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$$1) \delta_3(t \otimes s) = \delta_3 t \otimes s + t \otimes \delta_3 s. \quad 2) [\delta_3, \delta_2] t = \delta_{[3,2]} t.$$

1). По определению производной Ли:

$$\delta_3 t = t'(x'') - t(x'), \quad \delta_3 s = s'(x'') - s(x')$$

$$\begin{aligned} \delta_3(t \otimes s) &= t'(x'') \otimes s'(x'') - t(x') \otimes s'(x') + t'(x'') \otimes s'(x') - t(x') \otimes s'(x'') = (t'(x'') - t(x')) \otimes s'(x'') + \\ &+ t(x'') \otimes (s'(x'') - s(x')) = (\delta_3 t \otimes s + t \otimes \delta_3 s)(x'') \Rightarrow \delta_3(t \otimes s) = \delta_3 t \otimes s + t \otimes \delta_3 s. \end{aligned}$$

2). Для доказательства докажем для составных частей любого тензора: ковариантных векторов (+ правило Лейбница для правой части).

I. Ковектор (1-форма)

$$\alpha = \alpha_\mu dx^\mu, \text{ известно, что } \delta_3 \alpha_\mu = \partial_\lambda \alpha_\mu \xi^\lambda + \alpha_\lambda \partial_\mu \xi^\lambda$$

$$\begin{aligned} [\delta_3, \delta_2] \alpha_\mu &= \delta_3(\partial_\lambda \alpha_\mu \eta^\lambda + \alpha_\lambda \partial_\mu \eta^\lambda) - \delta_2(\partial_\lambda \alpha_\mu \xi^\lambda + \alpha_\lambda \partial_\mu \xi^\lambda) = \partial_x (\partial_\lambda \alpha_\mu \eta^\lambda + \alpha_\lambda \partial_\mu \eta^\lambda) \xi^\mu + \\ &+ (\partial_\lambda \alpha_x \eta^\lambda + \alpha_\lambda \partial_x \eta^\lambda) \partial_\mu \xi^\mu - \partial_x (\partial_\lambda \alpha_\mu \xi^\lambda + \alpha_\lambda \partial_\mu \xi^\lambda) \eta^\mu - (\partial_\lambda \alpha_x \xi^\lambda + \alpha_\lambda \partial_x \xi^\lambda) \partial_\mu \eta^\mu = \\ &= \partial_x \partial_\lambda \alpha_\mu \eta^\lambda \xi^\mu + \partial_\lambda \alpha_\mu \partial_x \eta^\lambda \xi^\mu + \partial_x \alpha_\lambda \partial_\mu \eta^\lambda \xi^\mu + \alpha_\lambda \partial_x \partial_\mu \eta^\lambda \xi^\mu + (\partial_\lambda \alpha_x \eta^\lambda + \alpha_\lambda \partial_x \eta^\lambda) \partial_\mu \xi^\mu - \\ &- \partial_x \partial_\lambda \alpha_\mu \xi^\lambda \eta^\mu - \partial_\lambda \alpha_\mu \partial_x \xi^\lambda \eta^\mu - \partial_x \alpha_\lambda \partial_\mu \xi^\lambda \eta^\mu - \alpha_\lambda \partial_x \partial_\mu \xi^\lambda \eta^\mu - (\partial_\lambda \alpha_x \xi^\lambda + \alpha_\lambda \partial_x \xi^\lambda) \partial_\mu \eta^\mu = \\ &= \partial_\lambda \alpha_\mu (\partial_x \eta^\lambda \xi^\mu - \partial_x \xi^\lambda \eta^\mu) + \alpha_\lambda (\partial_x \partial_\mu (\eta^\lambda \xi^\mu) - \partial_x \partial_\mu (\xi^\lambda \eta^\mu)) + \alpha_\lambda \partial_x \eta^\lambda \partial_\mu \xi^\mu - \\ &- \alpha_\lambda \partial_x \xi^\lambda \partial_\mu \eta^\mu = \delta_{[3,2]} \alpha_\mu \end{aligned}$$

II. Вектор

$$\alpha = a^\mu \partial_\mu, \text{ известно, что } \delta_3 a = [\xi, a]$$

$$\begin{aligned} [\delta_3, \delta_2] a &= \delta_3 [\eta, a] - \delta_2 [\xi, a] = [\xi, [\eta, a]] - [\eta, [\xi, a]] = -[[\eta, a], \xi] - [a, [\xi, \eta]] = \\ &= // \text{ тождество Якоби } // = [[\xi, \eta], a] = \delta_{[3,2]} a \end{aligned}$$

III. Правило Лейбница

для левой части доказано в предыдущих задачах.

$$[\delta_3, \delta_2](t \otimes s) = \delta_3(\delta_2 t \otimes s + t \otimes \delta_2 s) - \delta_2(\delta_3 t \otimes s + t \otimes \delta_3 s) =$$

$$\begin{aligned}
 & \left( \delta_3, \delta_2 \right) (t \otimes S) = \delta_3 (\delta_2 (t \otimes S)) - \delta_2 (\delta_3 (t \otimes S)) \\
 & = \delta_3 \delta_2 t \otimes S + \delta_2 t \otimes \delta_3 S + \delta_3 t \otimes \delta_2 S + t \otimes \delta_3 \delta_2 S - \delta_2 \delta_3 t \otimes S - \delta_2 t \otimes \delta_3 S - \\
 & - \delta_2 t \otimes \delta_3 S - t \otimes \delta_2 \delta_3 S = [\delta_3, \delta_2] t \otimes S + t \otimes [\delta_3, \delta_2] S.
 \end{aligned}$$

