Discrete Structures CSCI 2824 – Spring 2019
Instructor : Rachel Cox
Author : Alexander Hawkins
1st March 2019

1.a. $P \times (Q \cap R) = (P \times Q) \cap (P \times R)$ Discrete Structures Alex Hawkins

Suppose the pairs (x, y) is an ordered pair of elements.

$(x, y) \in (P \times (Q \cap R))$

$x \in P \wedge y \in Q \cap R$ Def. Cartesian Product

$x \in P \wedge (y \in Q \wedge y \in R)$ Def. Intersection

$(x \in P \wedge y \in Q) \wedge (x \in P \wedge y \in R)$ Def. Idempotent Laws

$(x, y) \in (P \times Q) \wedge (x, y) \in (P \times R)$ Def. Cartesian Product

$(x, y) \in (P \times Q \cap P \times R)$ Def. Intersection

Thus, $P \times (Q \cap R) \subseteq (P \times Q) \cap (P \times R)$

1.a. $(P \times Q) \cap (P \times R)$

Suppose (x, y) is an ordered pair of elements.

$(x, y) \in (P \times Q \cap P \times R)$

$[(x, y) \in (P \times Q)] \wedge [(x, y) \in (P \times R)]$

$(x \in P) \wedge (y \in Q) \wedge (x \in P) \wedge (y \in R)$

$(x \in P) \wedge (y \in Q) \wedge (y \in R)$

$(x \in P) \wedge (y \in Q \cap R)$

$(x, y) \in (P \times Q \cap R)$

Thus, $(P \times Q) \cap (P \times R) \subseteq (P \times Q \cap R)$

1.b. $P \times (Q \cap R) = (P \times Q) \cap (P \times R)$

Suppose the pair (x, y) is an ordered pair of elements.

$P \times (Q \cap R) = \{(x,y) \mid (x,y) \in P \times (Q \cap R)\}$ Def. Cartesian Product

$= \{(x,y) \mid x \in P \wedge y \in (Q \cap R)\}$

$= \{(x,y) \mid x \in P \wedge y \in Q \wedge y \in R\}$

$= \{(x,y) \mid (x \in P \wedge y \in Q) \wedge (x \in P \wedge y \in R)\}$

Def. Idempotent Laws

$= \{(x,y) \mid [(x,y) \in (P \times Q)] \wedge [(x,y) \in (P \times R)]\}$

Def. Cartesian Product

$= \{(x,y) \mid (x,y) \in (P \times Q \cap P \times R)\}$

Def. Intersection

Thus, $P \times (Q \cap R) \subseteq (P \times Q) \cap (P \times R)$

Def. Subset

Discrete Structures CSCI 2824 – Spring 2019
Instructor : Rachel Cox
Author : Alexander Hawkins
1st March 2019

1.c.i. $A - B = \{x \mid x \in A \land x \notin B\}$ $\quad E$ = Set of all even integers

P = Set of all positive integers

$E - P = \{x \mid x \in E \land x \notin P\}$ Def. Difference

$= \{x \mid x \in E \land x \in \overline{P}\}$ Def. Complement

$x \in (E \cap \overline{P})$ Def. Intersection $\quad E = \{\ldots, -4, -2, 0, \cancel{2}, \cancel{4}, \ldots\}$

$\overline{P}$ = Set of all negative integers $\quad P = \{1, \cancel{2}, 3, \cancel{4}, \ldots\}$

$E - P = \{0, -2, -4, -6, -8, \ldots\}$

1.c.ii. $P \cup N =$ $\quad P$ = Set of all positive integers

$\{\ldots, -3, -2, -1, 1, 2, 3, \ldots\}$ $\quad N$ = Set of all negative integers

1.c.iii. $D - E = \{x \mid x \in D \land x \notin E\}$ $\quad D$ = Set of all odd integers

Def. Difference $\quad E$ = Set of all even integers

$\{x \mid x \in D \land x \in \overline{E}\}$ Def. Complement $\quad \overline{E}$ = Set of all odd integers

$x \in (D \cap \overline{E})$ Def. Intersection $\quad D - E = \{\ldots, -3, -1, 1, 3, \ldots\}$

1.c.iv. $\overline{U}$ = Set of all non-integers $\quad U$ = Set of all integers

$\overline{U} = \{\ \}$

2.a. i. finite $A - B = \mathbb{N} - B = \{0\}$ I = Irrationals

ii. countably infinite $A - B = \mathbb{R} - I = \mathbb{Q}$

iii. Uncountably infinite $A - D = (1, 2)$ {All uncountably infinite.

i. Suppose B = A set of all positive integers.

iii. $A = (1, 2)$

$D = (3, 4)$

2.b.

3.1111111...
3.1100110...
3.1001001...
3.0101101...

$$b_i = \begin{cases} 0 & \text{if } a_{ii} \neq 0 \\ 1 & \text{if } a_{ii} = 0 \end{cases}$$

$$M = 3.0010\ldots$$

Since the number of each index is not equivalent to the number at each diagonal step, the interval $[3, 4]$ is uncountable. The new number M is real and unique and we can repeat M infinite times. Therefore, there are infinite numbers between the interval $[3, 4]$.

2.c. $\mathbb{R} = \mathbb{I} + \mathbb{Q}$

$\mathbb{R}$ = Real numbers (Uncountably infinite)
$\mathbb{I}$ = Irrational numbers (Uncountably infinite)
$\mathbb{Q}$ = Rational numbers (Countably infinite)

Suppose $\mathbb{I}$ is countably infinite.

Countably infinite ($\mathbb{I}$) + Countably infinite ($\mathbb{Q}$) will result to a countably infinite set. However, $\mathbb{R}$ is Uncountably infinite. Therefore, we have a contradiction.

3. a) $a_n = 2^n a_{n-n} - 2^n - 2^{n-1} - 2^{n-2} \dots - 2^3 - 2^2 - 2$

$= 2^n a_0 - 2^n - 2^{n-1} - 2^{n-2} \dots - 2^3 - 2^2 - 2$

$= -2^n - 2^n - 2^{n-1} - 2^{n-2} \dots - 2^3 - 2^2 - 2$

$= -2^n - (2^n + 2^{n-1} + 2^{n-2} + \dots + 2^3 + 2^2 + 2)$

$= -2^n - \underbrace{\sum_{i=1}^{n} 2^i}_{\text{Geometric}}$

3.c. $a_n = 5(-1)^n - n + 2$

$a_{n-1} = 5(-1)^{n-1} - (n-1) + 2$

$a_{n-2} = 5(-1)^{n-2} - (n-2) + 2$

Goal: $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$

$a_{n-1} + 2a_{n-2} + 2n - 9 =$

$(5[-1]^{n-1} - [n-1] + 2) +$

$2(5[-1]^{n-2} - [n-2] + 2) + 2n - 9 =$

$5[-1]^{n-1} - [n-1] + 2 +$

$10[-1]^{n-2} - 2[n-2] + 4 + 2n - 9 =$

$5[-1]^{n-1} - n + \cancel{1} + 2 + 10[-1]^{n-2}$

$- \cancel{2n} + \cancel{4} + \cancel{4} + \cancel{2n} - \cancel{9} =$

$5[-1]^{n-1} + 10[-1]^{n-2} - n + 2 =$

$-5(-1)^{n-2} + 10(-1)^{n-2} - n + 2 =$

$5(-1)^n - n + 2 = a_n$

3.b. $a_n = (n+2)\, a_{n-1}$

$a_{n-1} = (n+2)(n+1)\, a_{n-2}$

$a_{n-2} = (n+2)(n+1)(n)\, a_{n-3}$

$a_{n-3} = (n+2)(n+1)(n)(n-1)\, a_{n-4}$

$a_{n-4} = (n+2)(n+1)(n)(n-1)(n-2)\, a_{n-5}$

$a_{n-n} = (n+2)(n+1) \dots 5 \cdot 4 \cdot 3\, a_{n-n}$

$= \dfrac{(n+2)!}{2} \cdot a_0$

$a_0 = 3$

Discrete Structures CSCI 2824 – Spring 2019
Instructor : Rachel Cox
Author : Alexander Hawkins
1st March 2019

4.a. Suppose $n = -y$

$$f(0,-y) = 2(0)-(-y)$$
$$f(0,-y) = y$$

ex.

$f(0,-2) = 2$ $\quad$ $f(0,1) = -1$

$f(0,-1) = 1$ $\quad$ $f(0,2) = -2$

$f(0,0) = 0$ $\quad$ $f(0,3) = -3$

Since every integer in the output maps to some integer, the function ($f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $f(m,n) = 2m-n$) is onto.

4.b. $f(m,n) = m^2 - n^2$

$$f(-2,-2) = (-2)^2 - (-2)^2 = -4-4 = 0$$
$$f(3,-1) = 3^2 - (-1)^2 = 9+1 = 8$$
$$f(5,3) = 5^2 - 3^2 = 25-9 = 14$$
$$f(-4,4) = (-4)^2 - 4^2 = 16-16 = 0$$

Since every integer in the output maps to some integer, the function ($f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ where $f(m,n) = m^2 - n^2$) is onto.

4.c. Function: The set of all residents of Colorado have their preferred residency. Each Colorado resident has some sort of residency (onto) but some residents may be in the same residency (not one-to-one).

Discrete Structures CSCI 2824 – Spring 2019
Instructor : Rachel Cox
Author : Alexander Hawkins
1st March 2019

4.d. Function: The set of all residents of Colorado have their own Social Security Number. This function is one-to-one because for each resident, each resident has their own unique Social Security Number. The function is also onto because every Social Security Number (in this function) maps to a resident.