CSCI 2824
Spring 2019

Write **clearly**:

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Section number: 002
Assignment: Homework 2

Read the following:

- This cover sheet must be included as the first page for all written homework submissions to CSCI 2824.
- Fill out all of the fields above.
- Submit your written homework assignment to the electronic dropbox. You will receive graded feedback through the same mechanism.
- If you type up your homework assignment using MS Word or LaTeX, then you can earn one extra credit point per homework assignment. You **must** use properly formatted equations and nice-looking text in order to be eligible for this extra credit point. If you type it up and do not format equations properly or do not use the cover sheet (for example), you might still lose the style/neatness points.
- By submitting this assignment, you are agreeing that you have abided by the **CU Honor Code**, and that this submission constitutes your own original work.

1.

p = Aramis is a Knight
q = Bertrand is a Knight
r = Charleston is a Knight

Aramis says: "Bertrand is a Knave." : $\neg q$

Bertrand says: "Charleston is a Knave OR I am a Knight, but not both." : $\neg r \veebar q$

Charleston says: "Aramis is a Knight and Bertrand is a Knave." : $p \wedge \neg q$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg r \veebar q$	$p \wedge \neg q$	$p \Leftrightarrow q$	$q \Leftrightarrow \neg r \veebar q$	$r \Leftrightarrow p \wedge \neg q$	$(p \Leftrightarrow \neg q) \wedge (q \Leftrightarrow \neg r \veebar q) \wedge (r \Leftrightarrow$
T	T	T	F	F	F	T	F	F	T	F	F
T	T	F	F	F	T	F	F	F	F	T	F
T	F	T	F	T	F	F	T	T	T	T	T
T	F	F	F	T	T	T	T	T	F	F	F
F	T	T	T	F	F	T	F	T	T	F	F
F	T	F	T	F	T	F	F	T	F	T	F
F	F	T	T	T	F	F	F	F	T	F	F
F	F	F	T	T	T	T	F	F	F	T	F

Result: Aramis is a Knight, Bertrand is a Knave, and Charleston is a Knight.

2.a.i.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$((p \to q) \wedge (q \to r)) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Tautology

2. a. ii. $((p \to q) \wedge (q \to r)) \to (p \to r)$

$p \to q \equiv \neg p \vee q$ RBI

$q \to r \equiv \neg q \vee r$ RBI

$p \to r \equiv \neg p \vee r$

$((p \to q) \wedge (q \to r)) \to (p \to r) \equiv$

$((\neg p \vee q) \wedge (\neg q \vee r)) \to (\neg p \vee r)$ RBI

$\equiv \neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r)$ RBI

$\neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r)$ RBI

$\equiv (\neg p \vee q) \vee \neg(\neg q \vee r) \vee (\neg p \vee r)$ De Morgan

$\equiv (p \wedge \neg q) \vee (q \wedge \neg r) \vee (\neg p \vee r)$ Double Negation / De Morgan

$\equiv (p \wedge \neg q) \vee (q \wedge \neg r) \vee (\neg p \vee r)$ Associativity

$\equiv (p \wedge \neg q) \vee (\neg p \vee r) \vee (q \wedge \neg r)$ Commutativity

$\equiv (p \vee \neg p \vee r) \wedge (\neg q \vee r \vee \neg p) \vee (\neg r \wedge q)$ Distributive

$\equiv T \wedge (\neg p \vee \neg q \vee r) \wedge (q \vee \neg q \vee \neg p \vee r)$ Distributive

$q \vee \neg q \equiv T$ $\quad \vee \wedge \neg r \equiv T$ Negation

$\equiv T \wedge T \wedge T$

Satisfiable because all True
Similarly, like the Truth Table.

2.b. $(p \to q) \to r$ $p \to (q \to r)$

p	q	r	$p \to q$	$(p \to q) \to r$	$q \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	★ F	F	★ T
F	F	T	T	T	T	T
F	F	F	T	★ F	T	★ T

$(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent because their truth tables are all not the same. On rows 6 and 8, the values differ.

3. a. $\forall x P(x)$

$P(5) \wedge P(6) \wedge P(7) \wedge P(8)$

b. $\neg \exists x P(x)$

$\neg P(5) \wedge \neg P(6) \wedge \neg P(7) \wedge \neg P(8)$

c. $\neg \forall x P(x)$

$\neg P(5) \vee \neg P(6) \vee \neg P(7) \vee \neg P(8)$

4. $163 \rightarrow Odd \rightarrow 1 \rightarrow \frac{163-1}{3} = 54$

$54 \rightarrow Even \rightarrow 0 \rightarrow 54/3 = 18$

$18 \rightarrow Even \rightarrow 0 \rightarrow 18/3 = 6$

$6 \rightarrow Even \rightarrow 0 \rightarrow 6/3 = 2$

$2 \rightarrow Even \rightarrow \rightarrow 2/3 =$

$(163)_{10} = (20001)_3$

5. a. $\therefore T(V) \vee \neg T(S)$

$\therefore T(P) \rightarrow \neg T(V)$

$\therefore T(B) \Leftrightarrow (T(L) \wedge T(P))$

$\therefore \neg T(P)$

b. $(T(V) \vee \neg T(S)) \wedge (T(P) \rightarrow \neg T(V)) \wedge (T(B) \Leftrightarrow (T(L) \wedge T(P))) \wedge (\neg T(P))$

All statements need to be true.

Suppose $T(V)$ is True, $\neg T(P)$ would need to be True for $T(P) \rightarrow \neg T(V)$ to be satisfied.

Suppose $T(B)$ is false. $T(L)$ would need to be false for $T(B) \Leftrightarrow (T(L) \wedge T(P))$ to be satisfied.

The Gang went to Venice.

c. MARS $\wedge$.*