Упражнение 1

1). $U(x) = \begin{cases} \infty, & x<0 \\ Fx, & x \geq 0 \end{cases}$

$$\psi(x) = \frac{1}{\sqrt{p(x)}} \cos\left(\int_b^x p(y)dy - \frac{\pi}{4}\right), \quad p(x) = \sqrt{2m(E - Fx)}$$

$$\psi(0) = 0 \Rightarrow \int_a^b p(y)dy + \frac{\pi}{4} = \pi n - \frac{\pi}{2} \Rightarrow E_n = \left(\frac{2\pi F}{2\sqrt{2m}}\left(n - \frac{3}{4}\right)\right)^{2/3}$$

2). $U(r) = -\frac{e}{r}, \quad e = m = 0$

$$\psi_{e,m}(r, \theta, \varphi) = R_E(r)\, Y_{e,m}(\theta, \varphi), \quad R(r)\, r = u(r)$$

Ур-е Шрёдингера $-\frac{\hbar^2}{2m} u''(r) - \frac{e^2}{r} u(r) = E\, u(r)$.

при $r \to 0 \Rightarrow \frac{\hbar^2}{2m} u''(r) + \frac{e^2}{r} u(r) = 0$. $\Rightarrow$ решения при $r \to 0$ с $E \neq 0$ ассимптотически эквивалентны решению с $E = 0$ (его мы получили в 5.2):

$$\psi(r, \theta, \varphi) = \frac{J_{2e+1}\left(2\sqrt{\frac{2M\alpha}{\hbar^2} r}\right)}{\sqrt{\frac{2M\alpha}{\hbar^2} r}} Y_{e,m}(\theta, \varphi)$$

Сошьем решения:

$$J_\nu(z) \approx \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{\pi\nu}{2} - \frac{\pi}{2}\right), \quad z \to \infty.$$

$$u(r) \approx r^{1/4} \cos\left(2\sqrt{\frac{2M\alpha}{\hbar^2} r} - \frac{\pi}{4} - \frac{\pi}{2}\right)$$

$$u(r) = \frac{1}{\sqrt{p(r)}} \cos\left(\int_0^r p(r)dr + \varphi\right)$$

$$\int_0^r p(r)\,dr = \sqrt{2m}\int_0^r \sqrt{E+\frac{e^2}{r}}\,dr \approx 2\sqrt{2me^2 r} \Rightarrow \phi = -\frac{3\pi}{4} \Rightarrow$$

$\Rightarrow$ условия квантования $\int_0^b p(r)\,dr = \pi\hbar n$

$$\int_0^b p(r)\,dr = \sqrt{2me^2}\int_0^b \sqrt{\frac{1}{r}-\frac{1}{b}}\,dr = \sqrt{2me^2}\int_0^\infty \frac{dq\, q^2}{(b+q^2)^2}(-2b^2) = -\frac{\pi}{2}\sqrt{\frac{2me^2}{b}}$$

$$\Rightarrow \quad E_n = -\frac{me^4}{2\hbar^2 n^2}$$

Упражнение 2.

a). $$-\frac{\hbar^2}{2M}u''(r) + V(r)u(r) = E\,u(r); \quad V(r) = U(r) + \frac{(l+1)l}{2Mr^2}$$

Одномерное кв-е Б-3.

$$\int_{r_{min}^l}^{r_{max}^l} \sqrt{2M\left(E_{n,l,m} - V(r)\right)}\,dr = \hbar\pi\left(n+\frac{1}{2}\right).$$

$N_{E,l}$ - макс n: $E_{n,l,m} \le E \Rightarrow N(E) = \sum_m \sum_{l=0}^{l_{max}} N_{E,l} =$

$$= \sum_{l=0}^{l_{max}} (2l+1) N_{E,l} \approx \int_0^{l_{max}} dl\,(2l+1)\frac{\sqrt{2M}}{\pi\hbar}\int_{r_{min}^l}^{r_{max}^l} \sqrt{E - U(r) - \frac{l(l+1)}{2Mr^2}}\,dr =$$

$$= -\frac{\sqrt{2M}}{\pi\hbar}\frac{4M}{3}\int_{r_{min}}^{r_{max}} dr\, r^2\left(E - U(r) - \frac{l(l+1)}{2Mr^2}\right)^{3/2}\Bigg|_{l=0}^{l=l_{max}} \approx \text{// пренебрегая // верхним пределом} =$$

$$= \frac{2(2M)^{3/2}}{3\pi\hbar}\int_{r_{min}}^{r_{max}} r^2\left(E-U(r)\right)^{3/2} dr.$$

δ). $N(E) = \int d^3 r \int (d^3 p)\, \Theta(E - E_p) = \int d^3 r \int_0^{p_{max}} dp\, \frac{4\pi p^2}{(2\pi\hbar)^3} =$

$$= \frac{2(2M)^{3/2}}{3\pi\hbar} \int_{r_{min}}^{r_{max}} r^2 (E - U(r))^{3/2} dr.$$