№ 1

$$G^R(t,r) = \frac{\delta(t-r)}{4\pi r}$$

$$\Box G^R(t,r) = \eta^{\mu\nu}\partial_\nu\partial_\mu\left(\frac{\delta(t-r)}{4\pi r}\right) = \frac{\ddot{\delta}(t-r)}{4\pi r} - \Delta\left(\frac{\delta(t-r)}{4\pi r}\right)$$

$$\Delta\left(\frac{\delta(t-r)}{4\pi r}\right) = \frac{\Delta\delta(t-r)}{4\pi r} + \frac{\delta(t-r)}{4\pi}\Delta\frac{1}{r} + 2\,\frac{\nabla\delta(t-r)}{4\pi}\cdot\nabla\left(\frac{1}{r}\right) = \left\{\Delta\frac{1}{r} = -4\pi\delta(\vec{r})\right\} =$$

$$= \left\{\delta^{ij}\frac{\partial}{\partial x^j}\frac{\partial}{\partial x^i}\left(\delta(t-r)\right) = \frac{\partial}{\partial x^j}\left(\delta'(t-r)\frac{\partial}{\partial x^i}(r)\right) = \delta''(t-r)\frac{\partial r}{\partial x^j}\frac{\partial}{\partial x^i}(r) + \delta'(t-r)\frac{\partial^2}{\partial x^i\partial x^j}(r)\right\} =$$

$$= \sum_i\left\{\frac{\delta''(t-r)}{4\pi r}\left(\frac{x^i}{r}\right)^2 + \frac{\delta'(t-r)}{4\pi r}\frac{x^{i\,2}}{r^3} - \frac{\delta'(t-r)}{4\pi r^2} + 2\,\frac{\delta'(t-r)}{4\pi r^2}\left(\frac{x^i}{r}\right)^2 - \delta(t-r)\,\delta(\vec{r})\right\} =$$

$$= \frac{\delta''(t-r)}{4\pi r} - \frac{3\delta'(t-r)}{4\pi r^2} + \frac{3\delta'(t-r)}{4\pi r^2} - \delta(t-r)\,\delta(\vec{r}) = \frac{\delta''(t-r)}{4\pi r} - \delta(t-r)\,\delta(\vec{r}) \Rightarrow$$

$$\Rightarrow \frac{\ddot{\delta}(t-r)}{4\pi r} - \Delta\left(\frac{\delta(t-r)}{4\pi r}\right) = \frac{\delta''(t-r)}{4\pi r} - \frac{\delta''(t-r)}{4\pi r} + \delta(t-r)\,\delta(\vec{r}) = \delta(t-r)\,\delta(\vec{r})$$

сравним $\delta(t-r)\,\delta(\vec{r})$ и $\delta(t)\,\delta(\vec{r})$.

$$\left.\begin{aligned} &1).\ \int dt\,d^3r\ f(t,r)\,\delta(t-r)\,\delta(\vec{r}) = \int d^3r\ f(r,r)\,\delta(\vec{r}) = f(0,0), \\ &2).\ \int dt\,d^3r\ f(t,r)\,\delta(t)\,\delta(\vec{r}) = f(0,0) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \delta(t-r)\,\delta(\vec{r}) = \delta(t)\,\delta(\vec{r}) \quad \text{в } D'\{\text{пр-во распределений}\}$$

ч.т.д.