ITCS 6156/8156 Fall 2023
Machine Learning

Ensemble Methods

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Some content in the slides is based on Dr. Raquel Urtasun's lecture

An alternative derivation of AdaBoost

- Just write down the right cost function and optimize each parameter to minimize it
  - stagewise additive modeling (Friedman et. al. 2000)
- At each step employ the exponential loss function for classifier m

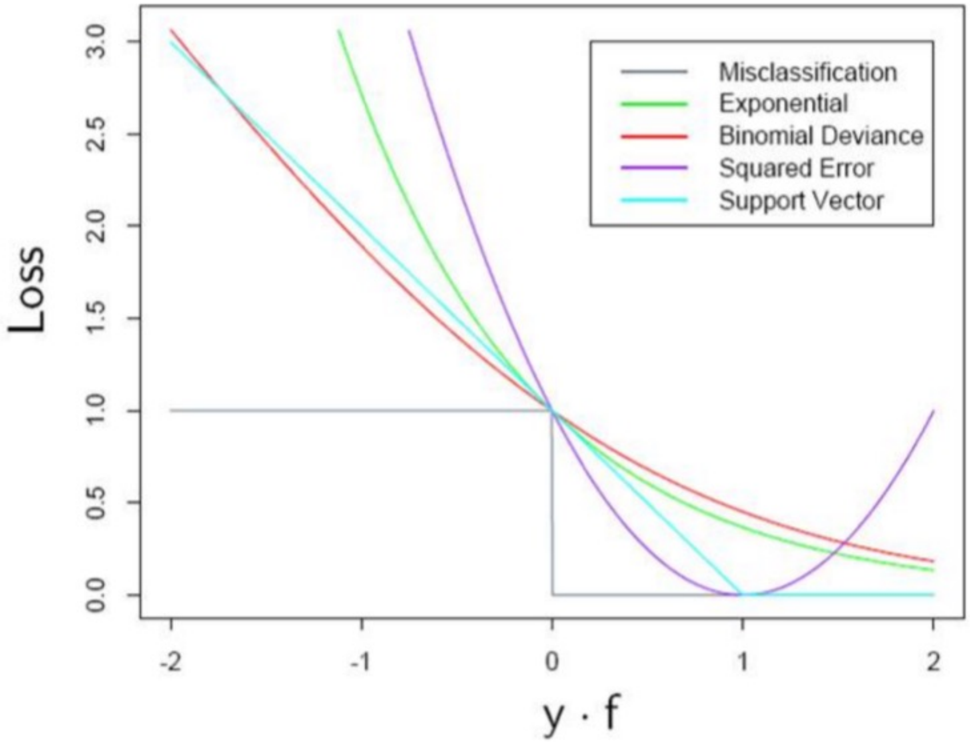
$$E = \sum_{n=1}^{N} \exp\{-t^{(n)} f_m(\mathbf{x}^{(n)})\}$$

- Real-valued prediction by committee of models up to m

$$f_m(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^{m} \alpha_i y_i(\mathbf{x})$$

- We want to minimize E w.r.t. α_m and the parameters of the classifier $y_m(\mathbf{x})$
- We do this in a sequential manner, one classifier at a time

Loss Functions



- Misclassification: $0/1$ loss
- Exponential loss: $\exp(-t \cdot f(x))$ (AdaBoost)
- Squared error: $(t - f(x))^2$
- Soft-margin support vector (hinge loss): $\max(0, 1 - t \cdot y)$

Learning classifier m using exponential loss

- At iteration m, the energy is computed as

$$E = \sum_{n=1}^{N} \exp\{-t^{(n)} f_m(\mathbf{x}^{(n)})\}$$

with

$$f_m(\mathbf{x}) = \frac{1}{2}\sum_{i=1}^{m} \alpha_i y_i(\mathbf{x}) = \frac{1}{2}\alpha_m y_m(\mathbf{x}) + \frac{1}{2}\sum_{i=1}^{m-1} \alpha_i y_i(\mathbf{x})$$

- We can compute the part that is relevant for the m-th classifier

$$\begin{aligned} E_{relevant} &= \sum_{n=1}^{N} \exp\left(-t^{(n)} f_{m-1}(\mathbf{x}^{(n)}) - \frac{1}{2}t^{(n)}\alpha_m y_m(\mathbf{x}^{(n)})\right) \\ &= \sum_{n=1}^{N} w_n^m \exp\left(-\frac{1}{2}t^{(n)}\alpha_m y_m(\mathbf{x}^{(n)})\right) \end{aligned}$$

with $w_n^m = \exp\left(-t^{(n)} f_{m-1}(\mathbf{x}^{(n)})\right)$

Continuing the derivation

$$
\begin{aligned}
E_{relevant} &= \sum_{n=1}^{N} w_n^m \exp\left(-t^{(n)} \frac{\alpha_m}{2} y_m(\mathbf{x}^{(n)})\right) \\
&= e^{-\frac{\alpha_m}{2}} \sum_{right} w_n^m + e^{\frac{\alpha_m}{2}} \sum_{wrong} w_n^m \\
&= \underbrace{\left(e^{\frac{\alpha_m}{2}} - e^{\frac{-\alpha_m}{2}}\right)}_{multiplicative\ \ constant} \underbrace{\sum_n w_n^m [t^{(n)} \neq y_m(\mathbf{x}^{(n)})]}_{wrong\ \ cases} + \underbrace{e^{-\frac{\alpha_m}{2}} \sum_n w_n^m}_{unmodifiable}
\end{aligned}
$$

- The second term is constant w.r.t. $y_m(\mathbf{x})$
- Thus we minimize the weighted number of wrong examples

AdaBoost Algorithm

- Input: $\{\mathbf{x}^{(n)}, t^{(n)}\}_{n=1}^{N}$, and WeakLearn: learning procedure, produces classifier $y(\mathbf{x})$
- Initialize example weights: $w_n^m(\mathbf{x}) = 1/N$
- For m=1:M
  - $y_m(\mathbf{x}) = WeakLearn(\{\mathbf{x}\}, \mathbf{t}, \mathbf{w})$, fit classifier by minimizing

$$J_m = \sum_{n=1}^{N} w_n^m [y_m(\mathbf{x}^n) \neq t^{(n)}]$$

  - Compute unnormalized error rate

$$\epsilon_m = \frac{J_m}{\sum w_n^m}$$

  - Compute classifier coefficient $\alpha_m = \log \frac{1-\epsilon_m}{\epsilon_m}$
  - Update data weights

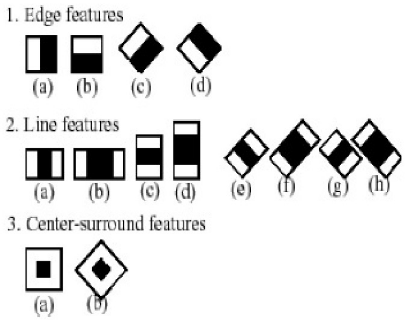
$$w_n^{m+1} = w_n^m \exp\left(-\frac{1}{2} t^{(n)} \alpha_m y_m(\mathbf{x}^{(n)})\right)$$

- Final model

$$Y(\mathbf{x}) = sign(y_M(\mathbf{x})) = sign(\sum_{m=1}^{M} \alpha_m y_m(\mathbf{x}))$$

An impressive example of boosting

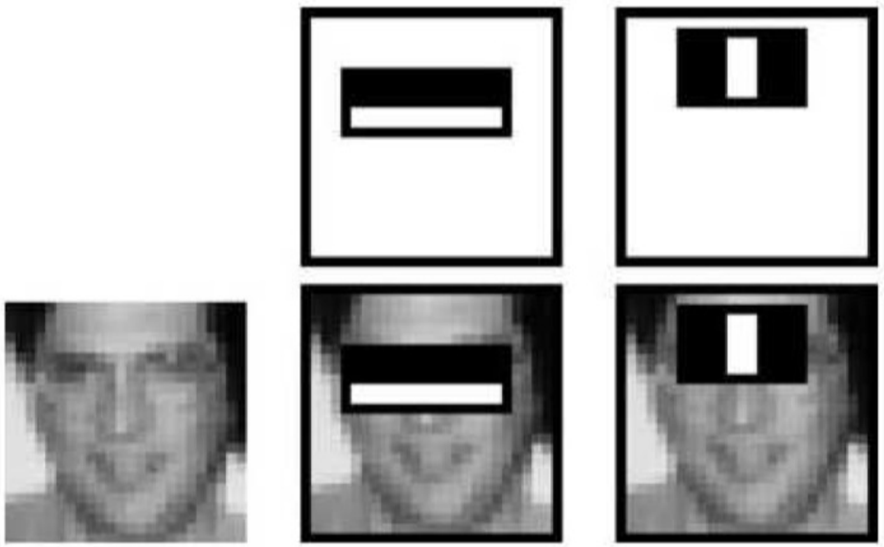
- Viola and Jones created a very fast face detector that can be scanned across a large image to find the faces.



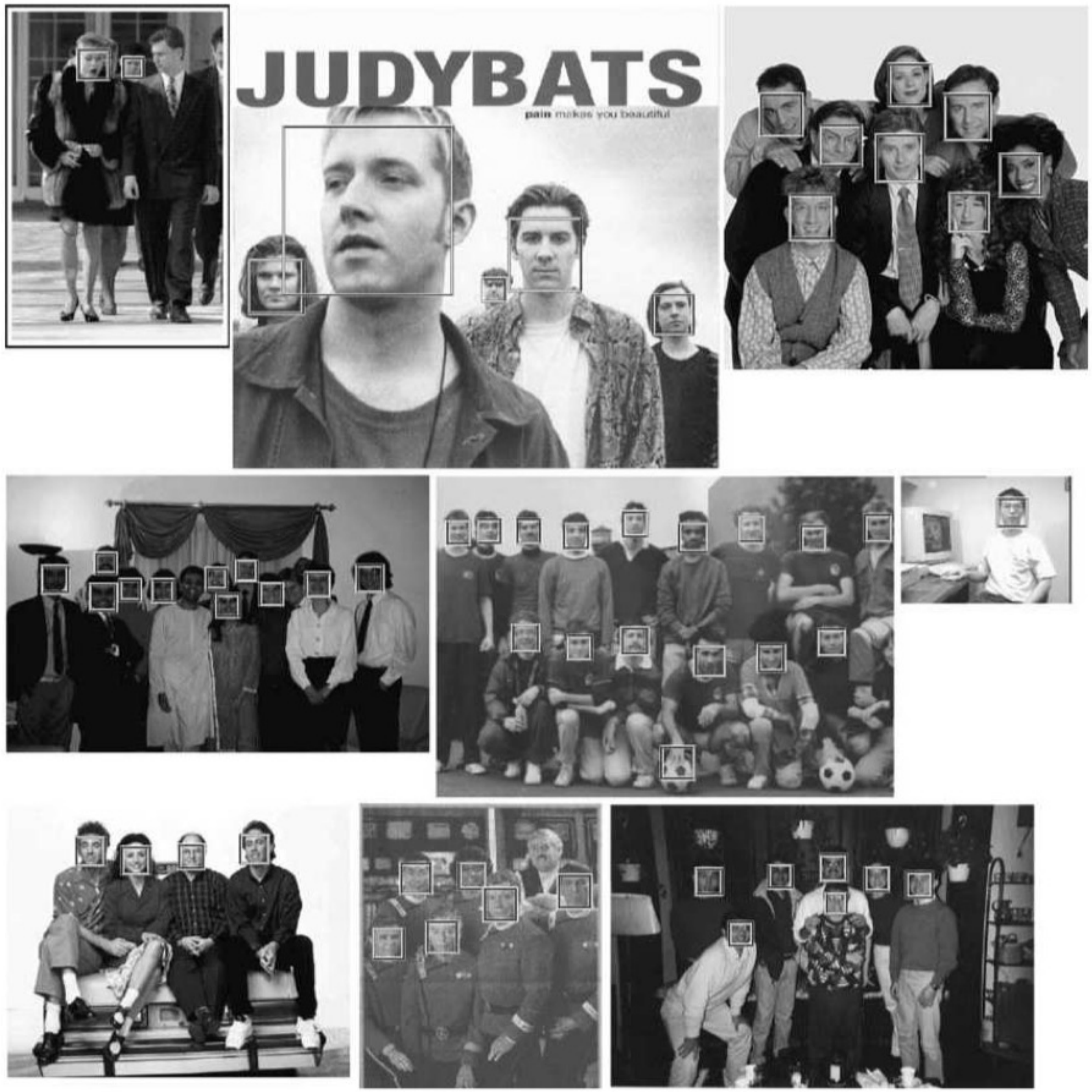
- The base classifier/weak learner just compares the total intensity in two rectangular pieces of the image.
  - There is a neat trick for computing the total intensity in a rectangle in a few operations.
    - So its easy to evaluate a huge number of base classifiers and they are very fast at runtime.
  - The algorithm adds classifiers greedily based on their quality on the weighted training cases.

AdaBoost in Face Detection

- Famous application of boosting: detecting faces in images
- Two twists on standard algorithm
  - Pre-define weak classifiers, so optimization=selection
  - Change loss function for weak learners: false positives less costly than misses

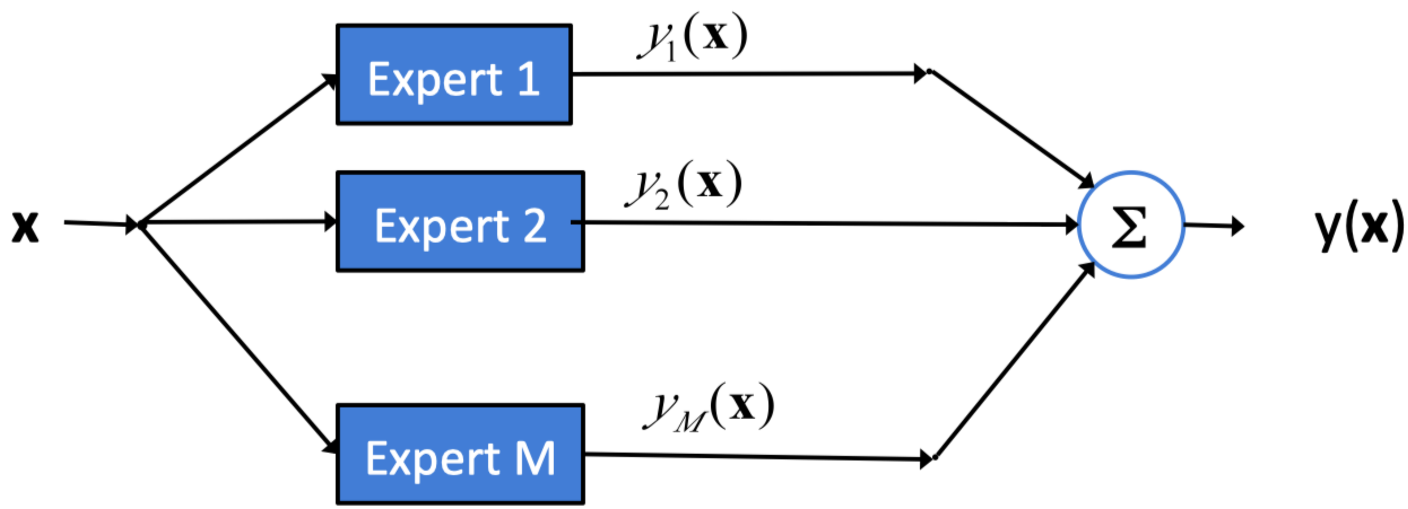


AdaBoost Face Detection Results



Ensemble Learning: Boosting and Bagging

- Experts cooperate to predict output

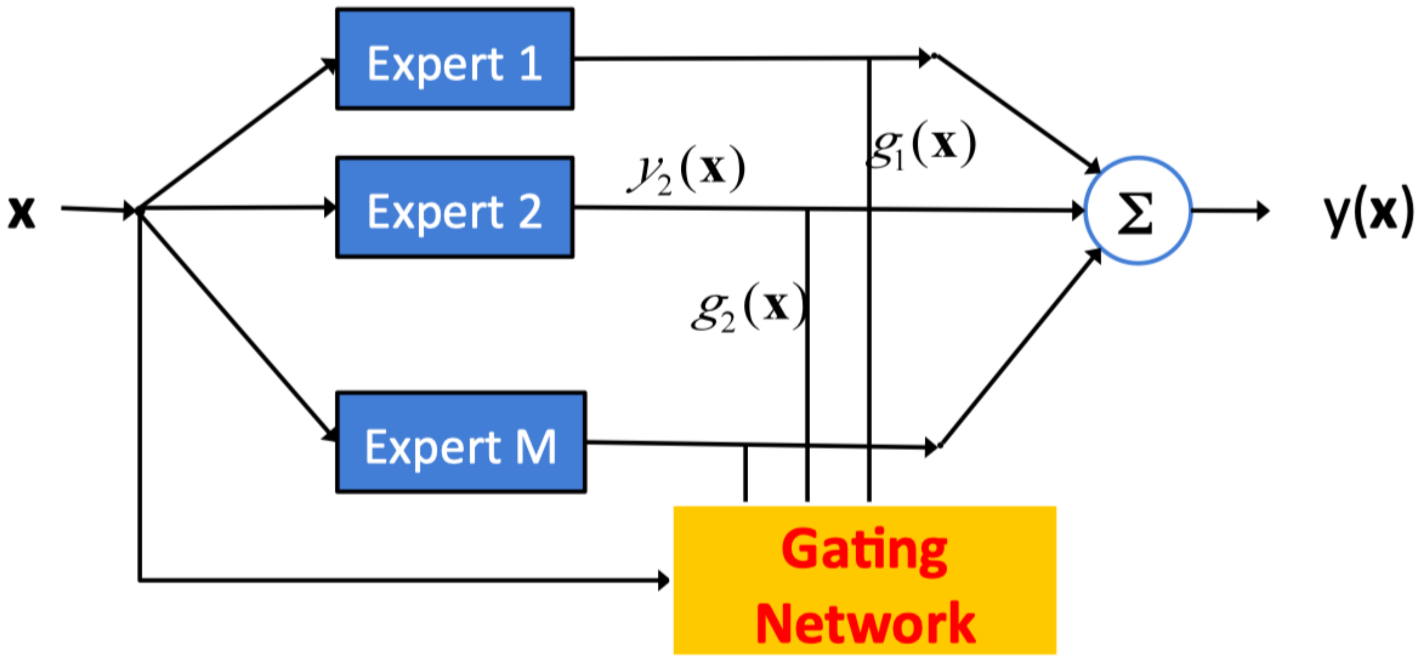


- Vote of each expert has consistent weight for each test example

$$y(\mathbf{x}) = \sum_m g_m y_m(\mathbf{x})$$

Mixture of Experts

- Weight of each expert is not constant – depends on input **x**



- Gating network encourages specialization (local experts) instead of cooperation

$$y(\mathbf{x}) = \sum_m g_m(\mathbf{x}) y_m(\mathbf{x})$$

Mixture of Experts: Summary

1. Cost function designed to make each expert estimate desired output independently
2. Gating network softmax over experts: stochastic selection of who is the true expert for given input
3. Allow each expert to produce distribution over outputs

Cooperation vs. Specialization

- Consider a regression problem
- To encourage cooperation, we can train to reduce discrepancy between average of predictors with target

$$E = (t - \frac{1}{M} \sum_m y_m(\mathbf{x}))^2$$

- This can overfit badly. It makes the model much more powerful than training each predictor separately
- Leads to odd objective: consider adding models/experts sequentially
  - if its estimate for t is too low, and the average of other models is too high, then model m encouraged to lower its prediction

Cooperation vs. Specialization

- To encourage specialization, train to reduce the average of each predictor's discrepancy with target

$$E = \frac{1}{M}\sum_m (t - y_m(\mathbf{x}))^2$$

- Use a weighted average: weights are probabilities of picking that "expert" for the particular training case

$$E = \frac{1}{M}\sum_m g_m(\mathbf{x})(t - y_m(\mathbf{x}))^2$$

- Gating output is softmax of $z = U\mathbf{x}$

$$g_m(\mathbf{x}) = \frac{\exp(z_m(\mathbf{x}))}{\sum_i \exp(z_i(\mathbf{x}))}$$

- We want to estimate the parameters of the gating as well as the classifier y_m

Derivatives of Simple Cost Function

- Look at derivatives to see what cost function will do

$$E = \frac{1}{M}\sum_{m} g_m(\mathbf{x})(t - y_m(\mathbf{x}))^2$$

- For gating network, increase weight on expert when its error is less than average error of experts

$$\begin{aligned}\frac{\partial E}{\partial y_m} &= \frac{1}{M} g_m(\mathbf{x})(t - y_m(\mathbf{x})) \\ \frac{\partial E}{\partial z_m} &= \frac{1}{M} g_m(\mathbf{x}) \left[(t - y_m(\mathbf{x}))^2 - E\right]\end{aligned}$$

Mixture of Experts: Final Cost Function

- Can improve cost function by allowing each expert to produce not just a single value estimate, but a distribution
- Result is a mixture of experts model:

$$p(t|MOE) = \sum_m g_m(\mathbf{x}) \mathcal{N}(t|y_m(\mathbf{x}), \Sigma)$$

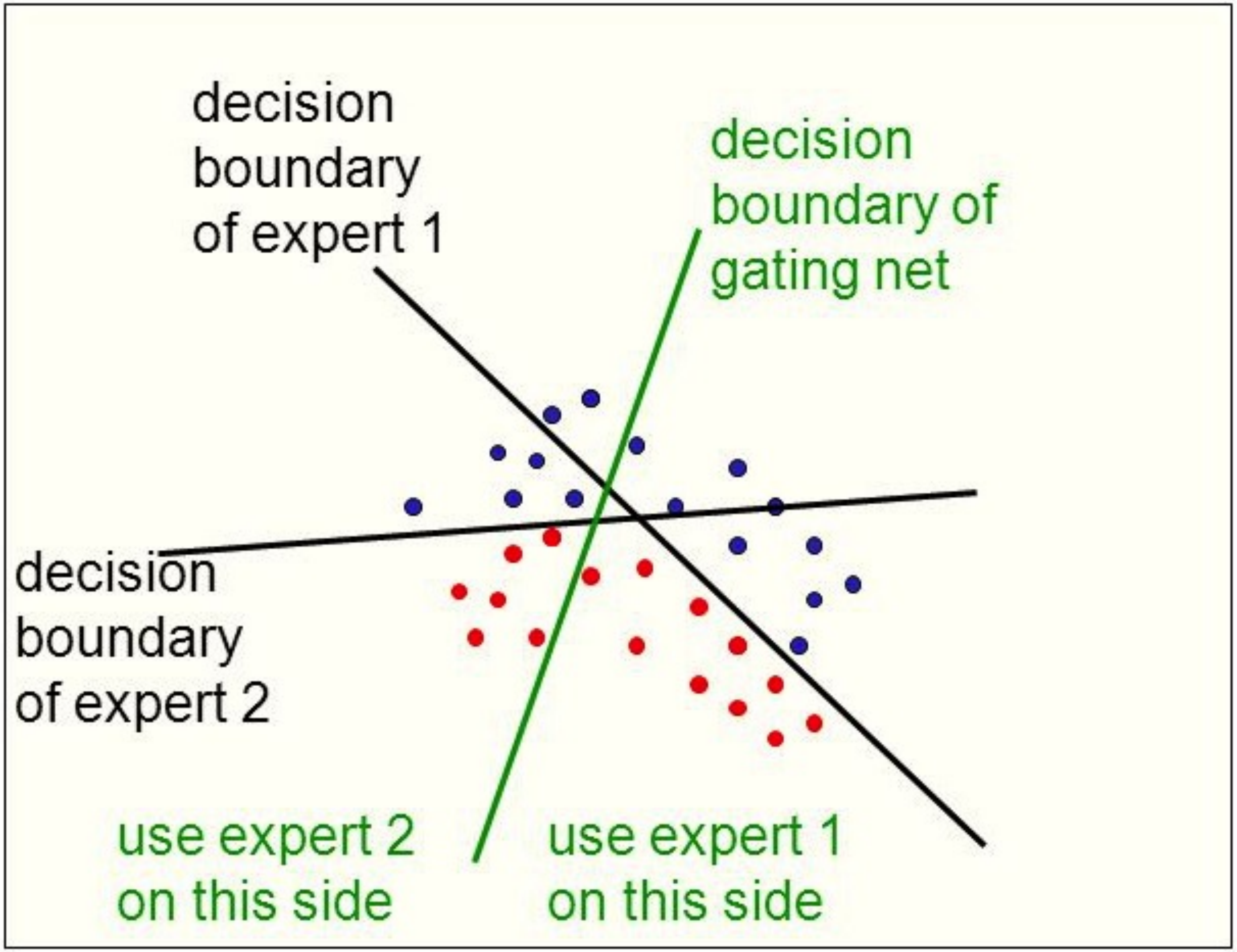
- Optimize minus log-likelihood:

$$-\log p(t|MOE) = -\log \sum_m g_m(\mathbf{x}) \exp\left(-\frac{1}{2}||t - y_m(\mathbf{x})||^2\right)$$

- Gradient: Error weighted by posterior probability of the expert

$$\frac{\partial E}{\partial y_m} = -2 \frac{g_m(\mathbf{x}) \exp\left(-\frac{1}{2}||t - y_m(\mathbf{x})||^2\right)}{\sum_i g_i(\mathbf{x}) \exp\left(-\frac{1}{2}||t - y_i(\mathbf{x})||^2\right)} (t - y_m(x))$$

Mixture of Experts: Example



Mixture of Experts: Summary

- Cost function designed to make each expert estimate desired output independently
- Gating network softmax over experts: stochastic selection of who is the true expert for given input
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Ensemble methods: Summary

- Differ in training strategy, and combination method
  - Parallel training with different training sets

    Bagging (bootstrap aggregation) – train separate models on overlapping training sets, average their predictions
  - Sequential training, iteratively re-weighting training examples so current classifier focuses on hard examples: boosting
  - Parallel training with objective encouraging division of labor: mixture of experts
- Notes:
  - Differ in: training strategy; selection of examples; weighting of components in final classifier

Questions?

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