№ 5

1). $\delta_\xi (t \otimes s) = \delta_\xi t \otimes s + t \otimes \delta_\xi s$. 2) $[\delta_\xi, \delta_\eta] t = \delta_{[\xi,\eta]} t$.

1). По опр-ю производной Ли:

$$\delta_\xi t = t'(x'^{\cdot}) - t(x'), \quad \delta_\xi s = s'(x'^{\cdot}) - s(x')$$

$$\delta_\xi (t \otimes s) = t'(x'^{\cdot}) \otimes s'(x') - t(x'^{\cdot}) \otimes s'(x'^{\cdot}) + t'(x'^{\cdot}) \otimes s'(x'^{\cdot}) - t(x'^{\cdot}) \otimes s'(x'^{\cdot}) = \left(t'(x'^{\cdot}) - t(x'^{\cdot})\right) \otimes s'(x'^{\cdot}) +$$

$$+ t(x'^{\cdot}) \otimes \left(s'(x'^{\cdot}) - s(x'^{\cdot})\right) = \left(\delta_\xi t \otimes s + t \otimes \delta_\xi s\right)(x'^{\cdot}) \Rightarrow \delta_\xi (t \otimes s) = \delta_\xi t \otimes s + t \otimes \delta_\xi s.$$

2). Для доказательства докажем для составных частей любого тензора: ковекторов и векторов (+ правило Лейбница для правой части).

I. ковектор (1-форма).

$\alpha = \alpha_\mu dx^\mu$, известно, что $\delta_\xi \alpha_\mu = \partial_\lambda a_\mu \xi^\lambda + a_\lambda \partial_\mu \xi^\lambda$

$$[\delta_\xi, \delta_\eta] \alpha_\mu = \delta_\xi (\partial_\lambda a_\mu \eta^\lambda + \alpha_\lambda \partial_\mu \eta^\lambda) - \delta_\eta (\partial_\lambda a_\mu \xi^\lambda + \alpha_\lambda \partial_\mu \xi^\lambda) = \partial_\varkappa (\partial_\lambda a_\mu \eta^\lambda + \alpha_\lambda \partial_\mu \eta^\lambda) \xi^\varkappa +$$

$$+ (\partial_\lambda a_\varkappa \eta^\lambda + \alpha_\lambda \partial_\varkappa \eta^\lambda) \partial_\mu \xi^\varkappa - \partial_\varkappa (\partial_\lambda a_\mu \xi^\lambda + \alpha_\lambda \partial_\mu \xi^\lambda) \eta^\varkappa - (\partial_\lambda \alpha_\varkappa \xi^\lambda + \alpha_\lambda \partial_\varkappa \xi^\lambda) \partial_\mu \eta^\varkappa =$$

$$= \underline{\partial_\varkappa \partial_\lambda a_\mu \eta^\lambda \xi^\varkappa} + \partial_\lambda a_\mu \partial_\varkappa \eta^\lambda \xi^\varkappa + \underline{\partial_\varkappa \alpha_\lambda \partial_\mu \eta^\lambda \xi^\varkappa} + \alpha_\lambda \partial_\varkappa \partial_\mu \eta^\lambda \xi^\varkappa + (\underline{\partial_\lambda a_\varkappa \eta^\lambda} + \alpha_\lambda \partial_\varkappa \eta^\lambda) \partial_\mu \xi^\varkappa -$$

$$- \underline{\partial_\varkappa \partial_\lambda a_\mu \xi^\lambda \eta^\varkappa} - \partial_\lambda a_\mu \partial_\varkappa \xi^\lambda \eta^\varkappa - \underline{\partial_\varkappa a_\lambda \partial_\lambda \xi^\lambda \eta^\varkappa} - \alpha_\lambda \partial_\varkappa \partial_\mu (\xi^\lambda) \eta^\varkappa - (\underline{\partial_\lambda \alpha_\varkappa \xi^\lambda} + \alpha_\lambda \partial_\varkappa \xi^\lambda) \partial_\mu \eta^\varkappa =$$

$$= \partial_\lambda a_\mu (\partial_\varkappa \eta^\lambda \xi^\varkappa - \partial_\varkappa \xi^\lambda \eta^\varkappa) + \alpha_\lambda (\partial_\varkappa \partial_\mu (\eta^\lambda) \xi^\varkappa - \partial_\varkappa \partial_\mu (\xi^\lambda) \eta^\varkappa) + a_\lambda \partial_\varkappa \eta^\lambda \partial_\mu \xi^\varkappa -$$

$$- \alpha_\lambda \partial_\varkappa \xi^\lambda \partial_\mu \eta^\varkappa = \delta_{[\xi,\eta]} \alpha_\mu$$

II. вектор

$a = a^\mu \partial_\mu$, известно, что $\delta_\xi a = [\xi, a]$

$$[\delta_\xi, \delta_\eta] a = \delta_\xi [\eta, a] - \delta_\eta [\xi, a] = [\xi, [\eta, a]] - [\eta, [\xi, a]] = -[[\eta, a], \xi] - [[a, \xi], \eta] =$$

$$= // \text{тождество Якоби} // = [[\xi, \eta], a] = \delta_{[\xi,\eta]} a$$

III. правило Лейбница

для левой части доказано в пред пункте задачи.

$$[\delta_\xi, \delta_\eta](t \otimes s) = \delta_\xi (\delta_\eta t \otimes s + t \otimes \delta_\eta s) - \delta_\eta (\delta_\xi t \otimes s + t \otimes \delta_\xi s) =$$

$$[\delta_\xi, \delta_\eta](t \otimes s) = \delta_\xi(\delta_\eta t \otimes s + t \otimes \delta_\eta s) - \delta_\eta(\delta_\xi t \otimes s + t \otimes \delta_\xi s)$$

$$= \delta_\xi \delta_\eta t \otimes s + \cancel{\delta_\eta t \otimes \delta_\xi s} + \cancel{\delta_\xi t \otimes \delta_\eta s} + t \otimes \delta_\xi \delta_\eta s - \delta_\eta \delta_\xi t \otimes s - \cancel{\delta_\xi t \otimes \delta_\eta s} -$$

$$- \cancel{\delta_\eta t \otimes \delta_\xi s} - t \otimes \delta_\eta \delta_\xi s = [\delta_\xi, \delta_\eta] t \otimes s + t \otimes [\delta_\xi, \delta_\eta] s.$$