Задача 2

$$\hat{H}(t) = \begin{pmatrix} \alpha t & \gamma \\ \gamma & -\alpha t \end{pmatrix} = \underbrace{\begin{pmatrix} \alpha t & 0 \\ 0 & -\alpha t \end{pmatrix}}_{H_0} + V, \quad \alpha \gg \gamma^2$$

$$|\psi_{(t)}\rangle = \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix}$$

$$|\psi_1(t\to-\infty)|^2 = 1$$

$$|\psi_2(t\to+\infty)|^2 - ?$$

$$i\hbar\partial_t \psi = H_0 \psi \Rightarrow \psi_{(t)} = \begin{pmatrix} C_1 e^{-i\alpha \frac{t^2}{2}} \\ C_2 e^{i\alpha\frac{t^2}{2}} \end{pmatrix}.$$

т.к. $|\psi_1(t\to-\infty)|^2 = 1 \Rightarrow \psi_i = \begin{pmatrix} e^{-i\alpha\frac{t^2}{2}} \\ 0 \end{pmatrix}$ $\quad \psi_f(t) = \begin{pmatrix} 0 \\ e^{i\alpha t^2/2} \end{pmatrix}. \Rightarrow$

$$\Rightarrow V_{fi} = \psi_f^+ V \psi_i = \gamma e^{-i\alpha t^2} \Rightarrow$$

$$\Rightarrow W_{1\to2} = \left| \int_{\mathbb{R}} \gamma e^{-i\alpha t^2} dt \right|^2 = \frac{\pi\gamma^2}{\alpha}.$$

Задача 3

$$\hat{H}(t) = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2} + e\mathcal{E}(t)\hat{x}, \quad \mathcal{E}(t) = \mathcal{E}_0 e^{-t^2/\tau^2}$$

$$e^2\mathcal{E}_0^2 \ll m\omega^3$$

$$V_{nm} = \langle n|\hat{V}|m\rangle = \frac{e\,\mathcal{E}_0 e^{-t^2/\tau^2}}{\sqrt{2m\omega}} \begin{cases} \sqrt{n}, & m = n-1 \\ \sqrt{n+1}, & m = n+1 \end{cases}$$

С помощью итераций получаем:

$$\left.\begin{aligned} i\partial_t \psi_1^{(1)} &= V_{10}^I \psi_0 \\ &\vdots \\ i\partial_t \psi_n^{(n)} &= V_{n\,n-1}^I \psi_{n-1}^{(n-1)} \end{aligned}\right\} \Rightarrow W_{0\to1} = \frac{\pi^2 e^2 \tau^2 \mathcal{E}_0^2}{2m\omega} e^{-\frac{\tau^2\omega^2}{2}}.$$

$$W_{0\to 2} = \left|(-i)^2 \sqrt{2!} \left(\frac{e\mathcal{E}_0}{\sqrt{2m\omega}}\right)^2 \int_{-\infty}^{\infty} e^{-\tau_1^2/\tilde{\tau}^2 - i\omega\tau_1} \int_{-\infty}^{\tau_1} e^{-t^2/\tilde{\tau}^2 - i\omega t} dt \right|^2 =$$

$$= \left|(-i)\frac{\sqrt{2}}{2} \left(\frac{e\mathcal{E}_0}{\sqrt{2m\omega}}\right)^2 \left(\int_{-\infty}^{\infty} e^{-\frac{\tau_1^2}{\tilde{\tau}^2} + i\omega\tau_1} d\tau_1\right)^2\right|^2 = \frac{1}{2}\left(\frac{\pi e^2 \mathcal{E}_0^2 \tilde{\tau}^2}{2m\omega}\right)^n e^{-\tilde{\tau}^2\omega^2}$$

$$W_{0\to n} = \left|(-i)^n \frac{\sqrt{n!}}{n!} \left(\frac{e\mathcal{E}_0}{\sqrt{2m\omega}}\right)^n \left(\int_{-\infty}^{\infty} e^{-\frac{\tau_1^2}{\tilde{\tau}^2} + i\omega\tau_1} d\tau_1\right)^n\right|^2 = \frac{1}{n!}\left(\frac{\pi e^2 \mathcal{E}_0^2 \tilde{\tau}^2}{2m\omega}\right)^n e^{-\frac{\tilde{\tau}^2\omega^2 n}{2}}$$

φ-на для

$$\Psi_n(t\to\infty) = \left(\frac{ie\mathcal{E}_0}{\hbar\sqrt{2m\omega}}\right)^n \sqrt{n!} \int_{-\infty}^{\infty} d\tau_1\, e^{-\frac{\tau_1^2}{\tilde{\tau}^2} + i\omega\tau_1} \int_{-\infty}^{\tau_1} d\tau_2\, e^{-\frac{\tau_2^2}{\tilde{\tau}^2} + i\omega\tau_2} *$$

$$* \dots * \int_{-\infty}^{\tau_{n-1}} d\tau_n\, e^{-\frac{\tau_n^2}{\tilde{\tau}^2} + i\omega\tau_n} = 2^{1-n} \left(\frac{ie\mathcal{E}_0}{\hbar\sqrt{2m\omega}}\right)^n \sqrt{n!} \int_{\mathbb{R}^n} dt^n\, e^{-t^2/\tilde{\tau}^2} e^{i\omega\sum_{i=1}^{n} t_i}$$

Задача 1

$$\hat{V} = -e\mathcal{E}\hat{z}\cos(\omega t), \quad \omega = \frac{3}{4}R_x + \varepsilon, \quad e\mathcal{E}a, \varepsilon \ll R_x, T.$$

$P_{n=2}$ - ? $\qquad \hat{H}_0 = \frac{\hat{p}^2}{2m} - \frac{e^2}{r}$

$P_{n=1}$ - ?

$$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = (\hat{H}_0 + \hat{V}(t))|\psi(t)\rangle.$$

Без возмущения:

$$|\psi_1(t) = e^{-\frac{iR_y t}{\hbar}}|\psi_1\rangle \ ; \ |\psi_2(t)\rangle = e^{-\frac{iR_y t}{4\hbar}}|\psi_2\rangle$$

Нестац. теория возмущений.

$$i\hbar\frac{\partial\psi_2}{\partial t} = V_{21}\,\psi_1(t) \Rightarrow i\hbar\frac{\partial\psi_2}{\partial t} = -e\,\mathcal{E}\cos(\omega t)\,e^{\frac{3i}{4}\frac{R_y t}{\hbar}} *$$

$$* \langle\psi_2|r\cos\theta|\psi_1\rangle\,\psi_1(t) = \Big\{ \langle\psi_2|r\cos\theta|\psi_1\rangle = \int d\Omega\, r^3\cos\theta\, R_{2,1}Y_{1,0}R_{1,0}Y_{0,0} =$$

$$= //\text{остальные } \psi_2 \text{ нечётные по } \theta// = \frac{2^6\sqrt{2}\,a}{3^4}\Big\} = -e\,\mathcal{E}\,(e^{\omega t}+e^{-\omega t})\frac{2^5\sqrt{2}\,a}{3^4} *$$

$$* e^{\frac{3i}{4\hbar}R_y t}\,\psi_1(t) \approx -e\,\mathcal{E}\,e^{-\frac{i\varepsilon t}{\hbar}}\frac{2^5\sqrt{2}}{3^4}\psi_1(t)$$

Аналогично

$$i\hbar\frac{\partial\psi_1}{\partial t} = -e\,\mathcal{E}\,e^{\frac{i\varepsilon t}{\hbar}}\frac{2^5\sqrt{2}}{3^4}\psi_2(t)$$

Решая систему подстановкой и интегрированием получаем.

$$\psi_2(t) = -i\,e^{-i\frac{\varepsilon}{2}t}\frac{C}{\sqrt{\varepsilon^2+C^2}}\sin\left(\frac{\sqrt{\varepsilon^2+C^2}}{2\hbar}t\right), \quad C = \frac{2^6\sqrt{2}}{3^4}e\,\mathcal{E}\,a. \Rightarrow$$

$$\Rightarrow P_{n=2}(T) = \frac{C^2}{\varepsilon^2+C^2}\sin^2\left(\frac{\sqrt{\varepsilon^2+C^2}}{2\hbar}T\right)$$

$$P_{n=1}(T) = 1 - P_{n=2}(T).$$