EP3101 Mini Project: Numerical Simulation of Foucault Pendulum

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1 Introduction

This project simulates the motion of a Foucault pendulum numerically using a finite difference method, accounting for the Coriolis force due to Earth's rotation. The simulation is performed for a pendulum at different latitudes and earth's angular velocity.

2 Parameters and Setup

The parameters used in the simulation are:

- Latitude: 25°
- Pendulum length (l): 1.0 m
- Pendulum mass (m): 1.0 kg
- Gravitational acceleration (g): 9.81 m/s^2
- Time step (Δt): 0.1 s
- Maximum simulation time (T): 3600 s
- Earth's angular velocity (ω_0): 360/86400 deg/s

3 Methodology

The motion of the pendulum is modeled using finite difference approximations. Let $x(t)$ and $y(t)$ be the horizontal displacements of the pendulum. The numerical integration is performed using the following recurrence relations:

$$y_i = c_1 y_{i-2} + c_2 y_{i-1} + c_3 x_{i-2} + c_4 x_{i-1}$$

$$x_i = c_1 x_{i-2} + c_2 x_{i-1} - c_3 y_{i-2} - c_4 y_{i-1}$$

where c_1, c_2, c_3, c_4 are coefficients depending on the system constants and simulation parameters.

4 MATLAB Code

The following MATLAB code was used to simulate the Foucault pendulum:

```
%% PARAMETERS
LATITUDE = 25;
ANGVAL = 360 / 86400;
G = 9.81;
dt = 0.1;
l = 1.0;
m = 1.0;
T = 3600;

%% CONVERSIONS
lat = deg2rad(LATITUDE);
av = deg2rad(ANGVAL);

%% EFFECTIVE ROTATION COMPONENT
Omeg = av * sin(lat);

%% FINITE DIFFERENCE COEFFICIENTS
phi = (G*dt^2/l - 2);
mu_p = Omeg^2 * dt^2 + 1;
mu_n = Omeg^2 * dt^2 - 1;

c1 = mu_n / mu_p;
c2 = -phi / mu_p;
c3 = 2 * Omeg * dt / mu_p;
c4 = Omeg * dt * phi / mu_p;

%% TIME VECTOR
t = 0:dt:T;
N = length(t);

%% INITIAL CONDITIONS
x = zeros(N, 1);
y = zeros(N, 1);

x(1) = 1.0;
x(2) = 1.0;
y(1) = 0.0;
y(2) = 0.0;

%% NUMERICAL INTEGRATION USING FINITE DIFFERENCE
for i = 3:N
    y(i) = c1 * y(i-2) + c2 * y(i-1) + c3 * x(i-2) + c4 * x(i-1);
    x(i) = c1 * x(i-2) + c2 * x(i-1) - c3 * y(i-2) - c4 * y(i-1);
end

%% PLOT RESULTS
figure;
plot(x, y, 'b', 'LineWidth', 1.5);
grid on;
axis equal;
xlabel('X Displacement (m)');
ylabel('Y Displacement (m)');
title('Foucault Pendulum Trajectory');
```

5 Simulation Results

5.1 Case $\omega_1 = \omega_0$

NOTE: Here ω_0 is the true angular velocity of earth.

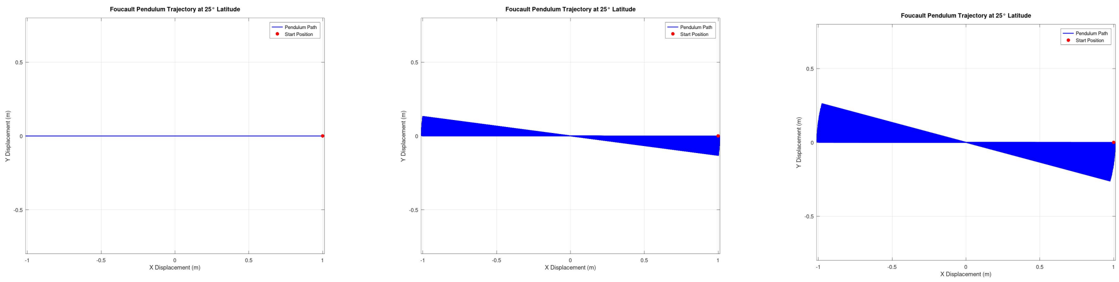


Figure 1: Pendulum trajectories *(top-view)* for ω_1 with latitudes $\theta = 0°, 30°, 90°$.

5.2 Case $\omega_2 = 10\omega_0$

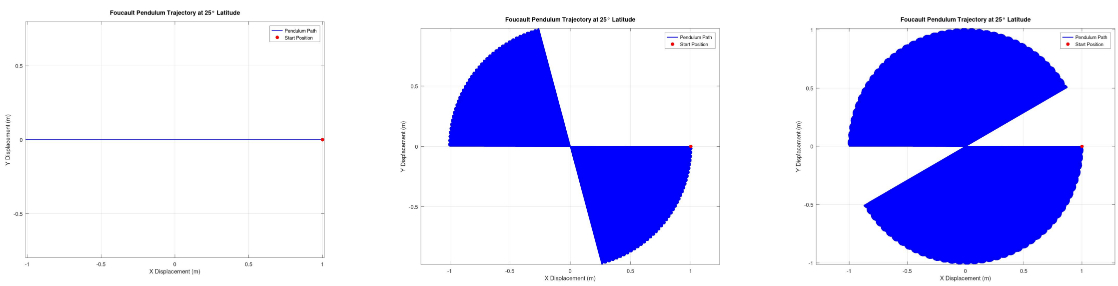


Figure 2: Pendulum trajectories *(top-view)* for ω_2 with latitudes $\theta = 0°, 30°, 90°$.

6 Calculations

The numerical calculations for the coefficients are:

$$
\begin{aligned}
\phi &= \frac{G\Delta t^2}{l} - 2 \\
\mu_p &= \Omega^2 \Delta t^2 + 1 \\
\mu_n &= \Omega^2 \Delta t^2 - 1 \\
c_1 &= \frac{\mu_n}{\mu_p}, \quad c_2 = -\frac{\phi}{\mu_p}, \quad c_3 = \frac{2\Omega\Delta t}{\mu_p}, \quad c_4 = \frac{\Omega\Delta t\phi}{\mu_p}
\end{aligned}
$$

7 Calculation Steps

$$\frac{x_{i+1}+x_{i-1}-2x_i}{\Delta t^2}+\frac{9x_i}{2}=\frac{\cancel{2}\Omega}{\cancel{2}\Delta t}(y_{i+1}-y_{i-1}) \qquad (1.1)$$

$$\boxed{x_{i+1}+x_{i-1}+\phi x_i=\cancel{2}\Omega\Delta t\,(y_{i+1}-y_{i-1})}$$

$$x_{i+1}=(-x_{i-1}-\phi x_i-\Omega\Delta t\,y_{i-1})+(\Omega\Delta t\,y_{i+1})$$

$$x_{i+1}=(-x_{i-1}-\phi x_i-\Omega\Delta t\,y_{i-1})+\Omega\Delta t\,(-y_{i-1}-\phi y_i-\Omega\Delta t\,x_{i+1}+\Omega\Delta t\,x_{i-1})$$

$$x_{i+1}=(-x_{i-1}-\phi x_i-\Omega\Delta t\,y_{i-1})-\Omega\Delta t\,y_{i-1}-\Omega\Delta t\,\phi y_i-\Omega^2\Delta t^2x_{i+1}+\Omega^2\Delta t^2x_{i-1}$$

$$(1+\Omega^2\Delta t^2)x_{i+1}=(-x_{i-1}+\Omega^2\Delta t^2x_{i-1}-\phi x_i-2\Omega\Delta t\,y_{i-1}-\Omega\Delta t\,y_i$$

$$(1+\Omega^2\Delta t^2)x_{i+1}=(\Omega^2\Delta t^2-1)x_{i-1}-\phi x_i-2\Omega\Delta t\,y_{i-1}-\Omega\Delta t\,y_i\phi$$

Say $\Omega^2\Delta t^2+1=M_+$

$\Omega^2\Delta t^2-1=M_-$

Figure 3: Step 1

1.2

$$M_+ x_{i+1} = M_- x_{i-1} - \phi x_i - 2\Omega \Delta t\, y_{i-1} - \Omega \Delta t\, \phi\, y_i$$

$$x_{i+1} = \frac{M_-}{M_+} x_{i-1} - \frac{\phi}{M_+} x_i - \frac{2\Omega \Delta t}{M_+} y_{i-1} - \frac{\Omega \Delta t\, \phi}{M_+} y_i$$

thus $x_{i+1} = f(x_{i-1}, x_i, y_{i-1}, y_i)$

$$\phi = \left(\frac{g\Delta t^2}{\ell} - 2\right) \qquad M_+ = \Omega^2 \Delta t^2 + 1$$

$$M_- = \Omega^2 \Delta t^2 - 1$$

Figure 4: Step 2

$$\frac{y_{i+1}+y_{i-1}-2y_i}{\Delta t^2}+\frac{g y_i}{l} = -\frac{2\Omega}{2\Delta t}\left(x_{i+1}-x_{i-1}\right)$$

$$y_{i+1}+y_{i-1}+\underbrace{\left(\frac{g\Delta t^2}{l}-2\right)}_{\phi} y_i = -\frac{\Omega\Delta t^2}{\Delta t}\left(x_{i+1}-x_{i-1}\right)$$

$$\boxed{y_{i+1} = -y_{i-1}-\phi y_i-\Omega\Delta t\left(x_{i+1}-x_{i-1}\right)}$$

$$y_{i+1} = -y_{i-1}-\phi y_i+\Omega\Delta t\, x_{i-1}-\Omega\Delta t\left(-x_{i-1}-\phi x_i-\Omega\Delta t\, y_{i-1}+\Omega\Delta t\, y_{i+1}\right)$$

$$y_{i+1} = -y_{i-1}+\Omega^2\Delta t^2 y_{i-1}-\phi y_i+\Omega\Delta t\, x_{i-1}+\Omega\Delta t\, x_{i-1}+\Omega\Delta t\,\phi\, x_i - \Omega^2\Delta t^2 y_{i+1}$$

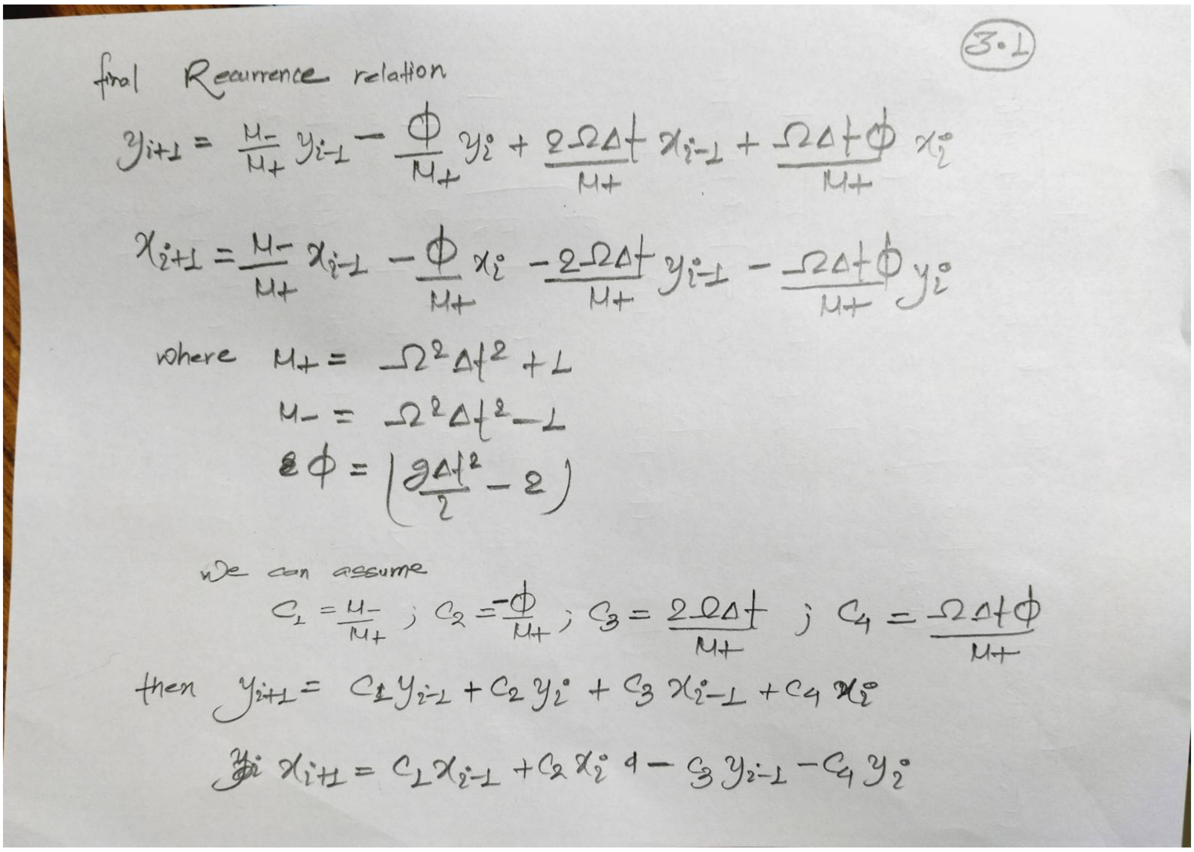
$$\left(1+\Omega^2\Delta t^2\right) y_{i+1} = \left(\Omega^2\Delta t^2-1\right) y_{i-1}-\phi y_i+2\Omega\Delta t\, x_{i-1}+\Omega\Delta t\,\phi\, x_i$$

$$M_+ y_{i+1} = M_- y_{i-1}-\phi y_i+2\Omega\Delta t\, x_{i-1}+\Omega\Delta t\,\phi\, x_i$$

$$y_{i+1} = \frac{M_-}{M_+} y_{i-1}-\frac{\phi y_i}{M_+}+\frac{2\Omega\Delta t}{M_+} x_{i-1}+\frac{\Omega\Delta t\,\phi}{M_+} x_i$$

thus $y_i = g\left(y_{i-1}, y_i, x_{i-1}, x_i\right)$

Figure 5: Step 3



3.1

final Recurrence relation

$$y_{i+1} = \frac{\mu_-}{\mu_+} y_{i-1} - \frac{\phi}{\mu_+} y_i + \frac{2\Omega\Delta t}{\mu_+} x_{i-1} + \frac{\Omega\Delta t\phi}{\mu_+} x_i$$

$$x_{i+1} = \frac{\mu_-}{\mu_+} x_{i-1} - \frac{\phi}{\mu_+} x_i - \frac{2\Omega\Delta t}{\mu_+} y_{i-1} - \frac{\Omega\Delta t\phi}{\mu_+} y_i$$

where $\mu_+ = \Omega^2\Delta t^2 + 1$

$\mu_- = \Omega^2\Delta t^2 - 1$

$2\phi = \left(\frac{g\Delta t^2}{2} - 2\right)$

we can assume

$C_1 = \frac{\mu_-}{\mu_+}$; $C_2 = \frac{-\phi}{\mu_+}$; $C_3 = \frac{2\Omega\Delta t}{\mu_+}$; $C_4 = \frac{\Omega\Delta t\phi}{\mu_+}$

then $y_{i+1} = C_1 y_{i-1} + C_2 y_i + C_3 x_{i-1} + C_4 x_i$

$x_{i+1} = C_1 x_{i-1} + C_2 x_i - C_3 y_{i-1} - C_4 y_i$

Figure 6: Step 4

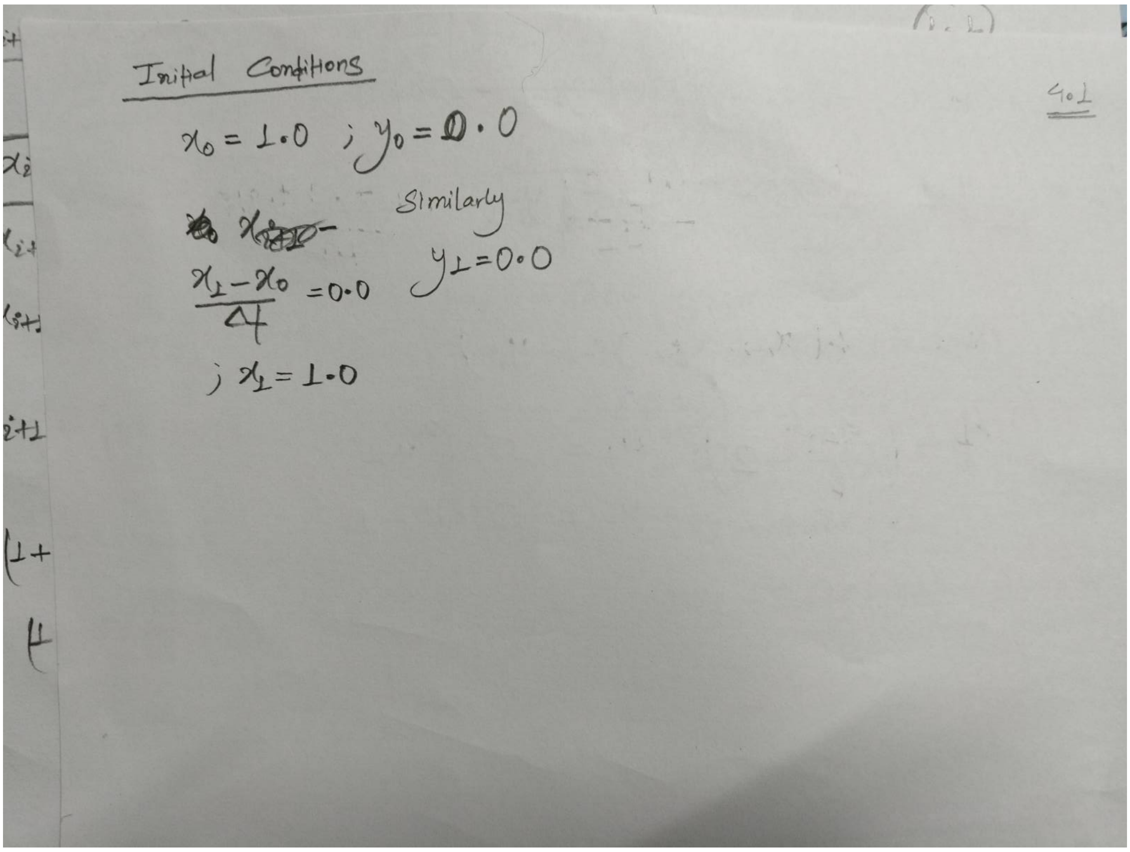


Figure 7: Step 5

8 Conclusion

The simulation successfully demonstrates the motion of the Foucault pendulum and its precession due to Earth's rotation. The finite difference method accurately approximates the trajectory. The angle swept by the pendulum plane increases with latitude angle, and it also increases with increasing ω.