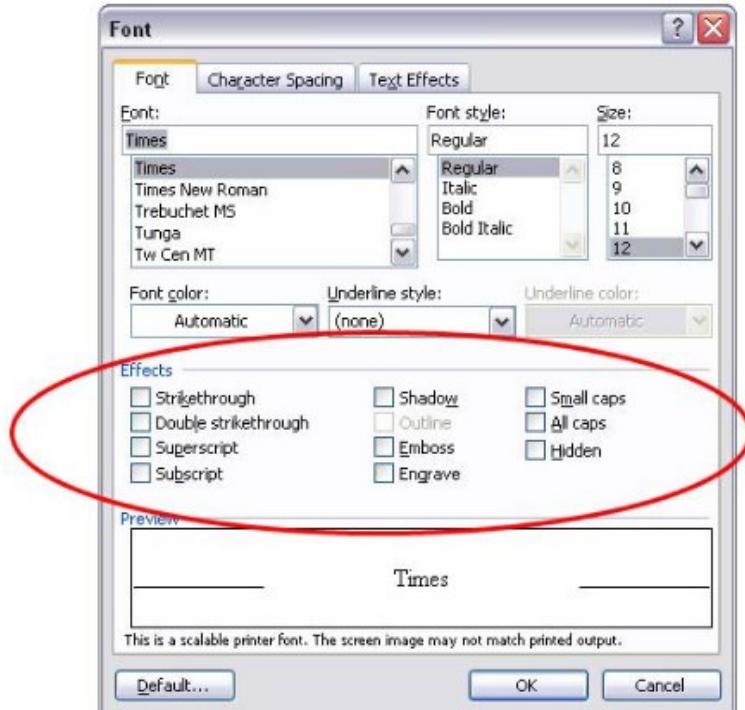
Executing a billion tests might be impractical for many software applications

Exhaustive testing is not possible!

Common Problem

<https://csrc.nist.gov/CSRC/media/Presentations/Combinatorial-Methods-for-System-and-Software-Testing.pdf>

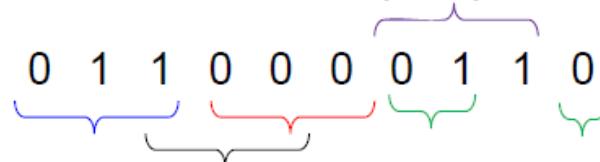
- Microsoft word has ten text effects
- Check button (on/off)
- All combinations $2^10 = 1024$ tests



Common Problem

https://csrc.nist.gov/CSRC/media/Presentations/Combinatorial-Methods-for-System-and-Software-Test/images-media/4_software-testing_kuhn.pdf

- There are $\binom{10}{3} = 120$ 3-way interactions.
- Each triple has $2^3 = 8$ settings: 000, 001, 010, 011, ..
- $120 \times 8 = 960$ combinations
- Each test exercises many triples:



Common Problem

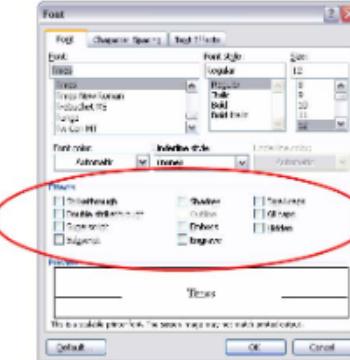
https://csrc.nist.gov/CSRC/media/Presentations/Combinatorial-Methods-for-System-and-Software-Testing/images_media/1_software_testing_kuhn.pdf

All triples in only 13 tests, covering $\binom{10}{3} 2^3 = 960$ combinations

Each row is a test:

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |

Each column is a parameter:





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Module 8d: Test Design Process

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Combinatorial Test Design Process

- Billions of test cases are not good!
- Instead of testing all possible combinations, find a subset that satisfy some well-defined combination strategies.
- Not every factor contributes to every fault – a fault is often the result of the interactions of a few factors

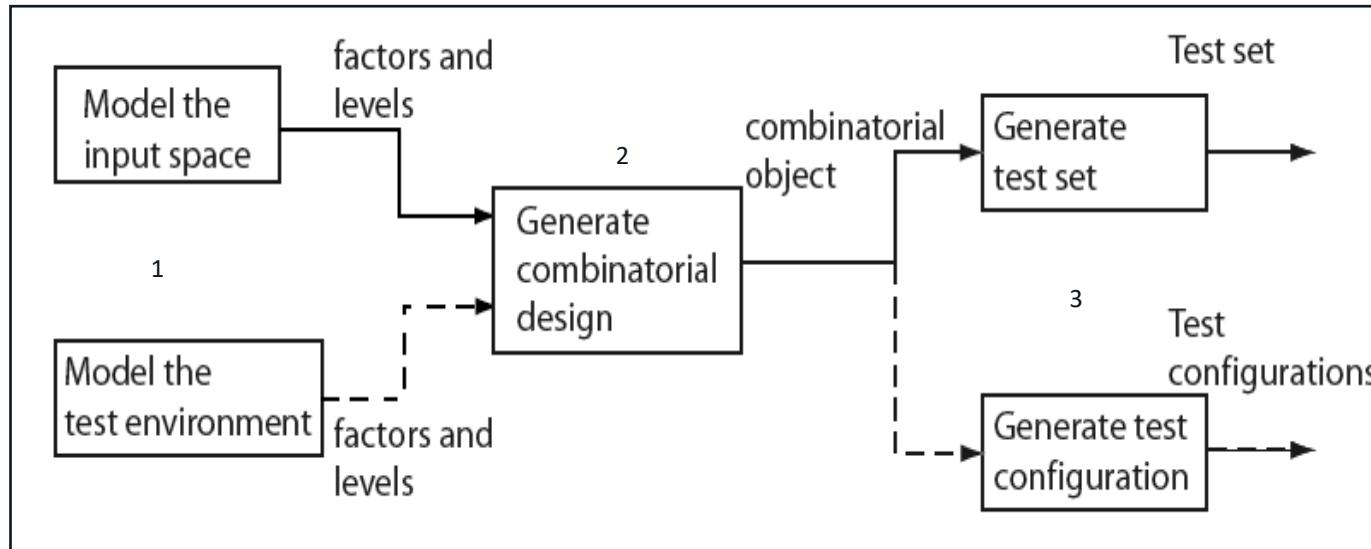


Diagram from Mathur, 2014



CSI3105: Software Testing Module 8e: Fault Types

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Interaction Faults

Introduce some more terminology

Revisited the concept of insanely big input spaces/ exhaustive testing and its feasibility

The crux of todays lecture..

Combinatorial design helps find Interaction Faults

Fault Models (from slides, Mathur (2014))

Faults aimed at by the combinatorial design techniques are known as **interaction faults**.

We say that an interaction fault is **triggered** when a certain combination of $t \geq 1$ input values causes the program containing the fault to enter an invalid state.

And always

- this invalid state must propagate to a point in the program execution where it is observable and hence is said to **reveal** the fault.

Faults triggered by some value of an input variable, i.e. $t=1$, regardless of the values of other input variables, are known as **simple** faults.

For $t=2$, the faults are known as **pairwise interaction** faults.

In general, for any arbitrary value of t , the faults are known as **t -way interaction** faults.

Fault Models (from slides, Mathur (2014))

Interactions e.g., failure occurs

```
if enemy_distance < 10
```

```
{  
    // do faulty AI attack code  
} // 1-way interaction
```

```
if enemy_distance < 10 AND enemy_health > 80
```

```
{  
    // do faulty AI attack code  
} // 2-way interaction
```

```
if enemy_distance < 10 AND enemy_health > 80 AND my_health > 80
```

```
{  
    // do faulty AI attack code  
} // 3-way interaction
```

Pairwise Interaction Fault - Example from Mathur (2014)

Error due to the interaction between factors X and Y

Program Specification:

Program requires 3 inputs x, y, z .

- x is assigned a value from the set $\{x_1, x_2, x_3\}$,
- variable y a value from the set $\{y_1, y_2, y_3\}$ and,
- variable z a value from the set $\{z_1, z_2\}$.

The program outputs:

- the function $f(x, y, z)$ when $x = x_1$ and $y = y_2$,
- function $g(x, y)$ when $x = x_2$ and $y = y_1$,
- **function $f(x, y, z) - g(x, y)$ when $x = x_1$ and $y = y_1$** ,
- and function $f(x, y, z) + g(x, y)$ when $x = x_2$ and $y = y_2$

```
1 begin
2   int x, y, z;
3   input (x, y, z);
4   if(x==x1 and y==y2)
5     output (f(x, y, z));
6   else if(x==x2 and y==y1)
7     output (g(x, y));
8   else
9     output (f(x, y, z)+g(x, y)) ← This statement is not protected correctly.
10 end
```

Pairwise Interaction Fault - Example from Mathur (2014)

Error due to the interaction between factors X :

```
1 begin
2   int x, y, z;
3   input (x, y, z);
4   if(x==x1 and y==y2)
5     output (f(x, y, z));
6   else if(x==x2 and y==y1)
7     output (g(x, y));
8   else
9     output (f(x, y, z)+g(x, y)) ← This statement is not protected correctly.
10  end
```

Program Specification:

The program outputs:

- the function $f(x, y, z)$ when $x = x_1$ and $y = y_2$,
- function $g(x, y)$ when $x = x_2$ and $y = y_1$,
- **function $f(x, y, z) - g(x, y)$ when $x = x_1$ and $y = y_1$** ,
- and function $f(x, y, z) + g(x, y)$ when $x = x_2$ and $y = y_2$

Program contains one error.

It must output $f(x, y, z) - g(x, y)$ when $x = x_1$ and $y = y_1$ and $f(x, y, z) + g(x, y)$ when $x = x_2$ and $y = y_2$

Error is revealed when value of $f(x, y, z) + g(x, y)$ not equal to $f(x, y, z) - g(x, y)$ when $x = x_1$ and $y = y_1$ for any value of z

3-way Interaction Fault - Example from Mathur (2014)

```
1 begin
2   int x, y, z, p;
3   input (x, y, z);
4   p=(x+y)*z; ← This statement must be p=(x-y)*z
5   if(p≥0)
6     output (f(x, y, z));
7   else
8     output (g(x, y));
9 end
```

Possible inputs
-1,-1, 0
-1, -1, 1
-1, 0, 0
-1, 0, 1
1, -1, 0
1, -1, 1
1, 0, 0
1, 0, 1

Program Specification:

- The program takes 3 variables: $x \in \{-1, 1\}$, $y \in \{-1, 0\}$
- and $z \in \{0, 1\}$.

The fault is triggered by all inputs such that **$x+y \neq x-y$ and $z \neq 0$.**

However, the fault is revealed only by the following two of the eight possible input combinations: $x=1, y=-1, z=1$ and $x=-1, y=-1, z=1$.



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Module 8f: Pairwise approach

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Pairwise approach

We now know what interaction faults are

How do we target them?

- Pairwise approach!

Pairwise approach – Algorithms and Tools

Several algorithms for generating a combinatorial object for test cases

- Latin Squares
- Orthogonal arrays
- In-parameter Order (IPO) procedure

Example Tools

- CATS (Constrained Array Test Systems)
- AETG (Telecordia Web-based system)
- CTS (Combinatorial Test Services) – IBM
- Test Vector Generator

Latin Squares

Let S be a finite set of n symbols. A Latin square of order n is an $n \times n$ matrix such that no symbol appears more than once in a row and column. The term "Latin square" arises from the fact that the early versions used letters from the Latin alphabet A, B, C, etc. in a square arrangement.

A B

B A

1 2 3

2 3 1

3 1 2

B A

A B

2 3 1

1 2 3

3 1 2

$S=\{A, B\}$. Latin squares of order 2.

2 1 3

3 2 1

1 3 2

$S=\{1, 2, 3\}$. Latin

squares of order 3.

Larger Latin Squares

Larger Latin squares of order n can be constructed by creating a row of n distinct symbols. Additional rows can be created by [permuting](#) the first row.

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 2 | 3 | 4 | 1 |
| 3 | 4 | 1 | 2 |
| 4 | 1 | 2 | 3 |

| | | | |
|---|---|---|---|
| 2 | 3 | 4 | 1 |
| 3 | 4 | 1 | 2 |
| 4 | 1 | 2 | 3 |

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
|---|---|---|---|

For example, here is a Latin square M of order 4 constructed by cyclically rotating the first row and placing successive rotations in subsequent rows.

Modulo arithmetic and Latin Squares

A Latin square of order $n > 2$ can also be constructed easily by doing modulo arithmetic. For example, the Latin square M of order 4 given below is constructed such that $M(i, j) = i + j \pmod{4}$, $1 \leq (i, j) \leq 4$.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 3 | 0 | 1 |
| 2 | 3 | 0 | 1 | 2 |
| 3 | 0 | 1 | 2 | 3 |
| 4 | 1 | 2 | 3 | 0 |

Mutually orthogonal Latin squares

Up next...

Mutually Orthogonal Latin Squares (MOLS)

Let M_1 and M_2 be two Latin squares, each of order n . Let $M_1(i, j)$ and $M_2(i, j)$ denote, respectively, the elements in the i th row and j th column of M_1 and M_2 .

We now create an $n \times n$ matrix L from M_1 and M_2 such that the $M(i, j)$ is $M_1(i, j)M_2(i, j)$, i.e. we simply juxtapose the corresponding elements of M_1 and M_2 .

If each element of L is unique, i.e. it appears exactly once in L , then M_1 and M_2 are said to be **mutually orthogonal** Latin squares of order n .

MOLS: Example

There are no MOLS of order 2. MOLS of order 3 follow.

$$M_1 = \begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{matrix}$$

$$M_2 = \begin{matrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{matrix}$$

$$L = \begin{matrix} 12 & 23 & 31 \\ 21 & 32 & 13 \\ 33 & 11 & 22 \end{matrix}$$

Juxtaposing the corresponding elements gives us L. Its elements are unique and hence M1 and M2 are MOLS.

MOLS: How many of a given order?

MOLS(n) is the set of MOLS of order n . When n is prime, or a power of prime, MOLS(n) contains $n-1$ mutually orthogonal Latin squares. Such a set of MOLS is a [complete](#) set.

MOLS do not exist for $n=2$ and $n=6$ but they do exist for all other values of $n > 2$.

Numbers 2 and 6 are known as [Eulerian numbers](#) after the famous mathematician Leonhard Euler (1707-1783). The number of MOLS of order n is denoted by $N(n)$. When n is prime or a power of prime, $N(n)=n-1$.

MOLS: How many of a given order?

- We can use MOLS to help us in pair wise test case design!
- If the number of levels of a factor is greater than 2, brilliant..
- What happens if the number of levels is 2 and no MOLs exist?



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Module 8g: SAMNA technique

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Pairwise designs

In this video we will look at a simple technique to generate a subset of factor combinations from the complete set. Each combination selected generates at least one test input or test configuration for the program under test.

Only [2-valued](#), or [binary](#), factors are considered. Each factor can be at one of two levels.
This assumption will be relaxed later.

Pairwise designs: Example

Suppose that a program to be tested requires 3 inputs, one corresponding to each input variable. Each variable can take only one of two distinct values.

Considering each input variable as a factor, the total number of factor combinations is 2^3 . Let X, Y, and Z denote the three input variables and $\{X_1, X_2\}$, $\{Y_1, Y_2\}$, $\{Z_1, Z_2\}$ their respective sets of values. All possible combinations of these three factors follow.

| | |
|-------------------|-------------------|
| (X_1, Y_1, Z_1) | (X_1, Y_1, Z_2) |
| (X_1, Y_2, Z_1) | (X_1, Y_2, Z_2) |
| (X_2, Y_1, Z_1) | (X_2, Y_1, Z_2) |
| (X_2, Y_2, Z_1) | (X_2, Y_2, Z_2) |

Pairwise designs: Reducing the combinations

Now suppose we want to generate tests such that each **pair** appears in at least one test.

There are 12 such pairs: (X_1, Y_1) , (X_1, Y_2) , (X_1, Z_1) , (X_1, Z_2) , (X_2, Y_1) , (X_2, Y_2) ,
 (X_2, Z_1) , (X_2, Z_2) , (Y_1, Z_1) , (Y_1, Z_2) , (Y_2, Z_1) , and (Y_2, Z_2) . The following **four** combinations cover all pairs:

$$\begin{array}{ll} (X_1, Y_1, Z_2) & (X_1, Y_2, Z_1) \\ (X_2, Y_1, Z_1) & (X_2, Y_2, Z_2) \end{array}$$

The above design is also known as a **pairwise** design. It is a **balanced** design because each value occurs exactly the same number of times. *There are several sets of four combinations that cover all 12 pairs.*

Example: ChemFun applet

A Java applet [ChemFun](#) allows its user to create an in-memory database of chemical elements and search for an element. The applet has 5 inputs listed after the next slide with their possible values.

We refer to the inputs as **factors**. For simplicity we assume that each input has exactly two possible values.

Example: ChemFun applet

Welcome to CS 177 /178 Programming with Multimedia Objects
Fall 2004
Chemical Element Fun

Create Element

Type element name here.

Show Element

Type element symbol here.

Type element atomic number here.

Type properties here.

The screenshot shows a Java applet titled "Chemical Element Fun". It features a sidebar on the left with two buttons: "Create Element" and "Show Element". The main content area contains four input fields for element properties. The first field is labeled "Type element name here.", the second "Type element symbol here.", the third "Type element atomic number here.", and the fourth "Type properties here.". Below each input field is a horizontal scroll bar. The background of the applet is yellow, and the overall layout is clean and organized.

Example: ChemFun applet: Factor identification

| Factor | Name | Levels | Comments |
|--------|---------------|--------------------|-----------------------------|
| 1 | Operation | {Create, Show} | Two buttons |
| 2 | Name | {Empty, Non-empty} | Data field, string expected |
| 3 | Symbol | {Empty, Non-empty} | Data field, string expected |
| 4 | Atomic number | {Invalid, Valid} | Data field, data typed > 0 |
| 5 | Properties | {Empty, Non-empty} | Data field, string expected |

5 factors, each with 2 levels.

$$2^5 = 32 \text{ tests}$$

We can reduce this to 6 tests

Example: ChemFun applet: Factor identification

| Factor | Name | Levels | Comments |
|--------|---------------|--------------------|-----------------------------|
| 1 | Operation | {Create, Show} | Two buttons |
| 2 | Name | {Empty, Non-empty} | Data field, string expected |
| 3 | Symbol | {Empty, Non-empty} | Data field, string expected |
| 4 | Atomic number | {Invalid, Valid} | Data field, data typed > 0 |
| 5 | Properties | {Empty, Non-empty} | Data field, string expected |

Map to levels to either 0 or 1's

A test string could be 0, 0, 0, 1, 1

Test case input = Create, Empty, Empty, Valid, Non-empty

ChemFun applet: Input/Output



Input: n=5 factors

Output: A set of factor combinations such that all pairs of input values are covered.

ChemFun applet: Step 1

Variable **k** represents the number of 1's in our test string

Compute the **smallest** integer **k** such that $n \leq 2^{k-1}$

Input: n=5 factor

If $k = 2$

$$2^{k-1} = 3$$

3 isn't greater or equal too n (n = 5)

If $k = 3$

$$2^{k-1} = 5$$

5 is greater or equal too n (n = 5)

ChemFun applet: Step 1

This formula shows us how many binary strings of length n , can have k ones

k = 3 (3 ones in our test string)

n = 5 factors (how many inputs)

$$\binom{n}{k} = \frac{!n}{!(n-k) !k}$$

k = 3, n = 5

$$\frac{!5}{!(5-3)!3}$$

$$\frac{120}{12} = 10$$

For k=3 we have S₅= 10

ChemFun applet: Step 2

Select any subset of **n** strings from S_{2k-1} . We have, $k=3$ and we have the following strings in the set S_5 ($s_k - 1 = 5$) AND **n = 5**

| | 1 | 2 | 3 | 4 | 5 |
|----|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 1 |
| 2 | 0 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 0 | 0 |
| 4 | 1 | 0 | 1 | 1 | 0 |
| 5 | 0 | 1 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 | 1 | 0 |
| 7 | 1 | 0 | 1 | 0 | 1 |
| 8 | 0 | 1 | 0 | 1 | 1 |
| 9 | 1 | 1 | 0 | 0 | 1 |
| 10 | 1 | 0 | 0 | 1 | 1 |

We select first five of the 10 strings in. S_5 .

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 1 |
| 2 | 0 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 0 | 0 |
| 4 | 1 | 0 | 1 | 1 | 0 |
| 5 | 0 | 1 | 1 | 0 | 1 |

ChemFun applet: Step 3

Append 0's to the end of each selected string. This will increase the size of each string from $2k-1$ to $2k$.

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 1 |
| 2 | 0 | 1 | 1 | 1 | 0 |
| 3 | 1 | 1 | 1 | 0 | 0 |
| 4 | 1 | 0 | 1 | 1 | 0 |
| 5 | 0 | 1 | 1 | 0 | 1 |



| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 2 | 0 | 1 | 1 | 1 | 0 | 0 |
| 3 | 1 | 1 | 1 | 0 | 0 | 0 |
| 4 | 1 | 0 | 1 | 1 | 0 | 0 |
| 5 | 0 | 1 | 1 | 0 | 1 | 0 |

ChemFun applet: Step 4

Each combination is of the kind (X_1, X_2, \dots, X_n) , where the value of each variable is selected depending on whether the bit in column i , $1 \leq i \leq n$, is a 0 or a 1.

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 2 | 0 | 1 | 1 | 1 | 0 | 0 |
| 3 | 1 | 1 | 1 | 0 | 0 | 0 |
| 4 | 1 | 0 | 1 | 1 | 0 | 0 |
| 5 | 0 | 1 | 1 | 0 | 1 | 0 |

ChemFun applet: Step 4 (contd.)

The following factor combinations by replacing the 0s and 1s in each column by the corresponding values of each factor.

| Factor | Name | Levels | Comments |
|--------|---------------|--------------------|-----------------------------|
| 1 | Operation | {Create, Show} | Two buttons |
| 2 | Name | {Empty, Non-empty} | Data field, string expected |
| 3 | Symbol | {Empty, Non-empty} | Data field, string expected |
| 4 | Atomic number | {Invalid, Valid} | Data field, data typed > 0 |
| 5 | Properties | {Empty, Non-empty} | Data field, string expected |

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|-----------|-----------|-----------|-----------|-----------|---------|
| 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 2 | 0 | 1 | 1 | 1 | 0 | 0 |
| 3 | 1 | 1 | 1 | 0 | 0 | 0 |
| 4 | 1 | 0 | 1 | 1 | 0 | 0 |
| 5 | 0 | 1 | 1 | 0 | 1 | 0 |
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | Create | Create | Show | Show | Show | Create |
| 2 | Empty | Non-empty | Non-empty | Non-empty | Empty | Empty |
| 3 | Non-empty | Non-empty | Non-empty | Empty | Empty | Empty |
| 4 | Valid | Invalid | Valid | Valid | Invalid | Invalid |
| 5 | Empty | Non-empty | Non-Empty | Empty | Non-empty | Empty |

ChemFun applet: tests

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|-----------|-----------|-----------|-----------|-----------|---------|
| 1 | Create | Create | Show | Show | Show | Create |
| 2 | Empty | Non-empty | Non-empty | Non-empty | Empty | Empty |
| 3 | Non-empty | Non-empty | Non-empty | Empty | Empty | Empty |
| 4 | Valid | Invalid | Valid | Valid | Invalid | Invalid |
| 5 | Empty | Non-empty | Non-Empty | Empty | Non-empty | Empty |



$t_1 : < \text{Button} = \text{Create}, \text{Name} = "", \text{Symbol} = 'C',$
 $\text{Atomic number} = 6, \text{Properties} = "" >$

ChemFun applet: All tests

$$T = \{ \begin{aligned} t_1 : & \text{ < Button = Create, Name = "", Symbol = 'C',} \\ & \text{Atomic number = 6, Properties = "" >} \\ t_2 : & \text{ < Button = Create, Name = "Carbon", Symbol = 'C',} \\ & \text{Atomic number = -6, Properties = "Non-metal" >} \\ t_3 : & \text{ < Button = Show, Name = "Hydrogen", Symbol = 'C',} \\ & \text{Atomic number = 1, Properties = "Non-metal" >} \\ t_4 : & \text{ < Button = Show, Name = "Carbon", Symbol = 'C',} \\ & \text{Atomic number = 6, Properties = "" >} \\ t_5 : & \text{ < Button = Show, Name = "", Symbol = "",} \\ & \text{Atomic number = -6, Properties = "Non-metal" >} \\ t_6 : & \text{ < Button = Create, Name = "", Symbol = "",} \\ & \text{Atomic number = -6, Properties = "" >} \end{aligned} \}$$

Recall that the total number of combinations is 32. Requiring only pairwise coverage reduces the tests to 6.



CSI3105: Software Testing

Module 8h: MOLs Approach

Creative
thinkers
made here.

MOLS: How many of a given order?

- What happens if the number of levels is greater than 2?
- MOLs!

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 2 | 3 | 4 | 1 |
| 3 | 4 | 1 | 2 |
| 4 | 1 | 2 | 3 |

4 levels? Map to 1 - 4

Combinatorial Testing : PD MOLS

- Example 1:
 - Program with three input parameters as follows:
 - $X_1 = \{p, q, r\}$, $X_2 = \{a, b\}$ and $X_3 = \{1,2,3,4\}$,
 - all combinations would involve $3 * 2 * 4 = 24$ test cases.
 - $|X_3| = 4$, $|X_1| = 3$, $|X_2| = 2$

Combinatorial Testing : PD MOLS

$X_1 = \{p, q, r\}$, $X_2 = \{a, b\}$ and $X_3 = \{1, 2, 3, 4\}$,

| Block | X3 | X1 | X2 |
|-------|----|----|----|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |

Combinatorial Testing : PD MOLS

$X_1 = \{p, q, r\}$, $X_2 = \{a, b\}$ and $X_3 = \{1, 2, 3, 4\}$,

| Block | X3 | X1 | X2 |
|-------|----|----|----|
| 1 | 1 | | |
| | 1 | | |
| | 1 | | |
| | 1 | | |
| 2 | 2 | | |
| | 2 | | |
| | 2 | | |
| | 2 | | |
| 3 | 3 | | |
| | 3 | | |
| | 3 | | |
| | 3 | | |
| 4 | 4 | | |
| | 4 | | |
| | 4 | | |
| | 4 | | |

Combinatorial Testing : PD MOLS

X1 = {p, q, r}, X2 = {a, b} and X3 = {1,2,3,4},

| Block | X3 | X1 | X2 |
|-------|----|----|----|
| 1 | 1 | 1 | |
| | 1 | 2 | |
| | 1 | 3 | |
| | 1 | 4 | |
| 2 | 2 | 1 | |
| | 2 | 2 | |
| | 2 | 3 | |
| | 2 | 4 | |
| 3 | 3 | 1 | |
| | 3 | 2 | |
| | 3 | 3 | |
| | 3 | 4 | |
| 4 | 4 | 1 | |
| | 4 | 2 | |
| | 4 | 3 | |
| | 4 | 4 | |

Combinatorial Testing : PD MOLS

$X_1 = \{p, q, r\}$, $X_2 = \{a, b\}$ and $X_3 = \{1, 2, 3, 4\}$,

| Block | X3 | X1 | X2 |
|-------|----|----|----|
| 1 | 1 | 1 | 1 |
| | 1 | 2 | 3 |
| | 1 | 3 | 4 |
| | 1 | 4 | 2 |
| 2 | 2 | 1 | 2 |
| | 2 | 2 | 4 |
| | 2 | 3 | 3 |
| | 2 | 4 | 1 |
| 3 | 3 | 1 | 3 |
| | 3 | 2 | 1 |
| | 3 | 3 | 2 |
| | 3 | 4 | 4 |
| 4 | 4 | 1 | 4 |
| | 4 | 2 | 2 |
| | 4 | 3 | 1 |
| | 4 | 4 | 3 |

Combinatorial Testing : PD MOLS

$X_1 = \{p, q, r\}$, $X_2 = \{a, b\}$ and $X_3 = \{1, 2, 3, 4\}$,

| Block | X3 | X1 |
|-------|----|----|
| 1 | 1 | 1 |
| | 1 | 2 |
| | 1 | 3 |
| | 1 | 4 |

| | 1 | 2 | 3 | 4 |
|----|---|---|---|---|
| X2 | 3 | 4 | 1 | 2 |
| 1 | 4 | 3 | 2 | 1 |
| 3 | 4 | 2 | 1 | 4 |
| 4 | 2 | 1 | 4 | 3 |

| | | | |
|---|---|---|---|
| 2 | 2 | 1 | 2 |
| | 2 | 2 | 4 |
| | 2 | 3 | 3 |
| | 2 | 4 | 1 |

| |
|---|
| 2 |
| 4 |
| 3 |
| 1 |

| | | | |
|---|---|---|---|
| 3 | 3 | 1 | 3 |
| | 3 | 2 | 1 |
| | 3 | 3 | 2 |
| | 3 | 4 | 4 |

| |
|---|
| 3 |
| 1 |
| 2 |
| 4 |

| | | | |
|---|---|---|---|
| 4 | 4 | 1 | 4 |
| | 4 | 2 | 2 |
| | 4 | 3 | 1 |
| | 4 | 4 | 3 |

| |
|---|
| 4 |
| 2 |
| 1 |
| 3 |

Combinatorial Testing : PD MOLS

$X_1 = \{p, q, r\}$, $X_2 = \{a, b\}$ and $X_3 = \{1, 2, 3, 4\}$,

| Block | X_3 | X_1 | X_2 |
|-------|-------|-------|-------|
| 1 | 1 | 1 | 1 |
| | 1 | 2 | 4 |
| | 1 | 3 | 2 |
| | 1 | 1 | 3 |
| 2 | 2 | 1 | 2 |
| | 2 | 2 | 1 |
| | 2 | 3 | 2 |
| | 2 | 1 | 1 |
| 3 | 3 | 1 | 1 |
| | 3 | 2 | 1 |
| | 3 | 3 | 2 |
| | 3 | 1 | 2 |
| 4 | 4 | 1 | 2 |
| | 4 | 2 | 2 |
| | 4 | 3 | 1 |
| | 4 | 1 | 1 |

Combinatorial Testing : PD MOLS

X1 = {p, q, r}, X2 = {a, b} and X3 = {1,2,3,4},

| Block | X3 | X1 | X2 |
|-------|----|----|----|
| 1 | 1 | p | a |
| | 1 | q | a |
| | 1 | r | b |
| | 1 | p | b |
| 2 | 2 | p | b |
| | 2 | q | a |
| | 2 | r | b |
| | 2 | p | a |
| 3 | 3 | p | a |
| | 3 | q | a |
| | 3 | r | b |
| | 3 | p | b |
| 4 | 4 | p | b |
| | 4 | q | b |
| | 4 | r | a |
| | 4 | p | a |

Combinatorial Testing : PD MOLS

$X_1 = \{p, q, r\}$, $X_2 = \{a, b\}$ and $X_3 = \{1, 2, 3, 4\}$,

| Block | X3 | X1 | X2 |
|-------|-----------|-----------|-----------|
| 1 | 1 | p | a |
| | 1 | q | a |
| | 1 | r | b |
| 2 | 2 | p | b |
| | 2 | q | a |
| | 2 | r | b |
| 3 | 3 | p | a |
| | 3 | q | a |
| | 3 | r | b |
| 4 | 4 | p | b |
| | 4 | q | b |
| | 4 | r | a |

all combinations would involve $3 \times 2 \times 4 = 24$ test cases
 We now have 12 test

Combinatorial Testing : PD MOLS

Connection = {nbn, adsl, 4g}, Printer = {local, network} and OS= {XP, win 7, win 8, win 10},

| Block | OS | Con | Printer |
|-------|--------|------|---------|
| 1 | XP | nbn | local |
| | XP | adsl | local |
| | XP | 4g | network |
| 2 | win 7 | nbn | network |
| | win 7 | adsl | local |
| | win 7 | 4g | network |
| 3 | win 8, | nbn | local |
| | win 8, | adsl | local |
| | win 8, | 4g | network |
| 4 | win 10 | nbn | network |
| | win 10 | adsl | network |
| | win 10 | 4g | local |

Factors, Levels and Factor Combination (from slides,

| Factor | Levels |
|---------|------------------------------|
| Size | Large Medium Small |
| Topping | Chicken Veg Vegan Beef |
| Address | Valid Invalid |

| Factor | Levels |
|---------|---------------|
| Size | 1 2 3 |
| Topping | 1 2 3 4 |
| Address | 1 2 |

$3 * 4 * 2 = 24$ combinations

Combinatorial Testing : PD MOLS

Size = {1, 2, 3}, Address = {1, 2} and Size = {1,2,3,4},



| Block | Topping | Size | Address |
|-------|---------|------|---------|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |

Combinatorial Testing : PD MOLS

Size = {p, q, r}, Address = {a, b} and Topping = {1,2,3,4},

| Block | Topping | Size | Address |
|--------------|---------|------|---------|
| 1 | 1 | | |
| | 1 | | |
| | 1 | | |
| | 1 | | |
| 2 | 2 | | |
| | 2 | | |
| | 2 | | |
| | 2 | | |
| 3 | 3 | | |
| | 3 | | |
| | 3 | | |
| | 3 | | |
| 4 | 4 | | |
| | 4 | | |
| | 4 | | |
| | 4 | | |

Combinatorial Testing : PD MOLS

Size = {p, q, r}, Address = {a, b} and Topping = {1,2,3,4},

| Block | Topping | Size | Address |
|--------------|---------|------|---------|
| 1 | 1 | 1 | |
| | 1 | 2 | |
| | 1 | 3 | |
| | 1 | 4 | |
| 2 | 2 | 1 | |
| | 2 | 2 | |
| | 2 | 3 | |
| | 2 | 4 | |
| 3 | 3 | 1 | |
| | 3 | 2 | |
| | 3 | 3 | |
| | 3 | 4 | |
| 4 | 4 | 1 | |
| | 4 | 2 | |
| | 4 | 3 | |
| | 4 | 4 | |

Combinatorial Testing : PD MOLS

Size = {p, q, r}, Address = {a, b} and Topping = {1,2,3,4},

| Block | Topping | Size | Address | |
|-------|---------|------|---------|---------|
| 1 | 1 | 1 | 1 | 1 2 3 4 |
| | 1 | 2 | 3 | 3 4 1 2 |
| | 1 | 3 | 4 | 4 3 2 1 |
| | 1 | 4 | 2 | 2 1 4 3 |
| 2 | 2 | 1 | 2 | |
| | 2 | 2 | 4 | |
| | 2 | 3 | 3 | |
| | 2 | 4 | 1 | |
| 3 | 3 | 1 | 3 | |
| | 3 | 2 | 1 | |
| | 3 | 3 | 2 | |
| | 3 | 4 | 4 | |
| 4 | 4 | 1 | 4 | |
| | 4 | 2 | 2 | |
| | 4 | 3 | 1 | |
| | 4 | 4 | 3 | |

Combinatorial Testing : PD MOLS

Block

Size = {1, 2, 3}, Address = {1, 2, 3, 4} and Topping = {1, 2, 3, 4}

| | Topping | Size |
|---|---------|------|
| 1 | 1 | 1 |
| | 1 | 2 |
| | 1 | 3 |
| | 1 | 4 |
| 2 | 2 | 1 |
| | 2 | 2 |
| | 2 | 3 |
| | 2 | 4 |
| 3 | 3 | 1 |
| | 3 | 2 |
| | 3 | 3 |
| | 3 | 4 |
| 4 | 4 | 1 |
| | 4 | 2 |
| | 4 | 3 |
| | 4 | 4 |

Address 1

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 |
| 3 | 3 | 4 | 1 | 2 |
| 4 | 4 | 3 | 2 | 1 |
| 2 | 2 | 1 | 4 | 3 |
| 4 | 1 | | | |
| 3 | | | | |
| 2 | | | | |
| 1 | | | | |
| 4 | | | | |
| 3 | | | | |
| 2 | | | | |
| 1 | | | | |
| 4 | | | | |
| 3 | | | | |
| 2 | | | | |
| 1 | | | | |
| 4 | | | | |
| 3 | | | | |
| 2 | | | | |
| 1 | | | | |
| 4 | | | | |
| 3 | | | | |
| 2 | | | | |
| 1 | | | | |

Combinatorial Testing : PD MOLS

Size = {1, 2, 3}, Address = {1, 2} and Topping = {1,2,3,4},

| Block | Topping | Size | Address |
|-------|---------|------|---------|
| 1 | 1 | 1 | 1 |
| | 1 | 2 | 3 |
| | 1 | 3 | 4 |
| | 1 | 1 | 1 |
| 2 | 2 | 1 | 2 |
| | 2 | 2 | 1 |
| | 2 | 3 | 2 |
| | 2 | 1 | 1 |
| 3 | 3 | 1 | 1 |
| | 3 | 2 | 1 |
| | 3 | 3 | 2 |
| | 3 | 1 | 2 |
| 4 | 4 | 1 | 2 |
| | 4 | 2 | 2 |
| | 4 | 3 | 1 |
| | 4 | 1 | 1 |

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 3 | 4 | 1 | 2 |
| 1 | 4 | 3 | 2 |
| 2 | 2 | 1 | 4 |
| 2 | 1 | 4 | 3 |

Combinatorial Testing : PD MOLS

Size = {1, 2, 3}, Address = {1, 2} and Topping = {1,2,3,4},

| Block | Topping | Size | Address |
|-------|---------|--------|---------|
| 1 | Chicken | Large | valid |
| | Chicken | Medium | valid |
| | Chicken | Small | invalid |
| | Chicken | Large | invalid |
| 2 | Veg | Large | invalid |
| | Veg | Medium | valid |
| | Veg | Small | invalid |
| | Veg | Large | valid |
| 3 | Vegan | Large | valid |
| | Vegan | Medium | valid |
| | Vegan | Small | invalid |
| | Vegan | Large | invalid |
| 4 | Beef | Large | invalid |
| | Beef | Medium | invalid |
| | Beef | Small | valid |
| | Beef | Large | valid |

Combinatorial Testing : PD MOLS

Size = {1, 2, 3}, Address = {1, 2} and Topping = {1,2,3,4},

| Block | Topping | Size | Address |
|-------|---------|--------|---------|
| 1 | Chicken | Large | valid |
| | Chicken | Medium | valid |
| | Chicken | Small | invalid |
| 2 | Veg | Large | invalid |
| | Veg | Medium | valid |
| | Veg | Small | invalid |
| 3 | Vegan | Large | valid |
| | Vegan | Medium | valid |
| | Vegan | Small | invalid |
| 4 | Beef | Large | invalid |
| | Beef | Medium | invalid |
| | Beef | Small | valid |

Combinatorial Testing : PD MOLS

| | | | |
|-----|---------|--------|---------|
| t1 | Chicken | Large | valid |
| t2 | Chicken | Medium | valid |
| t3 | Chicken | Small | invalid |
| t4 | Veg | Large | invalid |
| t5 | Veg | Medium | valid |
| t6 | Veg | Small | invalid |
| t7 | Vegan | Large | valid |
| t8 | Vegan | Medium | valid |
| t9 | Vegan | Small | invalid |
| t10 | Beef | Large | invalid |
| t11 | Beef | Medium | invalid |
| t12 | Beef | Small | valid |

Combinatorial Testing : PD MOLS

T1<Chicken, Large, "14 Walter RD, Morley 6052"> expected = \$14.95

T2 <Chicken, Medium, "14 Walter RD, Morley 6052"> expected = \$12.95

T3<Chicken , small, invalid> expected = No sale

T4<Veg, Large, invalid> expected = No sale

T5<Veg, Medium, "14 Walter RD, Morley 6052"> expected = \$11.95

T6<Veg, Small, invalid> expected = No sale

t7<Vegan, Large, "14 Walter RD, Morley 6052"> > expected = \$17.95

T8< Vegan, Medium, "14 Walter RD, Morley 6052"> expected = \$13.95

T9< Vegan , Small, invalid> expected = No sale

T10< Beef , Large, invalid> expected = No sale

T11< Beef , Medium , invalid> expected = No sale

T12< Beef , Small, "14 Walter RD, Morley 6052"> expected = \$9.95



CSI3105: Software Testing Module 8i: Test Coverage

Creative
thinkers
made here.

Combinatorial Testing : Coverage

All combinations

- Test cases where every possible combinations of values of the parameters must be covered.
- **Example 1:** Program with three input parameters as follows: $X_1 = \{p, q, r\}$, $X_2 = \{a, b\}$ and $X_3 = \{1,2,3,4\}$, all combinations would involve $3*2*4 = 24$ test cases.

Each Choice

- At least one test case for each value associated with the parameter.
- Using Example 1, a test set this criterion would be: $\{(p,a,1) (q,b,2) (r,a,3)(p,b,4)\}$

Pairwise

- Given any two parameters, at least one test case for every combinations of their respective values
- Covered in previous slides

t-wise

- Given any t parameters, at least one test case for every combinations of values of the t parameters.

Combinatorial Testing : Coverage

Base Choice Coverage

- One of the possible values of each parameter is picked as the *base choice*
- A base test is defined by using the base choice for each parameter
- Hold all base choices constant, except for one which is substituted then with non-base choice of that respective parameter
- **Example 1:** Program with three input parameters as follows: $X_1 = \{p, q, r\}$, $X_2 = \{a, b\}$ and $X_3 = \{1,2,3,4\}$, all combinations would involve $3*2*4 = 24$ test cases
- Using Example 1 and base choices for: $X_1 = \{p\}$, $X_2 = \{a\}$ and $X_3 = \{1\}$, test cases would be: $\{(p,a,1) (q,a,1) (r,a,1) \{(p,b,1) (p,a,2) (p,a,3) (p,a,4)\}\}$

Multiple Base Choices Coverage

- At least one or more base choices are designated for each of the parameters.
- Generation of base test cases same as before