№1

$$\psi_{00} = -2\frac{r_g}{r}, \quad \psi_{0i} = 0, \quad \psi_{ik} = 0$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \psi_{\mu\nu} - \frac{\eta_{\mu\nu}}{d-2}\psi\Big|_{d=4} = \eta_{\mu\nu} + \psi_{\mu\nu} + \eta_{\mu\nu}\frac{r_g}{r} \Rightarrow$$

$$\Rightarrow g_{\mu\nu} = \begin{pmatrix} 1-\frac{r_g}{r} & 0 & 0 & 0 \\ 0 & -1-\frac{r_g}{r} & 0 & 0 \\ 0 & 0 & -r^2\left(1+\frac{r_g}{r}\right) & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta\left(1+\frac{r_g}{r}\right) \end{pmatrix}$$

Ур-е Г-Я для безмассовой частицы при $\theta = \pi/2$

$$\left(1-\frac{r_g}{r}\right)^{-1}\left(\frac{\partial S}{\partial t}\right)^2 - \left(1+\frac{r_g}{r}\right)^{-1}\left(\frac{\partial S}{\partial r}\right)^2 - \frac{1}{r^2}\left(1+\frac{r_g}{r}\right)^{-1}\left(\frac{\partial S}{\partial \varphi}\right)^2 = 0$$

$$S = S_t + S_r + S_\varphi \Rightarrow \begin{matrix} S_\varphi = \alpha_\varphi \varphi \\ S_t = h\, t \end{matrix} \Rightarrow S_r = \pm\int \underbrace{\sqrt{h^2\frac{\left(1+\frac{r_g}{r}\right)}{\left(1-\frac{r_g}{r}\right)} - \frac{\alpha_\varphi^2}{r^2}}}_{\text{"}A(r)} \, dr \Rightarrow$$

$$\Rightarrow \beta_\varphi = \frac{\partial S}{\partial \alpha_\varphi} = \varphi \pm \int \frac{-\alpha_\varphi dr}{r^2 A(r)} \Rightarrow \varphi = \varphi_0 \pm \int \frac{\alpha_\varphi\, dr}{r^2 A(r)} \Rightarrow$$

$$\Rightarrow \Phi = 2\int_{r_{min}}^{\infty} \frac{dr}{\sqrt{\frac{\left(1+\frac{r_g}{r}\right)}{\left(1-\frac{r_g}{r}\right)\rho^2} - \frac{1}{r^2}}} \qquad \frac{h^2}{\alpha_\varphi^2} = \frac{1}{\rho^2}$$

r_{min} из условия на ближ. т.: $\frac{dr}{d\varphi} = 0 \Leftrightarrow A(r) = 0 \Leftrightarrow$

$$\Leftrightarrow 1 - \left(\frac{r_g}{r_{min}}\right)^2 = \frac{\rho^2}{r_{min}^2}\left(1-\frac{r_g}{r_{min}}\right)^2 \Rightarrow r_{min} = \sqrt{\rho^2 - r_g^2} - r_g \approx \rho - r_g$$

$$\Phi = 2\int_{r_{min}}^{\infty} \frac{dr}{r^2\sqrt{\left(1+\frac{2r_g}{r}\right)\frac{1}{\rho^2} - \frac{1}{r^2}}} = 2\int_0^{\frac{1}{r_{min}}} \frac{d\xi}{\sqrt{(1+2r_g\xi)\rho^{-2} - \xi^2}} =$$

$$= 2\int_0^{1/r_{min}} \frac{d\xi}{\sqrt{\rho^{-2} + r_g^2\rho^{-4} - r_g^2\rho^{-4} + 2r_g\rho^{-2}\xi - \xi^2}} = 2\int_0^{1/r_{min}} \frac{d\left(\xi - r_g\rho^{-2}\right)}{\sqrt{\rho^{-2}\left(1+r_g^2\rho^{-2}\right) - \left(\xi - r_g\rho^{-2}\right)^2}} =$$

$$= 2\arcsin \frac{\xi - r_g \rho^{-2}}{\rho^{-1}\sqrt{1 + r_g^2 \rho^{-2}}}\Bigg|_0^{\frac{1}{\rho - r_g}} \simeq 2\arcsin\left(\xi\rho - r_g \rho^{-1}\right)\Bigg|_0^{1/(\rho - r_g)} =$$

$$= 2\left(\arcsin\left(\frac{1}{1-\frac{r_g}{\rho}} - \frac{r_g}{\rho}\right) - \arcsin\left(-\frac{r_g}{\rho}\right)\right) \simeq 2\left(\frac{\pi}{2} + \frac{r_g}{\rho}\right) = \pi + \frac{2r_g}{\rho}$$

$\Rightarrow$ Отклонение луча от прямой: $\underbrace{\Theta = \Phi - \pi = \frac{2r_g}{\rho} = \frac{4GM}{\rho}}$.