№1

Найдем угловую скорость по определению:

$$\omega = \frac{d\varphi}{dt} = \left\{ \begin{array}{l} \varphi = \varphi_0 \pm J \int_{r_0}^{r} \frac{dr}{(1-\frac{r_g}{r})\sqrt{F(E,J,r)}}, \; d\varphi = \pm \frac{J\,dr}{(1-\frac{r_g}{r})\sqrt{F(E,J,r)}} \\ t = t_0 \pm E \int_{r_0}^{r} \frac{dr}{r^2\sqrt{F(E,J,r)}}, \; dt = \pm \frac{E\,dr}{r^2\sqrt{F(E,J,r)}} \end{array} \right\} =$$

$$= \pm \frac{J\,(1-\frac{r_g}{r})}{E\cdot r^2}$$

№2

$$\theta = \pi/2 \Rightarrow ds^2 = \left(1-\frac{r_g}{r}\right)(dx^+)^2 - 2\,dx^+dr - r^2 d\varphi^2 \Rightarrow$$

$$\Rightarrow g^{\mu\nu} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & -1+\frac{r_g}{r} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2\sin^2\theta} \end{pmatrix}$$

ур-е Г-Я:

$$g^{\mu\nu}(\partial_\mu S)(\partial_\nu S) = m^2 \Rightarrow -\left(\frac{\partial S}{\partial r}\right)^2\left(1-\frac{r_g}{r}\right) - 2\left(\frac{\partial S}{\partial x^+}\right)\left(\frac{\partial S}{\partial r}\right) - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2$$

$$\left.\begin{array}{l} \frac{\partial S}{\partial x^+} = -h \Rightarrow S_{x^+} = -h x^+ \\ \frac{\partial S}{\partial \varphi} = \alpha_\varphi \Rightarrow S_\varphi = \alpha_\varphi \cdot \varphi \end{array}\right\} \Rightarrow \left(\frac{\partial S}{\partial r}\right)^2\left(1-\frac{r_g}{r}\right) - 2h\left(\frac{\partial S}{\partial r}\right) + \frac{\alpha_\varphi^2}{r^2} + m^2 = 0$$

$$\Rightarrow \text{решая кв. ур-е: } \partial_r S = \frac{h \pm \sqrt{\overbrace{h^2 - \left(1-\frac{r_g}{r}\right)\left(m^2 + \frac{\alpha_\varphi^2}{r^2}\right)}^{=F(h,\alpha_\varphi,r)}}}{\left(1-\frac{r_g}{r}\right)} \Rightarrow$$

$$\Rightarrow S_r = \int_{r_0}^{r} \left(1-\frac{r_g}{r}\right)^{-1}\left(h \pm \sqrt{F(h,\alpha_\varphi,r)}\right) dr$$

$$\bullet\ \frac{\partial S}{\partial h} = x_0^+ = x^+ + \int_{r_0}^{r}\left(1-\frac{r_g}{r}\right)^{-1}\left(1 \pm \frac{h}{\sqrt{F(h,\alpha_\varphi,r)}}\right)dr \Rightarrow x^+ = x_0^+ - \int_{r_0}^{r} \frac{dr}{\left(1-\frac{r_g}{r}\right)}\left(1 \pm \frac{h}{\sqrt{F(h,\alpha_\varphi,r)}}\right) =$$

$$= x_0^+ + \left(r + r_g \ln\left|\frac{r}{r_g} - 1\right|\right)\Big|_{r_0}^{r} \pm \int_{r_0}^{r} \frac{dr}{1-\frac{r_g}{r}} \frac{h}{\sqrt{F(h,\alpha_\varphi,r)}}$$

$$\bullet\ \frac{\partial S}{\partial \alpha_\varphi} = \varphi_0 = \varphi \pm \int_{r_0}^{r} \frac{1}{\left(1-\frac{r_g}{r}\right)\cdot r^2} \frac{\left(1-\frac{r_g}{r}\right)\alpha_\varphi\,dr}{\sqrt{F(h,\alpha_\varphi,r)}} \Rightarrow \varphi = \varphi_0 \mp \int_{r_0}^{r} \frac{\alpha_\varphi\,dr}{r^2\sqrt{F(h,\alpha_\varphi,r)}}$$

Учитывая, что $x^+ - r = t$, то видим, что если $h = E$ и $\alpha_\varphi = J$, то решения совпадают с решениями Г-Я в метрике Шварцшильда.

А в уравнении на x^+ есть нерег. часть $\left(r + r_g \ln\left|\frac{r}{r_g} - 1\right|\right)\Big|_{r_0}^{r}$

Посмотрим на интеграл в x^+:

$$\int_{r_0}^{r} \frac{dr \cdot E'}{\left(1 - \frac{r_g}{r}\right)\sqrt{E'^2 - \left(1 - \frac{r_g}{r}\right)\left(m^2 + \frac{S^2}{r^2}\right)}} = \int_{r_0}^{r} \frac{dr}{\left(1 - \frac{r_g}{r}\right)}\left(1 + \left(1 - \frac{r_g}{r}\right)\frac{m^2 + \frac{S^2}{r_g^2}}{2E'^2} + O(r - r_g)\right) =$$

$$= \left(r + r_g \ln\left|\frac{r}{r_g} - 1\right|\right)\Big|_{r_0}^{r} + \left(\frac{m^2 + \frac{S^2}{r_g^2}}{2E'^2} + O(1)\right)(r - r_g) \Rightarrow$$

$\Rightarrow$ при выборе