№ 1

Часть 2 // ибо с ней были проблемы //.

На любых решениях $\ddot{x}^\lambda + \Gamma^\lambda_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = \frac{e}{m} F^\lambda{}_\varkappa \dot{x}^\varkappa$ вып-ся $\frac{d}{d\tau}\left(g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu\right) = 0$. $\Rightarrow$

$$\frac{d}{\tau}\left(g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu\right) = 2 g_{\lambda\rho}\ddot{x}^\lambda \dot{x}^\rho + \dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu \partial_\lambda g_{\mu\nu}$$

$$2 g_{\lambda\rho}\left(\frac{e}{m} F^\lambda{}_\varkappa \dot{x}^\varkappa - \Gamma^\lambda_{\mu\nu}\dot{x}^\mu \dot{x}^\nu\right)\dot{x}^\rho + \dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu \partial_\lambda g_{\mu\nu} = \left\{ g_{\lambda\rho} F^\lambda{}_\varkappa \dot{x}^\varkappa = F_{\rho\varkappa}\dot{x}^\varkappa \right\} =$$

$$= \dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu \partial_\lambda g_{\mu\nu} - 2 g_{\lambda\rho}\Gamma^\lambda_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \dot{x}^\rho = \dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu \partial_\lambda g_{\mu\nu} - 2 g_{\lambda\rho}\frac{g^{\lambda\theta}}{2}\left(\partial_\mu g_{\theta\nu} +\right.$$

$$\left. + \partial_\nu g_{\theta\mu} - \partial_\theta g_{\mu\nu}\right)\dot{x}^\rho \dot{x}^\mu \dot{x}^\nu = \dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu \partial_\lambda g_{\mu\nu} - \delta^\theta_\rho\left(\partial_\mu g_{\theta\nu} + \partial_\nu g_{\theta\mu} - \partial_\theta g_{\nu\mu}\right) *$$

$$* \dot{x}^\mu \dot{x}^\nu \dot{x}^\rho = \dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu \partial_\lambda g_{\mu\nu} - \partial_\mu g_{\lambda\nu}\dot{x}^\mu \dot{x}^\nu \dot{x}^\lambda + \partial_\nu g_{\lambda\mu}\dot{x}^\lambda \dot{x}^\nu \dot{x}^\mu - \partial_\lambda g_{\mu\nu}\dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu = 0.$$

№ 3

$$\begin{cases} \dot{p}_\mu = -\alpha\left(\partial_\mu g^{\lambda\nu}\left(p_\lambda + e A_\lambda\right) + 2e g^{\lambda\nu}\partial_\mu A_\lambda\right)\left(p_\nu + e A_\nu\right). \\ \dot{x}^\mu = 2\alpha g^{\mu\nu}\left(p_\nu + e A_\nu\right) \\ \alpha = \mathrm{const} \end{cases}$$

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↓
$$\Rightarrow \ddot{x}^\lambda + \Gamma^\lambda_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = \frac{e}{m} F^\lambda{}_\varkappa \dot{x}^\varkappa.$$

$$\ddot{x}^\mu = \left[2\alpha g^{\mu\nu}\left(p_\nu + e A_\nu\right)\right]^{\cdot} = 2\alpha\, \dot{x}^\lambda \partial_\lambda g^{\mu\nu}\left(p_\nu + e A_\nu\right) + 2\alpha g^{\mu\nu}\left(\dot{p}_\nu + e\dot{x}^\lambda \partial_\lambda A_\nu\right)$$

Подставим в $\ddot{x}^\mu$ $\dot{p}_\mu$ и $\dot{x}^\mu$:

$$\ddot{x}^\mu = (2\alpha)^2 \partial_\lambda g^{\mu\nu}(p_\nu + eA_\nu)\, g^{\lambda\varphi}(p_\varphi + eA_\varphi) - 2\alpha^2 g^{\mu\nu}\Big(\partial_\nu g^{\lambda\varkappa}(p_\lambda + eA_\lambda) +$$

$$+ 2e g^{\lambda\varkappa}\partial_\nu A_\lambda\Big)(p_\varkappa + eA_\varkappa) + (2\alpha)^2 g^{\mu\nu} g^{\lambda\varkappa}(p_\varkappa + eA_\varkappa)\partial_\lambda A_\nu =$$

$$= (2\alpha)^2 \partial_\lambda g^{\mu\nu} g^{\lambda\varphi}(p_\nu + eA_\nu)(p_\varphi + eA_\varphi) - 2\alpha^2 g^{\mu\nu}\partial_\nu g^{\lambda\varkappa}(p_\lambda + eA_\lambda)(p_\varkappa + eA_\varkappa) -$$

$$- 4\alpha^2 e g^{\mu\nu} g^{\lambda\varkappa}\partial_\nu A_\lambda (p_\varkappa + eA_\varkappa) + 4\alpha^2 e g^{\mu\nu} g^{\lambda\varkappa}\partial_\lambda A_\nu (p_\varkappa + eA_\varkappa) =$$

$$= (p_\varkappa + eA_\varkappa)(p_\lambda + eA_\lambda)\, 2\alpha^2\Big[2\partial_\nu g^{\mu\lambda} g^{\nu\varkappa} - g^{\mu\nu}\partial_\nu g^{\lambda\varkappa}\Big] + e4\alpha^2 (p_\varkappa + eA_\varkappa) g^{\mu\nu} g^{\lambda\varkappa} *$$

$$*(\partial_\lambda A_\nu - \partial_\nu A_\lambda) = \frac{\dot{x}^\theta \dot{x}^\xi}{2}\Big[2 g_{\theta\varkappa} g_{\xi\lambda}\partial_\nu g^{\mu\lambda} g^{\nu\varkappa} - g_{\theta\varkappa} g_{\xi\lambda} g^{\mu\nu}\partial_\nu g^{\lambda\varkappa}\Big] +$$

$$+ 2\alpha e g_{\rho\varkappa}\dot{x}^\rho g^{\lambda\varkappa} g^{\mu\nu} F_{\lambda\nu} = \frac{\dot{x}^\theta \dot{x}^\xi}{2}\Big[2\partial_\nu g^{\mu\lambda} g_{\xi\lambda}\delta^\nu_\theta - g^{\mu\nu} g_{\theta\varkappa} g_{\xi\lambda}\partial_\nu g^{\lambda\varkappa}\Big] +$$

$$+ 2\alpha e \dot{x}^\rho F_\rho{}^\mu$$

посмотрим на слагаемое

$$\dot{x}^\theta \dot{x}^\xi\big(2\partial_\nu g^{\mu\lambda} g_{\xi\lambda}\delta^\nu_\theta - g^{\mu\nu} g_{\theta\varkappa} g_{\xi\lambda}\partial_\nu g^{\lambda\varkappa}\big) =$$

$$= \dot{x}^\theta \dot{x}^\xi\big(2\partial_\theta g^{\mu\nu} g_{\xi\lambda} - g^{\mu\nu} g_{\theta\varkappa} g_{\xi\lambda}\partial_\nu g^{\lambda\varkappa}\big) = // \partial_\nu(g_{\xi\lambda} g^{\lambda\varkappa}) = g_{\xi\lambda}\partial g^{\lambda\varkappa} +$$

$$\partial_\nu \delta^\varkappa_\xi = 0$$

$$+ g^{\lambda\varkappa}\partial g_{\xi\lambda} // = \dot{x}^\theta \dot{x}^\xi\big(2\partial_\theta g^{\mu\nu} g_{\xi\lambda} + g^{\mu\nu} g_{\theta\varkappa} g^{\lambda\varkappa}\partial_\nu g_{\xi\lambda}\big) =$$

$$= \dot{x}^\theta \dot{x}^\xi\big(2\partial_\theta g^{\mu\lambda} g_{\xi\lambda} + g^{\mu\nu}\partial_\nu g_{\xi\theta}\big) = // \partial_\theta g^{\mu\lambda} g_{\xi\lambda} = -g^{\mu\lambda}\partial_\theta g_{\xi\lambda} // =$$

$$= \dot{x}^\theta \dot{x}^\xi g^{\mu\lambda}\big(-2\partial_\theta g_{\xi\lambda} + \partial_\lambda g_{\xi\theta}\big) = -2\dot{x}^\theta \dot{x}^\xi \frac{g^{\mu\lambda}}{2}\big(\partial_\theta g_{\xi\lambda} + \partial_\xi g_{\theta\lambda} -$$

$$- \partial_\lambda g_{\xi\theta}\big) = -\dot{x}^\theta \dot{x}^\xi \Gamma^\mu_{\theta\xi} \Rightarrow$$

$\Rightarrow$ Собирая все вместе $\Rightarrow$ $\ddot{x}^\mu + \dot{x}^\nu \dot{x}^\xi \Gamma^\mu_{\nu\xi} = 2\alpha e \dot{x}^\rho F_\rho{}^\mu \Rightarrow$

$\Rightarrow$ при $2\alpha = -\frac{1}{m}$ получаем, то что надо.

№ 4

Выберем произ... $p(\ldots)$ ...

... произв. т. $p=(\tau_0,\sigma_0)$ на пов-ти φ. И выберем новый базис $x'_0=\tau$, $x'_1=\sigma$, а остальные координаты остаются те же. В этом базисе $\dot\varphi(p)=\partial_0$, $\varphi'(p)=\partial_1$ $\Rightarrow$

$$\Rightarrow \begin{cases} \nabla_{\dot\varphi}\varphi' = \Gamma^{\lambda}_{01}\partial_\lambda \\ \nabla_{\varphi'}\dot\varphi = \Gamma^{\lambda}_{10}\partial_\lambda \end{cases}$$, т.к. кручение 0, то $\nabla_{\dot\varphi}\varphi' = \nabla_{\varphi'}\dot\varphi$.

Это верно для $\forall$ т. p пов-ти $\varphi(\sigma,\tau)$.