Homework 4 ID: 105070938 Alex Hawkins 002

1. $\forall x(P(x) \vee Q(x)) \overset{(1)}{\equiv} \forall x P(x) \vee \forall x Q(x)$ Distribution law

$\overset{(2)}{\equiv} P(c) \vee Q(c)$ Universal Instantiation

$\forall x(\neg Q(x) \vee S(x)) \overset{(3)}{\equiv} \forall x \neg Q(x) \vee \forall x S(x)$ Distribution

$\overset{(4)}{\equiv} \neg \exists x Q(x) \vee \forall x S(x)$ De Morgan

$\overset{(5)}{\equiv} \neg Q(c) \vee S(c)$ Universal Instantiation

$\forall x(R(x) \rightarrow \neg S(x)) \overset{(6)}{\equiv} \forall x R(x) \rightarrow \forall x \neg S(x)$ Distribution

$\overset{(7)}{\equiv} \forall x R(x) \rightarrow \neg \exists x S(x)$ De Morgan

$\overset{(8)}{\equiv} R(c) \rightarrow \neg S(c)$ Universal Instantiation

$\exists x \neg P(x) \overset{(9)}{\equiv} \neg \forall x P(x)$ De Morgan

$\overset{(10)}{\equiv} \neg P(c)$ Existential Instantiation

(11) $Q(c)$ using 2 and 10 Disjunctive Syllogism

(12) $S(c)$ using 5 and 11 Disjunctive Syllogism

(13) $\neg R(c)$ using 8 and 12 Modus Tollens

$\exists x \neg R(x)$ is True using 13

Existential Generalization

2.a

$$\frac{a_1 + a_2}{2} = \text{some number } \bar{a}$$

$$\frac{a_1 + a_2 + a_3}{3} = \text{some number } \bar{a}$$

$$\underbrace{\text{average}} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

Suppose $a_1 + a_2 + a_3 + \ldots a_n$ is less than $a \cdot n$

$$\frac{a_1 + a_2 + \ldots + a_n}{n} < \frac{a \cdot n}{n}$$

$$\frac{a_1 + a_2 + \ldots + a_n}{n} < a$$

$$\text{let } \bar{a} = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

$$\bar{a} < a$$

Using proof by contradiction, we have proved our claim which means there is at least one number greater or equal to $\bar{a}$.

2.b Suppose n is odd

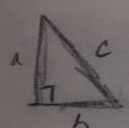
$n = 2k+1$

$3(2k+1)+2 = 6k+3+2 = 6k+5 = 6k+4+1 = 2(3k+2)+1$ let $l = 3k+2$

$= 2l+1$ for some number l

Using proof by contradiction we've proven our claim which means $3n+2$ is even.

2.c



$a^2 + b^2 = c^2$

Suppose $\sqrt{2} = a$

Suppose $\sqrt{2} = b$

$\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = c$

$\sqrt{2+2} = c$

$\sqrt{4} = c$

$2 = c \Leftarrow$ Rational Number

Utilizing a counterexample, we've disproved our claim which means the third side does not need to be irrational.

4 a. Suppose n is even

$n = 2k$

$(2k)^2 - 6(2k) + 5 =$

$4k^2 - 12k + 5 =$

$4k^2 - 12k + 4 + 1 =$

$4(k^2 - 3k + 1) + 1 =$

let $m = k^2 - 3k + 1$

$4m + 1$

Since $n^2 - 6n + 5$ is in an odd form $4m+1$, we've proven our claim that n is even using a Direct Proof.

b. Suppose n is even $\neg q \rightarrow \neg p$

$n = 2k$

$2(2k)^2 + 3(2k) + 1 =$

$2(4k^2) + 6k + 1 =$

$8k^2 + 6k + 1 =$

$(8k^2 + 6k) + 1 =$

$2k(4k+3) + 1 =$

let $m = 4k+3$ for some integer k

$2km + 1$ for some integer k and m

Since $2n^2 + 3n + 1$ is in the odd form $2km+1$, we've proven our claim using a Contrapositive proof.

3. $N = 100a + 10b + 1c = 3k \quad a+b+c = 3k$

$N = 100a + 10b + c = 3k$

$a+b+c = 3k$

$N = a+b+c + 99a + 9b = 100a + 10b + c$

$N = 3k + 99a + 9b$ let $k = n + 33a + 3b$

$N = 3(k + 33a + 3b)$

$N = 3k$

By Direct Proof, we have proven our claim that N is divisible by 3.

5. $|x-8|$

$x<8 \quad x>8$

$x=8$

Cases

$x + |x-8| \geq 8$

$\cancel{x} + 8 - \cancel{x} \geq 8$

$8 \geq 8$

Case 1

$x<8$

True under this assumption for all x less than 8

$x + |x-8| \geq 8$

$8 + |8-8| \geq 8$

$8 + |0| \geq 8$

$8 \geq 8$

Case 2

$x = 8$

True under this assumption for x equal to 8

$x + |x-8| \geq 8$

$x + x - 8 \geq 8$

$2x - 8 \geq 8$

$+8 \quad +8$

$\frac{2x}{2} \geq 16$

$x \geq 8$

Case 3

$x > 8$

True under this assumption for all x greater than 8