Artificial Intelligence

VBDS1402

Unit 1 Module 3

Stochastic Games

Stochastic Games

A **stochastic game** is a generalization of both **Markov Decision Processes (MDPs)** and **repeated games**.

It models situations where:

- **Multiple players** interact,
- The outcome of each action depends on **both players' choices** and **random events**, and

The game can move through **different states over time**.

Introduced by **Lloyd Shapley (1953)**, stochastic games describe strategic decision-making in **uncertain environments** – where players must make optimal moves despite the presence of **chance**.

Structure of a Stochastic Game

Component	Description
Players	Usually two (but can be more). Each player has their own strategy and goal.
States (S)	A finite set of possible game positions or situations. Each state represents a stage of the game.
Actions (A_1, A_2, ...)	Each player chooses an action at every state.
Transition Probability (P)	Determines the probability of moving from one state to another based on both players' actions and chance.
Reward (R)	Payoff given to each player at a state, depending on the chosen actions.
Discount Factor (γ)	Determines how much future rewards are valued compared to immediate rewards.

How the Game Proceeds

The game starts in an **initial state (s_0)**.

Both players **choose actions (a_1, a_2)** simultaneously or sequentially.

The **next state** is determined **probabilistically** using the **transition function $P(s' \mid s, a_1, a_2)$**.

Each player receives a **reward $R(s, a_1, a_2)$**.

The game moves to the next state and continues until:

- A **terminal state** is reached, or
- The game continues indefinitely (in discounted or average-reward settings).

Examples of Stochastic Games

Backgammon

- Dice rolls determine possible moves – adds randomness to the strategy.
- Players still apply strategic reasoning but must adapt to dice outcomes.

Monopoly

- Dice rolls and random cards affect movement and income.
- Players plan investments but must account for unpredictable events.

Poker / Bridge

- Random card distribution influences play.
- Players use probability and strategy to handle uncertainty.

Snakes and Ladders

- Entirely stochastic (no player decision) – useful as a simple example of chance-based transitions.

Importance in Artificial Intelligence

Reinforcement Learning (RL)

When multiple agents learn in an uncertain environment.

Robotics

For planning where sensors or outcomes are uncertain.

Economics & Finance

For modeling competitive markets under uncertainty.

Autonomous Systems

When agents must cooperate or compete with partial information.

Snakes & Ladder as a Stochastic Game (Case Study)

Snakes and Ladders is a **classic board game** that perfectly illustrates the concept of a **stochastic game**, or a **game with chance moves**.

It is ideal for understanding **how randomness (chance)** affects state transitions and outcomes – a core feature of stochastic games.

Snakes & Ladder as a Stochastic Game (Case Study - Why It Is a Stochastic Game?**)**

- In **Snakes and Ladders**, players move based on **dice rolls**.
  The result of a dice roll is **random**, and hence, the **next position (state)** of the player is **not deterministic** – it depends on chance.
- This randomness in the **transition from one square to another** makes it a **stochastic process**.

Element	Description
Players	2 or more players
Board	100 numbered squares (1 → 100)
Start State	Square 1
Goal State	Square 100 (Finish)
Moves	Determined by rolling a 6-faced die (values 1–6)
Special Squares	- Ladders: Move the player up to a higher-numbered square- Snakes: Move the player down to a lower-numbered square

Snakes & Ladder as a Stochastic Game(Case Study- Game Setup)

Snakes & Ladder as a Stochastic Game(Case Study – State Representation)

In game theory and AI, we represent **each square** as a **state (S)**.

- **S = {1, 2, 3, ..., 100}**
- At each turn, the **player's current position** is the current **state**.
- The **dice roll** determines the **next state transition**.

Example:

- If the player is at square 5 and rolls a 3, they move to square 8.
- If square 8 has the bottom of a ladder leading to square 26, the player's new state becomes **26**.

The **transition probability** depends on the dice roll:

- There are **6 possible moves** (1–6).
- Each outcome has an **equal probability of 1/6**.
- Therefore, from any state s, there are up to **6 possible next states** s'.

Formally:

- **P(s' | s, a)** = 1/6,
 where *a* is rolling the die, and *s'* is the resulting square (after accounting for snakes/ladders).

Snakes & Ladder as a Stochastic Game(Case Study - Transition Model)

Snakes & Ladder as a Stochastic Game(Case Study - Effect of Snakes and Ladders)

- These special squares **alter the transition outcome** deterministically **after** the random dice roll.
- So, the **transition** is a **two-step stochastic process**:
- Randomly move 1–6 steps based on dice roll.
- Deterministically jump (up/down) if you hit a ladder or snake.

Type	Effect	Example
Ladder	Moves player upward to a higher state (benefit)	Ladder from 4 → 14
Snake	Moves player downward to a lower state (loss)	Snake from 17 → 7

Snakes & Ladder as a Stochastic Game(Case Study- State Transition Example**)**

Current State (S)	Dice Roll	Intermediate State	Ladder/Snake	Next State (S')	Probability
5	2	7	None	7	1/6
5	3	8	Ladder to 26	26	1/6
5	4	9	None	9	1/6
5	5	10	Snake to 2	2	1/6
5	6	11	None	11	1/6
5	1	6	None	6	1/6

This shows the **probabilistic nature** of movement – players cannot control where they land.

Snakes & Ladder as a Stochastic Game(Case Study - Rewards and Goal**)**

Component	Description
Reward Function (R)	Can be defined as +1 for reaching the goal (square 100), 0 otherwise.
Goal	Reach or exceed square 100 first.
Terminal State	Square 100 — game ends when any player reaches it.

Since there are no decisions to make (only dice rolls), the game demonstrates **pure stochastic transitions without strategic control**.

Snakes & Ladder as a Stochastic Game(Case Study - Formal Model)

Element	Description
States (S)	Squares 1–100
Actions (A)	Roll a die (only one action)
Transition Probability (P)	1/6 for each dice outcome
Reward (R)	+1 for reaching state 100, 0 otherwise
Players	Two or more
Nature (Chance Player)	Dice roll outcome decides next state

Snakes & Ladder as a Stochastic Game(Case Study-AI Perspective)

Snakes and Ladders is a **single-agent stochastic environment** (if only one player is considered).

It is **episodic** (each game ends at square 100),

Sequential (each move depends on the previous position),

Stochastic (random dice rolls),

Fully observable (all positions known).

If multiple players are included, it becomes a **multi-agent stochastic game** where:

Each agent has an equal chance of progress,

The winner is determined by who reaches the terminal state first.

Deterministic vs. Stochastic Comparison

Feature	Deterministic Game	Stochastic Game
Outcome of Action	Always predictable	Involves randomness
Transition	Fixed	Probabilistic
Example	Chess, Checkers	Backgammon, Monopoly
AI Challenge	Search space	Probability handling