№ 5

1). $S[x] = -\int_A^B m(x)\,ds = -\int_A^B d\tau\, m(x)\sqrt{u_\mu u^\mu} \Rightarrow L = -m(x)\sqrt{u_\mu u^\mu}$

Ур-я движения:

$$\frac{\partial L}{\partial x^\mu} = -\partial_\mu m(x)\sqrt{u^\mu u_\mu}; \quad \frac{\partial L}{\partial u^\mu} = -\frac{m(x)u_\mu}{\sqrt{u_\mu u^\mu}} \Rightarrow \frac{d}{d\tau}\frac{\partial L}{\partial u^\mu} = -\frac{m(x)}{\sqrt{u_\varkappa u^\varkappa}}\frac{du_\mu}{d\tau} - \partial_\nu m(x)\frac{u^\nu u_\mu}{\sqrt{u_\varkappa u^\varkappa}}$$

$\Rightarrow$ $m(x)\frac{du_\mu}{ds} = \partial_\mu m - u_\mu u^\nu \partial_\nu m(x)$

2). С другой стороны — $S[x] = -\int_A^B dt\, m(r,t)\sqrt{1-v^2} \Rightarrow L = -m(r,t)\sqrt{1-v^2} \Rightarrow \vec{p} = \frac{\partial L}{\partial v} = \frac{m(r,t)v}{\sqrt{1-v^2}} \Rightarrow$

$\Rightarrow \mathcal{H} = \vec{v}\vec{p} - L = \frac{m(r,t)}{\sqrt{1-v^2}} = \sqrt{m^2(r,t) + p^2}$

3). $m(x) = m_0 + U(x)$, $U(x) \ll m_0$

Ур-е движение станет $\Rightarrow m_0 \frac{du_\mu}{ds} = \partial_\mu U(x) - u_\mu u^\nu \partial_\nu U(x)$.

в нерел пределе $(v \ll 1)$

- $\mu = 0$ $\frac{d}{dt}\left(\frac{m_0 v^2}{2}\right) = \partial_0 U(x) - \frac{(\partial_0 U(x) + v^j \partial_j U(x))}{1-v^2} \Rightarrow \frac{dT}{dt} = -v^\alpha \partial_\alpha U(x)$ ← закон сохранения энергии.
- $\mu = i$ $-\frac{m_0}{1-v^2}\frac{dv_i}{dt} = \partial_i U(x) + \frac{v_i}{1-v^2}\left(\partial_0 U(x) + v^j \partial_j U(x)\right) \Rightarrow$ Если $v_i(\partial_0 U(x) + v^j \partial_j U(x)) \ll \partial_i U \Rightarrow$

$\Rightarrow m_0 \frac{dv_i}{dt} = -\partial_i U(x)$ ← 2-й закон Ньютона в потенциале $U(x)$.

Условие $v_i\left(\partial_0 U(x) + v^j \partial_j U(x)\right) \ll \partial_i U$