№8.4

$$16\pi G\, \chi^{\mu\nu\varkappa\lambda} = |g|\left(g^{\mu\nu} g^{\varkappa\lambda} - g^{\mu\varkappa} g^{\nu\lambda}\right)$$

I.

1).
$$\begin{cases} g^{\alpha\beta} = \eta^{\alpha\beta} - h^{(2)\alpha\beta} \\ |g| = 1 + \mathrm{tr}\, h^{(2)} - \frac{1}{2}\mathrm{tr}\, h^{(1)2} \end{cases}$$

$$16\pi G\, \chi^{\alpha\beta\gamma\delta} = \left(1 + \mathrm{tr}\, h^{(2)} - \frac{1}{2}\mathrm{tr}\, h^{(1)2}\right)\left(g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\gamma} g^{\beta\delta}\right) =$$

$$= \left(1 + \mathrm{tr}\, h^{(1)} - \frac{1}{2}\mathrm{tr}\, h^{(2)}\right)\left[\left(\eta^{\alpha\beta} - h^{(2)\alpha\beta}\right)\left(\eta^{\gamma\delta} - h^{(2)\gamma\delta}\right) - \left(\eta^{\alpha\gamma} - h^{(2)\alpha\gamma}\right)\left(\eta^{\beta\delta} - h^{(2)\beta\delta}\right)\right] = \left(1 + \mathrm{tr}\, h^{(2)} - \frac{1}{2}\mathrm{tr}\, h^{(1)2}\right)\Big[\eta^{\alpha\beta}\eta^{\gamma\delta} +$$

$$+ \cancel{h^{(2)\alpha\beta} h^{(2)\gamma\delta}} - \eta^{\alpha\beta} h^{(2)\gamma\delta} - h^{(2)\alpha\beta}\eta^{\gamma\delta} - \eta^{\alpha\gamma}\eta^{\beta\delta} -$$

$$- \cancel{h^{(2)\alpha\gamma} h^{(2)\beta\delta}} + \eta^{\alpha\gamma} h^{(2)\beta\delta} + h^{(2)\alpha\gamma}\eta^{\beta\delta}\Big] = \left(1 + \mathrm{tr}\, h^{(2)} - \frac{1}{2}\mathrm{tr}\, h^{(1)2}\right) *$$

$$* \left[\eta^{\alpha\beta}\eta^{\gamma\delta} - \eta^{\alpha\gamma}\eta^{\beta\delta}\right] - \eta^{\alpha\beta} h^{(2)\gamma\delta} - \eta^{\alpha\beta} h^{(2)\gamma\delta} + h^{(2)\alpha\gamma}\eta^{\beta\delta} +$$

$$+ h^{(2)\beta\delta}\eta^{\alpha\gamma}.$$

2).
$$\begin{cases} g^{ab} = -h^{(2)ab} \\ |g| = 1 + \mathrm{tr}\, h^{(2)} - \frac{1}{2}\mathrm{tr}\, h^{(1)2} \end{cases}$$

$$16\pi G\, \chi^{abcd} = |g|\left(g^{ab} g^{cd} - g^{ad} g^{bc}\right) = \left(1 + \mathrm{tr}\, h^{(2)} - \frac{1}{2}\mathrm{tr}\, h^{(1)2}\right) *$$

$$* \left[-h^{(2)ab}\left(\eta^{cd} - h^{(2)cd}\right) - \left(-h^{(2)db}\right)\left(\eta^{ac} - h^{(2)ac}\right)\right] =$$

$$= -h^{(2)\alpha\bar{b}}\eta^{\gamma\delta} + h^{(2)\bar{\delta}\bar{b}}\eta^{\alpha\gamma}$$

3). $$\begin{cases} g^{ab} = \eta^{ab} - h^{(1)\bar{ab}} + \left(h^{(1)2}\right)^{\bar{ab}} - h^{(2)\bar{ab}} \\ |g| = 1 + \mathrm{tr}\, h^{(2)} - \frac{1}{2}\mathrm{tr}\, h^{(1)2} \\ g^{\alpha b} = -h^{(2)\bar{\alpha b}} \end{cases}$$

$$16\pi G\, \chi^{ab\gamma\delta} = |g|\left(g^{ab}g^{\gamma\delta} - g^{a\gamma}g^{b\delta}\right) = \left(1 + \mathrm{tr}\, h^{(2)} - \frac{1}{2}\mathrm{tr}\, h^{(1)2}\right) \cdot$$

$$\cdot\left[\left(\eta^{ab} - h^{(1)\bar{ab}} + \left(h^{(1)2}\right)^{\bar{ab}} - h^{(2)\bar{ab}}\right)\left(\eta^{\gamma\delta} - h^{(2)\bar{\gamma}\bar{\delta}}\right) - h^{(2)\bar{\gamma}\bar{a}}\, h^{(2)\bar{\delta}\bar{b}}\right] =$$

$$= \left(1 + \mathrm{tr}\, h^{(2)} - \frac{1}{2}\mathrm{tr}\, h^{(1)2}\right)\eta^{ab}\eta^{\gamma\delta} + \eta^{\gamma\delta}\left[\left(h^{(1)2}\right)^{\bar{ab}} - h^{(1)\bar{ab}} - h^{(2)\bar{ab}}\right] - h^{(2)\bar{\gamma}\bar{\delta}}\eta^{ab}.$$

II. Продифф-ем.

2). Было на лекции

1). $$16\pi G\, \chi^{\alpha\beta\gamma\delta}{}_{,\gamma\delta} = \eta^{\alpha\beta}\Box\,\mathrm{tr}\, h^{(2)} - \frac{\eta^{\alpha\beta}}{2}\Box\,\mathrm{tr}\, h^{(1)2} -$$

$$\eta^{\alpha\gamma}\eta^{\beta\delta}\left(\mathrm{tr}\, h^{(2)} - \frac{\mathrm{tr}\, h^{(1)2}}{2}\right)_{,\gamma\delta} - \Box h^{(2)\bar{\alpha\beta}} - \eta^{\alpha\beta}\left(h^{(2)\bar{\gamma}\bar{\delta}}\right)_{,\gamma\delta} +$$

$$+ \eta^{\beta\delta}\left(h^{(2)\bar{\alpha}\bar{\gamma}}\right)_{,\gamma\delta} + \eta^{\alpha\gamma}\left(h^{(2)\bar{\beta}\bar{\delta}}\right)_{,\gamma\delta} = \left\{ \Box\,\mathrm{tr}\, h^{(2)} = \frac{3}{4}\Box\,\mathrm{tr}\, h^{(1)2}, \right.$$

$$\left. \Box h^{(2)\bar{\alpha\beta}} = \frac{1}{2}\left(\mathrm{tr}\left(h^{(1)2}\right)\right)^{\bar{\alpha\beta}} - \mathrm{tr}\, h^{(1),\bar{\alpha}}\, h^{(1),\bar{\beta}} \right\} = \frac{\eta^{\alpha\beta}}{4}\Box\,\mathrm{tr}\, h^{(1)2} -$$

$$- \left(\mathrm{tr}\, h^{(2)}\right)^{,\bar{\alpha\beta}} + \frac{\left(\mathrm{tr}\, h^{(1)2}\right)^{,\bar{\alpha\beta}}}{2} = \frac{1}{2}\left(\mathrm{tr}\left(h^{(1)2}\right)\right)^{,\bar{\alpha\beta}} + \frac{1}{2}\mathrm{tr}\, h^{(1),\bar{\alpha}}\, h^{(1),\bar{\beta}} -$$

$$- \eta^{\alpha\beta}\left(h^{(2)\bar{\gamma}\bar{\delta}}{}_{,\gamma\delta} + \left(h^{(2)\bar{\alpha}\bar{\gamma}}{}_{,\gamma}\right)^{\bar{\beta}} + \left(h^{(2)\bar{\beta}\bar{\delta}}{}_{,\delta}\right)^{\bar{\alpha}}\right) =$$

$$= \left\{ \text{калибровка } \left| h^{(2)\mu\nu} - \frac{\eta^{\mu\nu}}{2} tr\, h^{(2)} \right|_{,\nu} = 0 \right\} = \frac{\eta^{\alpha\beta}}{4} \Box\, tr\, h^{(1)2} -$$

$$- \cancel{\left| tr\, h^{(2)}, {}^{\bar{\alpha}\bar{\beta}} \right.} + \frac{1}{2} tr\, h^{(1),\bar{\alpha}}\, h^{(1),\bar{\beta}} - \eta^{\alpha\beta}\left(h^{(2)\bar{\gamma}\bar{\delta}}\right)_{,\gamma\delta} +$$

$$+ \cancel{\frac{tr\, h^{(2),\bar{\alpha}\bar{\beta}}}{2}} + \cancel{\frac{tr\, h^{(2),\bar{\beta}\bar{\alpha}}}{2}} = \frac{\eta^{\alpha\beta}}{4} \Box\, tr\, h^{(1)2} - \eta^{\alpha\beta}\frac{3}{4}\frac{\Box\, tr\, h^{(1)2}}{2} +$$

$$+ \frac{1}{2} tr\, h^{(1),\bar{\alpha}}\, h^{(1),\bar{\beta}} = -\frac{1}{8}\eta^{\alpha\beta} \Box\, tr\, h^{(1)2} + \frac{1}{2} tr\, h^{(1),\bar{\alpha}}\, tr\, h^{(1),\bar{\beta}} =$$

$$= 16\pi G\, \chi^{\alpha\beta\gamma\delta}{}_{,\gamma\delta}$$

3). $16\pi G\, \chi^{\alpha\beta\gamma\delta}{}_{,\gamma\delta} = \eta^{\alpha\beta}\left(\Box\, tr\, h^{(2)} - \frac{1}{2}\Box\, tr\, h^{(1)2}\right) -$ (здесь $\frac{3}{4}\Box tr\, h^{(1)2}$)

$$- \Box\, h^{(1)\bar{\alpha}\bar{\beta}} - \Box h^{(2)\alpha\beta} + \Box\, h^{(1)2\,\bar{\alpha}\bar{\beta}} - \eta^{\alpha\beta}\, h^{(2)\gamma\delta}{}_{,\gamma\delta} =$$

$$= \frac{\eta^{\alpha\beta}}{4} \Box\, tr\, h^{(1)2} - \cancel{\Box h^{(1)\bar{\alpha}\bar{\beta}}}\ (\to 0 \text{ на ур-х движения}) - \frac{1}{2}\Box\left(h^{(1)2}\right)^{\bar{\alpha}\bar{\beta}} + \Box\, h^{(1)2\,\bar{\alpha}\bar{\beta}} - \eta^{\alpha\beta}\frac{3\Box\, tr\, h^{(1)2}}{2\cdot 4} =$$

$$= -\frac{1}{8}\eta^{\alpha\beta} \Box\, tr\, h^{(1)2} + \frac{1}{2}\Box\, h^{(1)2\,\bar{\alpha}\bar{\beta}} = 16\pi G\, \chi^{\alpha\beta\gamma\delta}{}_{,\gamma\delta}$$

III. $h^{(1)}_{\alpha\beta} \longmapsto h^{(1)}_{\alpha\beta} + A_{\alpha\beta}$

2). остаётся

Производные по $h^{(1)}_{\alpha\beta}$ также не меняются, посмотрим на слагаемые $\left(h^{(1)2}\right)^{\bar{\alpha}\bar{\beta}}$ и $tr\, h^{(1)2}$:

$$\cdot \left(h^{(1)2}\right)^{\bar{\alpha}\bar{\beta}} \longmapsto \left(h^{(1)2}\right)^{\bar{\alpha}\bar{\beta}} + 2A^{\bar{\alpha}}_{\ \gamma}\left(h^{(1)}\right)^{\gamma\bar{\beta}} + \left(A^2\right)^{\bar{\alpha}\bar{\beta}}$$

$$\Box (h^{(1)2})^{\overline{\alpha\beta}} \mapsto \Box (h^{(1)2})^{\overline{\alpha\beta}} + 2 A^{\overline{\alpha}}{}_{\overline{\jmath}} \Box (h^{(1)})^{\overline{\jmath\beta}} = \Box (h^{(1)2})^{\overline{\alpha\beta}}$$

$\vdots$

0

- $\mathrm{tr}(h^{(1)2}) \longrightarrow \mathrm{tr}(h^{(1)2}) + 2 A_{\mu\nu} (h^{(1)})^{\mu\nu} + \mathrm{tr}(A^2)$,

$$\Box \left(\mathrm{tr}(h^{(1)2}) + 2 A_{\mu\nu} h^{(1)\mu\nu} + \mathrm{tr}(A^2) \right) = \Box \, \mathrm{tr}(h^{(1)2})$$