Advances in Engineering Software 195 (2024) 103694

Contents lists available at ScienceDirect

Advances in Engineering Software

journal homepage: www.elsevier.com/locate/advengsoft

Arctic puffin optimization: A bio-inspired metaheuristic algorithm for solving engineering design optimization☆

Wen-chuan Wang *, Wei-can Tian, Dong-mei Xu, Hong-fei Zang

College of Water Resources, North China University of Water Resources and Electric Power, Zhengzhou 450046, PR China

ARTICLE INFO

Keywords:
Arctic puffin optimization
Metaheuristic optimization algorithm
Engineering design
Constrained optimization
Swarm intelligence

ABSTRACT

In this paper, we innovatively propose the Arctic Puffin Optimization (APO), a metaheuristic optimization algorithm inspired by the survival and predation behaviors of the Arctic puffin. The APO consists of an aerial flight (exploration) and an underwater foraging (exploitation) phase. In the exploration phase, the Levy flight and velocity factor mechanisms are introduced to enhance the algorithm's ability to jump out of local optima and improve the convergence speed. In the exploitation phase, strategies such as the synergy and adaptive change factors are used to ensure that the algorithm can effectively utilize the current best solution and guide the search direction. In addition, the dynamic transition between the exploration and development phases is realized through the behavioral conversion factor, which effectively balances global search and local development. In order to verify the advancement and applicability of the APO algorithm, it is compared with nine advanced optimization algorithms. In the three test sets of CEC2017, CEC2019, and CEC2022, the APO algorithm outperforms the other compared algorithms in 72%, 70%, and 75% of the cases, respectively. Meanwhile, the Wilcoxon signed-rank test results and Friedman rank-mean statistically prove the superiority of the APO algorithm. Furthermore, on thirteen real-world engineering problems, APO outperforms the other compared algorithms in 85% of the test cases, demonstrating its potential in solving complex real-world optimization problems. In summary, APO proves its practical value and advantages in solving various complex optimization problems by its excellent performance.

1. Introduction

Optimization aims to find the optimal solution by maximizing or minimizing the output of the objective function under relevant constraints. As the complexity of real-world engineering problems continues to increase, the demand for optimization techniques becomes increasingly apparent. Traditional optimization methods such as gradient descent and the simplex method [1,2], while capable of providing exact solutions under specific conditions, often prove inadequate when faced with increasing problem complexity, diverse constraints, and multiple peaks. These methods require a prior understanding of the problem characteristics for effective solutions, and when dealing with large-scale problems, they consume significant computational resources. Moreover, these approaches are susceptible to the challenge of local optima, making it difficult to find the global optimum. This limitation renders such methods inadequate for addressing the demands of today's complex engineering problems.

In recent years, the continuous emergence of metaheuristic algorithms inspired by biological or physical phenomena has taken place [3-5]. Compared to traditional methods, metaheuristic algorithms have simple concepts and frameworks, eliminating the need for gradient update information. They can effectively find optimal solutions for highly complex constraints within a reasonable timeframe [6]. Consequently, these methods are better suited for addressing the intricate challenges in engineering optimization problems and have been widely applied to numerous practical engineering issues, including industrial engineering problems, feature selection, multi-level image segmentation, community detection, image classification, and other domains [7-18].

A well-designed metaheuristic algorithm should strike a balance between two primary features: global exploration and local exploitation. This equilibrium represents the algorithm's need for extensive exploration in the search space while seeking global optimal solutions. As valuable solutions are discovered during the search, the algorithm

☆ The source code of APO is publicly available at https://www.mathworks.com/matlabcentral/fileexchange/167521-arctic-puffin-optimization-apo.

* Corresponding author.
E-mail address: wangwenchuan@ncwu.edu.cn (W.-c. Wang).

https://doi.org/10.1016/j.advengsoft.2024.103694
Received 29 February 2024; Received in revised form 22 May 2024; Accepted 31 May 2024
Available online 14 June 2024

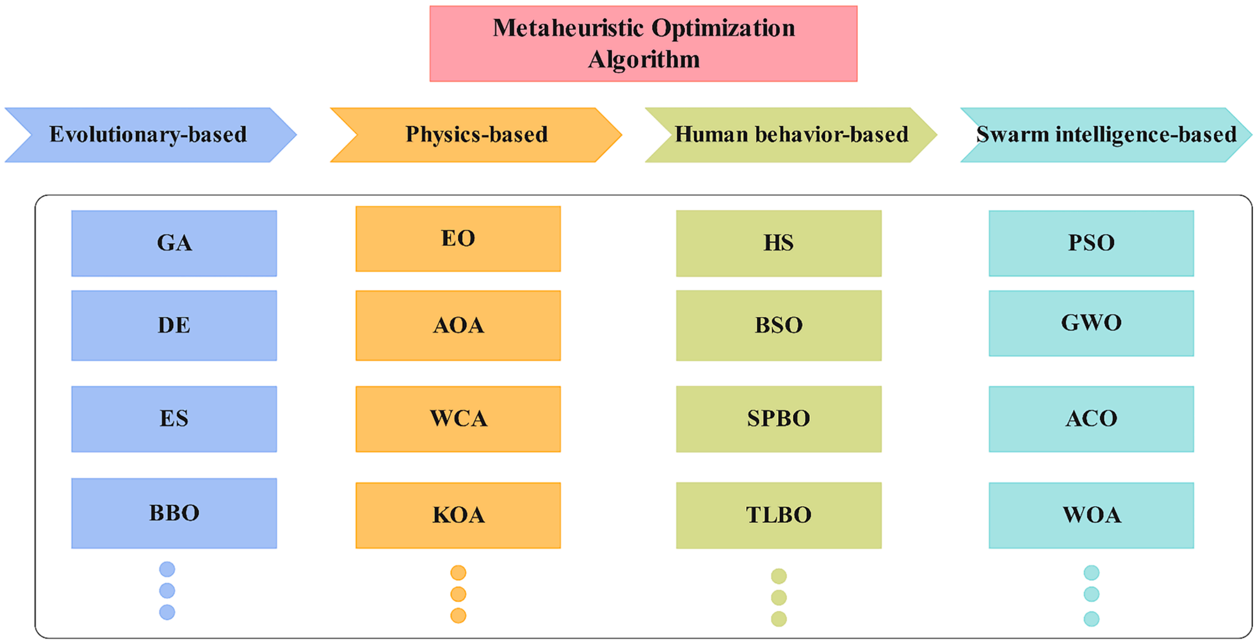

Fig. 1. Metaheuristic algorithm classification.

should delve into those regions to achieve local optimal solutions. This raises a critical question: How can we balance exploration and exploitation to make the algorithm more adept at escaping local optima, avoiding premature convergence, and converging rapidly to the global optimum? These characteristics become particularly crucial in optimization tasks involving multiple peaks and constraints. Metaheuristic algorithms need to flexibly explore the search space globally to discover potential solutions. Once valuable solution regions are identified, the algorithm should focus on local exploitation, intensifying the search for optimal local solutions. Such a balance is paramount for dealing with problem complexity and enhancing the algorithm's adaptability across diverse problem scenarios.

Although there are currently various optimization algorithms available, the continuous development of new metaheuristic algorithms is necessary due to the "no free lunch" (NFL) theorem [19]. This theorem suggests that the performance of different algorithms varies when solving optimization problems with different characteristics. A single algorithm cannot exhibit optimal performance across all problems. Furthermore, even if an algorithm shows improvement in one type of problem, its performance gains may be offset by another type of problem. This drives researchers to continuously seek out and develop new metaheuristic algorithms to meet the optimization requirements of different problem types and address the shortcomings of existing algorithms.

Therefore, based on the motivation above, this paper proposes the APO algorithm, a novel metaheuristic optimization algorithm. The algorithm simulates the behavior of puffins in two stages: aerial flight and underwater foraging, aiming to achieve a better balance between exploration and exploitation. The algorithm comprises several key operations, including aerial search and plunge predation during the aerial flight stage and gathering foraging, intensified search, and predator evasion during the underwater foraging stage. Through these steps, the algorithm is designed to explore the search space purposefully, striking a better balance between exploration and exploitation, thus avoiding falling into local optima. The specific contributions and innovations of this paper are outlined below.

- For the global optimization problem, this paper proposes a new APO algorithm for various optimization scenarios.
- In the aerial flight phase (exploration), introducing Levy flights and velocity factors improves the algorithm's ability to jump out of the local optimum. It increases its probability of finding a better solution in high-dimensional complex optimization problems.
- In the underwater foraging phase (exploitation), Levy flights, synergy factors, and adaptive change factors are used to ensure that the algorithm can effectively utilize the current best solution and guide the search direction.
- Establishing the behavioral conversion factor B with adaptive properties effectively balances the dynamic relationship between exploration and exploitation.
- The performance of the APO algorithm is validated using CEC2017, CEC2019, and CEC2022 functions, a total of 51 challenging benchmark functions, and compared and analyzed with nine well-known and advanced optimization algorithms.
- Comprehensive statistical analyses using two nonparametric tests, the Friedman rank-mean test, and the Wilcoxon signed-rank test, are performed to validate the robustness and superiority of the algorithm on different problem domains.
- Thirteen complex real-world engineering optimization problems in four domains are solved to assess and verify the generalizability and robustness of APO in a wide range of optimization scenarios.

The rest of the paper is outlined as follows: in Section 2, a review of the state-of-the-art literature on metaheuristic algorithms is presented, and an introduction to optimization algorithms in the field of improvement is provided. Through a comprehensive survey of existing research, this section provides the reader with an in-depth and comprehensive understanding of the status and trends in the field. Furthermore, Section 3 delves into a detailed description of the proposed APO algorithm. This includes an explanation of the algorithm's principles, key design elements, and how it mimics the habits of Arctic puffins to achieve a balance between global exploration and local exploitation. In addition, Section 4 discusses experimental data and test results from three test sets. Through data analysis, this section demonstrates the algorithm's performance in different scenarios and explores its superiority. Section 5 details the solution process of thirteen real-world optimization problems, using case studies to highlight the algorithm's effectiveness in solving real-world problems. Finally, Section 6 summarizes the paper's conclusions and provides insights into future

Table 1
A review of popular optimization algorithms.

Type	Algorithm	Inspiration	Advantage	Disadvantage
Evolutionary-based	GA [20]	Natural selection and genetic evolution.	Powerful exploration phase.	Poor exploitation phase.
	DE [21]	The theory of biological evolution.	Production solutions have a perfect variety.	Poor intensification component.
	ES [22]	The theory of biological evolution.	Having good parallelism and real-time performance.	Weak search capability.
	BBO [23]	The theory of biogeography.	Simplicity and efficiency	Sensitive to the curse of dimensionality
	EP [24]	The theory of biological evolution.	Capable of strong search ability to handle multi-objective problems.	Difficult parameter selection, long runtime.
Physics-based	EO [25]	Mass balance equation.	Strong global search capability, suitable for both continuous and discrete problems.	Unable to effectively balance exploration and exploitation stages.
	AOA [26]	Archimedes' principle.	Good robustness and global search capability.	Slow convergence speed.
	AOA [27]	Arithmetic operators are used in mathematical calculations.	Few parameters are capable of solving multi-dimensional problems.	It is easily trapped in local optima.
	WCA [28]	The water cycle process.	High computational efficiency and strong global optimization capability.	The algorithm is prone to getting stuck in local optima in later stages.
	KOA [29]	Kepler's Laws of Planetary Motion.	Capable of quickly converging to the global optimal solution.	The search capability is insufficient for complex nonlinear problems.
	BH [30]	Black hole phenomenon.	High optimization precision.	Prone to initialization influence.
	GSA [31]	Newton's laws of gravity and motion.	Good at finding the global optimum.	Slow convergence speed.
	YDSE [32]	Young's double-slit experiment.	Strong global search capability.	Many parameter configurations.
Human behavior-based	HS [33]	Impromptu composition with a music player.	Simple to implement, flexible.	Difficult parameter tuning.
	BSO [34]	The human brainstorming processes.	Strong practicality.	Lack of adaptability, easily influenced by the problem.
	SPBO [35]	Based on students' psychology.	Strong convergence ability	It is easy to fall into a local search.
	TLBO [36]	The impact of teachers on learners' output.	Strong convergence capability.	It is easy to fall into a local search.
	PO [37]	The multi-stage process of politics.	Strong global search capability.	Large computational load, slow convergence speed.
	ICA [38]	Inspired by imperial competitive behavior.	Strong global search capability.	Difficult parameter configuration.
Swarm intelligence-based	PSO [39]	The natural behavior of particle swarms.	Good ability of the intensification component.	Sensitivity to parameters
	GWO [6]	The leadership hierarchy and hunting mechanisms of gray wolves.	Strong convergence performance with few parameters.	Slow convergence speed in later stages.
	ACO [40]	Ants' behavior of releasing pheromones when searching for food.	Powerful exploitation phase.	Falling into the trap of local optimality in some problems
	WOA [41]	The feeding behavior of sperm whales.	Simplicity, efficiency, and static parameter-free.	It is easy to fall into the local optimum.
	HHO [42]	The cooperative behavior and hunting tactics of Harris's hawks.	There are a few parameters, simple and easy to implement.	Poor search capability on low-dimensional problems.
	BWO [43]	The life behaviors of beluga whales.	The structure is simple and easy to implement.	Difficulty in parameter setting.
	MSA [44]	The unique hunting behavior and cannibalistic behavior of praying mantises.	Strong search capability.	Weak search capability.
	ARO [45]	The survival strategies of rabbits.	Strong optimization capability and fast convergence speed.	Lack of robustness.
	GEO [46]	The instinct of hunting in golden eagles.	Strong global search capability.	Slow convergence speed.
	SMA [47]	The oscillation pattern of slime molds in the natural world.	Strong natural inspiration.	Difficult parameter selection, long runtime.
	PO [48]	The intelligence and life of Pumas.	Effective avoidance of local optima and an acceptable convergence rate.	Slow convergence speed in later stages.

research directions.

2. Related works

In many cases, metaheuristic algorithms have emerged as the preferred choice for solving optimization problems, especially when dealing with complex, intractable problems characterized by highly intricate search spaces that lack known polynomial time complexity algorithms. Metaheuristic algorithms are often designed based on natural phenomena. According to their internal mechanisms, these algorithms can be classified into several categories, as depicted in Fig. 1, mainly evolutionary-based, physics or mathematics-based, human behavior-based, and swarm intelligence-based algorithms. The following is a brief overview of these categories.

2.1. Evolutionary-based algorithms

Evolutionary metaheuristic optimization algorithms are designed based on the principles of biological evolution, simulating the processes of species evolution and genetic inheritance to iteratively optimize and search for the best solutions. In addition to representative algorithms such as genetic algorithms (GA) [20] and differential evolution algorithm (DE) [21], this category also includes evolutionary strategies (ES) [22], biogeography-based optimizers (BBO) [23], evolutionary programming (EP) [24], etc.

2.2. Physics-based algorithms

Algorithms in this category are inspired by the imitation of physical or mathematical principles, drawing inspiration mainly from the laws of motion in physics and mathematical models, providing insights for solving complex problems. Examples include equilibrium optimizer (EO) [25], archimedes optimization algorithm (AOA) [26], arithmetic optimization algorithm (aoa) [27], water cycle algorithm (wca) [28], kepler optimization algorithm (koa) [29], black hole algorithm (bh) [30], gravitational search algorithm (GSA) [31], Yang's Double-Slit Algorithm (YDSE) [32], etc.

2.3. Human behavior-based algorithms

Algorithms in this category draw inspiration from human behavior and social interaction. For example, harmony search (HS) [33] simulates the beauty of music and harmony, achieving global optimality by adjusting pitches. Brainstorm Optimization (BSO) takes inspiration from teamwork and creativity stimulation, simulating the brainstorming process to optimize the search [34]. Additionally, there are optimization

Fig. 2. Morphology of Arctic Puffins.

algorithms based on Student Psychology (SPBO) [35], Teaching–learning-based Optimization (TLBO) [36], Political Optimization (PO) algorithm [37], Imperialist Competitive Algorithm (ICA) [38], etc. These algorithms, based on human social behavior, offer unique inspiration for problem-solving.

2.4. Swarm intelligence-based algorithms

Swarm intelligence-based algorithms imitate collective behavior in nature, attempting to simulate the social and collaborative behavior of animals, insects, or birds. For instance, particle swarm optimization (PSO) emulates the foraging process of birds, where particles represent potential solutions and move based on their performance [39]. The Gray Wolf Algorithm (GWO) was developed as an optimized search method inspired by the activities of gray wolves in searching and hunting for prey [6]. ant colony optimization (ACO) is based on the behavior of ants releasing pheromones while searching for food, enabling the entire colony to find the optimal path [40]. The whale optimization algorithm (WOA) mimics the feeding behavior of humpback whales to adjust the search [41]. There are also the harris hawk algorithm (HHO) [42], beluga whale optimization (BWO) [43], mantis search algorithm (MSA) [44], artificial rabbit optimization (ARO) [45], golden eagle optimization (GEO) [46], slime mould algorithm (SMA) [47], puma optimizer algorithm (PO) [48], etc. These algorithms, based on collective behavior, draw inspiration from the intelligence of natural biological populations, providing a collective approach to problem-solving.

In order to provide a clearer overview of the properties of the above optimization algorithms, Table 1 reviews the advantages and disadvantages of each algorithm. Based on this overview, it can be observed that each optimization algorithm has different advantages and disadvantages. Among them, the lack of balance between the exploration and development phases, the tendency to fall into the local optimum dilemma, and the performance degradation on high-dimensional problems are still common problems appearing in the existing algorithms.

In order to better address these issues, in addition to proposing new optimization algorithms, researchers have been trying to propose improved versions of the algorithms in other areas and review them in this regard [49-54]. For example, Farhad Soleimanian Gharehchopogh analyzes the manta ray foraging optimization (MRFO) algorithm and its integration in different academic fields [55]. Hoda Zamani reviews and analyzes the Moth Flame Optimization algorithm and its variants to demonstrate the structural review, performance evaluation, and statistical analysis needed to improve the metaheuristic algorithm [56]. During this time, other researchers have proposed various effective mechanisms to improve the algorithm's overall performance [57-64].

Based on these studies, optimization algorithms can introduce a range of effective mechanisms to enhance their optimization-seeking performance. These mechanisms include increasing the diversity and randomness of the output solutions, accelerating the convergence to the optimal solution to explore the whole optimization space faster, and so on. Therefore, excellent optimization algorithms should introduce reasonable and practical intelligent mechanisms and strategies within a reasonable timeframe to balance the exploration and development phases of the algorithms to enhance their comprehensive performance further.

Synthesizing the new metaheuristic algorithms and their improved versions proposed in recent years, although they perform well in solving complex problems, they still have the potential for further development. By reviewing the related literature, these studies have motivated us to propose a newer and better-performing algorithm to cope with complex engineering problems in different domains and to find more effective solutions. Therefore, the APO algorithm proposed in this paper, as a supplement to the existing algorithms and a new direction of exploration, is expected to bring more in-depth development and breakthroughs in optimization algorithms.

3. Arctic puffin optimization

3.1. Inspiration

The Arctic puffin is a rare and small bird endemic to the Arctic. They normally inhabit the oceans, and these birds usually forage in groups or flocks with excellent fishing skills and flight abilities and work collaboratively to forage in the ocean [65]. Arctic puffins prefer to fly in clusters at the edge of the ocean, swimming at the surface and feeding on fish. Compared to other seabirds, the Arctic puffin has short, stubby wings and maintains flight at low altitudes by flapping its tiny wings at a high frequency. They can reach speeds of up to 88 kms per hour while airborne, flapping their wings four hundred times per minute, and can target food and dive for it in flight [66].

Moreover, the Arctic puffin maintains a remarkable balance between aerial flight and foraging in the water, as shown in Fig. 2. Their wings act as double paddles in swimming, serving as a power tool for fast diving in the water. Arctic puffins are exceptionally powerful predators, catching at least ten small fish per dive. Before diving, they use coordinated feeding behaviors, working with each other to increase their hunting efficiency while watching for behavioral signals from other puffins to find the best areas to catch. When diving, Arctic puffins utilize their uniquely designed beak with a rough tongue and an upper beak covered in sharp spines, allowing them to quickly grab prey. Immobilizing the prey in their mouths with the spines, they then continue to hunt and extend their dive time. In addition, when food resources are scarce around them, the Arctic puffin will flexibly adjust its underwater position to find more food sources and ensure its hunting efficiency.

The Arctic puffin prefers to stay in groups at sea or during flight activities. This collective and large-scale mixed lifestyle allows them to better survive in the Arctic. During their adventures in the Arctic, Arctic puffins will inevitably encounter danger. Once an enemy is spotted, the puffins will sound a warning and quickly fly in a flock through the air [67].

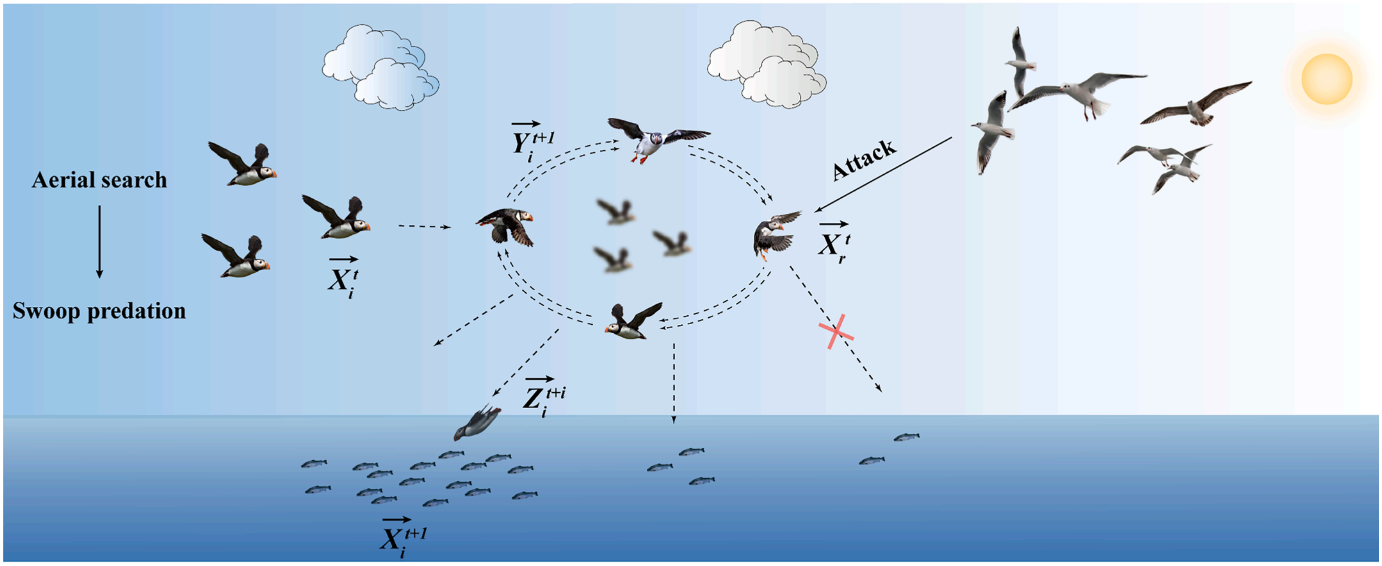

Fig. 3. Aerial flight stage of Arctic Puffins.

The APO algorithm originated as a survival strategy for the Arctic puffin in nature. It is briefly summarized below.

- The Arctic puffin uses formations or groups during flight to coordinate flight and improve flight efficiency and cooperative hunting opportunities.
- During flight, when large amounts of food resources are detected, Arctic puffins will quickly redirect their flight and use a dive-feeding strategy to capture prey quickly.
- On the water surface, Arctic puffins often round up schools of fish in a group-assisted manner to improve hunting efficiency. They also observe the behavior of other members to identify dive hotspots or food resources.
- Arctic puffins dynamically adjust their underwater position to find additional food sources when food resources are depleted around them.
- When a danger, such as a predator, is detected nearby, the Arctic puffin will quickly change its position and send a message to its conspecifics to avoid the danger.
- Notably, the randomness of Arctic puffin behavior underscores the motivation for the algorithm's design, as these five behaviors occur randomly in each individual in each iteration.

In summary, the specific behaviors of Arctic puffins in the air and on the water surface, including collective searching and swooping to feed during the flight phase, aggregating to forage, intensifying searching, and avoiding hazards during the underwater phase, form the basis for the design of the APO algorithms for solving the global optimization problem.

3.2. Mathematical model and basic principles

In this section, inspired by the survival behaviors of the Arctic Puffin, the APO algorithm is proposed. The mathematical model of APO consists of three main stages: population initialization, the aerial flight stage (exploration), and the underwater foraging stage (exploitation). Additionally, it involves the transition between these two strategies, induced by the behavioral transformation factor B of the puffin. The mathematical models for each stage are described as follows.

3.2.1. Initial population

Arctic puffins exhibit strong collectivism, whether during migration or at their habitat, always moving in groups and collaborating. Each Arctic Puffin represents a potential solution participating in the optimization. The generation process of initializing the population is described by the following equation:

$$\overrightarrow{X_i^t} = rand * (ub - lb) + lb, i = 1, 2, 3 \ldots N \tag{1}$$

where $\overrightarrow{X_i^t}$ represents the position of the i th Arctic Puffin; $rand$ generates a random number between 0 and 1; ub and lb represent the upper and lower bounds, respectively; N is the number of individuals in the population.

3.2.2. Aerial flight stage (Exploration)

Arctic puffins rely on unique strategies of flight and foraging to navigate their challenging existence. In their daily lives, they must adapt flexibly between the ocean and the air, meeting their nutritional needs and adjusting to diverse environments. During aerial navigation, puffins employ two crucial strategies to address distinct situations, as illustrated in Fig. 3. The first strategy involves aerial searching, while the second strategy involves plunge diving for prey. The varied behavioral strategies exhibited in these two stages demonstrate the adaptability of Arctic puffins in diverse scenarios, enabling their successful survival and reproduction. Subsequently, this paper will provide a detailed examination of these two strategies and elucidate their significance in the lives of Arctic puffins.

- **The first strategy is aerial search.**

Arctic puffins typically engage in coordinated flight in formations or groups, a collaborative action that enhances flight efficiency and opportunities for cooperative hunting. They maintain relatively low flying altitudes to facilitate the capture of potential underwater food resources. During this phase, they concentrate on scouting potential prey while remaining vigilant for potential predators in the vicinity. Under favorable conditions, or when predators are scarce, and fish populations are abundant, they adeptly accelerate towards the water surface to better capture their prey. The following are the position update equations associated with this strategy.

$$\overrightarrow{Y_i^{t+1}} = \overrightarrow{X_i^t} + \left(\overrightarrow{X_i^t} - \overrightarrow{X_r^t}\right) * L(D) + R \tag{2}$$

$$R = round(0.5 * (0.05 + rand)) * \alpha \tag{3}$$

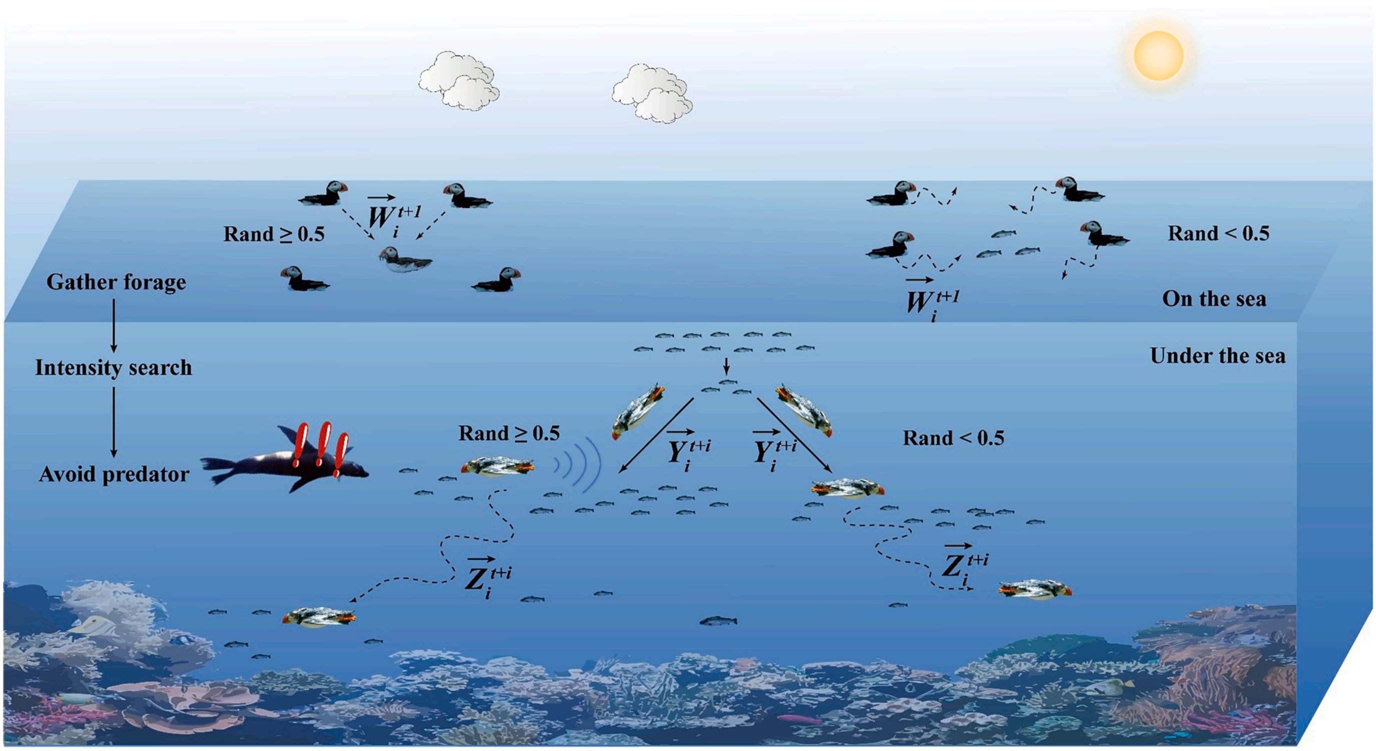

Fig. 4. Underwater foraging stage of Arctic Puffins.

$$\alpha \sim Normal(0, 1) \tag{4}$$

where r is a random integer between 1 and $N - 1$, excluding i; $\overrightarrow{X_i^t}$represents the current i th candidate solution in the population; $\overrightarrow{X_r^t}$ is a candidate solution randomly selected from the current population, with$\overrightarrow{X_i^t} \neq \overrightarrow{X_r^t}$; $L(D)$ denotes a random number generated through Levy flight; D is the dimensionality; αis a random number following a standard normal distribution.

During the aerial search strategy, Arctic puffins utilize the Levy flight coefficient, akin to their powerful wings, to alter their positions. Levy flight is renowned for its characteristic long-distance jumps, enabling puffins to efficiently explore food resources [68-71]. This flight search strategy aids them in rapidly covering vast oceanic areas, consistently maintaining proximity to potential food sources, and effectively searching for food resources. Simultaneously, when confronted with aerial predators like seagulls, puffin employ a rotational flight tactic to defend against adversaries, making it challenging for attackers to find a breakthrough. Thus, during the aerial search strategy for puffin, the crucial strategy is to identify suitable fishing locations. The flight strategy during this phase enhances the algorithm's global search capabilities, allowing it to better explore the problem space and increasing the chances of discovering the global optimal solution.

- **The second strategy is swooping predation.**

Swooping is a crucial strategy during Arctic puffin predation, as they rapidly redirect their flight to speed up food capture. This swooping strategy is essential in the face of other competitors, as they must ensure faster and more successful prey capture to survive. To model this swooping behavior, we introduced a velocity coefficient S to adjust the displacement of the puffin during the swoop. The following is the position update equation.

$$\overrightarrow{Z_i^{t+1}} = \overrightarrow{Y_i^{t+1}} * S \tag{5}$$

$$S = \tan((rand - 0.5) * \pi) \tag{6}$$

In this flight strategy, the Arctic puffin adjusts its displacement in the first stage by introducing a velocity coefficient S, where π is a mathematical constant equal to approximately 3.14. S plays a key role in regulating the speed and direction of Arctic puffin flight. Mathematically, S is a velocity coefficient that allows the puffin to flexibly adapt to different feeding needs by adjusting the magnitude and direction of its flight speed. With the parameter S, the algorithm more closely matches the aerial behavior of the puffin, making it more flexible in the face of competition and uncertainty and thus adapting to a more complex aerial environment. In addition, introducing the distributional properties of parameter S increases the algorithm's randomness and diversity and enhances the exploration ability of the Arctic puffin. As a result, the setup of the swooping predation phase further improves the search efficiency of the algorithm in the solution space. It enhances the adaptability and search ability of the algorithm when dealing with different contexts.

In general, within the flight stage of the APO, two key behavioral strategies are employed to enhance the algorithm's exploration process. The algorithm relies on the Levy flight coefficient to promote global search in the aerial flight strategy. By introducing the Levy flight coefficient, the algorithm introduces greater randomness and diversity, expanding the breadth of exploration. This strategy enables the APO to comprehensively explore the solution space, enhancing its broad search capability for potential solutions. Upon discovering suitable hunting grounds, the algorithm employs a swooping predation strategy to locate potential solutions through faster searches. The swooping predation strategy emphasizes an in-depth exploration of potential solutions in a shorter time, strengthening the algorithm's exploration capability. The APO rapidly and accurately focuses on possible optimal solutions through swooping predation, improving the algorithm's exploration speed and accuracy, and facilitating the search for high-quality solutions in the solution space.

To achieve optimal results in various scenarios, the algorithm chooses to merge candidate positions generated in both stages into a new solution. These solutions are then sorted by fitness, and the top N individuals are selected to form the new population. The equations are described as follows:

$$\overrightarrow{P_i^{t+1}} = \overrightarrow{Y_i^{t+1}} \cup \overrightarrow{Z_i^{t+1}} \tag{7}$$

$$new = sort\left(\overrightarrow{P_i^{t+1}}\right) \tag{8}$$

$$\overrightarrow{X_i^{t+1}} = new(1:N) \tag{9}$$

where *sort* is to sort the new population according to their fitness values, from small to large, to select the new population $\overrightarrow{X_i^{t+1}}$.

3.2.3. Underwater foraging stage (Exploitation)

The survival strategy of the Arctic puffin involves two crucial aspects: aerial flight and underwater foraging. Underwater foraging consists of three main strategies, each employed in specific contexts to enhance predation efficiency. These three strategies are gathering foraging, intensifying search, and avoiding predators, as illustrated in Fig. 4. The following sections will provide a detailed overview of these stages and explore their pivotal roles in the life of the Arctic Puffin.

- **Gathering foraging is the first underwater strategy.**

In the foraging behavior of Arctic puffins, they often adopt a collective strategy by gathering around schools of fish near the water's surface. This cooperative predation behavior enhances the efficiency and success rate of hunting. Cooperative foraging enables them to collaborate effectively, collectively surrounding and capturing schools of fish, thereby increasing the chances of successful predation. Moreover, puffins staying on the sea surface observe the behavior of other members to identify diving hotspots or food resources. The following equation describes the position update.

$$\overrightarrow{W_i^{t+1}} = \begin{cases} \overrightarrow{X_{r1}^t} + F * L(D) * \left(\overrightarrow{X_{r2}^t} - \overrightarrow{X_{r3}^t}\right) rand \geq 0.5 \\ \overrightarrow{X_{r1}^t} + F * \left(\overrightarrow{X_{r2}^t} - \overrightarrow{X_{r3}^t}\right) rand < 0.5 \end{cases} \tag{10}$$

where F represents the cooperative factor, adjusting the predation behavior of Arctic puffins. In this paper, $F = 0.5$. The variables r_1, r_2, r_3 are random integers between 1 and $N - 1$ (excluding i), and $\overrightarrow{X_{r1}^t}$, $\overrightarrow{X_{r2}^t}$, $\overrightarrow{X_{r3}^t}$ are candidate solutions randomly selected from the current population, and $r1 \neq r2 \neq r3, \overrightarrow{X_{r2}^t} \neq \overrightarrow{X_{r3}^t}$.

Two position equations describe the gathering foraging strategy to depict foraging behavior. In these equations, when $rand<0.5$, Arctic puffins engage in cooperative foraging with other puffins, utilizing the cooperative factor F to explore the surrounding environment through random movements, searching for food or other resources. The other equation, when $rand\geq 0.5$, represents a more complex food-seeking strategy. During the gathering foraging process, a puffin may follow other members and swiftly swim upon detecting a school of fish, altering its position to join a more advantageous predation group for better cooperative foraging. Hence, the Levy flight factor is employed again to enhance the puffin's ability for rapid predation, ensuring effective prey capture. This dual-strategy approach helps balance exploration and exploitation during optimization, contributing to a more comprehensive search of the solution space.

In the APO algorithm, the parameter F is designed as a synergistic factor, inspired by the collaboration and team predation exhibited by Arctic puffins in their foraging behavior. Arctic puffins often gather collectively around fish near the surface of the water and adopt an aggregation foraging strategy. This synergistic foraging behavior greatly improves the foraging efficiency and success rate. Therefore, the introduction of parameter F makes the algorithm closer to the behavioral patterns of animals in nature by simulating this collective, collaborative behavior. At the same time, a reasonable setting of the F value can enhance the diversity of the algorithm's solutions and development capability, significantly improving the algorithm's performance and efficiency by effectively balancing the exploration and development process of the algorithm. Through the sensitivity analysis of F, $F = 0.5$ was identified as a more optimal value. We discuss the sensitivity analysis of the parameters in detail later in the content.

- **Intensifying search is the second underwater strategy.**

As predation progresses, Arctic puffins may sense a depletion or exhaustion of food resources in their current foraging area after a certain period. To continue meeting their nutritional needs, they must alter their underwater positions to search for more fish or other underwater food sources. The position update equation for this stage is as follows:

$$\overrightarrow{Y_i^{t+1}} = \overrightarrow{W_i^{t+1}} * (1+f) \tag{11}$$

$$f = 0.1 * (rand - 1) * \frac{(T-t)}{T} \tag{12}$$

where T represents the total number of iterations, and t denotes the current iteration count. *Rand* is a random number that introduces some randomness to f. Eq. (11) describes this stage, representing variations built upon the initial position. In the second stage of the enhanced search process, the parameter $(1 + f)$ plays a key role, where f is an adaptive factor used to adjust the position of the Arctic puffin in the water. The design of this adaptive factor is inspired by the flexibility of puffins in adapting to their environment during foraging. Specifically, as the number of iterations increases, the parameter f is gradually adjusted so that the puffin can decide whether to change its position based on the progress and randomness of the search to find more abundant food resources.

This strategy ensures that Arctic puffins can flexibly adapt to the distribution of food in their environment, meeting their survival needs during predation. The algorithm dynamically explores the search space to find new solutions, contributing to its adaptability and developmental capabilities in the face of evolving problems and novel solutions.

- **Avoiding predators is the third underwater strategy.**

This strategy is employed to describe the behavior of Arctic puffins when they detect predators in their vicinity. They use a specific sound or call to alert other puffins, signaling the presence of danger. This call serves as a danger signal, triggering alertness in other puffins and prompting them to move away from the hazardous area. Simultaneously, when a predator is perceived to be nearby, Arctic puffins swiftly alter their position, rapidly swimming along a larger path toward a safe area to evade the danger. The following is the position update equation used for this strategy:

$$\overrightarrow{Z_i^{t+1}} = \begin{cases} \overrightarrow{X_i^t} + F * L(D) * \left(\overrightarrow{X_{r1}^t} - \overrightarrow{X_{r2}^t}\right) rand \geq 0.5 \\ \overrightarrow{X_i^t} + \beta * \left(\overrightarrow{X_{r1}^t} - \overrightarrow{X_{r2}^t}\right) rand < 0.5 \end{cases} \tag{13}$$

where β is a uniformly distributed random number between 0 and 1. In this strategy, especially when facing dangerous situations, there is a clever balancing mechanism. This mechanism involves two different modes of behavior: one is gradually avoiding danger, and the other is rapidly dodging danger. In the algorithm, this dual-behavior strategy simulates how to escape from local optima in different situations. When $rand \geq 0.5$ indicates a predator is close, Arctic puffins choose to dodge immediately, changing their position more substantially. This is a way of leveraging known information to enhance jumping ability, assisting the algorithm in finding better solutions in possible local optima. On the

Table 2
Sensitivity analysis of parameter F on the CEC2017 test set.

Functions	Index	$F = 0.1$	$F = 0.3$	$F = 0.5$	$F = 0.7$	$F = 0.9$
CEC17-F1	Average	3.16E+03	3.60E+03	2.92E+03	9.16E+03	7.73E+05
	Rank	2	3	1	4	5
CEC17-F3	Average	1.49E+04	4.81E+03	2.07E+03	2.80E+03	5.95E+03
	Rank	5	3	1	2	4
CEC17-F4	Average	5.00E+02	4.98E+02	4.95E+02	4.86E+02	5.04E+02
	Rank	4	3	2	1	5
CEC17-F5	Average	6.08E+02	5.68E+02	5.52E+02	5.55E+02	5.64E+02
	Rank	5	4	1	2	3
CEC17-F6	Average	6.11E+02	6.00E+02	6.00E+02	6.01E+02	6.04E+02
	Rank	5	2	1	3	4
CEC17-F7	Average	8.99E+02	8.10E+02	7.96E+02	8.13E+02	8.16E+02
	Rank	5	2	1	3	4
CEC17-F8	Average	8.98E+02	8.60E+02	8.48E+02	8.52E+02	8.54E+02
	Rank	5	4	1	2	3
CEC17-F9	Average	1.88E+03	1.01E+03	9.15E+02	9.20E+02	1.01E+03
	Rank	5	4	1	2	3
CEC17-F10	Average	4.16E+03	4.26E+03	4.65E+03	4.88E+03	5.24E+03
	Rank	1	2	3	4	5
CEC17-F11	Average	1.19E+03	1.15E+03	1.15E+03	1.15E+03	1.16E+03
	Rank	5	1	2	3	4
CEC17-F12	Average	1.16E+06	5.20E+05	2.01E+05	1.70E+05	3.83E+05
	Rank	5	4	2	1	3
CEC17-F13	Average	1.09E+04	1.43E+04	1.13E+04	6.65E+03	7.84E+03
	Rank	3	5	4	1	2
CEC17-F14	Average	1.84E+04	3.97E+03	1.49E+03	1.47E+03	1.46E+03
	Rank	5	4	3	2	1
CEC17-F15	Average	6.47E+03	3.52E+03	2.36E+03	1.92E+03	1.88E+03
	Rank	5	4	3	2	1
CEC17-F16	Average	2.51E+03	2.18E+03	2.21E+03	2.35E+03	2.21E+03
	Rank	5	1	3	4	2
CEC17-F17	Average	1.97E+03	1.80E+03	1.82E+03	1.83E+03	1.83E+03
	Rank	5	1	2	3	4
CEC17-F18	Average	2.65E+05	6.74E+04	3.85E+04	2.27E+04	1.80E+04
	Rank	5	4	3	2	1
CEC17-F19	Average	5.97E+03	7.10E+03	4.05E+03	2.16E+03	2.05E+03
	Rank	4	5	3	2	1
CEC17-F20	Average	2.22E+03	2.16E+03	2.14E+03	2.18E+03	2.17E+03
	Rank	5	2	1	4	3
CEC17-F21	Average	2.39E+03	2.35E+03	2.35E+03	2.36E+03	2.36E+03
	Rank	5	2	1	3	4
CEC17-F22	Average	2.30E+03	2.30E+03	2.30E+03	2.30E+03	2.31E+03
	Rank	3	2	1	4	5
CEC17-F23	Average	2.77E+03	2.71E+03	2.70E+03	2.71E+03	2.71E+03
	Rank	5	3	1	2	4
CEC17-F24	Average	2.96E+03	2.89E+03	2.88E+03	2.87E+03	2.88E+03
	Rank	5	4	2	1	3
CEC17-F25	Average	2.90E+03	2.89E+03	2.89E+03	2.89E+03	2.89E+03
	Rank	5	4	2	1	3
CEC17-F26	Average	4.43E+03	3.77E+03	3.92E+03	3.88E+03	4.29E+03
	Rank	5	1	3	2	4
CEC17-F27	Average	3.26E+03	3.23E+03	3.22E+03	3.24E+03	3.25E+03
	Rank	5	2	1	3	4
CEC17-F28	Average	3.23E+03	3.21E+03	3.21E+03	3.22E+03	3.22E+03
	Rank	5	2	1	3	4
CEC17-F29	Average	3.74E+03	3.50E+03	3.48E+03	3.55E+03	3.59E+03
	Rank	5	2	1	3	4
CEC17-F30	Average	1.03E+04	8.31E+03	9.10E+03	1.56E+04	3.02E+04
	Rank	3	1	2	4	5
Average ranking		4.48	2.79	1.83	2.52	3.38
Final ranking		5	3	1	2	4

other hand, when $rand < 0.5$ indicates that Arctic puffins choose to proactively avoid predators, their behavior tends to be more cautious. They evade potential danger by randomly changing their positions. This is an exploratory approach, allowing the algorithm to search the surrounding environment more delicately in hopes of discovering superior solutions.

Therefore, this dual-behavior strategy helps the algorithm maintain balance in different situations. It allows for cautious exploration of new areas while quickly adapting to known information, thereby enhancing the algorithm's global search capability and aiding in escaping local optima. This balancing mechanism is simulated in the algorithm to ensure robustness and adaptability.

In summary, the puffins employ different strategies during underwater foraging, including gathering forage, intense searching, and avoiding predators. These strategies may lead to different foraging outcomes under varying conditions. The algorithm chooses to merge the candidate positions from the three different position equations into a new solution to obtain optimal results in various situations. The solutions are sorted by fitness, and the top N individuals are selected. The equation is described as follows:

$$\overrightarrow{P_i^{t+1}} = \overrightarrow{W_i^{t+1}} \cup \overrightarrow{Y_i^{t+1}} \cup \overrightarrow{Z_i^{t+1}} \tag{14}$$

Table 3
Sensitivity analysis of parameter C on the CEC2017 test set.

Functions	Index	$C = 0.1$	$C = 0.3$	$C = 0.5$	$C = 0.7$	$C = 0.9$
CEC17-F1	Average	2.51E+08	7.84E+06	2.92E+03	1.32E+05	4.30E+04
	Rank	5	4	1	3	2
CEC17-F3	Average	2.31E+04	8.36E+03	2.07E+03	3.64E+03	3.12E+03
	Rank	5	4	1	3	2
CEC17-F4	Average	6.09E+02	5.11E+02	4.95E+02	4.92E+02	4.92E+02
	Rank	5	4	3	1	2
CEC17-F5	Average	7.08E+02	6.05E+02	5.52E+02	5.58E+02	5.58E+02
	Rank	5	4	1	3	2
CEC17-F6	Average	6.17E+02	6.08E+02	6.00E+02	6.03E+02	6.02E+02
	Rank	5	4	1	3	2
CEC17-F7	Average	9.64E+02	8.78E+02	7.96E+02	8.06E+02	7.99E+02
	Rank	5	4	1	3	2
CEC17-F8	Average	1.01E+03	9.00E+02	8.48E+02	8.49E+02	8.54E+02
	Rank	5	4	1	2	3
CEC17-F9	Average	1.73E+03	1.12E+03	9.15E+02	9.69E+02	9.88E+02
	Rank	5	4	1	2	3
CEC17-F10	Average	7.95E+03	6.99E+03	4.65E+03	4.62E+03	4.41E+03
	Rank	5	4	3	2	1
CEC17-F11	Average	1.31E+03	1.20E+03	1.15E+03	1.15E+03	1.16E+03
	Rank	5	4	1	2	3
CEC17-F12	Average	4.90E+06	5.34E+05	2.01E+05	3.05E+05	2.38E+05
	Rank	5	4	1	3	2
CEC17-F13	Average	2.25E+04	9.75E+03	1.13E+04	5.63E+03	6.14E+03
	Rank	5	3	4	1	2
CEC17-F14	Average	1.55E+03	1.48E+03	1.49E+03	1.45E+03	1.46E+03
	Rank	5	3	4	1	2
CEC17-F15	Average	3.18E+03	2.04E+03	2.36E+03	1.81E+03	1.80E+03
	Rank	5	3	4	2	1
CEC17-F16	Average	3.23E+03	2.52E+03	2.21E+03	2.17E+03	2.23E+03
	Rank	5	4	2	1	3
CEC17-F17	Average	2.15E+03	1.85E+03	1.82E+03	1.84E+03	1.81E+03
	Rank	5	4	2	3	1
CEC17-F18	Average	2.07E+04	1.76E+04	3.85E+04	1.56E+04	1.70E+04
	Rank	4	3	5	1	2
CEC17-F19	Average	3.12E+03	2.03E+03	4.05E+03	1.99E+03	1.98E+03
	Rank	4	3	5	2	1
CEC17-F20	Average	2.44E+03	2.18E+03	2.14E+03	2.15E+03	2.16E+03
	Rank	5	4	1	2	3
CEC17-F21	Average	2.50E+03	2.38E+03	2.35E+03	2.35E+03	2.35E+03
	Rank	5	4	1	3	2
CEC17-F22	Average	2.39E+03	2.32E+03	2.30E+03	2.30E+03	2.30E+03
	Rank	5	4	1	3	2
CEC17-F23	Average	2.86E+03	2.75E+03	2.70E+03	2.71E+03	2.71E+03
	Rank	5	4	1	2	3
CEC17-F24	Average	3.01E+03	2.89E+03	2.88E+03	2.88E+03	2.89E+03
	Rank	5	4	1	2	3
CEC17-F25	Average	2.96E+03	2.90E+03	2.89E+03	2.89E+03	2.89E+03
	Rank	5	4	1	3	2
CEC17-F26	Average	5.58E+03	4.15E+03	3.92E+03	4.25E+03	4.07E+03
	Rank	5	3	1	4	2
CEC17-F27	Average	3.33E+03	3.27E+03	3.22E+03	3.24E+03	3.23E+03
	Rank	5	4	1	3	2
CEC17-F28	Average	3.33E+03	3.25E+03	3.21E+03	3.22E+03	3.22E+03
	Rank	5	4	1	2	3
CEC17-F29	Average	4.21E+03	3.72E+03	3.48E+03	3.52E+03	3.51E+03
	Rank	5	4	1	3	2
CEC17-F30	Average	2.00E+05	4.70E+04	9.10E+03	2.42E+04	2.02E+04
	Rank	5	4	1	3	2
Average ranking		4.93	3.79	1.79	2.34	2.14
Final ranking		5	4	1	3	2

$$new = sort\left(\overrightarrow{P_i^{t+1}}\right) \tag{15}$$

$$\overrightarrow{X_i^{t+1}} = new(1 : N) \tag{16}$$

This comprehensive strategy allows the model to consider multiple foraging scenarios simultaneously, ultimately choosing the most suitable position as the optimal solution. This enhances the puffins' foraging and survival capabilities when facing different situations.

3.2.4. Behavior conversion factor B

In the APO algorithm, the Arctic puffin always tends to perform frequent aerial flights in the initial stage of the iteration to realize global search, while in the later stage of the iteration, it prefers to dive for food frequently to execute local exploitation. This search mechanism is inspired by the life habits of the Arctic puffin. In the early stage, the Arctic puffin is more inclined to search for suitable feeding waters, while in the later stage, it focuses on diving for food. Based on this behavioral pattern, the APO algorithm designs a behavioral transition coefficient B to achieve a smooth transition from global search to local exploitation. It is defined as follows.

Input : N, T, D, F and C

Output: the best $\overrightarrow{X_i^t}$ and its fitness value

Initialize N puffins, $\overrightarrow{X_i^t}$ $(i = 1, 2, \ldots.., N)$, using Eq.(1)

Evaluate each $\overrightarrow{X_i^t}$ and finding the one with the best fitness in $\overrightarrow{X_i^t}$

t=1

while (t<T)

    Calculate the behavior conversion factor B according to Eq.(10)

    if B>C %% Air flight period (Exploration phase)

        for i=1:N

            Update $\overrightarrow{Y_i^{t+1}}$ using Eq.(2) %% Air search

            Update $\overrightarrow{Z_i^{t+1}}$ using Eq.(5) %% Dive predation

            Select N excellent populations as the new population $\overrightarrow{X_i^{t+1}}$ using Eq.(7)-(9)

            Evaluate the puffins, $\overrightarrow{X_i^{t+1}}$, and replace $\overrightarrow{X_i^t}$ with, $\overrightarrow{X_i^{t+1}}$ if it is better.

            t=t+1

        end for

    else %% Diving and foraging period (Exploitation phase)

        for i=1:N

            Update $\overrightarrow{W_i^{t+1}}$ using Eq.(10) %% Gather for food

            Update $\overrightarrow{Y_i^{t+1}}$ using Eq.(11) %% Strengthen search

            Update $\overrightarrow{Z_i^{t+1}}$ using Eq.(13) %% Avoide predators

            Select N excellent populations as the new population $\overrightarrow{X_i^{t+1}}$ using Eq.(14)-(16)

        Evaluate the puffins, $\overrightarrow{X_i^{t+1}}$, and replace $\overrightarrow{X_i^t}$ with, $\overrightarrow{X_i^{t+1}}$ if it is better.

        t=t+1

        end for

    end if

end while

return $\overrightarrow{X_i^t}$

Fig. 5. Pseudocode of APO algorithm.

$$B = 2 * \log(1/rand) * (1 - t/T) \tag{17}$$

where *rand* is a random number in (0, 1), t and T are the current iteration number and the maximum iteration number, respectively. In the APO algorithm, B is an adaptively changing parameter, which is not arbitrarily selectable but is dynamically derived based on the ratio between the current iteration number and the maximum iteration number, and at the same time incorporates stochasticity to simulate the dynamic randomness of the Arctic puffin behavior. This design allows the value of B to be dynamically adjusted as iterations proceed, thus adapting to the search needs at different stages.

Meanwhile, in the APO algorithm, we introduce a parameter C, which compares B with C to determine the search strategy that should be executed at the current iteration stage. In this paper, the parameter C is set to 0.5, and the parameter determination part is discussed in later contents. The design of this comparison mechanism is inspired by the life habits of Arctic puffins, which can flexibly switch between aerial flight (exploration) and diving for foraging (exploitation) according to their needs. When the value of B is greater than C, the algorithm will enter the air flight phase. This means that the algorithm will deeply explore a wide area of the current solution space to find potential high-quality solutions. This strategy helps the algorithm avoid falling into a local optimum too early and ensures the solution space is fully explored. On the contrary, when the value of B is less than or equal to C, the algorithm enters the diving and foraging phase and shifts to performing a local exploitation strategy. At this point, the algorithm will perform a fine-grained search in the vicinity of the currently found high-quality solution to improve the accuracy of the solution further. This strategy helps the algorithm dig deeper into potential high-quality solutions and improves its convergence speed and accuracy.

Thus, the behavioral switching coefficient B plays a key role in achieving a balance between exploration and exploitation of the algorithm and corresponds to the switching between the two phases of life of the Arctic puffin: aerial flight and underwater foraging. By comparing it with parameter C, the algorithm can flexibly adapt the search strategy to different iteration stages. This design allows the algorithm to find an

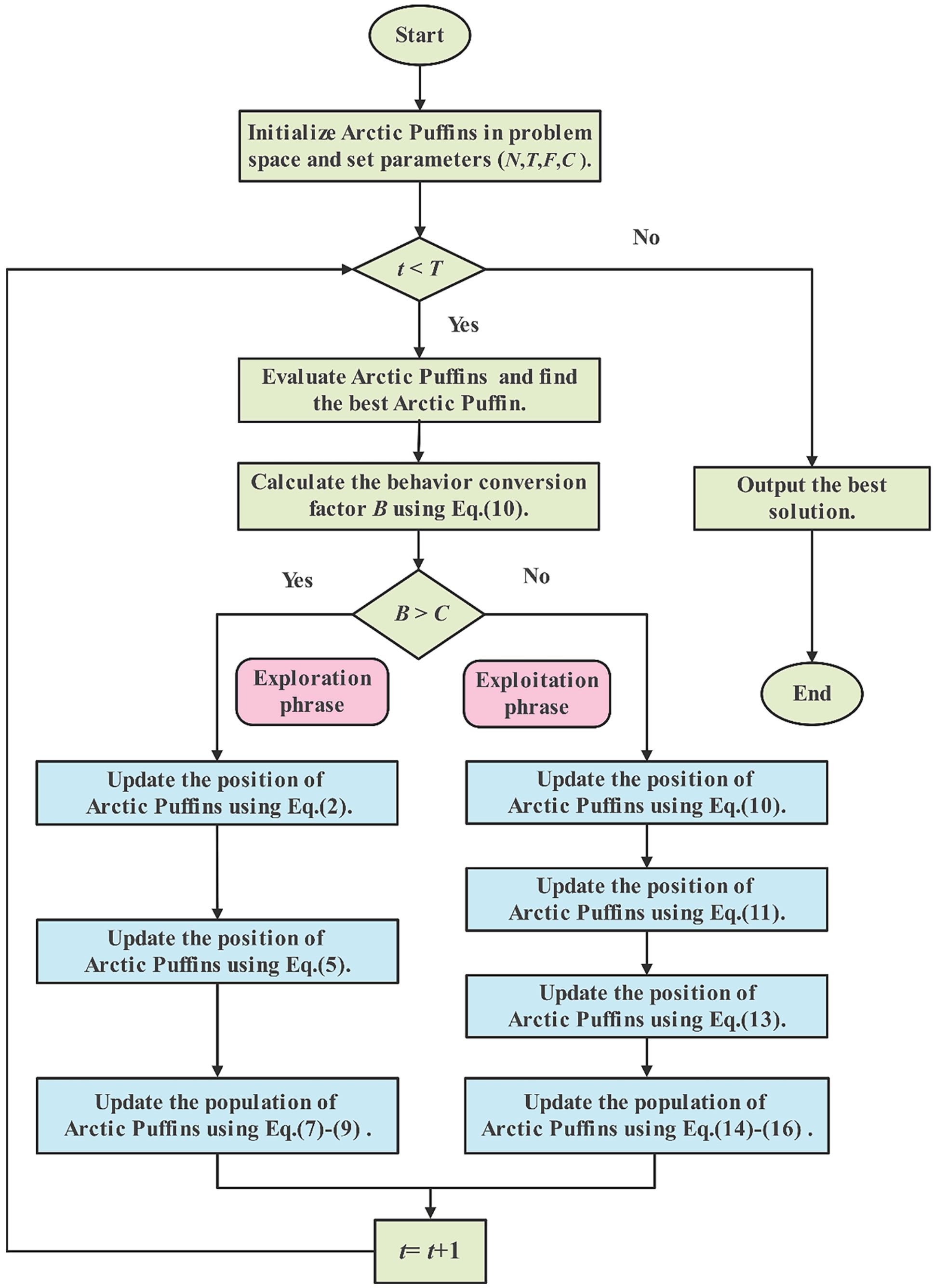

Fig. 6. Flowchart of APO algorithm.

appropriate balance between global search and local exploitation, which improves the performance and robustness of the algorithm.

3.3. *Parameter sensitivity analysis*

In order to gain a deeper understanding of the performance of the Arctic puffin algorithm, a parameter sensitivity analysis is conducted in this section, which focuses on the synergistic factor (F) and the parameter C. The synergistic factor F represents the extent to which the Arctic puffin adopts a synergistic strategy during the foraging process and predator avoidance. F and the Levy factor $L(D)$ together regulate the depth of the collective synergistic predation strategy adopted by the Arctic puffin during foraging and predator avoidance when avoiding predators. Positional change strategies when avoiding predators. Meanwhile, the size of parameter C directly affects the algorithm's balance between exploration (global search) and exploitation (local optimization).

In order to fully understand the impact of different parameter values on the algorithm's overall performance, the synergistic factor F and parameter C are tested in a wide range of experiments within a reasonable range. Some experiments were chosen to be conducted in the 30D case of the CEC2017 test set, and the average performance and the mean ranking results were collected for different parameters. The number of iterations is 1000, and the number of populations is 30. These results are summarized in Tables 2 and 3, which clearly show the trend of the impact of different parameter values on the algorithm's performance.

3.3.1. *Synergy factor f*

According to the results of the sensitivity analysis in Table 2, parameter F presents a clear influence on the performance of the Arctic Puffin algorithm. Specifically, the algorithm obtains the best performance when the value of F is 0.5, resulting in a first-ranking. In contrast, the algorithm also performs well when the values of F are 0.7 and 0.3, ranking second and third, respectively. This indicates that the algorithm can still find a solution efficiently within these values, albeit slightly inferior to $F = 0.5$. However, the algorithm's performance decreases significantly when the values of F are taken as 0.9 and 0.1, with rankings of fourth and fifth, respectively.

This suggests that the choice of parameter F is critical to the algorithm's performance and that inappropriate F values may lead to poorer performance. The combination of smaller F values and Levy flight factors can lead to under-exploration of the algorithm. In comparison, larger F values can lead to prematurely falling into local optimal solutions. Therefore, in both the aggregation foraging and predator avoidance phases, the parameter $F = 0.5$ enables the Puffin algorithm to strike a good balance between exploration and exploitation, which helps to search the solution space better. So, in this paper, making $F = 0.5$ is an appropriate choice to achieve a good balance in the optimization search process, which helps to search the solution space better.

3.3.2. *Parameter C*

The introduction of parameter C aims to determine the search strategy executed by the algorithm during the iteration process by comparing it with the behavioral conversion coefficient B to achieve a balanced transition between global search and local exploitation. In Table 3, we investigate the effect of parameter C on the comprehensive performance of the algorithm under different values. The experimental results show that the algorithm obtains the best performance and is ranked first when the value of C is 0.5. This is because, at this point, the algorithm can achieve a good balance between global search and local exploitation, both in terms of extensive exploration of the solution space and deep mining of high-quality solutions. In contrast, when C takes a larger value, such as 0.9 and 0.7, the algorithm's performance rankings are second and third, respectively. Although the performance improves on some functions, the algorithm's overall comprehensive performance decreases. The comprehensive performance is slightly inferior to the case when C is 0.5. When C takes smaller values, such as 0.3 and 0.1, the algorithm's performance decreases significantly, ranking fourth and fifth, respectively.

In summary, the choice of parameter C is crucial for the algorithm's performance. An appropriate value of C can enable the algorithm to achieve a good balance between global search and local exploitation and improve the performance and robustness of the algorithm. Therefore, in this paper, C is set to 0.5 to achieve the best algorithmic performance.

3.4. *Proposed algorithm: APO*

This section will provide a detailed presentation of the pseudocode and flowchart for the APO, aiming to offer a comprehensive understanding of the algorithm's implementation steps. Examining the pseudocode in Fig. 5 and the algorithm's flowchart in Fig. 6 facilitates a clearer comprehension of the specific implementation details of the APO.

3.5. *Algorithm complexity*

The time complexity of the APO algorithm primarily involves three processes: initialization of the population, fitness evaluation, and population update. The complexity of the initial population is $O(N \cdot D)$. The fitness evaluation within the main loop has a complexity of $O(N \cdot D \cdot T)$. The time complexity of the population update is $O(N \cdot D \cdot T)$. Therefore, the overall time complexity of APO is $O(N \cdot D \cdot T)$.

Table 4
Parameters of optimization algorithms.

Algorithm	Parameter	Value
AOA	α	5
	μ	0.5
AVOA	P_1	0.6
	P_2	0.4
	P_3	0.6
	alpha	0.8
	betha	0.2
	gamma	2.5
BWO	W_f	Decreases from [0.1 0.05]
CMAES	α	2
GTO	p	0.03
	Beta	3
	w	8
HHO	E_0	Changes form [−1, 1]
RSA	*Alpha*	0.1
	Beta	0.005
RUN	a	20
	b	12
SMA	z	0.03
APO	F	0.5
	C	0.5

4. Experimental results and discussions

To comprehensively assess the superiority and advancement of the APO, this section conducts comparative experiments, testing the APO against other state-of-the-art algorithms on diverse datasets. The content of this section is primarily divided into the following four parts: 4.1 Introduction to test functions and comparative algorithms; 4.2 CEC2017 test results; 4.3 CEC2019 test results; 4.4 CEC2022 test results.

4.1. *Introduction to test functions and comparative algorithms*

In this section, to comprehensively evaluate the performance of the APO, three different test sets are used: CEC2017, CEC2019, and CEC2022, comprising a total of 51 distinct test functions. There is a detailed introduction to each test set.

Firstly, the CEC2017 test set includes 29 test functions [72], excluding the F2 function. Compared to standard test functions, CEC2017 test functions are more complex, providing a more challenging evaluation of the algorithm's global optimization performance. Secondly, the CEC2019 test set [13], unlike other test sets, comprises 10 test functions with varying variable dimensions and search ranges. This adds a challenge to the algorithm's adaptability, testing its generalizability across different problems. Finally, the CEC2022 test set [73], the latest release, includes 12 test functions. The use of the CEC2022 test set aims to showcase the outstanding performance of the APO algorithm in handling emerging problems and challenges.

The three sets of test functions cover unimodal, multimodal, hybrid, and composition functions and different dimensionality tests. The unimodal function evaluates the algorithm's ability to find a single globally optimal solution, the multimodal function examines the algorithm's ability to handle multiple locally optimal solutions, and the hybrid function combines the properties of the unimodal and multimodal functions to test the algorithm's ability to perform both global and local searches. The composition function examines the robustness and adaptability of the algorithm. The specific characteristics of these test functions and the related introduction have been described in detail in Appendix.

Furthermore, in order to validate the state-of-the-art APO algorithms, nine recent and widely recognized optimization algorithms are selected for comparison in this paper. They are divided into three categories to observe the efficacy of APO for these three latest optimizers. These categories are as follows: The first category encompasses some of the most recently released algorithms, such as the African Vulture

Table 5
Comparison of results of various algorithms under the CEC2017 test set.

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
CEC17-F1	Average	5.56E+10	3.75E+03	5.02E+10	1.04E+04	6.46E+03	2.82E+07	4.56E+10	8.29E+03	1.27E+04	**2.92E+03**
	Best	3.97E+10	1.12E+02	4.01E+10	1.03E+02	1.01E+02	1.57E+07	3.54E+10	2.88E+03	3.96E+03	1.01E+02
	Std	9.09E+09	4.41E+03	4.26E+09	9.46E+03	7.52E+03	7.63E+06	5.04E+09	6.20E+03	6.63E+03	3.29E+03
	Rank	10	2	9	5	3	7	8	4	6	1
CEC17-F3	Average	8.12E+04	3.46E+04	7.66E+04	4.57E+05	6.24E+03	3.81E+04	7.97E+04	**1.50E+03**	6.67E+03	2.07E+03
	Best	6.32E+04	2.45E+04	6.41E+04	1.89E+05	7.03E+02	2.71E+04	7.10E+04	4.44E+02	8.77E+02	7.48E+02
	Std	8.16E+03	6.84E+03	6.29E+03	1.42E+05	4.55E+03	6.28E+03	4.89E+03	8.80E+02	4.59E+03	1.14E+03
	Rank	9	5	7	10	3	6	8	1	4	2
CEC17-F4	Average	1.36E+04	5.29E+02	1.22E+04	**4.15E+02**	4.91E+02	5.75E+02	9.71E+03	5.07E+02	5.03E+02	4.94E+02
	Best	6.83E+03	4.72E+02	9.92E+03	4.14E+02	4.00E+02	4.90E+02	4.77E+03	4.74E+02	4.70E+02	4.05E+02
	Std	4.48E+03	3.55E+01	1.32E+03	5.85E-01	2.77E+01	5.00E+01	2.90E+03	1.82E+01	1.80E+01	2.81E+01
	Rank	10	6	9	1	2	7	8	5	4	3
CEC17-F5	Average	8.74E+02	7.23E+02	9.27E+02	5.54E+02	6.87E+02	7.62E+02	9.21E+02	7.02E+02	6.27E+02	**5.52E+02**
	Best	8.09E+02	6.45E+02	8.89E+02	5.16E+02	6.04E+02	6.84E+02	8.67E+02	6.34E+02	5.70E+02	5.28E+02
	Std	3.74E+01	3.96E+01	1.70E+01	6.28E+01	4.23E+01	3.46E+01	2.47E+01	4.58E+01	2.94E+01	1.34E+01
	Rank	8	6	10	2	4	7	9	5	3	1
CEC17-F6	Average	6.78E+02	6.48E+02	6.91E+02	**6.00E+02**	6.47E+02	6.65E+02	6.86E+02	6.44E+02	6.10E+02	6.00E+02
	Best	6.65E+02	6.35E+02	6.84E+02	6.00E+02	6.23E+02	6.53E+02	6.72E+02	6.29E+02	6.03E+02	6.00E+02
	Std	7.00E+00	8.62E+00	3.40E+00	1.35E-02	9.38E+00	5.27E+00	6.88E+00	8.68E+00	4.14E+00	4.72E-02
	Rank	8	6	10	1	5	7	9	4	3	2
CEC17-F7	Average	1.38E+03	1.16E+03	1.39E+03	8.73E+02	1.08E+03	1.28E+03	1.38E+03	1.04E+03	8.70E+02	**7.96E+02**
	Best	1.29E+03	1.02E+03	1.33E+03	7.45E+02	9.58E+02	1.15E+03	1.30E+03	9.30E+02	7.98E+02	7.69E+02
	Std	4.36E+01	7.38E+01	3.21E+01	6.20E+01	8.05E+01	6.01E+01	4.10E+01	7.44E+01	4.68E+01	1.85E+01
	Rank	9	6	10	3	5	7	8	4	2	1
CEC17-F8	Average	1.12E+03	9.64E+02	1.14E+03	8.75E+02	9.50E+02	9.77E+02	1.14E+03	9.51E+02	9.28E+02	**8.48E+02**
	Best	1.07E+03	9.06E+02	1.11E+03	8.13E+02	9.09E+02	9.37E+02	1.10E+03	9.04E+02	8.76E+02	8.24E+02
	Std	2.92E+01	3.13E+01	1.37E+01	7.34E+01	2.58E+01	2.53E+01	1.50E+01	2.26E+01	2.99E+01	1.30E+01
	Rank	8	6	10	2	4	7	9	5	3	1
CEC17-F9	Average	7.45E+03	5.27E+03	1.11E+04	**9.00E+02**	3.79E+03	8.05E+03	1.04E+04	3.88E+03	3.70E+03	9.15E+02
	Best	5.76E+03	3.90E+03	9.24E+03	9.00E+02	2.69E+03	5.47E+03	8.60E+03	2.21E+03	1.77E+03	9.00E+02
	Std	9.87E+02	7.61E+02	8.53E+02	8.29E-02	7.66E+02	1.03E+03	1.09E+03	7.85E+02	9.32E+02	2.09E+01
	Rank	7	6	10	1	4	8	9	5	3	2
CEC17-F10	Average	7.52E+03	5.55E+03	8.68E+03	8.91E+03	5.53E+03	5.86E+03	8.27E+03	4.80E+03	4.73E+03	**4.65E+03**
	Best	6.70E+03	4.11E+03	8.12E+03	8.36E+03	4.00E+03	4.41E+03	7.39E+03	3.17E+03	3.39E+03	2.92E+03
	Std	5.91E+02	7.17E+02	2.85E+02	2.53E+02	8.91E+02	6.98E+02	4.01E+02	7.38E+02	6.51E+02	1.21E+03
	Rank	7	5	9	10	4	6	8	3	2	1
CEC17-F11	Average	1.03E+04	1.26E+03	7.07E+03	1.68E+04	1.24E+03	1.30E+03	8.66E+03	1.22E+03	1.28E+03	**1.15E+03**
	Best	5.32E+03	1.13E+03	4.34E+03	3.69E+03	1.14E+03	1.20E+03	6.13E+03	1.17E+03	1.17E+03	1.11E+03
	Std	3.20E+03	5.49E+01	1.21E+03	1.78E+04	5.73E+01	7.75E+01	1.28E+03	3.10E+01	6.94E+01	3.19E+01
	Rank	9	4	7	10	3	6	8	2	5	1
CEC17-F12	Average	1.29E+10	8.04E+06	1.02E+10	8.45E+06	6.83E+05	2.75E+07	1.42E+10	3.49E+06	4.31E+06	**2.01E+05**
	Best	7.70E+09	1.22E+06	7.15E+09	1.47E+06	6.31E+04	7.50E+06	5.07E+09	8.90E+05	4.62E+05	2.28E+04
	Std	2.87E+09	4.82E+06	1.72E+09	8.30E+06	6.36E+05	2.46E+07	3.04E+09	1.91E+06	2.91E+06	1.49E+05
	Rank	9	5	8	6	2	7	10	3	4	1
CEC17-F13	Average	1.12E+10	1.43E+05	6.51E+09	5.64E+06	1.99E+04	8.87E+05	1.26E+10	2.79E+04	4.05E+04	**1.13E+04**
	Best	1.11E+09	3.48E+04	1.94E+09	1.49E+06	2.69E+03	2.92E+05	3.77E+09	1.18E+04	9.28E+03	1.36E+03
	Std	5.23E+09	8.77E+04	2.59E+09	4.77E+06	2.17E+04	9.53E+05	7.73E+09	1.41E+04	2.86E+04	1.04E+04
	Rank	9	5	8	7	2	6	10	3	4	1
CEC17-F14	Average	1.85E+06	2.03E+05	2.87E+06	4.76E+05	4.02E+03	3.94E+05	4.84E+06	5.96E+03	9.46E+04	**1.49E+03**
	Best	3.65E+04	3.02E+03	3.70E+05	6.43E+04	1.57E+03	8.07E+03	6.86E+05	1.76E+03	8.66E+03	1.45E+03
	Std	1.90E+06	3.14E+05	1.65E+06	3.50E+05	4.43E+03	4.71E+05	4.20E+06	5.10E+03	6.53E+04	2.76E+01
	Rank	8	5	9	7	2	6	10	3	4	1
CEC17-F15	Average	2.77E+07	3.80E+04	2.63E+08	5.73E+06	6.51E+03	1.05E+05	4.93E+08	1.53E+04	2.17E+04	**2.36E+03**
	Best	1.64E+04	8.80E+03	5.27E+07	5.64E+05	1.73E+03	2.69E+04	1.19E+08	9.44E+03	2.11E+03	1.54E+03
	Std	6.90E+07	2.49E+04	1.14E+08	4.75E+06	6.58E+03	6.32E+04	1.59E+08	2.56E+03	1.68E+04	8.86E+02
	Rank	8	5	9	7	2	6	10	3	4	1
CEC17-F16	Average	5.52E+03	3.14E+03	5.46E+03	2.65E+03	2.74E+03	3.49E+03	5.37E+03	2.84E+03	2.51E+03	**2.21E+03**
	Best	3.91E+03	2.33E+03	4.64E+03	1.80E+03	2.19E+03	2.66E+03	4.44E+03	2.11E+03	1.85E+03	1.76E+03
	Std	1.11E+03	4.18E+02	4.41E+02	4.44E+02	3.06E+02	4.61E+02	7.19E+02	3.69E+02	3.30E+02	2.41E+02
	Rank	10	6	9	3	4	7	8	5	2	1
CEC17-F17	Average	4.10E+03	2.53E+03	4.09E+03	2.13E+03	2.33E+03	2.66E+03	4.90E+03	2.22E+03	2.34E+03	**1.82E+03**
	Best	2.34E+03	1.94E+03	3.15E+03	1.79E+03	1.88E+03	2.00E+03	3.19E+03	1.80E+03	1.92E+03	1.73E+03
	Std	1.45E+03	2.95E+02	4.80E+02	2.00E+02	1.68E+02	3.16E+02	1.85E+03	2.30E+02	2.14E+02	1.18E+02
	Rank	9	6	8	2	4	7	10	3	5	1
CEC17-F18	Average	2.63E+07	2.58E+06	3.77E+07	6.62E+06	7.66E+04	2.48E+06	4.83E+07	9.06E+04	1.53E+06	**3.85E+04**
	Best	1.13E+06	7.66E+04	8.03E+06	1.43E+06	2.17E+04	1.17E+05	5.33E+06	2.64E+04	2.85E+05	7.21E+03
	Std	2.61E+07	2.21E+06	1.86E+07	6.47E+06	6.40E+04	2.71E+06	3.84E+07	6.37E+04	1.22E+06	2.06E+04
	Rank	8	6	9	7	2	5	10	3	4	1
CEC17-F19	Average	2.79E+07	4.73E+04	3.52E+08	1.97E+06	8.43E+03	8.90E+05	7.53E+08	7.52E+03	2.47E+04	**4.05E+03**
	Best	1.82E+06	7.39E+03	1.21E+08	3.62E+05	2.05E+03	1.01E+05	1.45E+08	4.57E+03	2.04E+03	1.94E+03
	Std	6.16E+07	5.45E+04	1.64E+08	1.29E+06	9.78E+03	4.97E+05	6.58E+08	2.19E+03	1.88E+04	2.94E+03
	Rank	8	5	9	7	3	6	10	2	4	1
CEC17-F20	Average	2.82E+03	2.71E+03	2.98E+03	2.56E+03	2.51E+03	2.79E+03	3.04E+03	2.52E+03	2.57E+03	**2.14E+03**
	Best	2.53E+03	2.25E+03	2.71E+03	2.15E+03	2.21E+03	2.41E+03	2.83E+03	2.22E+03	2.12E+03	2.01E+03

(*continued on next page*)

Table 5 (*continued*)

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
	Std	1.91E+02	2.33E+02	1.23E+02	1.68E+02	1.61E+02	2.38E+02	1.28E+02	2.01E+02	1.92E+02	7.08E+01
	Rank	8	6	9	4	2	7	10	3	5	1
CEC17-F21	Average	2.67E+03	2.54E+03	2.71E+03	2.35E+03	2.46E+03	2.58E+03	2.70E+03	2.44E+03	2.41E+03	**2.35E+03**
	Best	2.56E+03	2.42E+03	2.49E+03	2.32E+03	2.40E+03	2.46E+03	2.62E+03	2.38E+03	2.35E+03	2.32E+03
	Std	4.68E+01	6.06E+01	5.05E+01	4.97E+01	3.92E+01	5.65E+01	5.15E+01	3.54E+01	3.53E+01	1.78E+01
	Rank	8	6	10	2	5	7	9	4	3	1
CEC17-F22	Average	8.76E+03	5.75E+03	8.49E+03	1.02E+04	3.09E+03	7.04E+03	8.64E+03	3.46E+03	5.95E+03	**2.30E+03**
	Best	7.32E+03	2.30E+03	7.51E+03	8.88E+03	2.30E+03	2.33E+03	7.01E+03	2.30E+03	2.30E+03	2.30E+03
	Std	7.49E+02	2.25E+03	5.34E+02	3.53E+02	1.81E+03	1.70E+03	1.01E+03	2.01E+03	9.39E+02	7.48E-01
	Rank	9	4	7	10	2	6	8	3	5	1
CEC17-F23	Average	3.51E+03	2.95E+03	3.33E+03	**2.69E+03**	2.88E+03	3.22E+03	3.30E+03	2.80E+03	2.76E+03	2.70E+03
	Best	3.21E+03	2.78E+03	3.26E+03	2.66E+03	2.77E+03	2.94E+03	3.17E+03	2.72E+03	2.73E+03	2.67E+03
	Std	1.47E+02	8.26E+01	4.19E+01	4.72E+01	7.60E+01	1.27E+02	7.18E+01	4.10E+01	2.62E+01	1.97E+01
	Rank	10	6	9	1	5	7	8	4	3	2
CEC17-F24	Average	3.86E+03	3.12E+03	3.57E+03	**2.85E+03**	3.04E+03	3.50E+03	3.42E+03	2.94E+03	2.92E+03	2.88E+03
	Best	3.48E+03	2.95E+03	3.42E+03	2.83E+03	2.92E+03	3.23E+03	3.28E+03	2.88E+03	2.87E+03	2.85E+03
	Std	2.13E+02	9.08E+01	6.55E+01	3.12E+01	7.41E+01	1.59E+02	1.48E+02	3.11E+01	2.92E+01	1.47E+01
	Rank	10	6	9	1	5	8	7	4	3	2
CEC17-F25	Average	5.71E+03	2.91E+03	4.40E+03	**2.88E+03**	2.91E+03	2.94E+03	4.84E+03	2.91E+03	2.89E+03	2.89E+03
	Best	4.51E+03	2.88E+03	4.16E+03	2.88E+03	2.88E+03	2.89E+03	3.83E+03	2.88E+03	2.88E+03	2.88E+03
	Std	7.31E+02	1.85E+01	1.30E+02	6.43E-02	2.34E+01	2.55E+01	6.21E+02	2.23E+01	1.61E+01	7.94E+00
	Rank	10	4	8	1	5	7	9	6	3	2
CEC17-F26	Average	1.04E+04	6.64E+03	1.07E+04	3.93E+03	5.07E+03	7.68E+03	1.02E+04	6.04E+03	4.90E+03	**3.92E+03**
	Best	8.83E+03	2.80E+03	9.47E+03	3.43E+03	2.80E+03	2.96E+03	8.53E+03	2.93E+03	4.23E+03	2.80E+03
	Std	9.77E+02	1.26E+03	4.19E+02	3.74E+02	1.60E+03	1.22E+03	8.77E+02	1.32E+03	3.95E+02	5.47E+02
	Rank	9	6	10	2	4	7	8	5	3	1
CEC17-F27	Average	4.70E+03	3.30E+03	3.96E+03	**3.20E+03**	3.30E+03	3.45E+03	3.84E+03	3.28E+03	3.23E+03	3.22E+03
	Best	3.94E+03	3.24E+03	3.69E+03	3.20E+03	3.21E+03	3.30E+03	3.44E+03	3.22E+03	3.20E+03	3.20E+03
	Std	4.15E+02	3.37E+01	1.64E+02	5.47E-05	6.78E+01	9.42E+01	3.05E+02	3.18E+01	1.68E+01	9.19E+00
	Rank	10	5	9	1	6	7	8	4	3	2
CEC17-F28	Average	6.74E+03	3.27E+03	6.43E+03	3.30E+03	3.24E+03	3.33E+03	6.35E+03	3.23E+03	3.26E+03	**3.21E+03**
	Best	4.86E+03	3.23E+03	5.78E+03	3.30E+03	3.19E+03	3.25E+03	5.23E+03	3.19E+03	3.21E+03	3.13E+03
	Std	9.93E+02	2.06E+01	3.03E+02	9.17E-05	3.40E+01	3.30E+01	8.13E+02	2.65E+01	4.77E+01	2.58E+01
	Rank	10	5	9	6	3	7	8	2	4	1
CEC17-F29	Average	8.07E+03	4.30E+03	6.85E+03	3.99E+03	4.26E+03	4.68E+03	6.43E+03	4.15E+03	3.86E+03	**3.48E+03**
	Best	4.89E+03	3.75E+03	6.00E+03	3.39E+03	3.64E+03	3.95E+03	4.83E+03	3.73E+03	3.50E+03	3.37E+03
	Std	2.59E+03	2.30E+02	5.48E+02	2.75E+02	3.21E+02	5.25E+02	1.09E+03	2.52E+02	1.87E+02	1.02E+02
	Rank	10	6	9	3	5	7	8	4	2	1
CEC17-F30	Average	1.89E+09	6.64E+05	9.40E+08	1.35E+06	1.22E+04	4.02E+06	2.91E+09	3.11E+05	5.21E+04	**9.10E+03**
	Best	1.21E+08	1.80E+05	3.55E+08	4.29E+05	6.00E+03	7.03E+05	1.23E+09	6.03E+04	1.02E+04	6.21E+03
	Std	1.34E+09	3.30E+05	3.48E+08	7.24E+05	4.29E+03	2.14E+06	1.01E+09	3.97E+05	3.10E+04	2.61E+03
	Rank	9	5	8	6	2	7	10	4	3	1
Average ranking		8.86	5.31	8.86	4.41	3.52	6.72	8.79	3.66	3.55	**1.31**
Final ranking		10	6	9	5	2	7	8	4	3	**1**

Algorithm (AVOA) [74], the beluga whale optimization algorithm (BWO) [43], the Artificial gorilla troop optimizer (GTO) [75], the slime mould algorithm (SMA) [47], and the reptile search algorithm (RSA) [76]. The second category covers some of the most widely researched and cited algorithms, such as the Harris Hawk Algorithm (HHO) [42], the arithmetic optimization algorithm (AOA) [27], and the Runge-Kuta Algorithm (RUN) [77]. The last category includes high-performance optimizers such as the CMAES [78] algorithm, which is the winner of the CEC competition. All of these competing algorithms are compared, thus further highlighting the competitiveness of the APO algorithm. The parameter settings for all algorithms are provided in Table 4 to achieve the best performance for all algorithms.

During the testing period, the number of groups for each algorithm is set to 30, and the maximum number of iterations is set to 1000. Each group of algorithms independently performs 30 optimization search tests for each test function, and the three evaluation metrics, average, best, and standard deviation(Std), are computed separately to assess the algorithmic performance and to calculate the Friedman mean rankings of the test data [79]. The best results are indicated in bold.

In order to fully demonstrate the benefits of the APO algorithm, this study also provides tests based on the same number of functional evaluations. These exhaustive test results and related data are included in the Appendix as supplementary material.

To ensure the fairness of the tests, all experiments were conducted using MATLAB 2022b on an Intel (R) Xeon (R) Gold 5218 CPU @ 2.30 GHz, 90 GB RAM, and Windows 10 operating system.

4.2. CEC2017 test results

In this section, 29 functions from CEC2017 are selected, covering four dimensions: 10, 30, 50, and 100. These functions possess more complex testing capabilities compared to standard test functions. The 29 functions can be categorized into four types: unimodal functions (F1, F3), simple multimodal functions (F4–F10), hybrid functions (F10–F19), and composition functions (F20–F29). Unimodal and simple multimodal functions assess the algorithm's exploitation and exploration capabilities. In contrast, composition and hybrid functions aim to test the algorithm's ability to avoid falling into local optima. In the experimental discussion, the dimensions of the functions are uniformly set to 30 for benchmark testing. Given the randomness of optimization algorithms, the results of each experiment will vary. Therefore, each algorithm is tested 30 times in this study, and the parameter settings for each algorithm are detailed in Table 4.

4.2.1. Results discussion

Table 5 shows the test results of the different algorithms on the CEC2017 test set, specifically the average, best value, and standard deviation of each algorithm after 30 independent runs, along with the corresponding mean rankings. The bolded experimental data are the best results. The analysis of the results in Table 5 indicates that the APO

Table 6
Wilcoxon rank sum test results of each algorithm under CEC2017.

Function	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA
CEC17-F1	1.73E-06	1.06E-04	1.73E-06	5.44E-01	1.06E-01	1.73E-06	1.73E-06	5.19E-02	2.30E-02
CEC17-F3	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.85E-03	1.73E-06	1.73E-06	1.73E-06	1.40E-02
CEC17-F4	3.41E-05	1.73E-06	2.05E-04	1.73E-06	1.73E-06	1.73E-06	3.93E-01	1.73E-06	1.73E-06
CEC17-F5	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F6	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F7	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F8	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F9	6.04E-03	1.73E-06	1.11E-03	1.73E-06	1.73E-06	7.52E-02	1.04E-02	1.73E-06	1.73E-06
CEC17-F10	4.11E-03	1.92E-06	7.50E-01	4.78E-01	2.88E-06	1.73E-06	5.86E-01	1.73E-06	1.73E-06
CEC17-F11	1.41E-01	1.73E-06	4.20E-04	2.70E-02	1.73E-06	1.73E-06	9.10E-01	1.73E-06	1.73E-06
CEC17-F12	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F13	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.20E-01	1.73E-06	1.73E-06	1.73E-06	2.35E-06
CEC17-F14	1.73E-06	1.92E-06	1.73E-06	1.73E-06	6.32E-05	1.92E-06	1.73E-06	1.38E-03	1.73E-06
CEC17-F15	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.48E-02	1.73E-06	1.73E-06	5.22E-06	3.16E-03
CEC17-F16	1.92E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F17	2.13E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	5.75E-06	1.73E-06	1.73E-06
CEC17-F18	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F19	1.73E-06	2.13E-06	1.73E-06	1.73E-06	3.00E-02	1.73E-06	1.73E-06	5.19E-02	4.86E-05
CEC17-F20	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F21	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F22	7.66E-01	2.37E-05	7.66E-01	1.40E-02	4.73E-06	1.96E-03	8.77E-01	1.92E-06	2.13E-06
CEC17-F23	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F24	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F25	2.13E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F26	1.40E-02	6.32E-05	1.11E-03	1.73E-06	2.35E-06	3.00E-02	1.48E-02	5.75E-06	1.73E-06
CEC17-F27	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC17-F28	1.15E-04	1.73E-06	3.18E-06	1.73E-06	1.73E-06	1.73E-06	8.47E-06	1.73E-06	1.73E-06
CEC17-F29	2.80E-01	1.73E-06	1.02E-05	1.73E-06	1.73E-06	1.73E-06	2.84E-05	1.73E-06	1.73E-06
CEC17-F30	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.85E-03	1.73E-06	1.73E-06	1.73E-06	1.73E-06
+/=/-	0/0/29	0/0/29	0/0/29	7/0/22	1/0/28	0/0/29	0/0/29	1/0/28	0/0/29

algorithm performs remarkably well among the twenty-nine functions. In twenty-one functions, the APO algorithm outperforms the comparison algorithms significantly, ranking first in the average value ranking, showcasing its position as the best optimization algorithm. Particularly in unimodal and simple multimodal functions, the APO algorithm excels in five functions, ranking first and demonstrating its outstanding performance. Although slightly inferior to the CMAES and GTO algorithms in the F4 function and ranked second in the F3, F6, and F9 functions. The APO algorithm ranks first in sixteen out of twenty functions for composition and hybrid functions, displaying its excellent competitiveness. In composition functions, APO ranks first, obtaining the best solution and highlighting its powerful exploration and exploitation capabilities. However, CMAES achieves the optimal average value in F23–F25 and F27. The APO algorithm achieves the best results and rankings in 73% of the functions. The second-ranked GTO and third-ranked SMA algorithms perform comparably, while the competition-winning CMAES algorithm excels in individual functions.

In addition, the results of the Wilcoxon rank-sum test presented in Table 6 indicate that the APO algorithm performs superiorly on the majority of test functions and exhibits significant differences. At a significance level of 0.05, $p < 0.05$ indicates a significant difference in the APO algorithm. The symbols "+/=/−" are used to represent the number of significant differences, where "+" and "−' respectively denote cases where the performance of the comparative algorithm is superior or inferior to that of APO. "=" indicates cases where the results of the comparative algorithm are similar to those of the APO algorithm. Overall, APO outperforms comparative algorithms on 72% of the test functions, and this improvement is statistically significant.

The iterative process curves, Figs. 7-9, demonstrate the difference in convergence ability performance between the APO algorithm and the comparison algorithm on CEC2017. By observing the downward trend of the curves, we can get an idea of the convergence speed of the algorithm and its ability to reach the optimal solution. The APO algorithm shows similar convergence ability in the unimodal and simple multimodal functions with other state-of-the-art algorithms. This suggests that the APO algorithm can effectively find the global optimal solution or the local optimal solution in these functions. It is worth mentioning that on the F3, F4, F6, and F9 functions, the APO algorithm's convergence speed and convergence ability are slightly inferior to that of the competition-winning algorithm CMAES, but it still performs well. In addition, the APO algorithm shows more excellent exploration and exploitation ability in combinatorial and hybrid functions. In most cases, the convergence accuracy and speed of the APO algorithm are better than those of the other compared algorithms, which indicates that compared to unimodal and simple multimodal functions, the APO algorithm has a stronger advantage in dealing with more complex testing problems.

In the box plot, the length of the box can reflect the degree of data dispersion. The shorter the box, the more concentrated the data, and the longer the box, the more dispersed the data and the less stable it is. Observing box plots Figs. 10–12, we can draw some conclusions. First, the box plots show that the APO, GTO, AVOA, and SMA algorithms have almost no outliers and obvious performance fluctuations, indicating high stability. Secondly, by looking at the box lengths, it can be noticed that in most cases, the APO algorithm has shorter and lower position box lengths. This indicates that the APO algorithm has higher solution accuracy and superior stability compared to other algorithms.

Radar Fig. 13 shows the rank mean ranking of different optimization algorithms on each test function, reflecting the algorithm's overall performance by the size of the coverage area. In the radar graph, each axis represents a test function, and the boundaries of the graph represent the average ranking of the algorithm on each function. Specifically in Fig. 13, F1 through F30 represent each test function of CEC2017, while 1 through 12 represent the mean rankings of the algorithms on these functions. The smaller the coverage area, the higher the comprehensive ranking of the algorithm on all test functions, and the better the overall performance. Therefore, we can intuitively compare the comprehensive performance differences of different algorithms through radar charts. Fig. 13 shows that the APO algorithm has the smallest coverage area and ranks first in the mean distribution of most functions. This indicates that, compared with the other nine algorithms, APO has an order of magnitude advantage under the same conditions, especially in combination

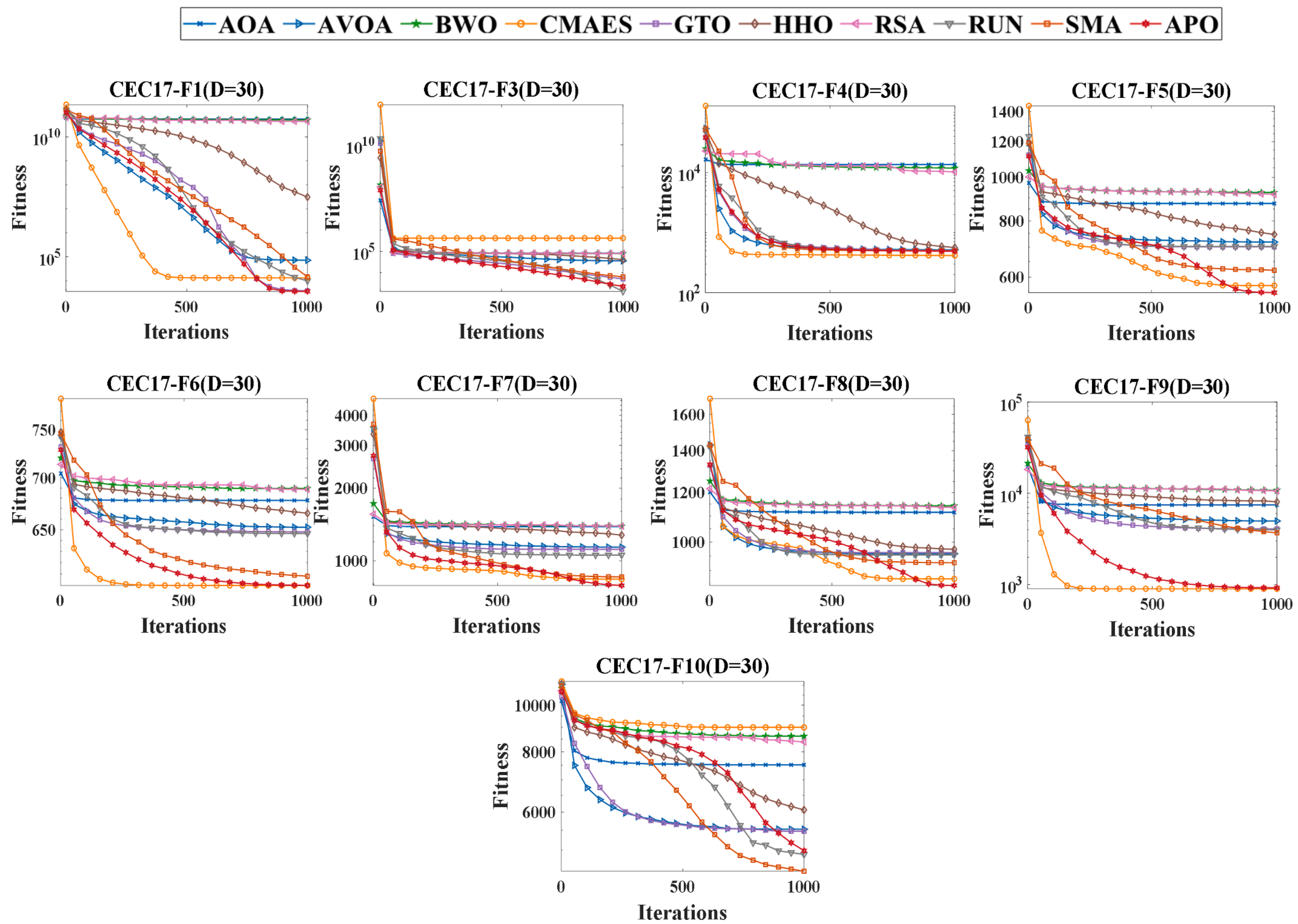

Fig. 7. Iterative convergence curves of APO and other algorithms on CEC2017 unimodal and simple multimodal functions (F1-F10).

and mixed functions. In addition, CMAES, GTO, and AVOA algorithms have shown relative superiority in individual functions (such as simple multimodal functions F3, F4, F6, and F9). However, APO algorithms have also performed well, ranking second or third.

In summary, the APO algorithm has demonstrated excellent convergence ability, stability, and solution accuracy on the CEC2017 test set. Although the performance of the APO algorithm is not as good as that of individual algorithms on a few simple multimodal functions, it has significant advantages on the whole, especially on composite functions and mixed functions, which further verifies the progressiveness and superiority of the APO algorithm on complex test problems.

4.2.2. Scalability testing

In order to test the optimization ability of APO for different dimensional functions, 10D, 30D, 50D, and 100D are selected for the scalability test. The population size $N = 30$, the maximum iteration number $T = 1000$, and the parameter settings of each algorithm are the same as those in the previous experiments. In order to explicitly verify the performance difference between the APO algorithm and the comparison algorithms, the Friedman test results on 29 functions are provided in Table 7, while the change process of the average fitness for some of the different classes of functions is shown in Fig. 14.

According to the experimental results, when the dimensionality increases from 10D, 30D, 50D, and 100D, the algorithm's solution accuracy and robustness decrease, which is because the increase in dimensionality makes the function more complicated, and more adjustments are needed for optimization. However, compared with the other nine comparative algorithms, the APO algorithm still has the highest solving accuracy for most of the functions, which verifies that the APO algorithm has better solving ability in both low- and high-dimensional cases. Although the APO algorithm is the second-best algorithm for the functions F5 and F8 in 50D, which is slightly lower than the CMAES algorithm, the APO algorithm outperforms the CMAES algorithm again in 100D. It is ahead of the other comparative algorithms. This further shows that APO has a competitive advantage in solving complex function optimization problems. Therefore, APO has strong optimization performance and robustness in solving low- and high-dimensional problems, and the APO algorithm can be used as a powerful tool for solving problems in any dimension.

4.3. CEC2019 test results

To further evaluate the performance of the proposed algorithm, APO, this section is tested and analyzed for CEC 2019. In CEC 2019, the F1-F3 functions have different dimension values and ranges, and the F4-F10 functions are 10-dimensional minimization problems. Since many of the CEC 2019 test functions are multimodal, it is very challenging. The APO algorithm was selected for comparison with nine other optimization algorithms (AOA, AVOA, BWO, CMAES, GTO, HHO, RSA, RUN, and SMA). The parameter settings of these algorithms are shown in Table 4. Table 8 records the average, best, and standard deviation of these ten algorithms after thirty runs and gives the corresponding ranking.

According to the results in Table 8, the APO algorithm performs remarkably well on most optimization problems. Out of the ten functions, the APO algorithm ranks first in a total of seven functions (F2, F4, F6-F10). Compared to other algorithms, the APO algorithm generally

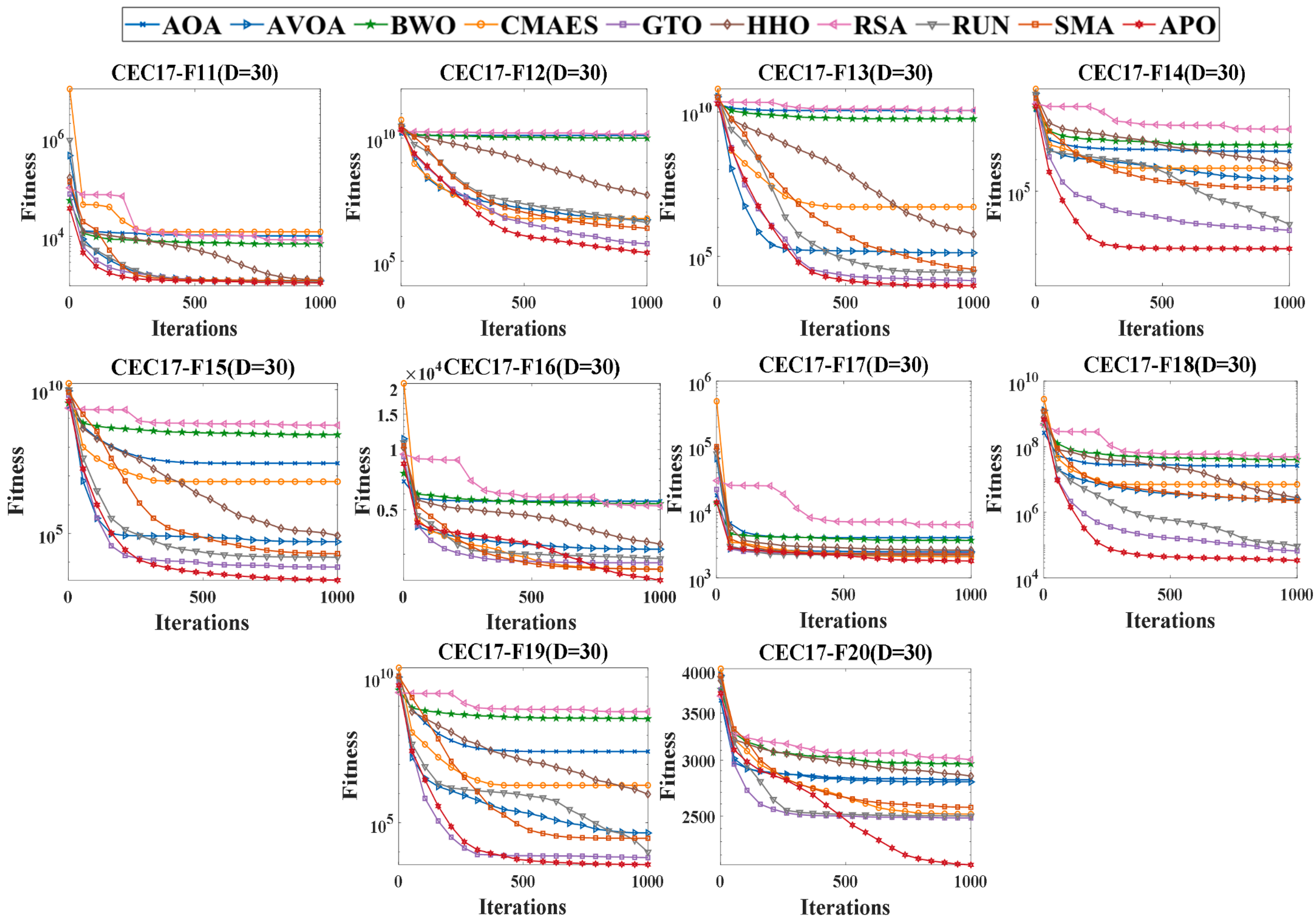

Fig. 8. Iterative convergence curve of APO and other algorithms on CEC2017 hybrid functions (F11-F20).

exhibits smaller standard deviations, indicating more stable performance. However, it ranks lower on function F1, not reaching the theoretical optimum. It ranks second on functions F3 and F5, where the GTO and CMAES algorithms take first place, respectively. Overall, the APO algorithm ranks in the top two in 90% of the test functions. Additionally, the APO algorithm ranks first in the overall average, demonstrating its outstanding competitiveness.

Moreover, the Wilcoxon rank-sum test was used to compare the significant differences between various advanced algorithms and APO. The notation "+/=/−" is used to indicate the number of significant differences. From Table 9, it can be observed that, compared to other algorithms, only two or fewer functions have better test results than the APO algorithm. Even the AOA algorithm failed to achieve better results than the APO algorithm, and only GTO performed better than APO in two functions, while other algorithms performed better in only one function. Overall, the APO algorithm demonstrates its ability to successfully escape local optima and search in a better solution space when solving complex and diverse testing problems like CEC2019.

It can be observed from Fig. 15 that the convergence tendency of each comparison algorithm gradually becomes smoother during the iteration process, showing different degrees of local shackles, thus verifying the balance between the exploration ability and exploitation ability of the APO algorithm. Although the APO algorithm fails to reach the theoretical optimal value in the test function F1, in most cases, the APO algorithm converges more accurately, finds better values from the beginning to the end of the iteration, and the convergence curve seldom has large inflection points, which means that it can jump out of the local optimal solution quickly.

The box plots of each algorithm are observed in Fig. 16, in which the APO algorithm has a more concentrated distribution of data results in most of the test functions, and the accuracy of the results is higher, which proves that the APO algorithm has stability and high efficiency. In addition, Fig. 18 shows the radar distribution plot of each algorithm on the CEC2019 function, which shows that the APO algorithm covers the smallest and most uniform area, indicating that it performs well in solving various complex test functions.

Fig. 17 illustrates the radar distribution of the algorithms on the CEC2019 functions, revealing the difference in their overall performance. Specifically in Fig. 17, F1, F3…F10 represent the ten test functions of CEC2019, while 1, 2, …, 12 represent the mean rankings of the algorithms on these functions. The APO algorithm has the smallest overall coverage area, suggesting that it performs better than the other algorithms on most of the test functions. However, it is worth noting that there is a more abrupt point in the plot, corresponding to the multimodal function F1, on which the APO algorithm has a lower ranking. In contrast, newly proposed algorithms like GTO and SMA perform better on such problems. This implies that the APO algorithm still has some challenges or limitations when dealing with specific multimodal function problems compared to the comparison algorithms.

4.4. CEC2022 test results

Following the evaluation and testing of the APO algorithm using the CEC2017 and CEC2019 test sets, where it demonstrated superior optimization capabilities, this section further validates the excellence of the APO algorithm using the latest CEC2022 test set. The test set consists of

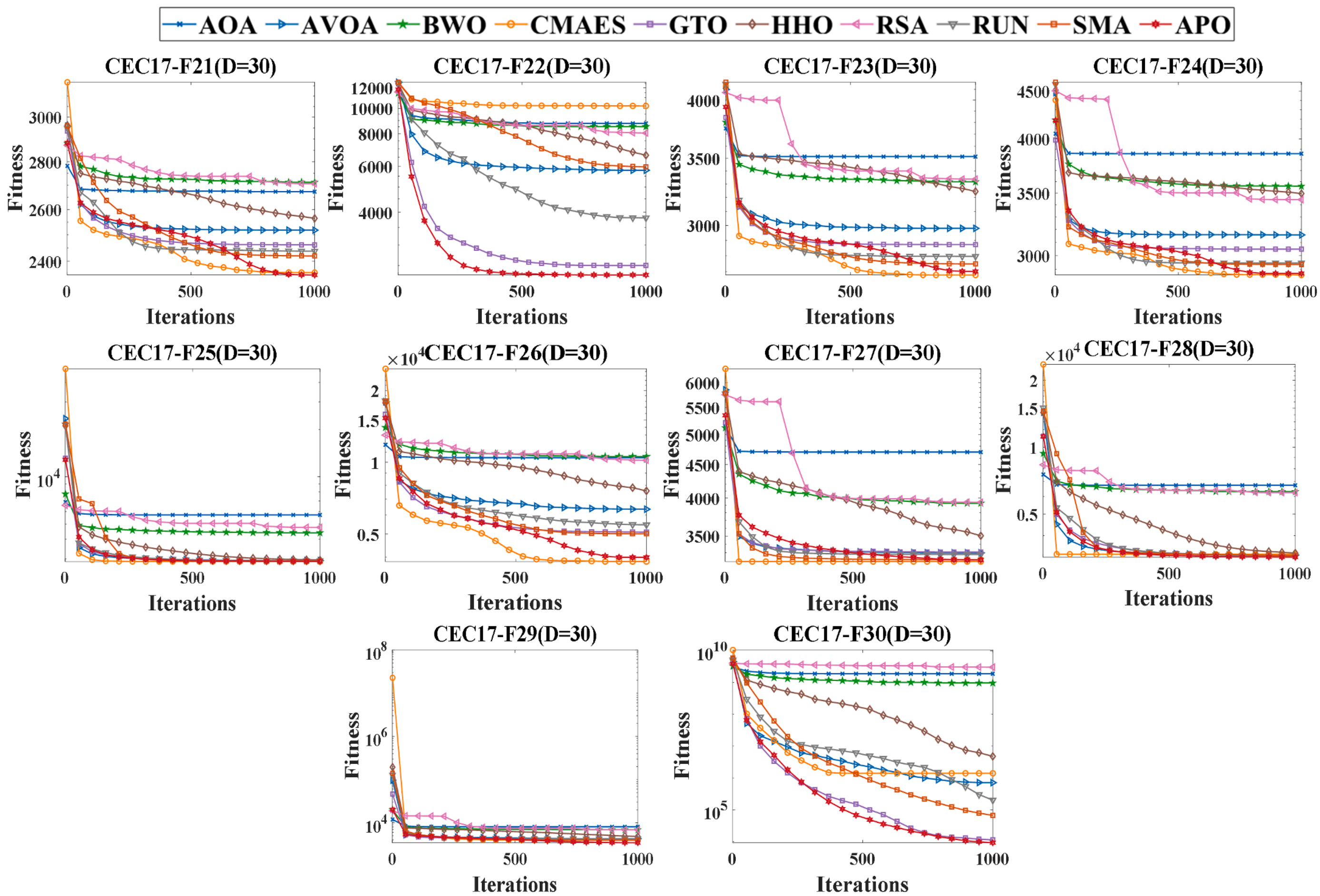

Fig. 9. Iterative convergence curve of APO and other algorithms on CEC2017 composition functions (F21-F30).

twelve functions categorized into unimodal functions (F1), basic functions (F2-F5), hybrid functions (F6-F8), and composition functions (F9-F12). The selected dimensionality is 10, and the algorithm is run thirty times with parameter settings detailed in Table 4. Table 10 records the averages, best values, standard deviations, and corresponding rankings after thirty runs for the ten algorithms.

As per the results in Table 10, the APO algorithm demonstrates a significant overall advantage in the CEC2022 test set, ranking first and achieving the best function values in nine out of the twelve functions. It ranks second in functions F2 and F9, with only a slight difference from the algorithms ranking higher. Following a comprehensive analysis, the Wilcoxon test is used again. Table 11 shows that the APO algorithm has significant differences compared to other algorithms, with an overall superiority in 75% of the functions.

Fig. 18 clearly shows the trend of the adaptation values of the ten algorithms in the evolutionary process. From the figure, it can be seen that APO converges faster than the other comparative algorithms, and the convergence curve is also smoother in general, with fewer times of falling into local extremes and a better ability to find optimization. When solving various types of complex problems, the APO algorithm shows better convergence ability and robustness.

Fig. 19 shows the box plot results of different algorithms. Observation reveals that APO, CMAES, GTO, RUN, and SMA have almost no outliers and significant performance fluctuations on most of the functions, indicating that they have high stability. Also, in function F9, APO has a higher box than CMAES; in function F12, there is a slight difference between SMA and GTO. However, overall, APO's box lengths are shorter and lower in most cases, indicating that APO has a clear advantage in both search capability and stability when compared to the other algorithms.

Radar Fig. 20 records the overall performance of each algorithm on different classes of CEC2022 functions. Specifically in Fig. 17, F1, F3… F12 represent the 12 test functions on CEC2019, while 1, 2…, and 12 represent the mean rankings of the algorithms on these functions. It can be observed that the APO algorithm has the smallest coverage area, further validating its superiority in terms of overall performance. This is followed by GTO, SMA, and RUN, whose coverage areas are close behind, highlighting the competitive nature of the newly proposed algorithms in recent years. In contrast, the differences in the coverage areas of the other algorithms are more obvious. This indicates that the APO algorithm has a significant advantage in terms of stability and performance when solving the CEC2022 test function and also reflects the emergence of new algorithms in the performance competition.

5. Engineering problems

Optimization algorithms ultimately need to be applied to effectively solve real-life optimization problems to maximize individual gains or minimize losses. Although APO algorithms present promising results in the test set evaluated, the results of the test function may not fully reflect the effectiveness of the algorithm's application in real-life scenarios.

Therefore, in addition to the analysis of the APO algorithms presented in the previous sections, a total of thirteen engineering problems in four domains will be used in this section to test the real-world constrained optimization-solving capabilities of the APO algorithms. Among these thirteen engineering cases, they cover mechanical engineering problems, industrial chemical processes, process synthesis and design problems, and parameter optimization of the Muskingum model,

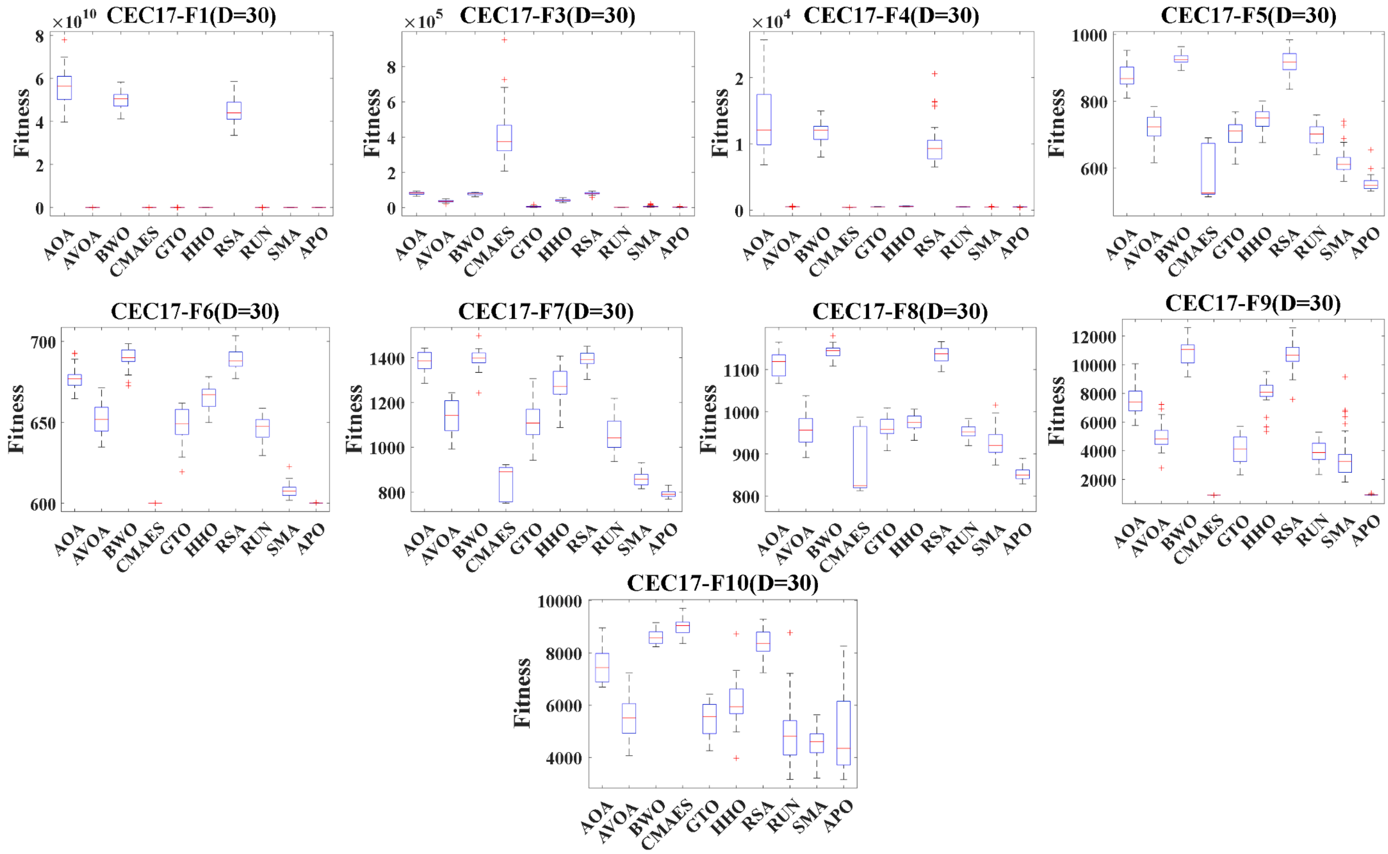

Fig. 10. Box plots of APO and other algorithms on CEC2017 unimodal and simple multimodal functions (F1-F10).

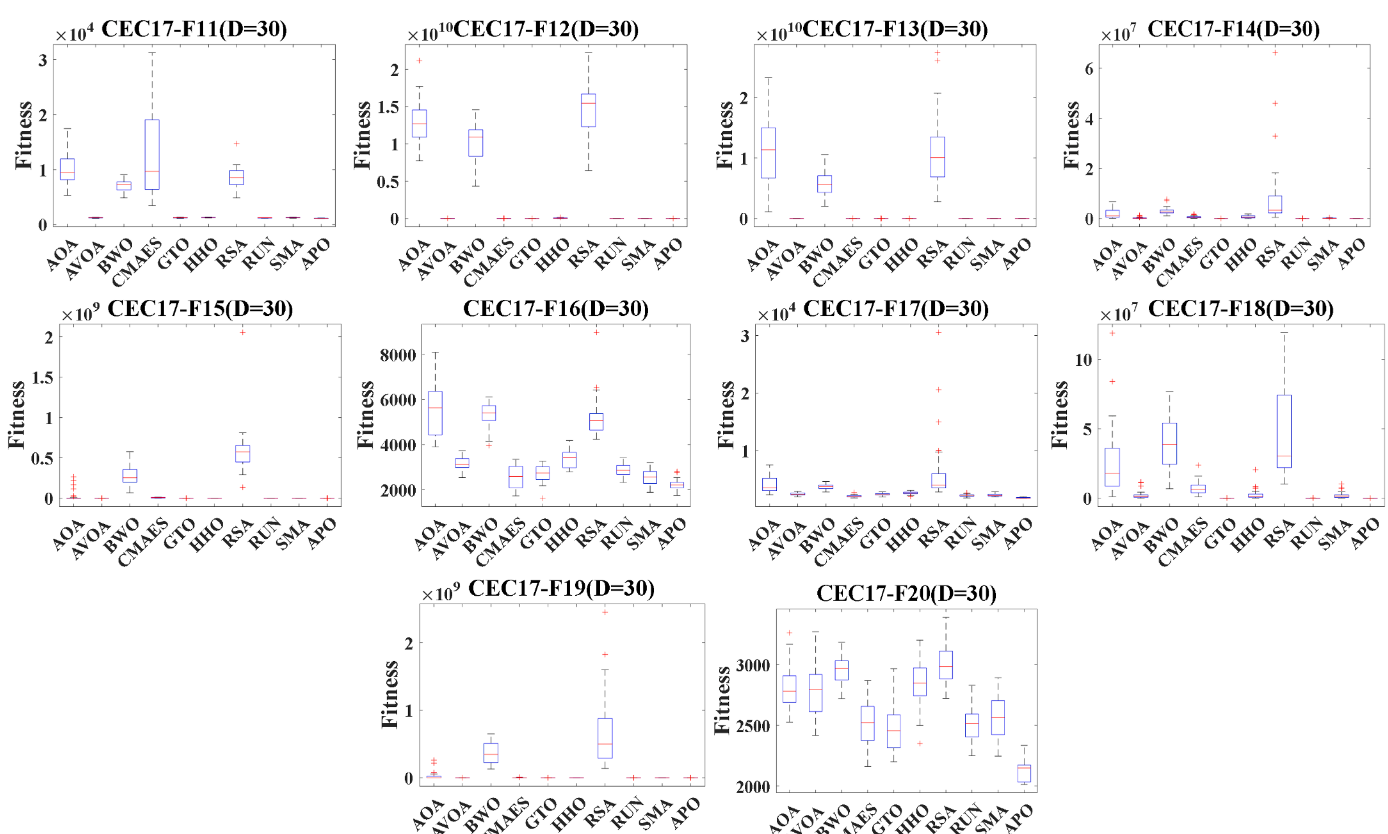

Fig. 11. Box plot of APO and other algorithms on CEC2017 hybrid functions (F11-F20).

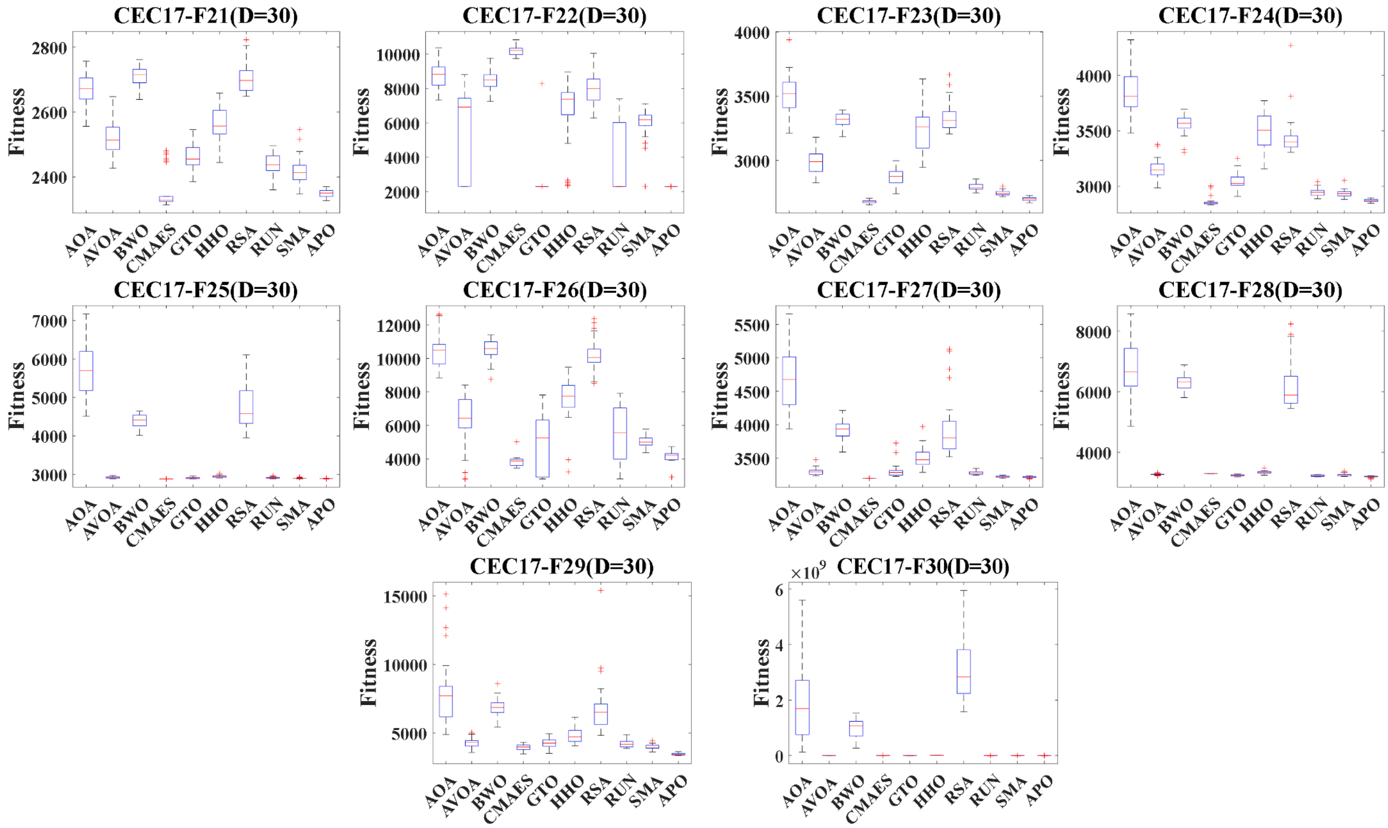

Fig. 12. Box plot of APO and other algorithms on CEC2017 composition functions (F21-F30).

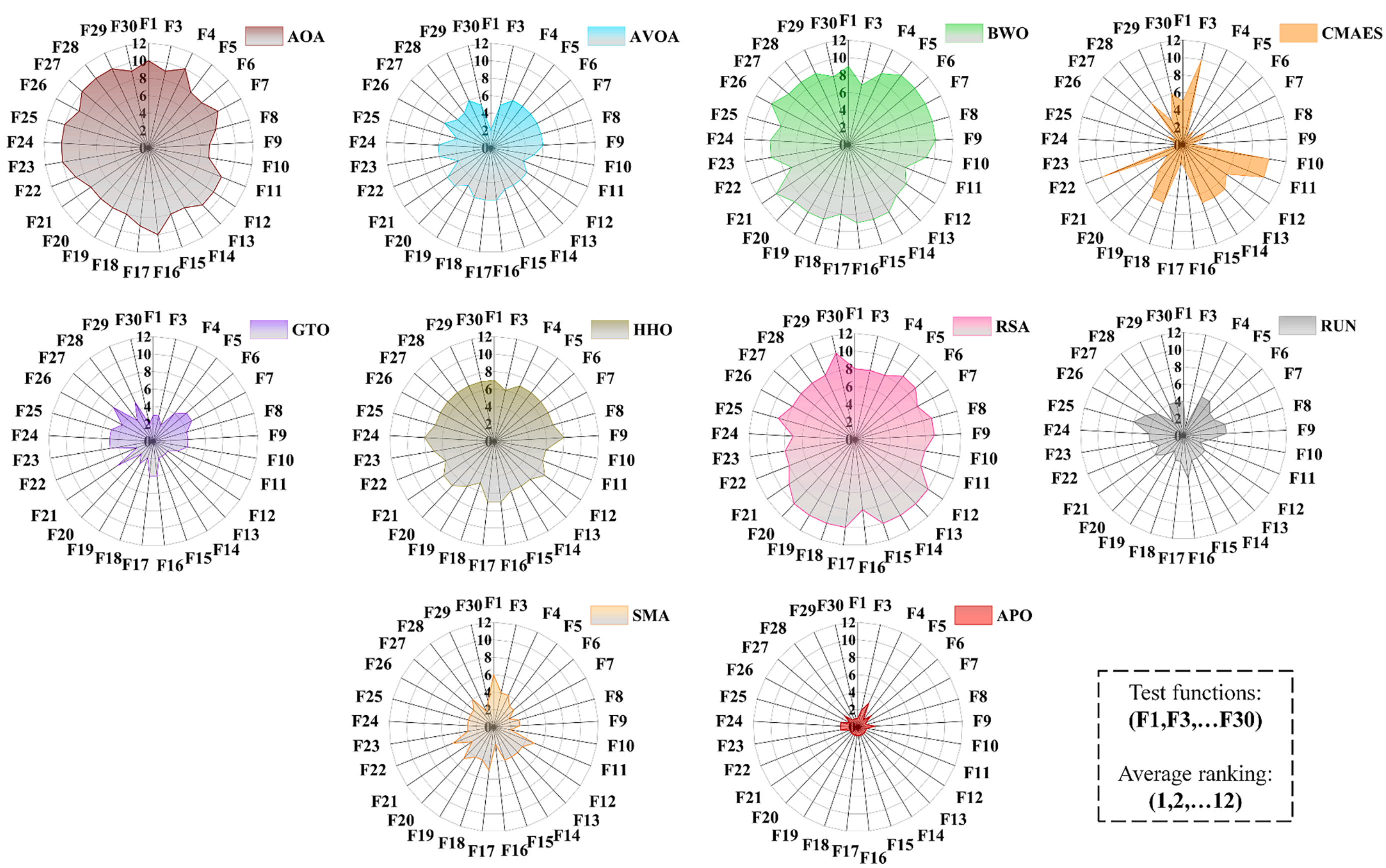

Fig. 13. Radar charts of various algorithms in the CEC2017 test set.

Table 7
The Friedman rank mean results of each algorithm in different dimensions of CEC2017.

Dimension	10D		30D		50D		100D	
Algorithms	Average ranking	Final ranking	Average ranking	Final ranking	Average ranking	Final ranking	Average ranking	Final ranking
AOA	8.52	9	8.75	9	9.07	10	8.85	10
AVOA	5.14	5	5.32	6	5.48	6	5.08	6
BWO	7.72	8	8.75	9	8.76	8	8.72	9
CMAES	5.66	6	4.38	5	3.62	4	4.58	5
GTO	3.19	2	3.62	2	3.83	5	3.72	4
HHO	6.72	7	6.68	7	6.52	7	6.45	7
RSA	9.41	10	8.68	8	8.76	8	8.35	8
RUN	3.52	3	3.72	4	3.59	2	3.55	3
SMA	4	4	3.62	2	3.59	2	3.35	2
APO	**1.12**	**1**	**1.48**	**1**	**1.79**	**1**	**2.35**	**1**

respectively. Among them, the mechanical engineering problems are challenging to test the algorithms due to their diverse constraints and objective functions, so six mechanical engineering problems are selected as cases to verify the feasibility and applicability of the algorithms. In addition, to more comprehensively assess the generalizability and robustness of the APO algorithm in a wider range of engineering optimization scenarios, we also selected three cases in the field of industrial chemical processes (RC02, RC04, RC05) and three cases in the field of process synthesis and design problems (RC08, RC10, RC11) from the real-world CEC2020 constrained problems [80]. Parameter optimization problems in the Muskingum model are highly nonlinear and complex, requiring algorithms with strong global search and local optimization capabilities to efficiently search the parameter space and find optimal solutions.

The selection of these real-world engineering problems covers practical applications in different fields. These engineering problems are more challenging than the CEC2017, CEC2019, and CEC2022 test sets. By discussing the potential challenges and limitations of the different problem domains, it will be possible to gain a more comprehensive understanding of the APO algorithms' performance in various real-world scenarios and provide a more nuanced view of their applicability in real-world engineering applications.

In order to evaluate the performance of APO algorithms, nine algorithms with the same test functions are selected for comparison testing, including AOA, AVOA, BWO, CMAES, GTO, HHO, RSA, RUN, and SMA. During the testing period, the number of populations is set to 30, and the maximum number of iterations is set to 1000. Each group of algorithms independently executes the test function thirty times for each test function. Optimization tests and the three evaluation metrics, average, best, and standard deviation (Std), are computed separately to evaluate the algorithms' performance, and each algorithm's final ranking is recorded. The best results will be indicated in bold. Each algorithm has the same parameter settings as listed in Table 4 and incorporates a penalty function to handle complex constraints.

5.1. Mechanical engineering problem

5.1.1. Tension/Compression spring design

The objective of this problem is to minimize the mass of the spring subjected to constraints on shear force, deflection, natural frequency, outer diameter, and other factors, as depicted in Fig. 21. Moreover, this problem requires solving while satisfying constraints related to shear force, deflection, natural frequency, outer diameter, and others. There are three design variables in this problem [81].

The experimental results are presented in Table 12. As shown in the table, the APO algorithm achieves the optimal solution for minimizing the objective function as $[X_1, X_2, X_3] = [0.0517, 0.3567, 11.2890]$, with the corresponding optimal value being 0.01266523. Furthermore, the APO algorithm outperforms other algorithms in terms of both average and standard deviation, demonstrating its effectiveness in solving practical engineering optimization design problems and further emphasizing the robustness of the APO algorithm.

5.1.2. Three-bar truss

The truss problem is a common architectural form widely used in bridges, buildings, and mechanical equipment. The design optimization problem of a truss structure involves adjusting parameters such as the dimensions, shapes, and connection methods of the truss members to achieve the lightest weight while meeting strength and stiffness requirements [82]. Fig. 22 illustrates the structure of this problem.

Using APO and the other nine optimization algorithms to solve the problem and run them independently thirty times, the results for each algorithm are presented in Table 13. It can be observed that both the APO and GTO algorithms achieved the optimal value and outperformed the other eight algorithms. However, compared to the GTO algorithm, APO has a better average value and also exhibits a superior standard deviation compared to the other nine algorithms. Therefore, in this problem, the APO algorithm demonstrates superior optimization capability and solution stability.

5.1.3. Speed reducer design

The reducer design problem is more complex compared to the previous two problems. It involves seven design variables [83], as illustrated in Fig. 23. The main objective is to minimize the weight of the reducer by adjusting the seven design variables while satisfying eleven constraints. Tables 14 and 15 present the results after multiple runs of APO and other algorithms.

It can be observed that the overall optimization results of the APO algorithm are significantly better than those of other algorithms, achieving the minimum weight for the reducer. This indicates that the APO algorithm can provide the optimal solution for the best reducer design, confirming the superior applicability and stability of the APO algorithm in the reducer design problem.

5.1.4. Gear design optimization

This problem is an actual optimization problem in mechanical engineering with relatively simple constraints and four variables [84]. The objective is to reduce the specific transmission cost of the gear system, as illustrated in Fig. 24.

To investigate the capability of the APO algorithm for this problem further, it was run thirty times and compared with other state-of-the-art optimization algorithms. The results are shown in Tables 16 and 17. It can be observed that the APO algorithm, along with the AVOA, CMAES, GTO, HHO, and RUN algorithms, achieved the optimal values in the minimum column. However, in terms of average values and standard deviations, APO reached the optimum, with values of 7.18E-11 and 2.10E-11, respectively. Therefore, the APO algorithm demonstrates better optimization precision and stability compared to other algorithms for this engineering problem.

5.1.5. Cantilever beam

The cantilever beam consists of five square-sectioned hollow units;

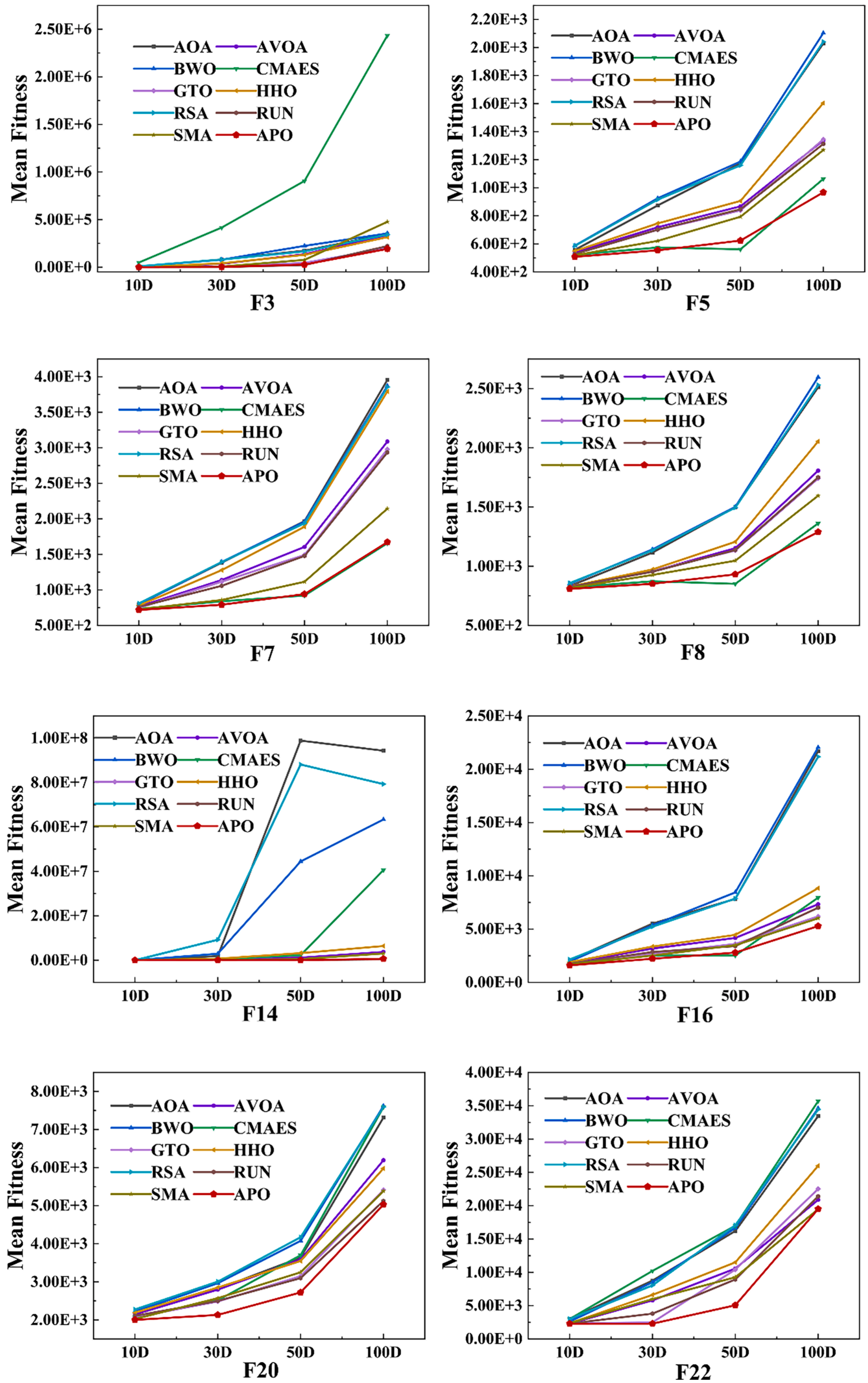

Fig. 14. APO scalability test analysis.

Table 8
Comparison of results of various algorithms under the CEC2019 test set.

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
CEC19-F1	Average	7.45E+04	**1.00E+00**	**1.00E+00**	9.47E+05	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	2.71E+00
	Best	1.00E+00	1.00E+00	1.00E+00	2.41E+04	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
	Std	3.84E+05	0.00E+00	0.00E+00	7.61E+05	0.00E+00	0.00E+00	0.00E+00	1.06E-15	0.00E+00	4.39E+00
	Rank	9	1	1	10	1	1	1	1	1	8
CEC19-F2	Average	9.60E+03	4.86E+00	5.00E+00	2.71E+03	4.39E+00	4.98E+00	4.99E+00	4.29E+00	4.95E+00	**4.17E+00**
	Best	3.18E+03	4.24E+00	4.99E+00	9.38E+02	4.21E+00	4.70E+00	4.87E+00	4.22E+00	4.26E+00	3.47E+00
	Std	2.69E+03	2.85E-01	1.41E-03	7.51E+02	2.53E-01	6.11E-02	3.27E-02	1.41E-01	1.83E-01	2.75E-01
	Rank	10	4	8	9	3	6	7	2	5	1
CEC19-F3	Average	9.83E+00	1.98E+00	4.58E+00	7.53E+00	**1.41E+00**	4.79E+00	7.20E+00	1.50E+00	3.73E+00	1.44E+00
	Best	7.15E+00	1.00E+00	2.42E+00	1.24E+00	1.41E+00	2.54E+00	5.48E+00	1.00E+00	1.41E+00	1.00E+00
	Std	7.99E-01	8.75E-01	9.89E-01	1.99E+00	4.35E-08	1.15E+00	8.93E-01	3.37E-01	2.49E+00	2.00E-01
	Rank	10	4	6	9	1	7	8	3	5	2
CEC19-F4	Average	4.65E+01	3.58E+01	7.70E+01	1.88E+01	2.94E+01	4.66E+01	7.99E+01	3.22E+01	2.13E+01	**8.23E+00**
	Best	2.39E+01	1.59E+01	5.87E+01	2.99E+00	9.95E+00	1.64E+01	5.31E+01	1.69E+01	5.98E+00	2.99E+00
	Std	1.51E+01	1.24E+01	9.00E+00	1.01E+01	1.32E+01	1.24E+01	1.56E+01	1.04E+01	8.89E+00	2.35E+00
	Rank	7	6	9	2	4	8	10	5	3	1
CEC19-F5	Average	7.55E+01	1.31E+00	5.20E+01	**1.00E+00**	1.39E+00	1.92E+00	8.50E+01	1.39E+00	1.34E+00	1.05E+00
	Best	2.72E+01	1.04E+00	2.86E+01	1.00E+00	1.05E+00	1.61E+00	4.39E+01	1.11E+00	1.12E+00	1.01E+00
	Std	2.48E+01	1.93E-01	1.04E+01	3.61E-03	2.96E-01	1.87E-01	2.48E+01	2.58E-01	1.49E-01	3.39E-02
	Rank	9	3	8	1	5	7	10	6	4	2
CEC19-F6	Average	1.02E+01	6.11E+00	1.04E+01	1.23E+00	4.83E+00	7.77E+00	1.03E+01	5.87E+00	4.85E+00	**1.00E+00**
	Best	8.39E+00	3.44E+00	8.43E+00	1.00E+00	2.42E+00	4.98E+00	8.43E+00	3.46E+00	1.70E+00	1.00E+00
	Std	9.66E-01	1.77E+00	7.83E-01	1.25E+00	1.33E+00	1.71E+00	9.13E-01	1.42E+00	1.56E+00	3.51E-13
	Rank	8	6	10	2	3	7	9	5	4	1
CEC19-F7	Average	1.33E+03	1.00E+03	1.64E+03	1.61E+03	9.41E+02	1.23E+03	1.77E+03	5.93E+02	6.94E+02	**5.11E+02**
	Best	8.48E+02	4.14E+02	1.22E+03	1.17E+03	4.99E+02	5.49E+02	1.47E+03	1.29E+02	3.77E+02	1.20E+02
	Std	2.62E+02	2.74E+02	1.87E+02	1.72E+02	3.12E+02	2.83E+02	1.97E+02	2.65E+02	2.12E+02	2.04E+02
	Rank	7	5	9	8	4	6	10	2	3	1
CEC19-F8	Average	4.65E+00	4.36E+00	4.69E+00	4.71E+00	4.03E+00	4.62E+00	4.92E+00	3.66E+00	3.76E+00	**3.26E+00**
	Best	3.53E+00	3.55E+00	4.25E+00	3.88E+00	2.87E+00	3.65E+00	4.50E+00	2.33E+00	2.82E+00	2.41E+00
	Std	4.46E-01	2.92E-01	1.71E-01	2.74E-01	4.29E-01	3.44E-01	1.54E-01	4.50E-01	4.18E-01	4.21E-01
	Rank	7	5	8	9	4	6	10	2	3	1
CEC19-F9	Average	2.39E+00	1.40E+00	3.08E+00	1.16E+00	1.24E+00	1.40E+00	2.76E+00	1.30E+00	1.26E+00	**1.11E+00**
	Best	1.35E+00	1.13E+00	2.17E+00	1.10E+00	1.09E+00	1.08E+00	2.00E+00	1.12E+00	1.14E+00	1.04E+00
	Std	7.20E-01	1.91E-01	4.26E-01	2.96E-02	1.05E-01	1.32E-01	5.23E-01	8.09E-02	8.74E-02	4.70E-02
	Rank	8	7	10	2	3	6	9	5	4	1
CEC19-F10	Average	2.11E+01	2.11E+01	2.14E+01	2.14E+01	1.83E+01	2.10E+01	2.14E+01	2.12E+01	2.04E+01	**1.35E+01**
	Best	2.09E+01	2.09E+01	2.12E+01	2.12E+01	3.31E+00	1.93E+01	2.11E+01	2.10E+01	1.13E+00	1.00E+00
	Std	5.01E-02	1.07E-01	8.02E-02	9.40E-02	6.36E+00	3.35E-01	1.02E-01	1.60E-01	3.65E+00	9.84E+00
	Rank	6	5	10	9	2	4	8	7	3	1
Average ranking		8.10	4.60	7.90	6.10	3.00	5.80	8.20	3.80	3.50	**1.90**
Final ranking		9	5	8	7	2	6	10	4	3	**1**

Table 9
Wilcoxon rank sum test results of each algorithm under CEC2019.

Function	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA
CEC19-F1	3.19E-01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
CEC19-F2	0.00E+00	1.30E-03	1.30E-03	0.00E+00	1.30E-03	1.30E-03	1.30E-03	1.30E-03	1.30E-03
CEC19-F3	1.60E-03	3.00E-04	1.40E-03	1.50E-03	3.00E-04	1.40E-03	1.50E-03	3.00E-04	8.00E-04
CEC19-F4	0.00E+00	0.00E+00	0.00E+00	3.00E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.40E-03
CEC19-F5	0.00E+00	3.00E-04	0.00E+00	1.00E-04	3.00E-04	3.00E-04	0.00E+00	3.00E-04	3.00E-04
CEC19-F6	1.70E-03	1.50E-03	4.10E-03	0.00E+00	1.50E-03	1.50E-03	4.40E-03	1.40E-03	1.00E-03
CEC19-F7	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
CEC19-F8	1.30E-03	1.30E-03	1.30E-03	1.30E-03	1.20E-03	1.30E-03	1.30E-03	8.00E-04	1.20E-03
CEC19-F9	3.00E-04	3.00E-04	5.00E-04	3.00E-04	3.00E-04	3.00E-04	4.00E-04	3.00E-04	3.00E-04
CEC19-F10	2.06E-01	2.06E-01	0.00E+00	0.00E+00	2.54E-01	1.99E-01	1.00E-04	5.98E-02	2.80E-01
+/=/-	0/0/10	1/0/9	1/0/9	1/0/9	2/0/8	1/0/9	1/0/9	1/0/9	1/0/9

its structure can be seen in Fig. 25. The objective of optimizing the cantilever beam design is to minimize the weight of the beam. The constraint is to satisfy a vertical displacement constraint, and it has five variables [84].

Tables 18 and 19 present the results of the APO algorithm and the other nine algorithms. The APO algorithm achieves a leading position in both the optimal and average values, ranking first. Additionally, APO attains the best standard deviation, which is 6.32E-16. This engineering application case further confirms the reliability and applicability of the APO algorithm in engineering cases.

5.1.6. *Step cone pulley*

This engineering problem aims to design the minimum-weight conical pulley, and its structure can be seen in Fig. 26. This problem has eleven constraints and five design variables, making it more challenging to assess the algorithm's solving capabilities [36]. Therefore, using the APO algorithm and nine other optimization algorithms for solving, the results are shown in Tables 20 and 21.

According to Table 20, it can be observed that both the APO algorithm and the GTO algorithm achieve the minimum value in the optimal solution, which is 1.64E+01. Regarding average values and standard deviations, the APO algorithm outperforms other advanced algorithms,

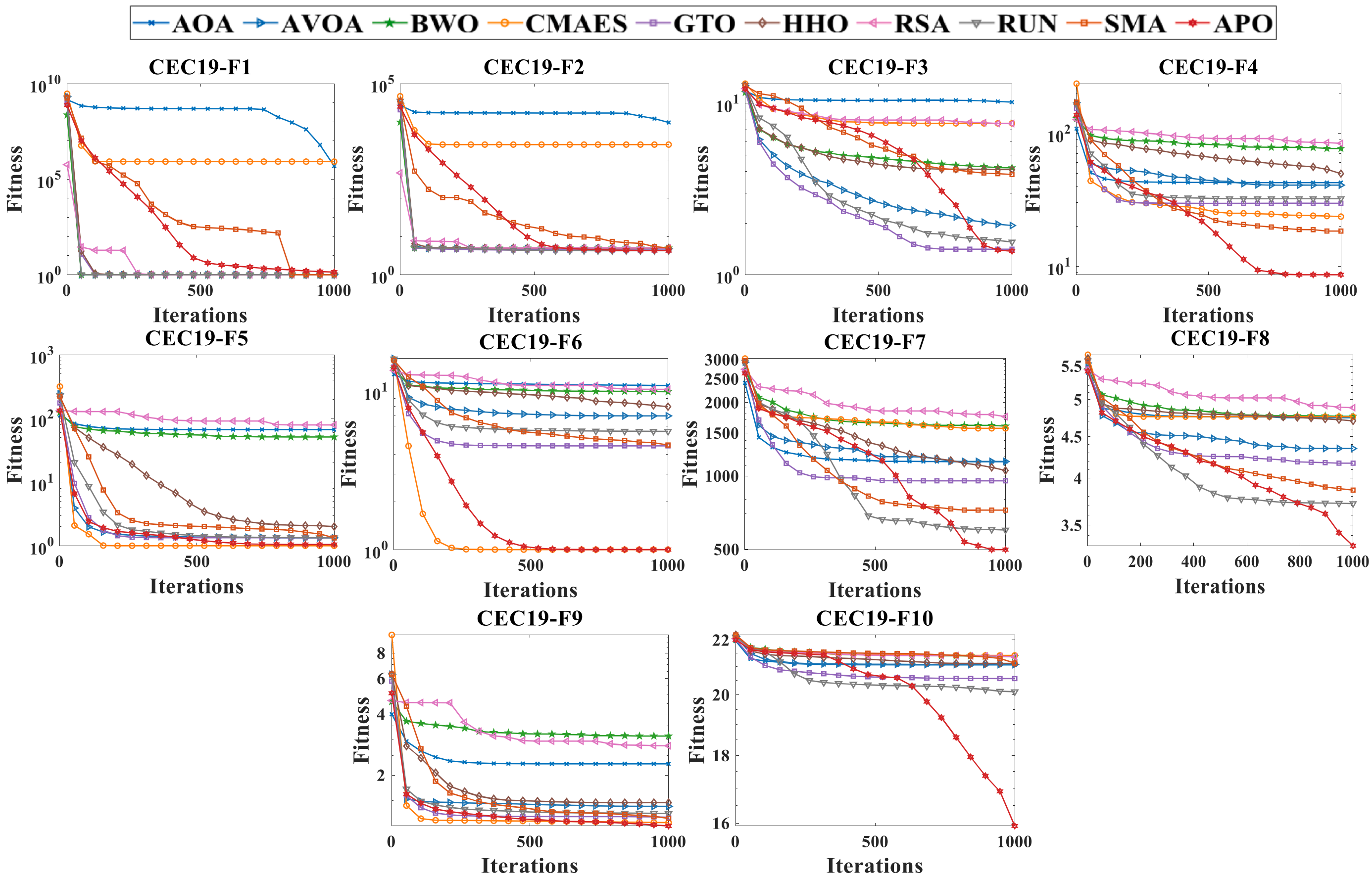

Fig. 15. Convergence process curve of APO and other algorithms in CEC2019.

demonstrating strong optimization capabilities and convergence stability. This further validates the APO algorithm's reliability and effectiveness in applying more complex engineering problems.

5.2. Industrial chemical processes

5.2.1. Heat exchanger network design (RC02)

In this typical heat exchange network design case, three nonlinear equation constraints and six linear equation constraints containing nonlinear objective functions need to be solved. Overall, the problem involves eleven variables and nine equational constraints, and these complex constraints make the problem more challenging to solve. The specific problem can be referred to in the literature [80] on RC02.

Tables 22 and 23 show the APO algorithm's objective function values and variable results along with the other nine algorithms for solving this problem. It is worth noting that the GTO algorithm ranks first in this problem, while the APO algorithm ranks second. Although the APO algorithm performed similarly to the GTO algorithm and outperformed the other compared algorithms in terms of optimal and mean values, the APO algorithm slightly underperformed the GTO algorithm in terms of standard deviation, indicating that it is not as stable in solving the problem as the GTO algorithm. This finding may imply that the APO algorithm may exhibit a certain degree of instability when dealing with this type of heat exchange network design constraint problem, and more optimization is needed to improve its performance.

5.2.2. Reactor network design (RC04)

The reactor network design problem focuses on optimizing the sequence of two continuous stirred tank reactors (CSTRs) to increase product concentration, thereby enhancing their quality and production efficiency. In the process of solving, it is necessary to consider six variables and satisfy four equality constraints and one inequality constraint. For specific questions, please refer to the introduction of RC04 in reference [80].

Tables 24 and 25 show the solution results of different algorithms for this problem. The APO, GTO, and SMA algorithms rank first, second, and third, respectively. Regarding optimal values, the performance of APO, GTO, SMA, and RUN algorithms is comparable, and they can all achieve equal objective function values when solving problems. However, the APO algorithm leads other algorithms regarding mean and standard deviation. This discovery indicates that the APO algorithm exhibits better optimization accuracy and stability when dealing with reactor network design problems in this field.

5.2.3. Haverly's pooling problem (RC05)

Haverley's Pooling Problem is a typical linear objective nonlinear constrained optimization problem that involves nine variables, four equality constraints, and two inequality constraints. For specific questions, please refer to the introduction of RC05 in reference [80].

Tables 26 and 27 show the APO algorithm's objective function values and variable results and the other nine algorithms in solving this problem. The results show that SMA, APO, and GTO algorithms rank first, second, and third. This indicates that the SMA algorithm outperforms other algorithms in this problem. However, APO and GTO algorithms perform similarly to SMA algorithms in terms of optimal solutions and are also competitive in average values, surpassing other algorithms. However, in terms of standard deviation, the APO algorithm is slightly inferior to the SMA algorithm, indicating that the stability of the APO algorithm is slightly inferior to the SMA algorithm when solving this problem.

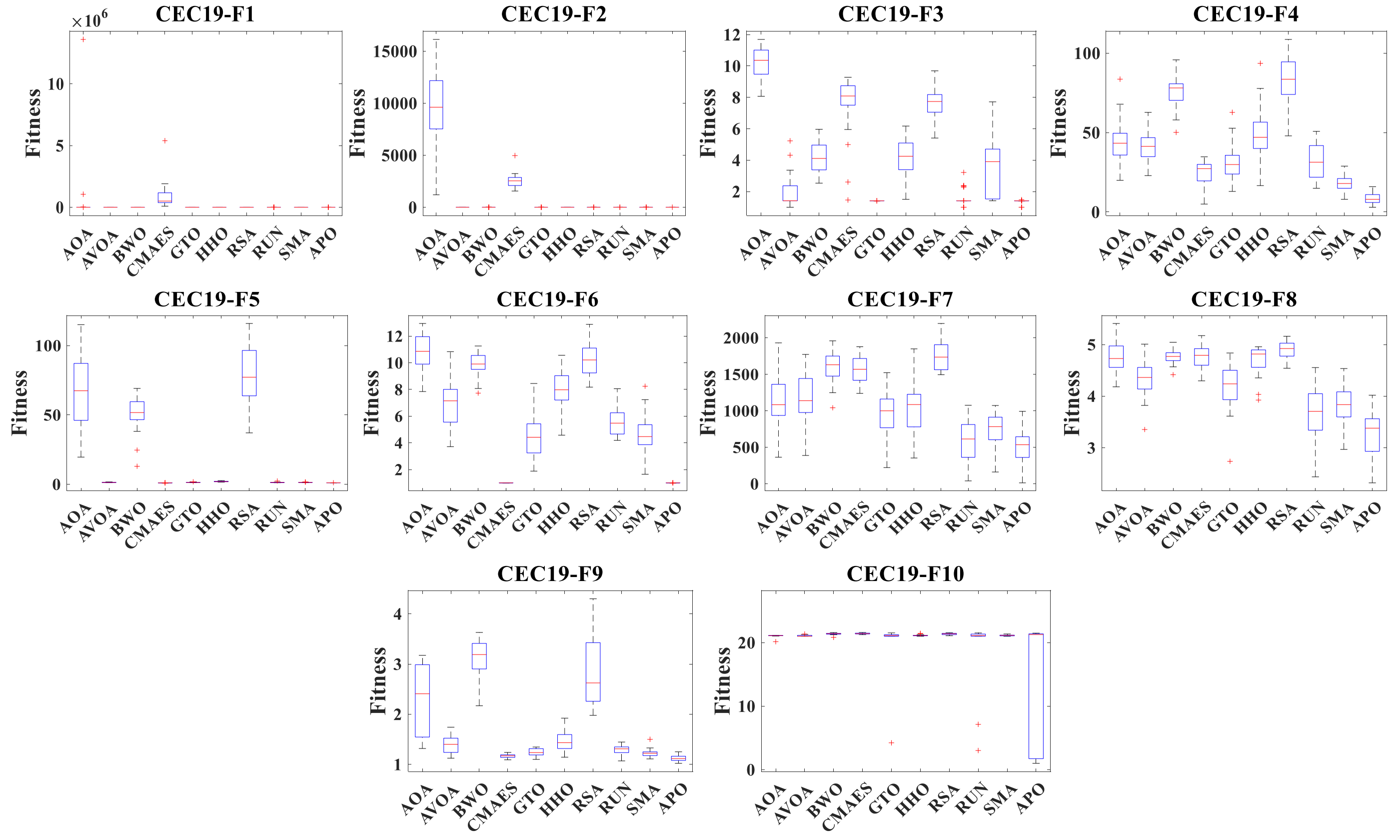

Fig. 16. Box plot of the test results of APO and other algorithms in CEC2019.

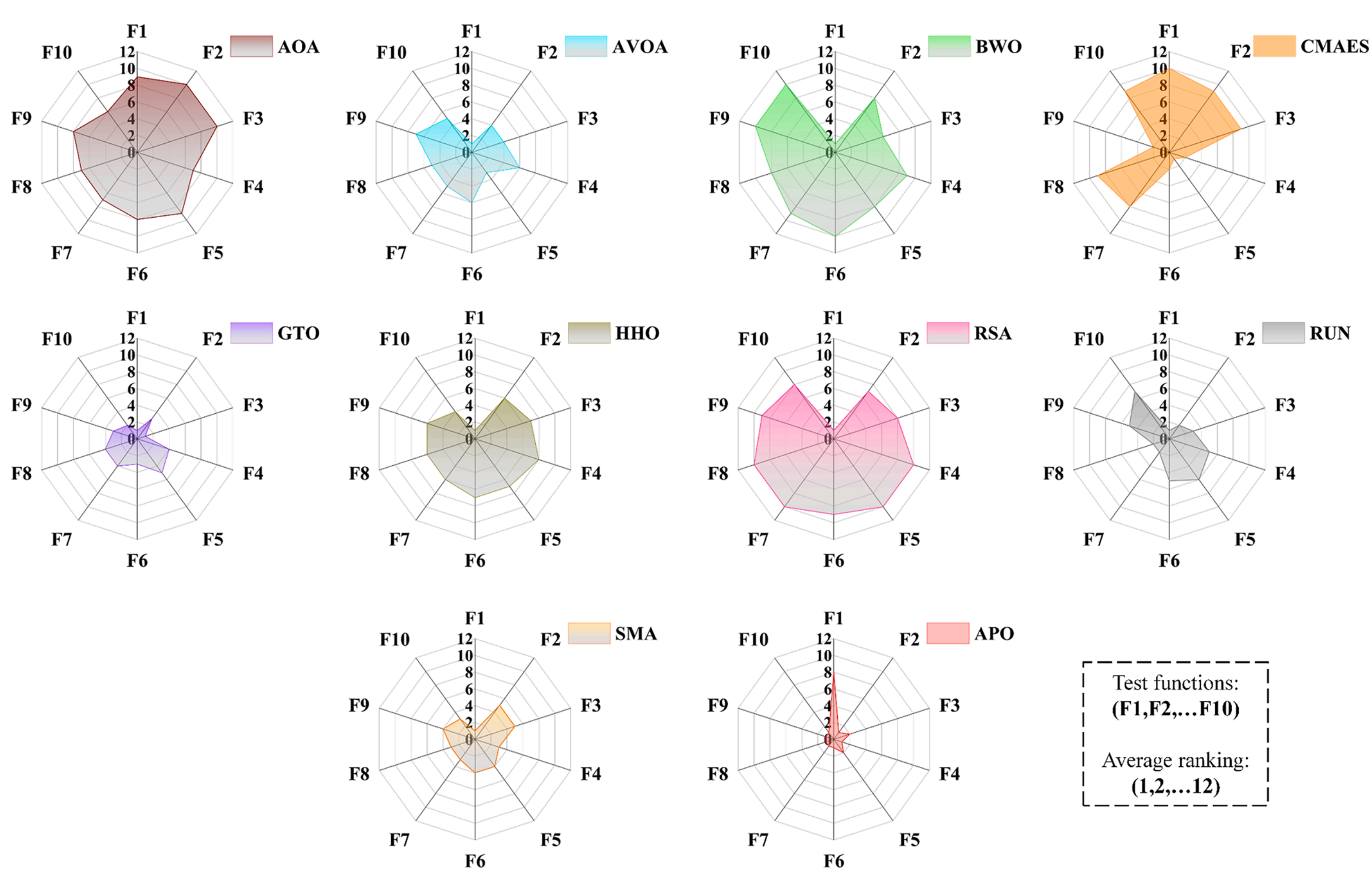

Fig. 17. Radar charts of each algorithm on the CEC2019 test set.

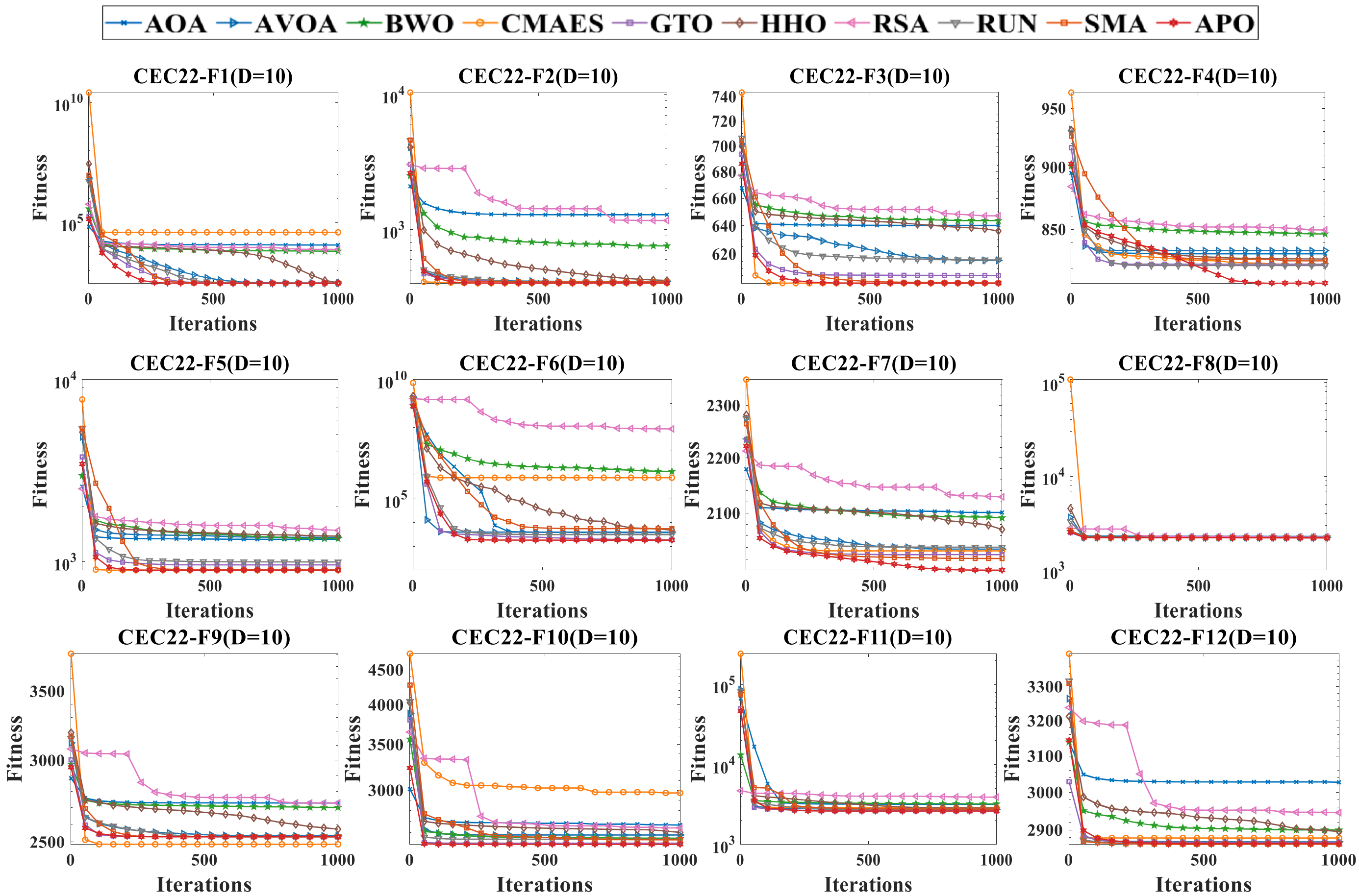

Fig. 18. Convergence process curves of APO and other algorithms on the CEC2022 test set.

5.3. Process synthesis and design problems

5.3.1. Process synthesis problem (RC08)

The process synthesis problem contains two nonlinear constraints and two variables and is relatively simple compared to previous engineering optimization problems. The specific problem can be referred to as the introduction of RC08 in the literature [80].

Table 28 shows the results of the APO algorithm and nine other algorithms for solving the objective function values and variables in this problem. The results show that all the algorithms obtain the exact optimal solution of 1.99. It shows that all algorithms perform similarly on this problem and can search for the optimal solution efficiently. Regarding mean and standard deviation, the AVOA, GTO, and APO algorithms lead the other algorithms and jointly rank first. This indicates that the APO algorithm has a significant average performance and stability advantage in solving this problem, further confirming its reliability and applicability in this engineering case.

5.3.2. Process flow sheeting problem (RC10)

The problem can be regarded as a nonconvex-constrained optimization problem containing three nonlinear constraints and three variables, which is relatively simple compared to other engineering optimization problems. The specific problem description can be found in the literature [80] about RC10.

Table 29 shows the results of solving objective function values and variables for the APO algorithm and the other nine algorithms on this problem. The results show that all the algorithms except AOA, BWO, and RSA obtained the exact optimal solution with a value of 1.05. In terms of mean and standard deviation, the APO algorithm leads the other compared algorithms in the first place. This indicates that the APO algorithm has a significant advantage in terms of average performance and stability in solving this problem, which further confirms that it can find a better solution in this engineering case and has a high value for engineering applications.

5.3.3. Two-reactor problem (RC11)

The primary aim of this problem is to choose one of the two reactors to optimize the production cost. The problem involves seven variables, four equation constraints, and four inequality constraints, which makes it somewhat complex and challenging compared to the previous two problems. A specific description of the problem can be found in the literature [80] about RC11.

Tables 30 and 31 show the objective function values and variable results of the APO algorithm and the other nine algorithms for solving this problem. It can be observed that GTO, RUN, SMA, and APO can obtain similar optimal solutions, and the average values of RUN and APO are ahead of the other algorithms. However, the standard deviation of APO is smaller than that of RUN, indicating that the algorithm has higher stability. This further demonstrates the robustness and stability of the APO algorithm in solving such problems.

5.4. Optimization of Muskingum model parameters

The Muskingum model is a hydrological model first proposed by McCarthy GT in 1938 that is widely used in the fields of flood forecasting and reservoir scheduling [85,86]. When performing flood path prediction, the parameter values of the model and its structure are crucial to the accuracy of flood forecasting. Therefore, accurately obtaining the parameters of the Muskingum model becomes a crucial task [11]. In order to improve the accuracy of flood forecasting, we will use the APO

Table 10
Comparison of results of various algorithms under the CEC2022 test set.

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
CEC22-F1	Average	1.16E+04	3.06E+02	6.15E+03	3.67E+04	3.00E+02	3.09E+02	8.86E+03	3.00E+02	3.00E+02	**3.00E+02**
	Best	3.82E+03	3.00E+02	3.90E+03	8.55E+03	3.00E+02	3.01E+02	4.46E+03	3.00E+02	3.00E+02	3.00E+02
	Std	5.05E+03	1.34E+01	1.12E+03	2.02E+04	1.91E-11	7.72E+00	2.20E+03	2.19E-03	2.46E-03	2.99E-14
	Rank	9	5	7	10	2	6	8	3	4	1
CEC22-F2	Average	1.18E+03	4.19E+02	7.75E+02	**4.01E+02**	4.06E+02	4.22E+02	9.72E+02	4.05E+02	4.10E+02	4.02E+02
	Best	5.38E+02	4.00E+02	5.38E+02	4.00E+02	4.00E+02	4.00E+02	5.65E+02	4.00E+02	4.05E+02	4.00E+02
	Std	4.68E+02	2.72E+01	1.20E+02	2.90E-01	3.19E+00	3.06E+01	4.35E+02	4.40E+00	1.32E+01	2.00E+00
	Rank	10	6	8	1	4	7	9	3	5	2
CEC22-F3	Average	6.41E+02	6.17E+02	6.47E+02	6.00E+02	6.05E+02	6.39E+02	6.47E+02	6.15E+02	6.00E+02	**6.00E+02**
	Best	6.20E+02	6.02E+02	6.35E+02	6.00E+02	6.00E+02	6.16E+02	6.33E+02	6.01E+02	6.00E+02	6.00E+02
	Std	7.79E+00	8.29E+00	5.92E+00	2.08E-06	5.02E+00	1.25E+01	7.67E+00	8.31E+00	1.69E-01	1.99E-09
	Rank	8	6	9	2	4	7	10	5	3	1
CEC22-F4	Average	8.31E+02	8.32E+02	8.47E+02	8.23E+02	8.23E+02	8.29E+02	8.48E+02	8.23E+02	8.26E+02	**8.09E+02**
	Best	8.17E+02	8.15E+02	8.34E+02	8.03E+02	8.08E+02	8.08E+02	8.32E+02	8.13E+02	8.05E+02	8.04E+02
	Std	8.92E+00	1.11E+01	4.78E+00	8.40E+00	6.56E+00	9.87E+00	8.59E+00	4.43E+00	1.05E+01	3.53E+00
	Rank	7	8	9	3	4	6	10	2	5	1
CEC22-F5	Average	1.38E+03	1.28E+03	1.39E+03	9.00E+02	1.01E+03	1.42E+03	1.54E+03	1.01E+03	9.08E+02	**9.00E+02**
	Best	1.02E+03	9.39E+02	1.21E+03	9.00E+02	9.01E+02	1.12E+03	1.24E+03	9.03E+02	9.00E+02	9.00E+02
	Std	1.81E+02	2.18E+02	1.06E+02	0.00E+00	1.02E+02	1.66E+02	1.59E+02	6.28E+01	2.19E+01	3.66E-14
	Rank	7	6	8	1	4	9	10	5	3	1
CEC22-F6	Average	4.02E+03	4.25E+03	1.81E+06	6.82E+05	1.88E+03	4.79E+03	6.09E+07	3.99E+03	5.58E+03	**1.82E+03**
	Best	1.91E+03	1.89E+03	1.94E+05	5.44E+04	1.81E+03	2.47E+03	1.12E+07	1.90E+03	1.91E+03	1.80E+03
	Std	1.35E+03	2.03E+03	1.50E+06	5.01E+05	4.83E+01	2.09E+03	3.30E+07	1.15E+03	2.18E+03	1.70E+01
	Rank	4	5	9	8	2	6	10	3	7	1
CEC22-F7	Average	2.11E+03	2.04E+03	2.10E+03	2.03E+03	2.03E+03	2.07E+03	2.13E+03	2.04E+03	2.02E+03	**2.00E+03**
	Best	2.05E+03	2.00E+03	2.06E+03	2.01E+03	2.00E+03	2.03E+03	2.08E+03	2.01E+03	2.00E+03	2.00E+03
	Std	5.28E+01	1.93E+01	1.77E+01	9.26E+00	8.35E+00	2.65E+01	2.48E+01	1.12E+01	3.77E+00	1.30E+00
	Rank	9	6	8	4	3	7	10	5	2	1
CEC22-F8	Average	2.30E+03	2.23E+03	2.24E+03	2.24E+03	2.22E+03	2.23E+03	2.34E+03	2.22E+03	2.22E+03	**2.20E+03**
	Best	2.22E+03	2.22E+03	2.23E+03	2.23E+03	2.21E+03	2.22E+03	2.23E+03	2.22E+03	2.20E+03	2.20E+03
	Std	9.50E+01	3.42E+00	4.88E+00	5.54E+00	3.32E+00	7.23E+00	4.69E+02	2.07E+00	6.12E+00	6.32E+00
	Rank	9	5	7	8	3	6	10	4	2	1
CEC22-F9	Average	2.73E+03	2.53E+03	2.69E+03	**2.49E+03**	2.53E+03	2.56E+03	2.73E+03	2.53E+03	2.53E+03	2.53E+03
	Best	2.64E+03	2.53E+03	2.62E+03	2.49E+03	2.53E+03	2.53E+03	2.61E+03	2.53E+03	2.53E+03	2.53E+03
	Std	6.70E+01	2.68E+01	2.60E+01	7.17E-01	5.93E-01	2.33E+01	5.66E+01	5.59E-04	3.47E-04	7.69E-13
	Rank	9	6	8	1	5	7	10	3	4	2
CEC22-F10	Average	2.68E+03	2.57E+03	2.56E+03	2.80E+03	2.52E+03	2.61E+03	2.71E+03	2.58E+03	2.52E+03	**2.50E+03**
	Best	2.51E+03	2.50E+03	2.51E+03	2.50E+03	2.50E+03	2.50E+03	2.52E+03	2.50E+03	2.50E+03	2.50E+03
	Std	1.21E+02	6.70E+01	6.10E+01	4.82E+02	4.61E+01	1.03E+02	1.48E+02	5.45E+01	4.55E+01	7.10E-02
	Rank	8	5	4	10	3	7	9	6	2	1
CEC22-F11	Average	3.37E+03	2.71E+03	3.23E+03	2.87E+03	2.68E+03	2.80E+03	3.89E+03	2.70E+03	2.71E+03	**2.61E+03**
	Best	2.80E+03	2.60E+03	2.88E+03	2.60E+03	2.60E+03	2.61E+03	3.02E+03	2.60E+03	2.60E+03	2.60E+03
	Std	3.14E+02	1.36E+02	2.65E+02	9.15E+01	1.58E+02	1.34E+02	3.77E+02	1.41E+02	1.60E+02	5.48E+01
	Rank	9	4	8	7	2	6	10	3	5	1
CEC22-F12	Average	3.02E+03	2.87E+03	2.90E+03	2.89E+03	2.86E+03	2.91E+03	2.94E+03	2.86E+03	2.86E+03	**2.86E+03**
	Best	2.89E+03	2.86E+03	2.87E+03	2.85E+03	2.86E+03	2.86E+03	2.88E+03	2.86E+03	2.86E+03	2.86E+03
	Std	6.14E+01	7.31E+00	1.77E+01	2.03E+01	1.80E+00	3.87E+01	7.52E+01	1.76E+00	1.24E+00	8.28E-01
	Rank	10	5	7	6	2	8	9	3	1	4
Average ranking		8.25	5.58	7.67	5.08	3.17	6.83	9.58	3.75	3.58	**1.42**
Final ranking		9	6	8	5	2	7	10	4	3	**1**

Table 11
Wilcoxon rank sum test results of each algorithm under CEC2022.

Function	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA
CEC22-F1	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.64E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC22-F2	1.73E-06	1.71E-06	1.73E-06	1.73E-06	1.68E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC22-F3	1.73E-06	1.73E-06	1.73E-06	1.62E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC22-F4	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC22-F5	1.73E-06	1.73E-06	1.73E-06	1.45E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC22-F6	5.29E-04	3.38E-03	1.73E-06	1.73E-06	1.73E-06	6.32E-05	1.73E-06	1.25E-04	1.64E-05
CEC22-F7	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC22-F8	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06	3.11E-05	1.73E-06	1.73E-06
CEC22-F9	4.29E-06	1.73E-06	1.73E-06	1.73E-06	1.55E-06	1.73E-06	1.73E-06	1.73E-06	1.73E-06
CEC22-F10	6.98E-06	1.73E-06	1.73E-06	2.18E-02	1.73E-06	2.13E-06	1.60E-04	1.73E-06	1.73E-06
CEC22-F11	5.22E-06	5.31E-05	1.73E-06	2.64E-03	1.89E-04	2.07E-02	1.73E-06	1.15E-04	2.83E-04
CEC22-F12	1.73E-06	4.07E-05	1.73E-06	1.97E-05	2.80E-01	2.13E-06	1.73E-06	3.18E-01	5.75E-06
+/=/-	0/0/12	0/0/12	0/0/12	2/1/9	1/0/11	0/0/12	0/0/12	1/0/11	1/0/11

algorithm to optimize the parameters of the nonlinear Muskingum model to improve the accuracy and predictive ability of the model. The mathematical representation of the nonlinear Muskingum model is as follows:

$$S_t = K[XI_t + (1-X)Q_t]^{\beta(u_t)} \tag{18}$$

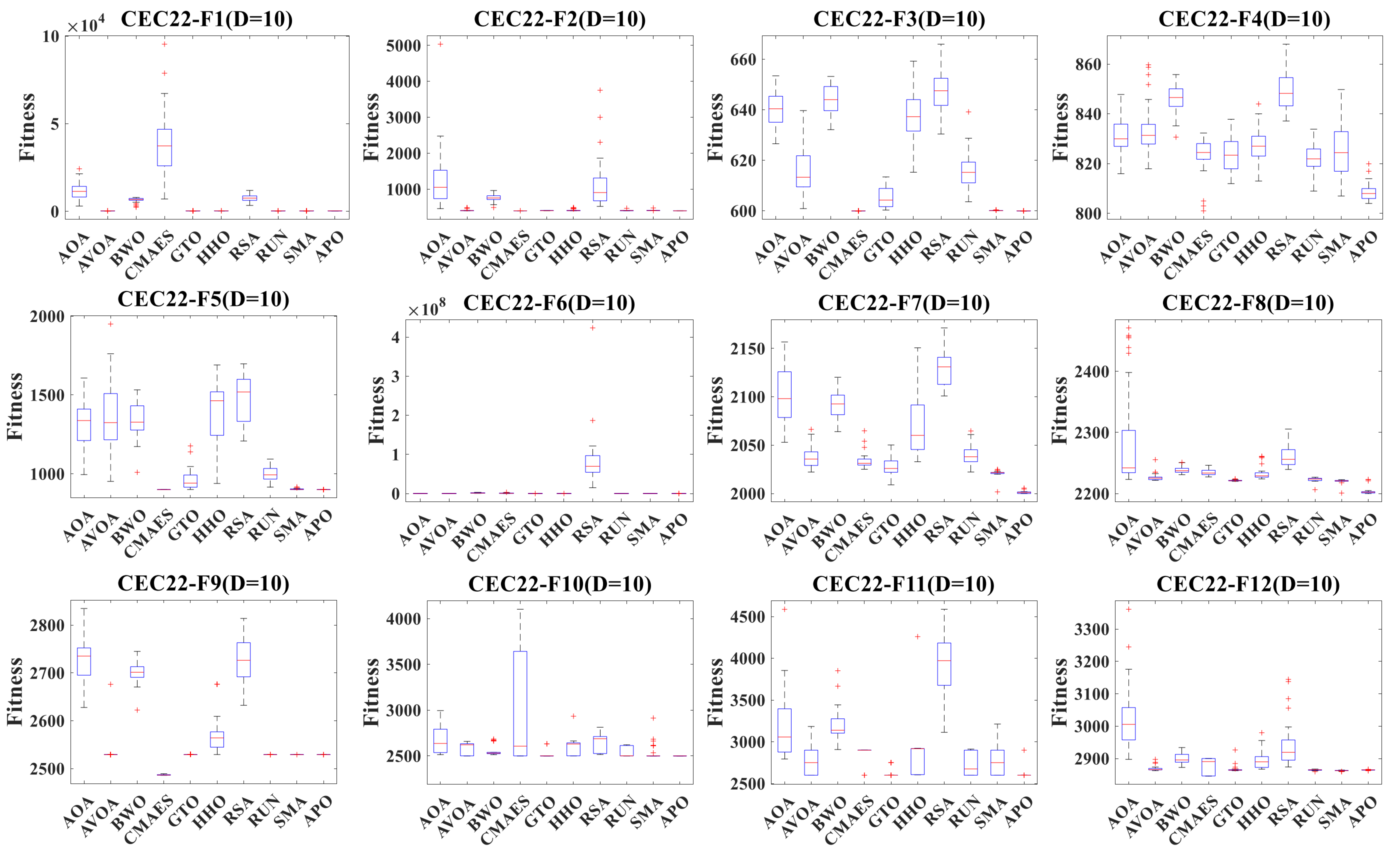

Fig. 19. Box plot of APO and other algorithms on the CEC2022 test set.

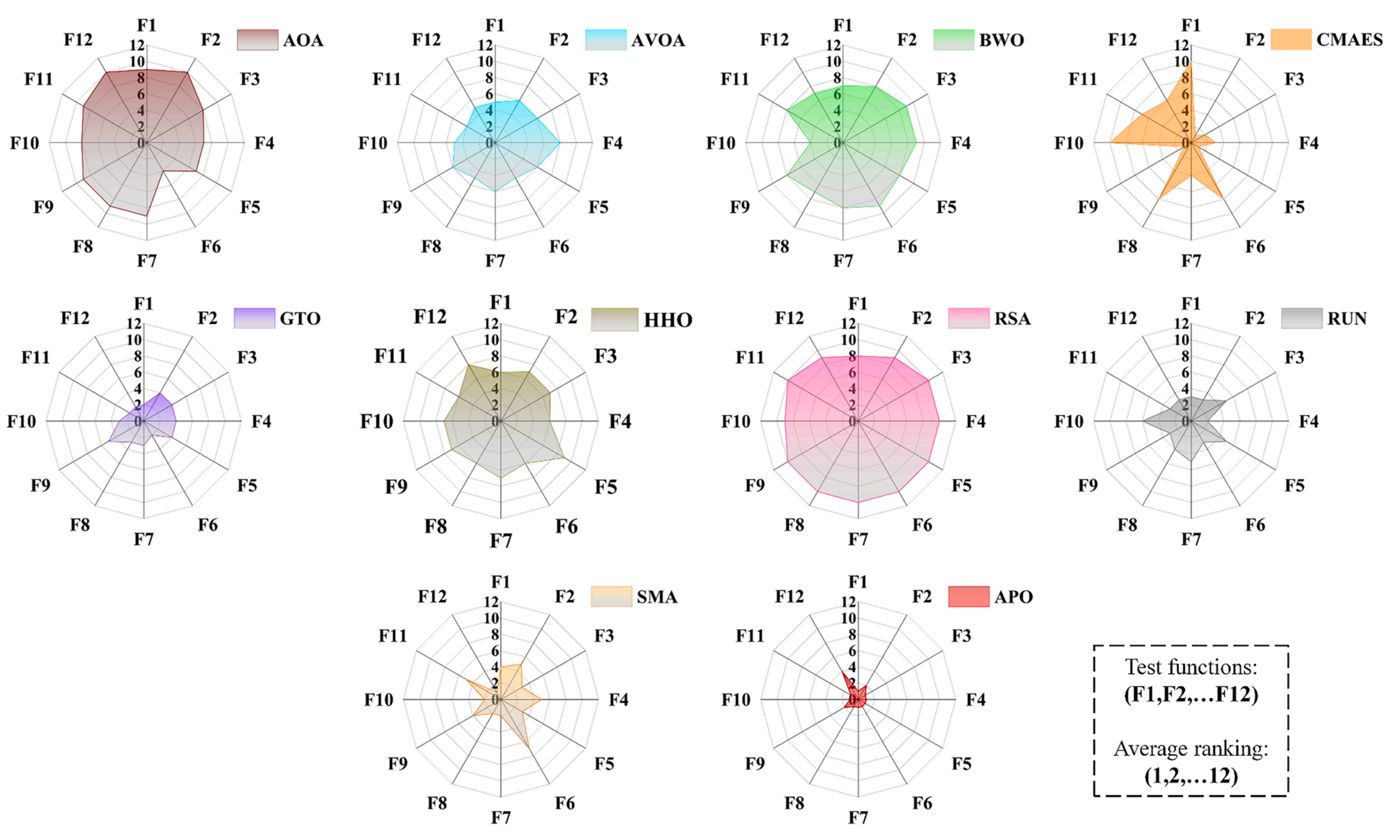

Fig. 20. Radar charts of various algorithms on the CEC2022 test set.

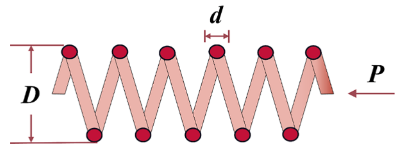

Fig. 21. Tension/compression spring design.

Table 12
Comparison of optimization results for tension/compression spring design.

Algorithm	Best	Average	Std	Rank	X_1	X_2	X_3
AOA	0.01314209	0.01570840	0.00646105	9	0.0500	0.3141	14.7350
AVOA	0.01266586	0.01325201	0.00058163	3	0.0519	0.3612	11.0300
BWO	0.01289897	0.01363306	0.00113981	7	0.0500	0.3172	14.2646
CMAES	0.01332763	0.01608058	0.00136760	10	0.0540	0.4076	9.2083
GTO	0.01266767	0.01270525	0.00003288	2	0.0521	0.3656	10.7860
HHO	0.01266526	0.01354770	0.00094471	5	0.0517	0.3577	11.2340
RSA	0.01319585	0.01547310	0.00558871	8	0.0500	0.3105	15.0000
RUN	0.01267022	0.01357481	0.00182890	6	0.0512	0.3444	12.0502
SMA	0.01266696	0.01345705	0.00110168	4	0.0514	0.3494	11.7306
APO	**0.01266523**	**0.01266565**	**0.00000147**	**1**	0.0517	0.3567	11.2890

$$\beta(u_t) = a + be^{-e^{cu_t}}$$
$$\begin{cases} u_t = I_t / I_{max} \\ I_{max} = \max\limits_{1\le t\le T} I_t \end{cases} \tag{19}$$

where *a, b*, and *c* are the model's optimized rate-setting parameters; *Imax* is the maximum inflow value of the river; I_t and Q_t represent the upstream and downstream flows at time *t*, respectively; *St* is the channel storage volume; *K* is the channel storage coefficient; and *X* is the specific gravity coefficient of the inflow and a particular section of the river.

After the initialization of the parameters is completed, the time periods are obtained. Assuming that the initial value of the flow is calculated to be equal to the inflow, i.e., Q_0=I_0, the initial river channel slot storage is obtained, and the calculation formula is as follows:

$$S_0 = K[XI_0 + (1-X)I_0]^{\beta(u_0)} \tag{20}$$

The equation for the cumulative tank storage for the next time period is expressed as follows:

$$S_{t+1} = S_t + \Delta t q_i \tag{21}$$

where k_i is the tank storage volume at time period *i*.

$$q_i = -\left(\frac{1}{1-X}\right)\left(\frac{S_i}{K}\right)^{\frac{1}{\beta(u_{i-1})}} + \left(\frac{1}{1-X}\right)I_i \tag{22}$$

Thus, the equation for the flow rate is derived and expressed as follows:

$$Q_{t+1} = \frac{1}{1-X}\left(\frac{S_{t+1}}{K}\right)^{1/\beta(u_t+1)} - \frac{X}{1-X}I_{t+1} \tag{23}$$

To verify the feasibility and effectiveness of the APO algorithm for parameter rate determination in nonlinear Muskingum models, we selected flood process data from Wilson (1960) in the UK. This data represents a smooth unimodal process line with obvious nonlinear relationships and is therefore widely used to verify the effectiveness of various parameter optimization estimation techniques in Muskingum models.

In order to comprehensively evaluate the performance of the APO algorithm when combined with the model, the same nine algorithms were selected and run 30 times for a solution. Using the sum of squared errors (SSQ) between measured and simulated discharge as the

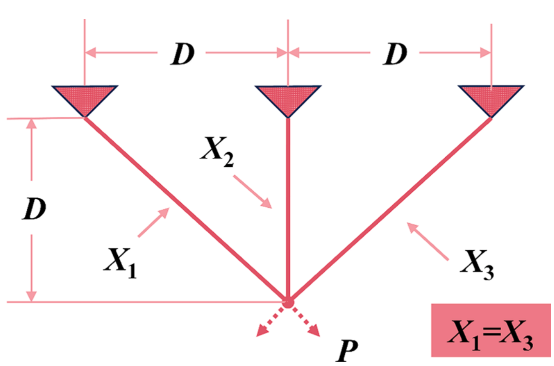

Fig. 22. Three-bar truss design model.

Table 13
Comparison of optimization results for three-bar truss.

Algorithm	Best	Average	Std	Rank	X_1	X_2
AOA	263.92234464	267.26523473	6.29E+00	9	0.7946	0.3916
AVOA	263.89584535	263.90584731	2.32E-02	3	0.7887	0.4081
BWO	263.96385520	264.42448721	3.08E-01	7	0.7819	0.4280
CMAES	263.89592112	264.03482849	9.63E-02	6	0.7885	0.4086
GTO	**263.89584338**	263.89584415	2.83E-06	2	0.7887	0.4082
HHO	263.89704912	263.97993490	1.25E-01	5	0.7874	0.4119
RSA	264.01582496	265.43383227	1.73E+00	8	0.7964	0.3877
RUN	263.89589324	263.92270218	7.89E-02	4	0.7884	0.4089
SMA	265.45741932	270.00799488	1.80E+00	10	0.8352	0.2923
APO	**263.89584338**	**263.89584338**	**2.99E-14**	**1**	0.7887	0.4082

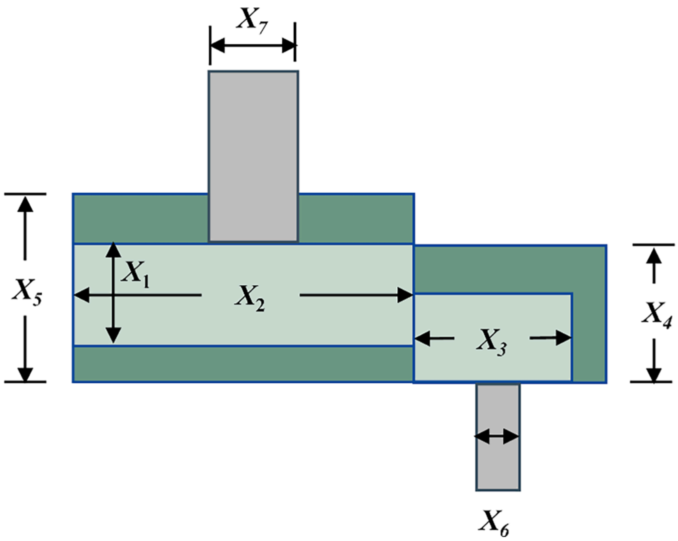

Fig. 23. Speed reducer design model.

Table 14
Comparison of optimization results for speed reducer design.

Algorithm	Best	Average	Std	Rank
AOA	3086.72560273	3156.05823763	4.50E+01	8
AVOA	2994.50164207	3000.13480073	4.91E+00	5
BWO	3025.09210562	3089.88246358	3.91E+01	7
CMAES	3011.85377335	3252.76304332	1.15E+02	10
GTO	2994.47106615	2998.30872623	6.95E+00	4
HHO	3008.11000749	3038.26023693	1.96E+01	6
RSA	3165.79741184	3240.70312050	5.35E+01	9
RUN	2994.52978679	2997.21653307	2.35E+00	3
SMA	2994.47139125	2994.47323312	1.90E-03	2
APO	**2994.47106615**	**2994.47106615**	**2.81E-10**	**1**

optimization objective function, we can verify the adaptability and superiority of the model in flood algorithms. The comparison of the results of the APO algorithm and other algorithms for optimizing the nonlinear Muskingum model is shown in Table 32.

Based on the data in the table, we can observe that the APO algorithm combined with the model achieved the best value, while the GTO algorithm and AVOA algorithm ranked second and third, respectively. This indicates that the performance of the proposed algorithm APO is superior to other algorithms, and by searching for appropriate parameters, the Muskingum model can make predictions more accurate. In addition, the standard deviation of other algorithms is relatively large, while the standard deviation of the APO algorithm combined with the model is 1.06E-10, indicating a smaller overall dispersion and better stability. Therefore, compared with other algorithms, it further highlights the superiority and stability of the APO algorithm in dealing with the optimization of Muskingum parameters in the field of flood forecasting.

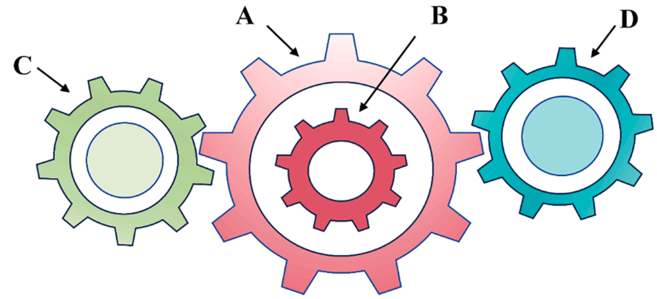

Fig. 24. Gear design optimization model.

Table 16
Comparison of optimization results for gear design optimization.

Algorithm	Best	Average	Std	Rank
AOA	1.17E-10	1.37E-08	1.22E-08	9
AVOA	2.70E-12	1.45E-09	1.46E-09	5
BWO	2.31E-11	5.42E-09	7.34E-09	6
CMAES	2.70E-12	6.71E-09	1.09E-08	7
GTO	2.70E-12	5.68E-10	5.11E-10	2
HHO	2.70E-12	1.11E-09	1.05E-09	4
RSA	9.92E-10	2.74E-08	2.96E-08	10
RUN	2.70E-12	6.60E-10	9.75E-10	3
SMA	2.31E-11	7.70E-09	1.00E-08	8
APO	**2.70E-12**	**7.18E-12**	**2.10E-11**	**1**

Table 17
Variable results for solving gear design optimization.

Algorithm	X_1	X_2	X_3	X_4
AOA	54.3640	22.1271	17.3992	47.8032
AVOA	42.9689	18.9426	16.1408	48.9999
BWO	42.9689	18.9426	16.1408	48.9999
CMAES	43.2202	15.9431	18.6771	49.3625
GTO	43.3640	19.4457	16.0233	49.2529
HHO	48.9315	15.7757	18.7550	43.0097
RSA	47.0228	25.8481	12.0000	46.4063
RUN	43.4936	19.0849	15.7123	48.9172
SMA	53.2049	19.9871	12.5419	33.5124
APO	49.0490	16.3319	19.2934	43.4194

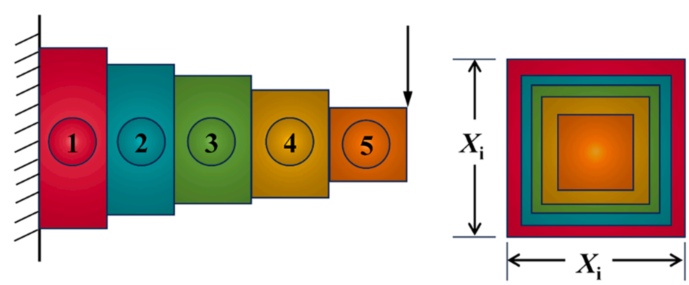

Fig. 25. Cantilever beam model.

Table 15
Variable results for solving speed reducer design.

Algorithm	X_1	X_2	X_3	X_4	X_5	X_6	X_7
AOA	3.6000	0.7000	17.0000	7.3000	8.3000	3.4833	5.2945
AVOA	3.5000	0.7000	17.0000	7.3008	7.7163	3.3502	5.2867
BWO	3.5029	0.7000	17.0000	7.3000	8.3000	3.3535	5.3112
CMAES	3.5000	0.7000	17.0000	7.8862	8.2529	3.3514	5.2868
GTO	3.5000	0.7000	17.0000	7.3000	7.7153	3.3502	5.2867
HHO	3.5037	0.7000	17.0000	7.6117	8.0211	3.3508	5.2907
RSA	3.6000	0.7000	17.0000	8.3000	8.3000	3.5458	5.3741
RUN	3.5000	0.7000	17.0000	7.3000	7.7171	3.3502	5.2867
SMA	3.5000	0.7000	17.0000	7.3000	7.7153	3.3502	5.2867
APO	3.5000	0.7000	17.0000	7.3000	7.7153	3.3502	5.2867

Table 18
Comparison of optimization results for cantilever beam.

Algorithm	Best	Average	Std	Rank
AOA	1.52301424	2.39361330	7.94E-01	10
AVOA	1.33995980	1.34007969	1.23E-04	4
BWO	1.34257395	1.35102869	5.37E-03	7
CMAES	1.33995839	2.12722325	8.27E-01	9
GTO	1.33995668	1.34000698	1.14E-04	3
HHO	1.34028467	1.34291049	2.19E-03	6
RSA	1.35661257	1.39609109	2.89E-02	8
RUN	1.33995655	1.33996268	8.64E-06	2
SMA	1.33998076	1.34018631	1.51E-04	5
APO	**1.33995636**	**1.33995636**	**6.32E-16**	1

Table 19
Variable results for solving cantilever beam.

Algorithm	X_1	X_2	X_3	X_4	X_5
AOA	5.1341	6.3478	6.9517	3.6487	2.3250
AVOA	6.0138	5.3079	4.4962	3.5083	2.1475
BWO	6.1774	5.4290	4.3001	3.4366	2.1726
CMAES	6.0210	5.3129	4.4912	3.4971	2.1515
GTO	6.0140	5.3110	4.4961	3.5000	2.1526
HHO	6.0228	5.3591	4.5325	3.4243	2.1402
RSA	6.1419	5.5541	4.1276	3.6271	2.2899
RUN	6.0171	5.3082	4.4958	3.5009	2.1517
SMA	6.0280	5.3220	4.4877	3.4992	2.1371
APO	6.0160	5.3092	4.4943	3.5015	2.1527

5.5. *Discussion of limitations*

Among thirteen engineering cases in four fields, the APO algorithm achieved first place in eleven practical problems, surpassing other comparative algorithms. Although performing well in most engineering cases, the APO algorithm still needs to improve when dealing with problems in the field of industrial chemical processes. Specifically, the GTO algorithm leads other algorithms in the RC02 problem, while the SMA algorithm outperforms other algorithms in the RC05 problem. In contrast, the APO algorithm ranks first in the RC04 problem in this field and second in the RC02 and RC05 problems. Although the APO algorithm can obtain the same optimal solution as the first-place algorithm, its mean and standard deviation performance is inferior to that of the GTO and SMA algorithms. This indicates that although the APO algorithm can achieve the best results when dealing with such problems, there are areas for improvement in stability and robustness. Therefore, APO may have the following potential limitations and challenges:

Specifically, when dealing with problems in industrial chemical

Table 20
Comparison of optimization results for step cone pulley.

Algorithm	Best	Average	Std	Rank
AOA	1.65E+95	2.68E+96	2.03E+96	8
AVOA	2.68E+72	1.36E+77	4.98E+77	3
BWO	3.49E+94	7.38E+95	8.76E+95	7
CMAES	3.72E+75	1.58E+98	1.33E+98	10
GTO	**1.64E+01**	1.73E+01	5.32E-01	2
HHO	5.63E+81	4.23E+91	1.37E+92	6
RSA	1.34E+97	7.03E+97	2.31E+97	9
RUN	4.59E+82	8.29E+86	2.05E+87	5
SMA	2.08E+79	3.96E+82	8.48E+82	4
APO	**1.64E+01**	**1.69E+01**	**2.62E-01**	1

Table 21
Variable results for solving step cone pulley.

Algorithm	X_1	X_2	X_3	X_4	X_5
AOA	40.6905	55.9192	74.4398	90.0000	90.0000
AVOA	40.6976	56.0035	74.6651	89.5179	88.6167
BWO	40.2284	55.3203	73.9596	88.3109	88.3109
CMAES	39.7117	54.6457	72.8551	87.3497	88.3306
GTO	38.8679	53.4837	71.3060	85.4940	89.5953
HHO	39.7150	54.6503	72.8611	87.3569	87.1656
RSA	41.2906	60.0000	76.0232	90.0000	90.0000
RUN	40.6283	55.9081	74.5380	89.3656	89.4947
SMA	40.8749	56.2476	74.9906	89.9078	87.9099
APO	39.1842	53.9193	71.8867	86.1897	88.2271

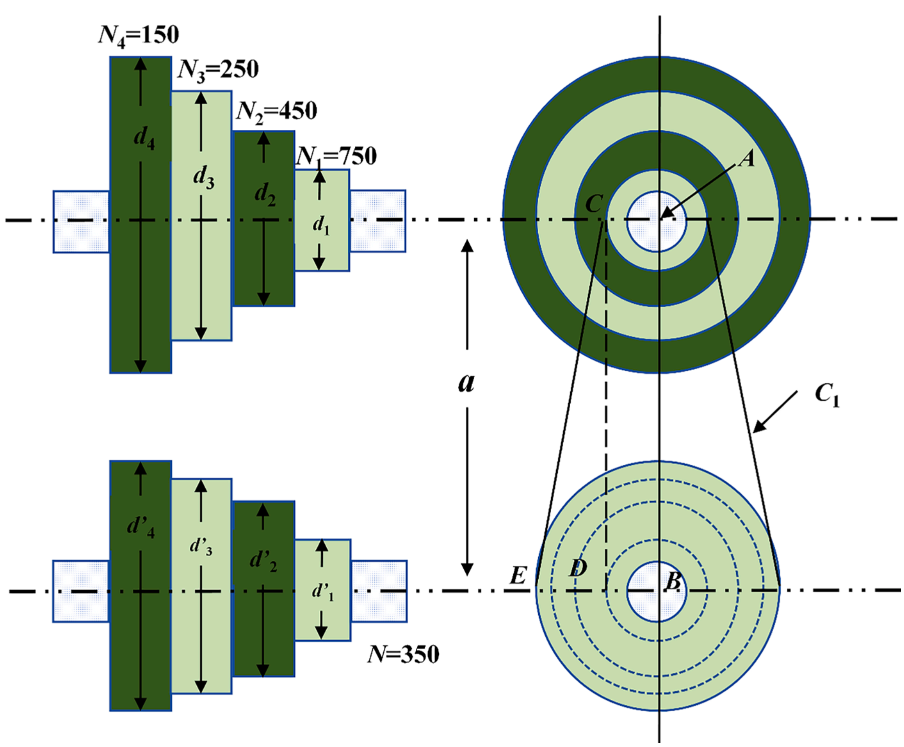

Fig. 26. Step cone pulley model.

Table 22
Comparison of the optimization results of each algorithm for solving RC02.

Algorithm	Best	Average	Std	Rank
AOA	8.07E+11	1.47E+12	4.80E+11	5
AVOA	1.54E+07	9.63E+08	1.12E+09	3
BWO	1.20E+13	3.57E+13	1.16E+13	7
CMAES	1.04E+04	3.63E+13	1.85E+13	8
GTO	**7.05E+03**	**7.05E+03**	**4.05E-09**	**1**
HHO	2.56E+09	1.65E+12	2.52E+12	6
RSA	7.23E+12	4.13E+13	2.33E+13	9
RUN	1.67E+04	8.51E+10	2.15E+11	4
SMA	2.31E+13	5.49E+13	7.61E+12	10
APO	**7.05E+03**	**7.05E+03**	1.55E-03	2

processes, the APO algorithm may exhibit a lack of stability over multiple runs, and this instability may be due to the uneven distribution of the initial population, which causes the search strategy to exhibit variation over multiple runs. In addition, when dealing with complex problems with high-dimensional search space, due to the stochastic nature of the algorithm, premature convergence or falling into a local optimum may occur during the iteration process, thus generating variations in different iterations and thus affecting the consistency of the algorithm. Moreover, the performance of the APO algorithm is strongly influenced by the initial parameters. The parameter settings in this paper are validated based on sensitivity analysis of multiple sets of test functions. However, the determination of the initial parameters may affect the algorithm's robustness in different scenarios in the face of problems in different domains, which is likewise a factor that needs to be further considered.

Therefore, in order to improve these limitations, the initial population distribution can be adjusted in subsequent studies to optimize the exploration and utilization mechanism, and reasonable parameters can be identified for more optimization and improvement to enhance the stability and robustness of the APO algorithm in a broader range of domains.

6. Conclusions and future work

This paper presents a new metaheuristic optimization algorithm, the Arctic Puffin Algorithm (APO). The algorithm draws on the Arctic puffin's group behavior and survival skills in aerial flight and diving hunting and enhances the search capability during the exploitation and exploration phases by integrating relevant adaptive strategies. With the help of behavioral transition factors, APO achieves a dynamic equilibrium between exploration and exploitation, thus providing near-global optimal solutions to complex optimization problems.

To validate the optimization search performance of APO, three challenging test function sets are selected for evaluation, namely CEC2017, CEC2019, and CEC2022. In addition, thirteen real-world engineering problems in four different domains are used to verify the generalizability and robustness of APO in a wide range of optimization scenarios. To demonstrate the advancement and competitiveness of APO, different classes of algorithms are selected for extensive comparisons, including recently proposed algorithms, highly cited algorithms, and competition-winning class algorithms, such as AOA, AVOA, BWO, CMAES, GTO, HHO, RSA, RUN, and SMA. The results show that the APO algorithm exhibits more vital competitive ability and stability in most cases. The APO algorithm obtained the best results and rankings in 73%, 70%, and 75% of the CEC2017, CEC2019, and CEC2022 test functions, respectively. The best results were achieved on eleven out of thirteen real-world engineering problems, with the best performance on the mechanical engineering problem, the process synthesis and design problem, and the Muskingum parameter optimization

Table 24
Comparison of the optimization results of each algorithm for solving RC04.

Algorithm	Best	Average	Std	Rank
AOA	−3.36E-01	−7.01E-02	1.16E-01	8
AVOA	−3.84E-01	−3.61E-01	3.47E-02	4
BWO	−3.88E-01	−3.12E-01	6.37E-02	5
CMAES	−3.68E-01	1.22E-01	5.03E-01	9
GTO	**−3.90E-01**	**−3.84E-01**	5.68E-03	2
HHO	−3.76E-01	−1.89E-01	1.32E-01	7
RSA	−4.30E-02	6.21E-01	2.60E+00	10
RUN	**−3.90E-01**	−2.86E-01	1.51E-01	6
SMA	**−3.90E-01**	−3.72E-01	2.45E-02	3
APO	**−3.90E-01**	**−3.84E-01**	**5.57E-03**	**1**

Table 25
Variable results of ten algorithms solving RC04.

Algorithm	X_1	X_2	X_3	X_4	X_5	X_6
AOA	1.0000	0.3987	0.0000	0.3883	0.0027	15.7628
AVOA	0.9849	0.4566	0.0187	0.3874	0.2964	11.9417
BWO	1.0000	0.3924	0.0000	0.3909	0.0000	16.0000
CMAES	0.8974	0.5389	0.0990	0.3715	1.9157	6.8444
GTO	0.9987	0.3923	0.0045	0.3928	0.0000	15.9749
HHO	0.9976	0.4504	0.0027	0.3781	0.0540	12.6649
RSA	1.0000	0.9562	0.0000	0.0489	0.0092	0.5245
RUN	0.9987	0.3923	0.0044	0.3928	0.0000	15.9749
SMA	0.9989	0.3925	0.0042	0.3927	0.0000	15.9608
APO	0.9987	0.3923	0.0045	0.3928	0.0000	15.9749

Table 26
Comparison of the optimization results of each algorithm for solving RC05.

Algorithm	Best	Average	Std	Rank
AOA	−3.69E+02	−3.55E+01	8.27E+01	7
AVOA	−1.56E+02	−7.98E+01	2.00E+01	5
BWO	−4.05E+01	−1.60E+01	1.11E+01	9
CMAES	3.35E+03	1.28E+04	6.02E+03	10
GTO	**−4.36E+02**	−1.05E+02	1.58E+02	3
HHO	−9.66E+01	−5.53E+01	1.36E+01	6
RSA	−5.94E+01	−3.24E+01	1.67E+01	8
RUN	−1.08E+02	−9.59E+01	9.56E+00	4
SMA	**−4.36E+02**	**−4.36E+02**	**1.60E+00**	1
APO	**−4.36E+02**	−4.35E+02	4.57E+00	2

Table 23
Variable results of ten algorithms solving RC02.

Algorithm	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
AOA	819,000.0000	1131,000.0000	2050,000.0000	0.0000	0.0000	0.0000	186.8949	300.0000	216.1992	285.4990	400.0000
AVOA	818,968.5177	1130,353.2636	2049,999.7040	0.0007	0.0129	0.0407	181.9195	294.9774	218.1032	286.9647	395.0000
BWO	819,000.0000	1131,000.0000	2050,000.0000	0.0481	0.0507	0.0443	157.4251	278.4781	226.1234	300.0000	400.0000
CMAES	818,999.9393	1130,999.9344	2049,999.9332	0.0411	0.0218	0.0264	181.9000	295.0000	218.1000	286.9000	395.0000
GTO	819,000.0000	1131,000.0000	2050,000.0000	0.0507	0.0507	0.0507	181.9000	295.0000	218.1000	286.9000	395.0000
HHO	817,529.3855	1128,966.1912	2046,312.4371	0.0507	0.0507	0.0507	182.0579	295.2570	218.3007	287.2900	395.2230
RSA	819,000.0000	1131,000.0000	2050,000.0000	0.0000	0.0000	0.0000	200.0000	300.0000	216.7520	300.0000	400.0000
RUN	818,999.9901	1130,999.9866	2049,999.9339	0.0122	0.0180	0.0203	181.9000	295.0001	218.1005	286.9002	395.0002
SMA	819,000.0000	1131,000.0000	2050,000.0000	0.0507	0.0507	0.0507	200.0000	300.0000	257.1836	300.0000	385.8261
APO	818,999.9999	1131,000.0000	2049,999.9998	0.0507	0.0507	0.0507	181.9000	295.0000	218.1000	286.9000	395.0000

Table 27
Variable results of ten algorithms solving RC05.

Algorithm	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
AOA	23.3368	200.0000	0.0000	100.0000	23.3368	100.0000	0.0000	100.0000	0.9897
AVOA	0.6425	59.5161	0.0000	20.5547	0.0037	35.1013	0.0000	22.4168	0.8844
BWO	0.0000	8.4417	0.0000	0.0000	0.0000	6.7056	0.0000	0.7941	0.3828
CMAES	62.2278	101.5117	13.9284	62.8876	29.0151	59.0648	46.4746	28.3583	1.2562
GTO	0.6429	200.0000	1.8571	95.1428	0.0000	100.0000	0.0000	98.5000	1.0189
HHO	0.7673	34.2162	0.0003	5.9950	0.1087	23.2878	0.0000	8.4938	0.6348
RSA	0.2961	14.7191	0.0000	0.0000	0.0000	11.4743	0.2312	1.1571	0.0009
RUN	0.6113	35.1625	0.0000	8.7463	0.0000	22.5892	0.0000	10.6262	0.7645
SMA	0.6472	200.0000	1.8626	95.1319	0.0000	100.0000	0.0000	98.4973	1.0189
APO	0.6429	200.0000	1.8571	95.1429	0.0000	100.0000	0.0000	98.5000	1.0189

Table 28
Comparison of optimization results for ten algorithms solving RC08.

Algorithm	Best	Average	Std	Rank	X_1	X_2
AOA	**1.99E+00**	2.00E+00	4.45E-02	9	0.4897	1.4900
AVOA	**1.99E+00**	**1.99E+00**	**6.78E-16**	**1**	0.4897	1.4900
BWO	**1.99E+00**	**1.99E+00**	1.27E-06	5	0.4897	1.4900
CMAES	**1.99E+00**	**1.99E+00**	5.99E-04	6	0.4897	1.1823
GTO	**1.99E+00**	**1.99E+00**	**6.78E-16**	**1**	0.4897	0.7441
HHO	**1.99E+00**	**1.99E+00**	1.66E-03	7	0.4897	0.7927
RSA	**1.99E+00**	2.01E+00	6.20E-02	10	0.4897	0.6930
RUN	**1.99E+00**	**1.99E+00**	1.83E-15	4	0.4897	1.0191
SMA	**1.99E+00**	**1.99E+00**	2.31E-03	8	0.4897	1.4898
APO	**1.99E+00**	**1.99E+00**	**6.78E-16**	**1**	0.4897	1.4898

problem. APO showed good competition in the industrial chemical process category, although it did not perform as well as GTO and SMA. In summary, the main contributions of this paper include:

- The APO algorithm improves its exploration and exploitation capabilities by introducing various strategies, such as Levy flight coefficients and synergy factor, that search the solution space comprehensively while digging the potential solution space more deeply. Especially when facing complex and diverse problems, one can focus on the possible optimal solutions more accurately and quickly.
- The behavioral conversion factor B in the APO algorithm exhibits adaptivity and dynamics, which effectively balance the dynamic relationship between exploration and exploitation and enhance the algorithm's robustness.
- The performance of APO is evaluated using three sets of challenging benchmark functions, CEC2017, CEC2019, and CEC2022, demonstrating superior results across a wide range of characteristics and complexities. Statistical validation using Friedman and Wilson-signed rank tests: APO receives first place, confirming significant superiority.
- APO demonstrated exceptional real-world utility by successfully solving thirteen engineering cases in four domains, notably mechanical engineering problems, process synthesis and design problems, and a leading comparison algorithm for parameter optimization of the Muskingum model.

The APO algorithm will be further explored and improved in future research. For example, considering the parameter settings in the algorithm, a version of APO with automatic parameter settings can be investigated. Meanwhile, the development of a multi-objective version of APO can be explored to effectively solve multi-objective optimization problems, further expanding the application scope and applicability of the algorithm. In addition, the APO algorithm can potentially solve many complex optimization problems in the real world. Future research can focus on identifying unsolved research problems, such as multilevel threshold image segmentation, botnet detection in the Internet of Things, etc. And consider applications in specific engineering fields, thus expanding the application of the APO algorithm to more fields. In conclusion, further improvement and application of the APO algorithm will provide a key impetus for advancing metaheuristic optimization algorithm technology and provide more stable and effective solutions to solve practical problems.

Ethical approval

This article does not contain any studies with human participants performed by any of the authors.

Table 30
Comparison of optimization results for ten algorithms solving RC11.

Algorithm	Best	Average	Std	Rank
AOA	1.06E+02	2.31E+02	6.31E+01	9
AVOA	9.92E+01	1.78E+02	6.19E+01	6
BWO	1.01E+02	1.52E+02	4.47E+01	4
CMAES	1.18E+02	2.14E+02	2.18E+01	8
GTO	**9.91E+01**	1.59E+02	5.51E+01	5
HHO	9.98E+01	1.84E+02	6.10E+01	7
RSA	2.34E+02	3.57E+02	1.63E+02	10
RUN	**9.91E+01**	**1.01E+02**	3.65E+00	2
SMA	**9.91E+01**	1.04E+02	4.07E+00	3
APO	**9.91E+01**	**1.01E+02**	3.59E+00	**1**

Table 29
Comparison of optimization results for ten algorithms solving RC10.

Algorithm	Best	Average	Std	Rank	X_1	X_2	X_3
AOA	1.23E+00	1.23E+00	8.77E-05	10	0.2143	−1.0000	−0.0393
AVOA	**1.05E+00**	1.19E+00	7.07E-02	5	0.9320	−2.0896	0.6929
BWO	1.09E+00	1.22E+00	2.47E-02	9	0.9363	−2.0797	0.9102
CMAES	**1.05E+00**	1.08E+00	5.30E-02	2	0.9320	−2.0896	1.0274
GTO	**1.05E+00**	1.15E+00	8.76E-02	3	0.9320	−2.0896	1.4525
HHO	**1.05E+00**	1.19E+00	7.03E-02	4	0.9320	−2.0897	1.4768
RSA	1.06E+00	1.20E+00	5.91E-02	7	0.9358	−2.0898	1.1809
RUN	**1.05E+00**	1.19E+00	7.07E-02	6	0.9320	−2.0896	1.1343
SMA	**1.05E+00**	1.21E+00	5.31E-02	8	0.9320	−2.0896	1.1274
APO	**1.05E+00**	**1.05E+00**	**2.26E-16**	**1**	0.9320	−2.0896	1.0761

Table 31
Variable results for 10 algorithms solving RC11.

Algorithm	X_1	X_2	X_3	X_4	X_5	X_6	X_7
AOA	14.4500	0.0000	2.8169	0.0000	1.4900	−0.0340	14.3881
AVOA	13.2902	0.0037	3.5987	0.0066	1.4523	0.1483	13.2686
BWO	13.7543	0.0000	3.2217	0.0000	1.4661	−0.1761	13.7650
CMAES	0.1576	14.3947	0.2278	4.9480	−0.0392	0.8711	14.5806
GTO	13.4088	0.0000	3.5119	0.0000	0.8407	0.3670	13.3838
HHO	14.1994	0.0000	3.0397	0.0000	1.4862	−0.4900	14.2038
RSA	10.1699	4.7684	2.7072	4.3960	1.0798	0.8090	14.9185
RUN	13.4088	0.0000	3.5119	0.0000	0.7390	0.0000	13.3838
SMA	13.4077	0.0000	3.5126	0.0000	1.1408	−0.0056	13.3827
APO	13.4088	0.0000	3.5119	0.0000	0.6840	0.0834	13.3838

Table 32
Comparison of results of ten algorithms for solving *SSQ*.

Algorithm	Best	Average	Std	Rank
AOA	3.14E+01	4.59E+01	1.08E+01	7
AVOA	2.15E+01	2.28E+01	4.59E+00	3
BWO	2.09E+01	2.84E+01	4.00E+00	5
CMAES	1.38E+02	1.59E+02	3.22E+01	9
GTO	2.08E+01	2.10E+01	3.02E+00	2
HHO	2.16E+01	3.49E+01	1.18E+01	6
RSA	3.21E+01	8.04E+01	2.77E+01	8
RUN	2.05E+01	2.45E+01	6.58E+00	4
SMA	2.54E+02	4.38E+02	5.00E+01	10
APO	**2.05E+01**	**2.05E+01**	**1.06E-10**	**1**

CRediT authorship contribution statement

Wen-chuan Wang: Writing – original draft, Methodology, Formal analysis, Conceptualization. **Wei-can Tian:** Writing – original draft, Methodology, Investigation, Data curation, Conceptualization. **Dong-mei Xu:** Writing – original draft, Methodology, Investigation. **Hong-fei Zang:** Investigation, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgments

The authors are grateful for the support of the special project for collaborative innovation of science and technology in 2021 (No: 202121206).

Appendix

SeeTables 33,34,35 for a description of the three test function sets.

Table 33
Details of CEC2017 benchmark functions.

Description	NO.	Functions	*Fi*
Unimodal functions	F1	Shifted and rotated bent cigar function	100
	F3	Shifted and rotated Zakharov function	300
Simple multimodal functions	F4	Shifted and rotated Rosen brock's function	400
	F5	Shifted and rotated Rastrigin's function	500
	F6	Shifted and rotated expanded Scaffer's F6 function	600
	F7	Shifted and rotated Lunacek Bi-Rastrigin function	700
	F8	Shifted and rotated Non-continuous Rastrigin's function	800
	F9	Hifted and rotated Levy function	900
	F10	Shifted and rotated Schwefel's function	1000
Hybrid functions	F11	Hybrid function 1 ($N = 3$)	1100
	F12	Hybrid function 2 ($N = 3$)	1200
	F13	Hybrid function 3 ($N = 3$)	1300
	F14	Hybrid function 4 ($N = 4$	1400
	F15	Hybrid function 5 ($N = 4$)	1500
	F16	Hybrid function 6 ($N = 4$)	1600
	F17	Hybrid function 6 ($N = 5$)	1700
	F18	Hybrid function 6 ($N = 5$)	1800
	F19	Hybrid function 6 ($N = 5$)	1900
	F20	Hybrid function 6 ($N = 6$)	2000
Composition functions	F21	Composition function 1 ($N = 3$)	2100
	F22	Composition function 2 ($N = 3$)	2200
	F23	Composition function 3 ($N = 4$)	2300
	F24	Composition function 4 ($N = 4$)	2400
	F25	Composition function 5 ($N = 5$)	2500
	F26	Composition function 6 ($N = 5$)	2600
	F27	Composition function 7 ($N = 6$)	2700
	F28	Composition function 8 ($N = 6$)	2800

(*continued on next page*)

Table 33 (*continued*)

Description	NO.	Functions	*Fi*
	F29	Composition function 9 ($N = 3$)	2900
	F30	Composition function 10 ($N = 3$)	3000

Table 34
Details of CEC2019 benchmark functions.

NO.	Functions	*Fi*	*D*
F1	Storn's Chebyshev Polynomial Fitting Problem	1	9
F2	Inverse Hilbert Matrix Problem	1	16
F3	Lennard-Jones Minimum Energy Cluster	1	18
F4	Rastrigin's Function	1	10
F5	Griewangk's Function	1	10
F6	Weierstrass Function	1	10
F7	Modified Schwefel's Function	1	10
F8	Expanded Schaffer's F6 Function	1	10
F9	Happy Cat Function	1	10
F10	Ackley Function	1	10

Table 35
Details of CEC2022 benchmark functions.

Description	NO.	Functions	*Fi*
Unimodal functions	F1	Shifted and full Rotated Zakharov Function	300
Basic functions	F2	Shifted and full Rotated Rosenbrock's Function	400
	F3	Shifted and full Rotated Expanded Schaffer's $f6$ Function	600
	F4	Shifted and full Rotated Non-Continuous Rastrigin's Function	800
	F5	Shifted and full Rotated Levy Function	900
Hybrid functions	F6	Hybrid Function 1 ($N = 3$)	1800
	F7	Hybrid Function 2 ($N = 6$)	2000
	F8	Hybrid Function 3 ($N = 5$)	2200
Composition functions	F9	Composition Function 1 ($N = 5$)	2300
	F10	Composition Function 2 ($N = 4$)	2400
	F11	Composition Function 3 ($N = 5$)	2600
	F12	Composition Function 4 ($N = 6$)	2700

Although the existing literature usually adopts the same number of iterations as a criterion for algorithm performance evaluation, equivalently, using the same number of function evaluations is also a valid criterion for measuring algorithm performance. Based on this concept, this paper follows the CEC test set criteria sets the maximum number of evaluations (MaxFes = 300,000) as a unified evaluation criterion, and conducts a series of comparison tests to further validate the performance of the APO algorithm. The rest of the relevant parameter settings are kept consistent with the experiments in the paper.

Table 36 shows the test results of each algorithm on the CEC2017 test set under the condition of the same number of evaluations. The results show that among the 24 tested functions, the APO algorithm ranks first in the average ranking, which is significantly better than the other compared algorithms and establishes its position as the best optimization algorithm. The APO algorithm is ranked first in both the unimodal and the simple multimodal functions, and it is second only to the RUN and the GTO algorithms in the hybrid functions F18 and F19. On the combination functions F25 and F28, the APO algorithm lags behind the SMA and RUN algorithms, respectively. On the rest of the functions, the APO algorithm is ranked first, obtaining the best results and rankings in 83% of the tested functions. Figs. 27-29 demonstrates the difference in convergence ability between the APO algorithm and the other comparative algorithms on CEC2017, where the APO algorithm achieves better convergence results on most functions, showing excellent exploration and development capabilities.

Table 37 presents the test results of each algorithm on the CEC2019 test set under the same number of evaluations. The APO algorithm ranked first in seven of the ten tested functions (F2, F4, F6-F10). Compared to other algorithms, the APO algorithm typically exhibits a smaller standard deviation, indicating a more stable performance. Despite a slightly lower ranking on function F1, which fails to reach the theoretical optimum, and falling behind the RUN and SMA algorithms on F3 and F7, respectively, the APO algorithm overall ranks in the top two for 90% of the tested functions. In addition, the APO algorithm ranked first in the overall average, highlighting its superior competitiveness. Fig. 30 further reveals the convergence trend of the compared algorithms during the iteration process, showing different degrees of localized bounding phenomena, which validates that the APO algorithm achieves a good balance between exploration and exploitation capabilities. Although the APO algorithm fails to reach the theoretical optimum in test function F1, in most cases, it can converge quickly and accurately, finding better solutions from the beginning to the end of the iteration, with a smooth convergence curve and few large fluctuations, indicating that it can jump out of the local optimum quickly.

Table 38 shows the test results of each algorithm on the CEC2022 test set under the condition of the same number of evaluations. According to Table 38, the APO algorithm shows a significant overall advantage on the CEC2022 test set, ranking first and achieving the best function value in eleven test functions. Despite a slightly lower ranking in function F12, the gap between it and the higher-ranked algorithms is minimal, and overall it performs well in 91% of the tested functions. Fig. 31 clearly shows the trend of the adaptation values of the ten algorithms during the evolutionary process, with the APO algorithm converging faster, having a smoother convergence curve, falling into fewer local extremes, and optimizing better than the other compared algorithms. When solving various complex problems, the APO algorithm shows excellent convergence performance and robustness.

In summary, even if the maximum number of evaluations is used as a criterion, the APO algorithm still shows a clear leading edge on most of the test functions, proving its potential and value as an efficient optimization algorithm.

Table 36
Comparison of results of various algorithms under the CEC2017 test set.

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
CEC17-F1	Average	4.47E+10	5.02E+03	4.63E+10	1.08E+11	6.14E+03	8.78E+06	4.16E+10	4.32E+03	8.47E+03	**3.63E+03**
	Best	3.46E+10	1.08E+02	4.02E+10	5.57E+10	2.69E+02	5.78E+06	3.09E+10	7.55E+02	1.40E+02	1.42E+02
	Std	5.68E+09	6.19E+03	3.25E+09	2.76E+10	6.29E+03	2.00E+06	5.89E+09	5.51E+03	6.83E+03	3.65E+03
	Rank	8	3	9	10	4	6	7	2	5	1
CEC17-F3	Average	7.75E+04	3.13E+02	7.17E+04	2.01E+05	3.00E+02	3.99E+03	7.31E+04	3.00E+02	3.00E+02	**3.00E+02**
	Best	5.67E+04	3.00E+02	5.62E+04	1.36E+05	3.00E+02	1.90E+03	5.65E+04	3.00E+02	3.00E+02	3.00E+02
	Std	7.60E+03	3.72E+01	5.23E+03	4.94E+04	5.18E-05	1.44E+03	6.69E+03	2.95E-08	7.78E-03	1.94E-13
	Rank	9	5	7	10	3	6	8	2	4	1
CEC17-F4	Average	9.78E+03	4.79E+02	1.02E+04	2.86E+04	4.76E+02	5.10E+02	8.41E+03	4.70E+02	4.90E+02	**4.26E+02**
	Best	5.69E+03	4.04E+02	8.16E+03	9.38E+03	4.00E+02	4.71E+02	4.03E+03	4.04E+02	4.76E+02	4.00E+02
	Std	2.45E+03	3.03E+01	1.18E+03	1.17E+04	2.54E+01	2.31E+01	2.53E+03	2.35E+01	7.34E+00	3.46E+01
	Rank	8	4	9	10	3	6	7	2	5	1
CEC17-F5	Average	8.03E+02	6.92E+02	8.96E+02	1.07E+03	6.87E+02	7.23E+02	8.89E+02	6.97E+02	5.85E+02	**5.54E+02**
	Best	7.33E+02	6.33E+02	8.53E+02	8.67E+02	6.18E+02	6.58E+02	8.33E+02	6.31E+02	5.56E+02	5.29E+02
	Std	3.04E+01	3.37E+01	2.22E+01	7.86E+01	4.85E+01	2.97E+01	2.81E+01	4.43E+01	1.99E+01	1.75E+01
	Rank	7	4	9	10	3	6	8	5	2	1
CEC17-F6	Average	6.66E+02	6.30E+02	6.85E+02	7.19E+02	6.45E+02	6.61E+02	6.80E+02	6.43E+02	6.01E+02	**6.00E+02**
	Best	6.48E+02	6.14E+02	6.76E+02	6.87E+02	6.26E+02	6.50E+02	6.65E+02	6.25E+02	6.00E+02	6.00E+02
	Std	6.04E+00	7.95E+00	3.51E+00	1.60E+01	8.24E+00	5.48E+00	5.64E+00	6.88E+00	9.35E-01	4.53E-03
	Rank	7	3	9	10	5	6	8	4	2	1
CEC17-F7	Average	1.32E+03	1.06E+03	1.35E+03	2.86E+03	1.09E+03	1.17E+03	1.35E+03	1.03E+03	8.32E+02	**7.89E+02**
	Best	1.22E+03	8.91E+02	1.27E+03	2.03E+03	9.34E+02	1.03E+03	1.27E+03	8.99E+02	7.85E+02	7.73E+02
	Std	4.93E+01	7.91E+01	3.76E+01	4.17E+02	6.72E+01	5.35E+01	3.66E+01	6.52E+01	2.38E+01	1.29E+01
	Rank	7	4	8	10	5	6	9	3	2	1
CEC17-F8	Average	1.04E+03	9.54E+02	1.13E+03	1.30E+03	9.50E+02	9.59E+02	1.11E+03	9.46E+02	8.87E+02	**8.50E+02**
	Best	9.81E+02	9.06E+02	1.09E+03	1.11E+03	8.98E+02	9.23E+02	1.08E+03	9.09E+02	8.58E+02	8.30E+02
	Std	3.65E+01	2.77E+01	1.37E+01	8.76E+01	3.42E+01	2.11E+01	1.57E+01	2.49E+01	2.47E+01	1.25E+01
	Rank	7	5	9	10	4	6	8	3	2	1
CEC17-F9	Average	5.75E+03	5.07E+03	1.02E+04	2.31E+04	4.08E+03	6.51E+03	9.34E+03	3.72E+03	2.35E+03	**9.18E+02**
	Best	3.51E+03	3.27E+03	8.76E+03	1.28E+04	2.97E+03	5.35E+03	7.97E+03	2.31E+03	9.21E+02	9.00E+02
	Std	8.82E+02	7.23E+02	6.59E+02	5.72E+03	6.70E+02	5.81E+02	6.99E+02	7.90E+02	1.30E+03	1.84E+01
	Rank	6	5	9	10	4	7	8	3	2	1
CEC17-F10	Average	6.55E+03	5.21E+03	8.29E+03	8.50E+03	5.57E+03	5.12E+03	7.72E+03	4.62E+03	4.32E+03	**4.14E+03**
	Best	5.18E+03	3.65E+03	7.08E+03	7.58E+03	4.20E+03	4.01E+03	6.80E+03	3.01E+03	2.43E+03	2.46E+03
	Std	5.39E+02	6.37E+02	4.02E+02	3.35E+02	7.00E+02	5.95E+02	3.51E+02	6.80E+02	6.44E+02	5.78E+02
	Rank	7	5	9	10	6	4	8	3	2	1
CEC17-F11	Average	3.70E+03	1.23E+03	5.91E+03	1.62E+04	1.23E+03	1.24E+03	6.94E+03	1.20E+03	1.24E+03	**1.14E+03**
	Best	1.68E+03	1.16E+03	3.38E+03	3.92E+03	1.16E+03	1.17E+03	4.99E+03	1.16E+03	1.15E+03	1.11E+03
	Std	1.36E+03	4.95E+01	9.00E+02	7.87E+03	5.83E+01	3.68E+01	1.55E+03	2.86E+01	5.25E+01	2.38E+01
	Rank	7	4	8	10	3	5	9	2	6	1
CEC17-F12	Average	8.16E+09	1.35E+06	8.70E+09	1.40E+10	3.24E+04	1.05E+07	1.28E+10	2.79E+05	1.16E+06	**2.01E+04**
	Best	4.01E+09	1.16E+05	5.74E+09	4.61E+09	4.62E+03	2.50E+06	3.89E+09	4.32E+04	1.44E+05	5.64E+03
	Std	2.56E+09	9.57E+05	1.39E+09	6.34E+09	1.77E+04	5.54E+06	3.36E+09	1.31E+05	9.42E+05	9.74E+03
	Rank	7	5	8	10	2	6	9	3	4	1
CEC17-F13	Average	3.77E+04	5.13E+04	3.97E+09	7.78E+09	1.94E+04	2.19E+05	8.72E+09	1.69E+04	3.27E+04	**9.04E+03**
	Best	2.34E+04	1.61E+04	1.76E+09	1.14E+09	1.72E+03	1.21E+05	2.61E+09	5.62E+03	3.55E+03	1.36E+03
	Std	1.55E+04	2.38E+04	1.05E+09	5.08E+09	2.28E+04	7.79E+04	5.94E+09	1.27E+04	2.66E+04	7.90E+03
	Rank	5	6	8	9	3	7	10	2	4	1
CEC17-F14	Average	3.11E+04	1.57E+04	1.33E+06	1.58E+06	1.96E+03	4.37E+04	2.61E+06	1.58E+03	3.38E+04	**1.50E+03**
	Best	3.19E+03	2.94E+03	1.58E+05	2.35E+05	1.47E+03	3.35E+03	3.16E+05	1.50E+03	3.32E+03	1.44E+03
	Std	2.90E+04	1.11E+04	8.59E+05	1.19E+06	6.35E+02	4.36E+04	2.29E+06	6.13E+01	1.65E+04	3.87E+01
	Rank	5	4	8	9	3	7	10	2	6	1
CEC17-F15	Average	2.46E+04	1.37E+04	1.49E+08	4.28E+08	4.47E+03	5.70E+04	4.68E+08	4.28E+03	2.61E+04	**2.80E+03**
	Best	1.48E+04	3.42E+03	2.08E+07	7.89E+07	1.64E+03	6.73E+03	3.15E+07	1.84E+03	1.86E+03	1.54E+03
	Std	1.18E+04	9.64E+03	8.82E+07	4.15E+08	3.54E+03	5.09E+04	1.76E+08	2.95E+03	1.50E+04	2.20E+03
	Rank	5	4	8	9	3	7	10	2	6	1
CEC17-F16	Average	4.45E+03	2.86E+03	4.98E+03	5.29E+03	2.74E+03	3.22E+03	4.76E+03	2.68E+03	2.43E+03	**2.25E+03**
	Best	3.12E+03	2.21E+03	4.33E+03	3.76E+03	2.11E+03	2.26E+03	3.78E+03	2.02E+03	1.75E+03	1.91E+03
	Std	7.49E+02	3.10E+02	3.17E+02	7.96E+02	3.77E+02	4.67E+02	7.61E+02	2.44E+02	3.33E+02	2.08E+02
	Rank	7	5	9	10	4	6	8	3	2	1
CEC17-F17	Average	2.80E+03	2.28E+03	3.51E+03	3.78E+03	2.31E+03	2.52E+03	4.21E+03	2.21E+03	2.10E+03	**1.78E+03**
	Best	2.19E+03	1.84E+03	2.67E+03	2.79E+03	1.83E+03	1.83E+03	3.09E+03	1.75E+03	1.82E+03	1.70E+03
	Std	3.26E+02	2.52E+02	3.31E+02	5.56E+02	2.33E+02	2.78E+02	1.91E+03	2.29E+02	1.61E+02	8.38E+01
	Rank	7	4	8	9	5	6	10	3	2	1
CEC17-F18	Average	1.00E+06	1.36E+05	1.90E+07	2.00E+07	2.19E+04	1.18E+06	3.40E+07	**7.41E+03**	3.25E+05	8.76E+03
	Best	3.66E+04	2.97E+04	2.19E+06	2.05E+06	2.06E+03	9.40E+04	7.13E+06	2.34E+03	7.56E+04	2.00E+03
	Std	1.26E+06	1.14E+05	1.06E+07	1.57E+07	1.62E+04	1.02E+06	3.18E+07	6.95E+03	2.74E+05	6.81E+03
	Rank	6	4	8	9	3	7	10	1	5	2
CEC17-F19	Average	1.11E+06	7.54E+03	1.79E+08	1.00E+09	**3.44E+03**	2.76E+05	6.00E+08	5.63E+03	3.57E+04	4.10E+03

(continued on next page)

Table 36 (*continued*)

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
	Best	8.48E+05	2.27E+03	6.01E+07	1.44E+08	1.96E+03	5.99E+04	1.56E+08	3.93E+03	2.10E+03	1.96E+03
	Std	1.22E+05	4.91E+03	9.06E+07	7.74E+08	2.05E+03	2.15E+05	7.06E+08	1.43E+03	2.13E+04	3.00E+03
	Rank	7	4	8	10	1	6	9	3	5	2
CEC17-F20	Average	2.74E+03	2.53E+03	2.87E+03	2.96E+03	2.53E+03	2.70E+03	2.84E+03	2.43E+03	2.43E+03	**2.12E+03**
	Best	2.51E+03	2.11E+03	2.65E+03	2.65E+03	2.26E+03	2.37E+03	2.47E+03	2.26E+03	2.09E+03	2.01E+03
	Std	1.64E+02	1.88E+02	8.64E+01	1.27E+02	1.74E+02	2.00E+02	1.26E+02	1.50E+02	1.76E+02	7.52E+01
	Rank	7	4	9	10	5	6	8	3	2	1
CEC17-F21	Average	2.61E+03	2.52E+03	2.68E+03	2.85E+03	2.45E+03	2.55E+03	2.68E+03	2.45E+03	2.40E+03	**2.35E+03**
	Best	2.52E+03	2.43E+03	2.58E+03	2.63E+03	2.37E+03	2.49E+03	2.60E+03	2.38E+03	2.36E+03	2.33E+03
	Std	5.00E+01	5.31E+01	3.71E+01	9.67E+01	3.70E+01	4.71E+01	5.02E+01	4.23E+01	2.22E+01	1.22E+01
	Rank	7	5	9	10	4	6	8	3	2	1
CEC17-F22	Average	8.33E+03	5.32E+03	8.01E+03	9.75E+03	3.08E+03	6.52E+03	8.04E+03	3.12E+03	5.71E+03	**2.30E+03**
	Best	7.01E+03	2.30E+03	6.90E+03	8.83E+03	2.30E+03	2.32E+03	6.88E+03	2.30E+03	4.78E+03	2.30E+03
	Std	5.87E+02	2.09E+03	4.11E+02	3.54E+02	1.80E+03	1.79E+03	8.77E+02	1.68E+03	4.56E+02	8.54E-01
	Rank	9	4	7	10	2	6	8	3	5	1
CEC17-F23	Average	3.43E+03	2.92E+03	3.26E+03	3.43E+03	2.89E+03	3.06E+03	3.23E+03	2.78E+03	2.74E+03	**2.70E+03**
	Best	3.14E+03	2.83E+03	3.16E+03	3.21E+03	2.77E+03	2.86E+03	3.11E+03	2.70E+03	2.70E+03	2.67E+03
	Std	1.29E+02	7.04E+01	3.99E+01	1.48E+02	7.98E+01	1.25E+02	8.31E+01	3.08E+01	2.65E+01	1.91E+01
	Rank	9	5	8	10	4	6	7	3	2	1
CEC17-F24	Average	3.79E+03	3.14E+03	3.47E+03	3.63E+03	3.03E+03	3.47E+03	3.39E+03	2.92E+03	2.93E+03	**2.87E+03**
	Best	3.40E+03	2.94E+03	3.29E+03	3.36E+03	2.90E+03	3.13E+03	3.22E+03	2.85E+03	2.89E+03	2.84E+03
	Std	2.08E+02	1.04E+02	8.49E+01	1.45E+02	8.48E+01	1.36E+02	1.54E+02	3.09E+01	1.99E+01	1.61E+01
	Rank	10	5	8	9	4	7	6	2	3	1
CEC17-F25	Average	4.50E+03	2.90E+03	4.26E+03	1.17E+04	2.90E+03	2.91E+03	4.63E+03	2.91E+03	**2.89E+03**	2.89E+03
	Best	3.74E+03	2.88E+03	3.92E+03	6.76E+03	2.88E+03	2.88E+03	3.72E+03	2.88E+03	2.88E+03	2.88E+03
	Std	4.75E+02	1.62E+01	1.23E+02	3.19E+03	2.02E+01	1.78E+01	6.07E+02	1.67E+01	1.34E+00	7.72E+00
	Rank	8	3	7	10	4	6	9	5	1	2
CEC17-F26	Average	9.57E+03	6.34E+03	1.00E+04	1.29E+04	4.82E+03	6.38E+03	9.91E+03	4.99E+03	4.64E+03	**3.88E+03**
	Best	8.05E+03	2.80E+03	8.99E+03	1.02E+04	2.80E+03	2.87E+03	8.44E+03	2.80E+03	4.09E+03	2.80E+03
	Std	8.94E+02	1.17E+03	4.33E+02	1.63E+03	1.66E+03	2.02E+03	9.49E+02	1.61E+03	2.78E+02	6.35E+02
	Rank	7	5	9	10	3	6	8	4	2	1
CEC17-F27	Average	4.52E+03	3.26E+03	3.83E+03	3.85E+03	3.28E+03	3.36E+03	3.69E+03	3.26E+03	**3.21E+03**	3.22E+03
	Best	3.83E+03	3.22E+03	3.63E+03	3.33E+03	3.21E+03	3.25E+03	3.43E+03	3.22E+03	3.19E+03	3.20E+03
	Std	3.50E+02	2.69E+01	1.07E+02	3.35E+02	6.45E+01	1.08E+02	2.07E+02	2.06E+01	9.86E+00	9.61E+00
	Rank	10	3	8	9	5	6	7	4	1	2
CEC17-F28	Average	5.96E+03	3.17E+03	6.04E+03	9.58E+03	3.18E+03	3.25E+03	6.38E+03	**3.10E+03**	3.23E+03	3.15E+03
	Best	4.12E+03	3.10E+03	5.65E+03	7.17E+03	3.10E+03	3.20E+03	4.62E+03	3.10E+03	3.10E+03	3.10E+03
	Std	7.69E+02	5.72E+01	1.94E+02	1.93E+03	6.70E+01	3.18E+01	8.94E+02	1.89E+01	3.93E+01	6.13E+01
	Rank	7	3	8	10	4	6	9	1	5	2
CEC17-F29	Average	6.38E+03	4.01E+03	6.10E+03	6.74E+03	4.15E+03	4.39E+03	6.01E+03	4.08E+03	3.74E+03	**3.43E+03**
	Best	4.57E+03	3.52E+03	5.19E+03	4.69E+03	3.53E+03	3.73E+03	4.83E+03	3.74E+03	3.38E+03	3.33E+03
	Std	1.00E+03	2.17E+02	3.81E+02	1.27E+03	2.99E+02	3.49E+02	7.70E+02	2.02E+02	1.73E+02	9.75E+01
	Rank	9	3	8	10	5	6	7	4	2	1
CEC17-F30	Average	1.65E+08	3.96E+04	6.29E+08	7.13E+08	9.67E+03	1.52E+06	2.53E+09	1.30E+04	1.69E+04	**7.37E+03**
	Best	4.63E+06	1.16E+04	2.33E+08	8.03E+07	5.75E+03	2.85E+05	1.01E+09	8.33E+03	9.22E+03	5.68E+03
	Std	6.45E+08	2.03E+04	2.38E+08	6.01E+08	2.79E+03	8.05E+05	9.81E+08	2.74E+03	4.19E+03	1.70E+03
	Rank	7	5	8	9	2	6	10	3	4	1
Average ranking		7.31	4.24	8.28	9.72	3.66	6.10	8.38	2.86	3.28	1.17
Final ranking		7	5	8	10	4	6	9	2	3	**1**

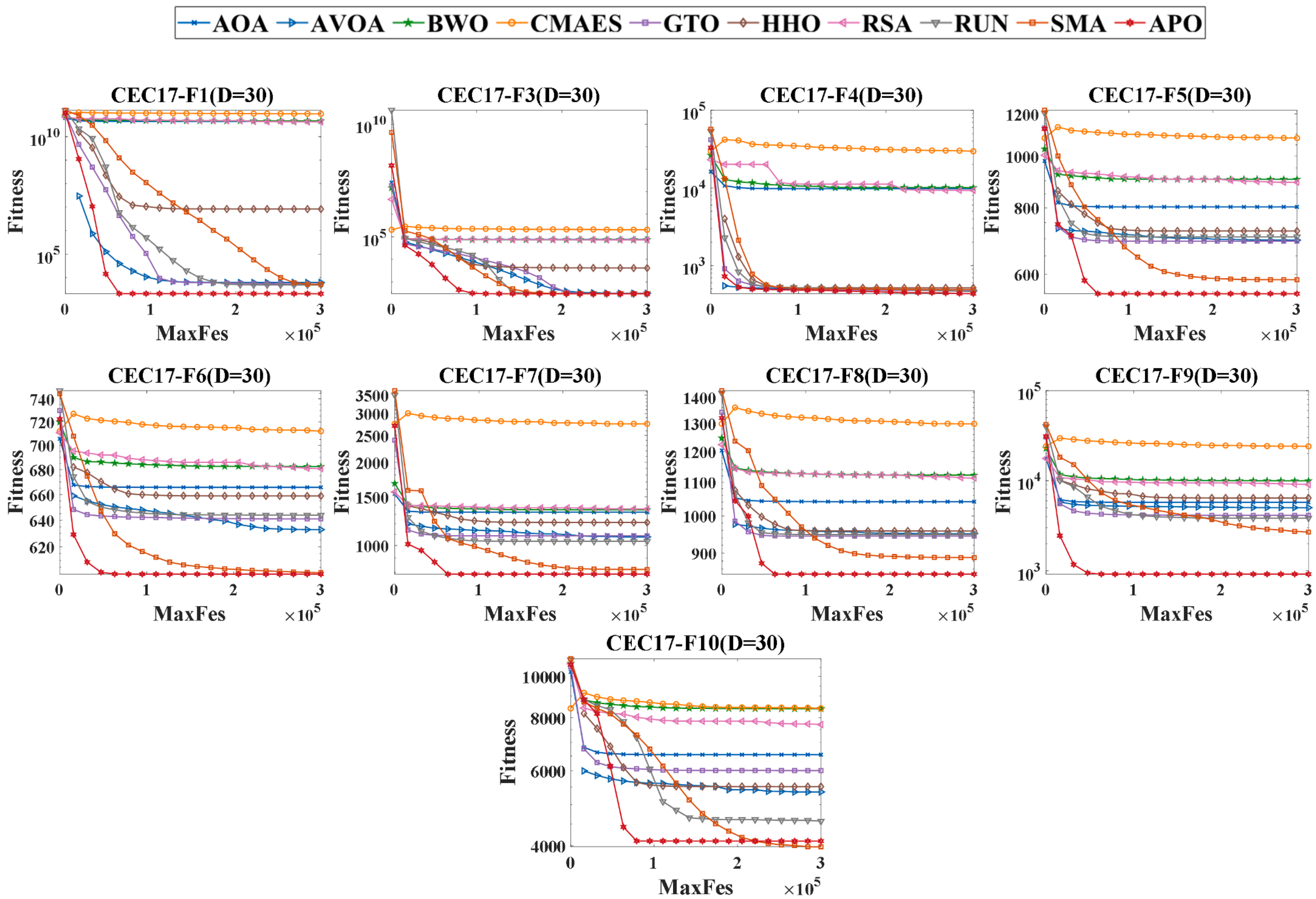

Fig. 27. Iterative convergence curves of APO and other algorithms on CEC2017 unimodal and simple multimodal functions (F1-F10).

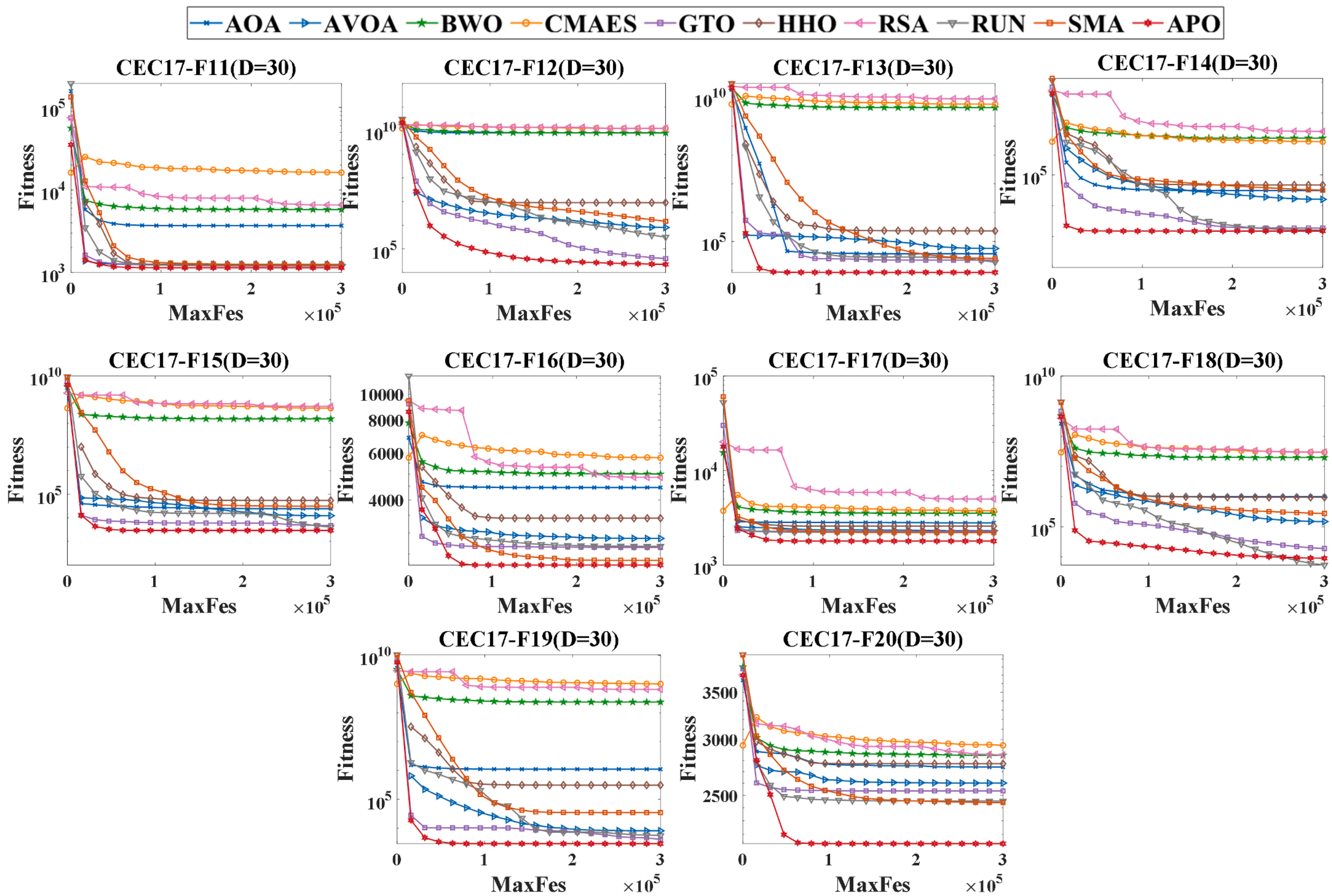

Fig. 28. Iterative convergence curve of APO and other algorithms on CEC2017 hybrid functions (F11-F20).

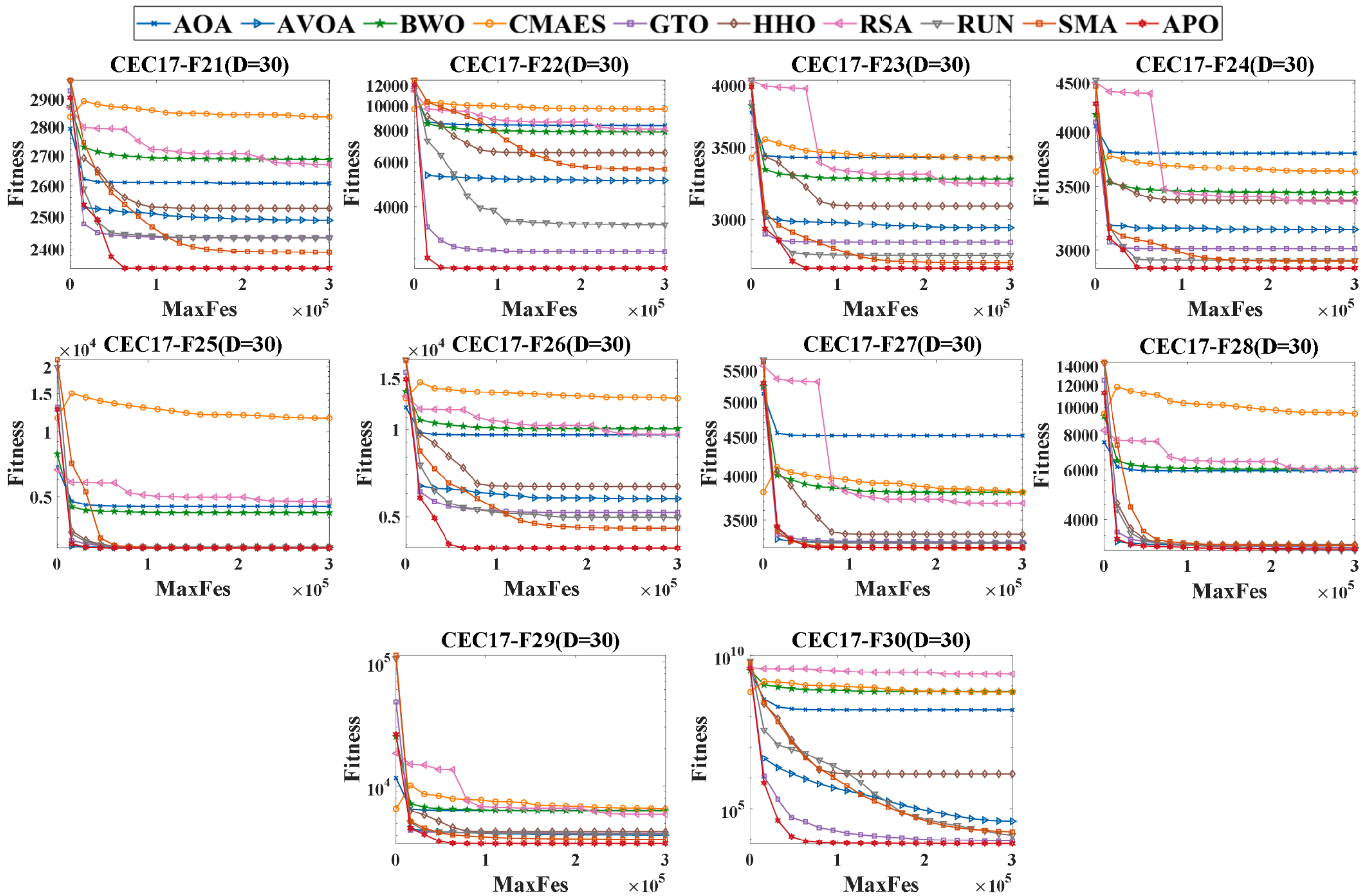

Fig. 29. Iterative convergence curve of APO and other algorithms on CEC2017 composition functions (F21-F30).

Table 37
Comparison of results of various algorithms under the CEC2019 test set.

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
CEC19-F1	Average	**1.00E+00**	**1.00E+00**	**1.00E+00**	8.82E+07	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	**1.00E+00**	1.00E+00
	Best	1.00E+00	1.00E+00	1.00E+00	3.11E+07	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00
	Std	3.73E-15	0.00E+00	0.00E+00	4.73E+07	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.79E-03
	Rank	1	1	1	10	1	1	1	1	1	9
CEC19-F2	Average	3.74E+03	4.73E+00	5.00E+00	9.12E+03	4.37E+00	4.90E+00	4.98E+00	4.27E+00	4.65E+00	**3.89E+00**
	Best	1.64E+01	4.22E+00	4.98E+00	4.80E+03	4.20E+00	4.42E+00	4.54E+00	4.22E+00	4.22E+00	3.26E+00
	Std	1.76E+03	3.58E-01	3.32E-03	2.16E+03	2.58E-01	1.75E-01	8.38E-02	4.26E-02	3.58E-01	3.25E-01
	Rank	9	5	8	10	3	6	7	2	4	1
CEC19-F3	Average	8.13E+00	1.40E+00	3.37E+00	1.08E+01	1.41E+00	2.52E+00	6.10E+00	**1.38E+00**	2.97E+00	1.40E+00
	Best	5.65E+00	1.00E+00	1.89E+00	9.94E+00	1.41E+00	1.41E+00	5.11E+00	1.00E+00	1.41E+00	1.00E+00
	Std	1.05E+00	7.47E-02	7.40E-01	3.00E-01	3.74E-14	1.06E+00	5.90E-01	1.04E-01	1.94E+00	7.47E-02
	Rank	9	3	7	10	4	5	8	1	6	2
CEC19-F4	Average	4.44E+01	2.62E+01	6.81E+01	8.66E+01	2.73E+01	3.99E+01	7.07E+01	2.81E+01	1.08E+01	**8.23E+00**
	Best	2.39E+01	9.95E+00	5.91E+01	5.28E+01	7.96E+00	1.60E+01	4.04E+01	8.96E+00	3.98E+00	2.99E+00
	Std	1.35E+01	9.08E+00	6.31E+00	1.67E+01	1.28E+01	1.07E+01	1.76E+01	9.76E+00	4.25E+00	2.35E+00
	Rank	7	3	8	10	4	6	9	5	2	1
CEC19-F5	Average	1.72E+01	1.26E+00	4.43E+01	6.39E+01	1.33E+00	1.82E+00	7.12E+01	1.19E+00	1.23E+00	**1.05E+00**
	Best	1.75E+00	1.05E+00	2.69E+01	2.60E+01	1.07E+00	1.15E+00	4.81E+01	1.03E+00	1.07E+00	1.01E+00
	Std	9.24E+00	1.88E-01	9.77E+00	2.43E+01	1.88E-01	3.97E-01	1.98E+01	1.28E-01	1.12E-01	3.39E-02
	Rank	7	4	8	9	5	6	10	2	3	1
CEC19-F6	Average	9.99E+00	5.72E+00	9.56E+00	9.76E+00	4.94E+00	6.75E+00	9.23E+00	5.65E+00	3.99E+00	**1.00E+00**
	Best	7.04E+00	1.80E+00	8.02E+00	8.86E+00	1.91E+00	3.03E+00	6.88E+00	3.97E+00	1.10E+00	1.00E+00
	Std	1.24E+00	1.99E+00	6.62E-01	4.73E-01	1.50E+00	1.84E+00	1.23E+00	9.53E-01	1.66E+00	0.00E+00
	Rank	10	5	8	9	3	6	7	4	2	1
CEC19-F7	Average	1.17E+03	7.85E+02	1.39E+03	1.48E+03	9.67E+02	1.02E+03	1.47E+03	5.61E+02	**4.50E+02**	5.11E+02
	Best	6.53E+02	3.69E+02	1.02E+03	1.15E+03	3.99E+02	5.34E+02	1.17E+03	2.17E+02	6.56E+01	1.20E+02
	Std	3.07E+02	2.43E+02	1.80E+02	1.63E+02	3.27E+02	2.93E+02	1.85E+02	2.06E+02	2.35E+02	2.04E+02
	Rank	7	4	8	10	5	6	9	3	1	2
CEC19-F8	Average	4.61E+00	3.93E+00	4.50E+00	4.63E+00	3.93E+00	4.40E+00	4.65E+00	3.77E+00	3.14E+00	**3.00E+00**

(continued on next page)

Table 37 (*continued*)

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
	Best	3.78E+00	3.27E+00	4.18E+00	4.31E+00	2.94E+00	3.72E+00	4.19E+00	2.66E+00	2.21E+00	2.17E+00
	Std	3.92E-01	2.97E-01	1.69E-01	1.65E-01	4.71E-01	2.89E-01	2.51E-01	4.07E-01	4.23E-01	3.88E-01
	Rank	8	4	7	9	5	6	10	3	2	1
CEC19-F9	Average	1.33E+00	1.31E+00	2.68E+00	3.62E+00	1.19E+00	1.40E+00	2.44E+00	1.28E+00	1.14E+00	**1.04E+00**
	Best	1.15E+00	1.08E+00	1.74E+00	2.63E+00	1.07E+00	1.15E+00	1.63E+00	1.13E+00	1.05E+00	1.01E+00
	Std	1.70E-01	1.44E-01	3.95E-01	5.64E-01	7.36E-02	1.58E-01	7.45E-01	7.47E-02	5.15E-02	2.71E-02
	Rank	6	5	9	10	3	7	8	4	2	1
CEC19-F10	Average	2.10E+01	2.10E+01	2.12E+01	2.13E+01	1.81E+01	2.10E+01	2.12E+01	1.93E+01	1.97E+01	**1.04E+01**
	Best	2.05E+01	2.10E+01	2.07E+01	2.12E+01	3.32E+00	2.09E+01	2.09E+01	2.16E+00	1.01E+00	1.00E+00
	Std	1.02E-01	6.09E-02	1.40E-01	5.81E-02	6.18E+00	4.73E-02	9.82E-02	5.30E+00	5.08E+00	1.02E+01
	Rank	6	7	9	10	2	5	8	3	4	1
Average ranking		7.00	4.10	7.30	9.70	3.50	5.40	7.70	2.80	2.70	2.00
Final ranking		7	5	8	10	4	6	9	3	2	**1**

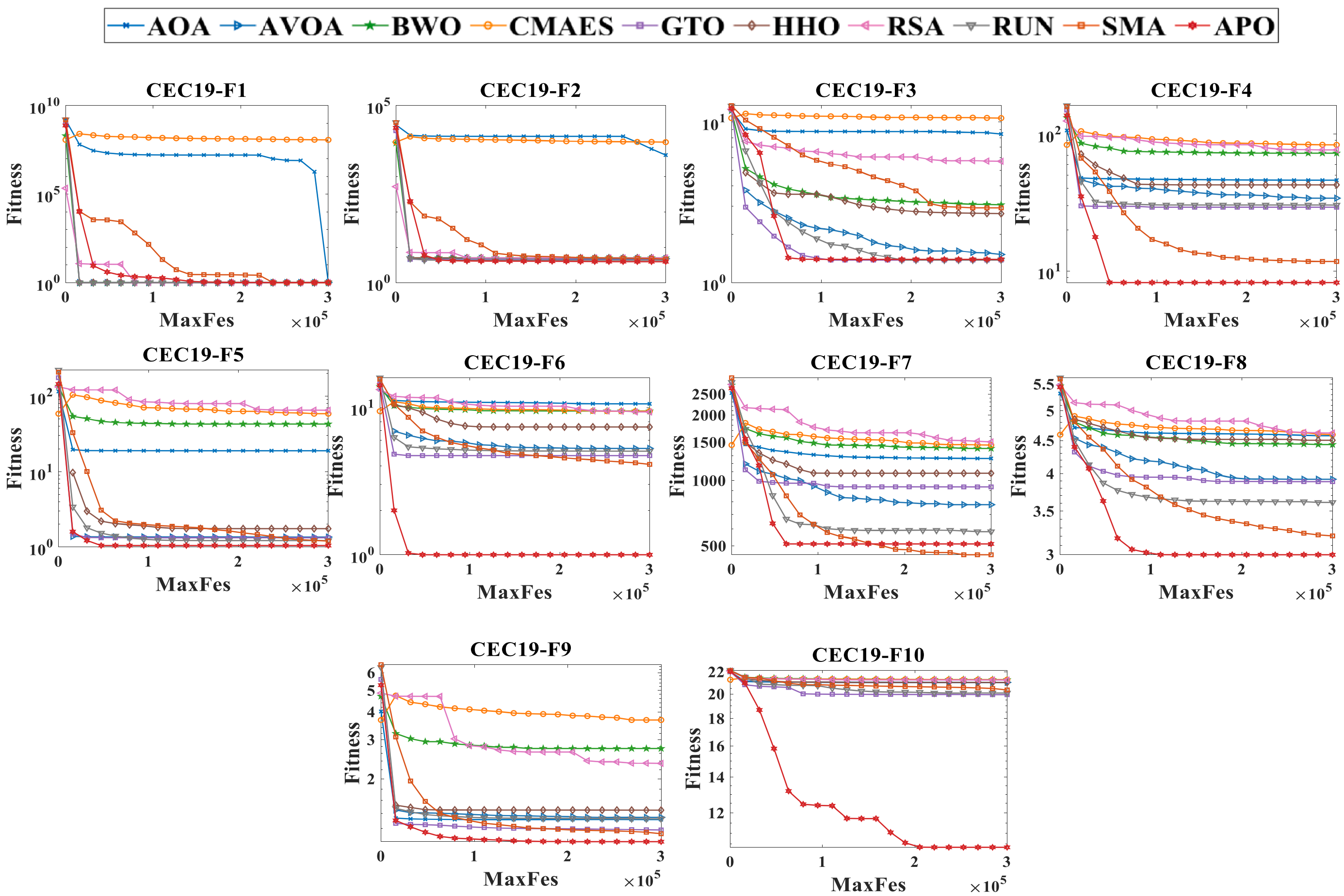

Fig. 30. Convergence process curve of APO and other algorithms in CEC2019.

Table 38
Comparison of results of various algorithms under the CEC2022 test set.

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
CEC22-F1	Average	4.07E+02	3.00E+02	4.51E+03	1.13E+04	3.00E+02	3.00E+02	6.27E+03	3.00E+02	3.00E+02	**3.00E+02**
	Best	3.00E+02	3.00E+02	2.13E+03	2.02E+03	3.00E+02	3.00E+02	2.24E+03	3.00E+02	3.00E+02	3.00E+02
	Std	2.13E+02	6.76E-14	7.81E+02	5.21E+03	9.80E-13	1.88E-01	1.52E+03	2.42E-12	8.95E-06	1.06E-14
	Rank	7	1	8	10	1	6	9	4	5	1
CEC22-F2	Average	4.36E+02	4.10E+02	6.33E+02	9.42E+02	4.05E+02	4.13E+02	7.99E+02	4.04E+02	4.07E+02	**4.02E+02**
	Best	4.00E+02	4.00E+02	5.19E+02	4.72E+02	4.00E+02	4.00E+02	5.21E+02	4.00E+02	4.04E+02	4.00E+02
	Std	3.45E+01	1.70E+01	7.87E+01	5.05E+02	3.12E+00	2.32E+01	2.95E+02	4.52E+00	2.20E+00	1.99E+00
	Rank	7	5	8	10	3	6	9	2	4	1
CEC22-F3	Average	6.40E+02	6.05E+02	6.38E+02	6.51E+02	6.05E+02	6.24E+02	6.42E+02	6.13E+02	6.00E+02	**6.00E+02**
	Best	6.21E+02	6.00E+02	6.26E+02	6.36E+02	6.01E+02	6.03E+02	6.33E+02	6.03E+02	6.00E+02	6.00E+02
	Std	7.88E+00	6.92E+00	5.58E+00	1.17E+01	3.33E+00	1.14E+01	4.41E+00	6.13E+00	1.09E-02	0.00E+00

(continued on next page)

Table 38 (*continued*)

Functions	Index	AOA	AVOA	BWO	CMAES	GTO	HHO	RSA	RUN	SMA	APO
	Rank	8	4	7	10	3	6	9	5	2	1
CEC22-F4	Average	8.29E+02	8.32E+02	8.42E+02	8.67E+02	8.24E+02	8.28E+02	8.44E+02	8.25E+02	8.19E+02	**8.09E+02**
	Best	8.14E+02	8.09E+02	8.33E+02	8.34E+02	8.06E+02	8.11E+02	8.29E+02	8.12E+02	8.06E+02	8.04E+02
	Std	8.37E+00	1.24E+01	3.96E+00	1.29E+01	7.49E+00	8.26E+00	7.58E+00	7.61E+00	9.58E+00	3.53E+00
	Rank	6	7	8	10	3	5	9	4	2	1
CEC22-F5	Average	1.35E+03	1.07E+03	1.29E+03	1.99E+03	9.75E+02	1.39E+03	1.35E+03	9.72E+02	9.00E+02	**9.00E+02**
	Best	1.01E+03	9.13E+02	1.14E+03	1.21E+03	9.02E+02	1.13E+03	1.10E+03	9.03E+02	9.00E+02	9.00E+02
	Std	1.39E+02	1.57E+02	6.99E+01	5.05E+02	9.26E+01	1.29E+02	1.38E+02	3.24E+01	2.43E-01	2.99E-14
	Rank	8	5	6	10	4	9	7	3	2	1
CEC22-F6	Average	3.91E+03	4.29E+03	3.90E+05	1.77E+07	1.82E+03	3.50E+03	4.18E+07	1.90E+03	5.56E+03	**1.81E+03**
	Best	1.91E+03	1.88E+03	5.26E+04	2.15E+05	1.80E+03	1.92E+03	1.13E+06	1.85E+03	1.93E+03	1.80E+03
	Std	1.31E+03	2.07E+03	2.62E+05	2.86E+07	2.14E+01	2.13E+03	1.63E+07	8.03E+01	1.80E+03	6.91E+00
	Rank	5	6	8	9	2	4	10	3	7	1
CEC22-F7	Average	2.10E+03	2.02E+03	2.08E+03	2.08E+03	2.03E+03	2.04E+03	2.11E+03	2.04E+03	2.02E+03	**2.00E+03**
	Best	2.05E+03	2.00E+03	2.05E+03	2.05E+03	2.00E+03	2.02E+03	2.06E+03	2.02E+03	2.00E+03	2.00E+03
	Std	3.22E+01	5.94E+00	1.25E+01	1.96E+01	9.08E+00	1.52E+01	2.38E+01	8.08E+00	3.60E+00	1.29E+00
	Rank	9	3	8	7	4	6	10	5	2	1
CEC22-F8	Average	2.29E+03	2.22E+03	2.23E+03	2.24E+03	2.22E+03	2.23E+03	2.38E+03	2.22E+03	2.22E+03	**2.20E+03**
	Best	2.22E+03	2.20E+03	2.23E+03	2.23E+03	2.20E+03	2.22E+03	2.23E+03	2.22E+03	2.20E+03	2.20E+03
	Std	8.51E+01	5.14E+00	2.21E+00	1.37E+01	3.90E+00	7.19E+00	7.56E+02	1.85E+00	5.13E+00	6.16E+00
	Rank	9	3	7	8	4	6	10	5	2	1
CEC22-F9	Average	2.61E+03	2.53E+03	2.67E+03	2.69E+03	2.53E+03	2.54E+03	2.70E+03	2.53E+03	2.53E+03	**2.53E+03**
	Best	2.56E+03	2.53E+03	2.65E+03	2.56E+03	2.53E+03	2.53E+03	2.60E+03	2.53E+03	2.53E+03	2.53E+03
	Std	2.30E+01	1.19E-13	1.63E+01	8.00E+01	0.00E+00	3.73E+01	5.09E+01	3.06E-10	2.03E-05	0.00E+00
	Rank	7	1	8	9	1	6	10	4	5	1
CEC22-F10	Average	2.63E+03	2.54E+03	2.53E+03	2.58E+03	2.53E+03	2.58E+03	2.67E+03	2.57E+03	2.50E+03	**2.50E+03**
	Best	2.50E+03	2.50E+03	2.51E+03	2.51E+03	2.50E+03	2.45E+03	2.51E+03	2.50E+03	2.50E+03	2.50E+03
	Std	9.94E+01	5.89E+01	2.65E+01	9.03E+01	5.35E+01	6.83E+01	1.32E+02	5.71E+01	5.38E-02	5.23E-02
	Rank	9	5	3	8	4	7	10	6	2	1
CEC22-F11	Average	2.76E+03	2.69E+03	3.06E+03	2.10E+04	2.68E+03	2.81E+03	3.64E+03	2.68E+03	2.76E+03	**2.61E+03**
	Best	2.60E+03	2.60E+03	2.87E+03	1.00E+04	2.60E+03	2.60E+03	3.10E+03	2.60E+03	2.60E+03	2.60E+03
	Std	1.12E+02	1.29E+02	1.69E+02	5.89E+03	1.50E+02	1.17E+02	3.36E+02	1.30E+02	1.94E+02	5.48E+01
	Rank	6	4	8	10	3	7	9	2	5	1
CEC22-F12	Average	2.98E+03	2.86E+03	2.88E+03	2.91E+03	2.86E+03	2.89E+03	2.92E+03	2.86E+03	**2.86E+03**	2.86E+03
	Best	2.88E+03	2.86E+03	2.87E+03	2.87E+03	2.86E+03	2.86E+03	2.87E+03	2.86E+03	2.86E+03	2.86E+03
	Std	5.79E+01	2.18E+00	8.51E+00	3.59E+01	2.78E+00	3.15E+01	6.28E+01	1.03E+00	1.61E+00	8.28E-01
	Rank	10	4	6	8	3	7	9	2	1	5
Average ranking		7.58	4.00	7.08	9.08	2.92	6.25	9.25	3.75	3.25	**1.33**
Final ranking		8	5	7	9	2	6	10	4	3	**1**

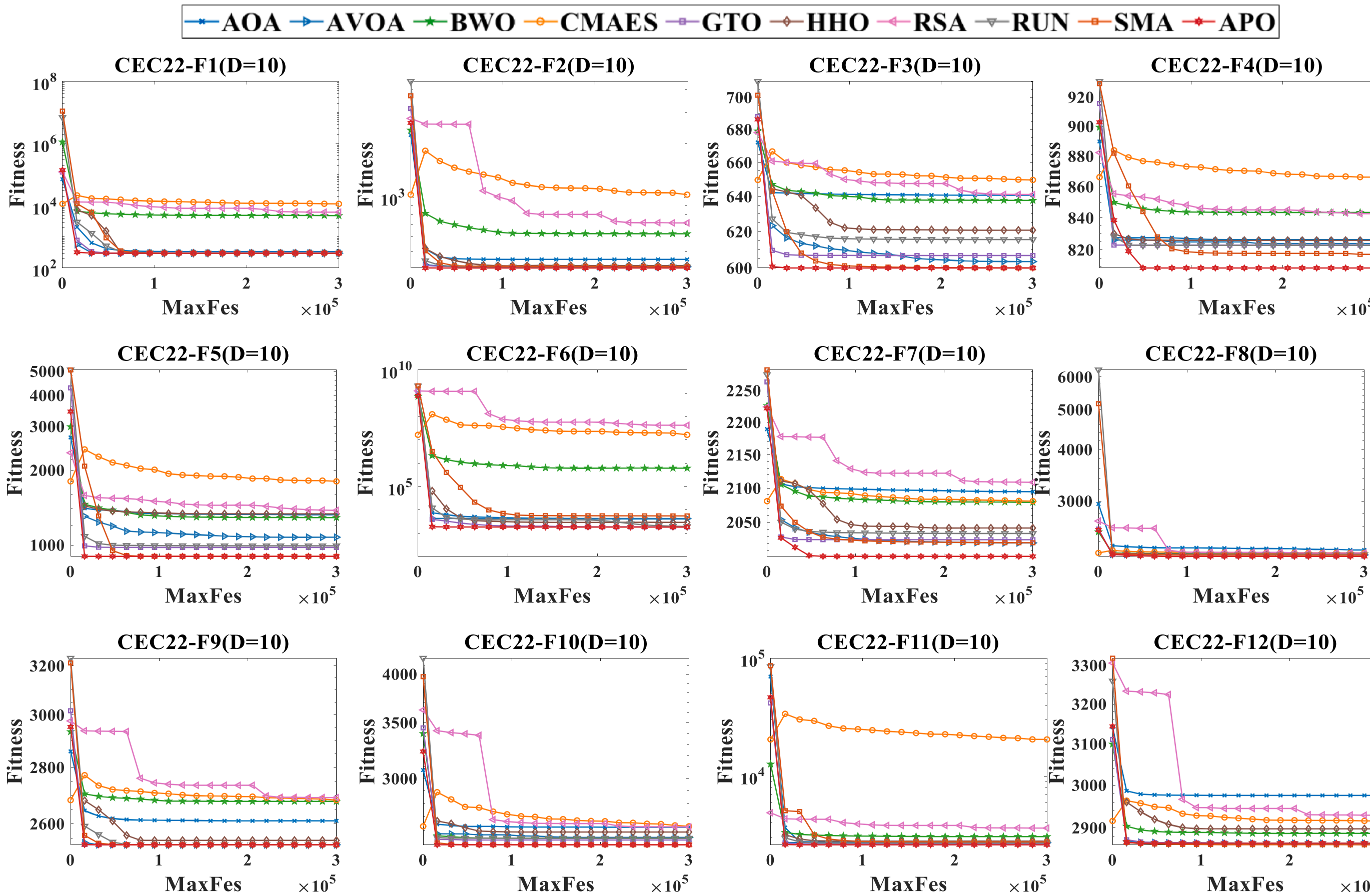

Fig. 31. Convergence process curves of APO and other algorithms on the CEC2022 test set.

References

[1] Nelder JA, Mead R. A simplex method for function minimization. Comput J 1965;7:308–13.

[2] Mandic DP. A generalized normalized gradient descent algorithm. IEEE Signal Proc Lett 2004;11:115–8.

[3] Ezugwu AE, Shukla AK, Nath R, Akinyelu AA, Agushaka JO, Chiroma H, et al. Metaheuristics: a comprehensive overview and classification along with bibliometric analysis. Artif Intell Rev 2021;54:4237–316.

[4] Guan Z, Ren C, Niu J, Wang P, Shang Y. Great wall construction algorithm: a novel meta-heuristic algorithm for engineer problems. Expert Syst Appl 2023;233:120905.

[5] Goodarzimehr V, Shojaee S, Hamzehei-Javaran S, Talatahari S. Special relativity search: a novel metaheuristic method based on special relativity physics. Knowl Based Syst 2022;257:109484.

[6] Mirjalili S, Mirjalili SM, Lewis A. Grey Wolf Optimizer. Adv Engineer Software 2014;69:46–61.

[7] Dhiman G, Kumar V. Seagull optimization algorithm: theory and its applications for large-scale industrial engineering problems. Knowl Based Syst 2019;165:169–96.

[8] Alweshah M, Khalaileh SA, Gupta BB, Almomani A, Hammouri AI, Al-Betar MA. The monarch butterfly optimization algorithm for solving feature selection problems. Neural Comput Applicat 2022;34:11267–81.

[9] Wang W, Tian W, Chau K-w, Xue Y, Xu L, Zang H. An improved bald eagle search algorithm with cauchy mutation and adaptive weight factor for engineering optimization. Comp Model Engineer Sci 2023;136:1603–42.

[10] Wang W, Tian W, Chau K, Zang H, Ma M, Feng Z, et al. Multi-reservoir flood control operation using improved bald eagle search algorithm with ε constraint method. Water (Basel) 2023.

[11] W-c Wang, W-c Tian, d-m Xu, K-w Chau, Ma Q, C-j Liu. Muskingum models' development and their parameter estimation: a state-of-the-art review. Water Res Managem 2023;37:3129–50.

[12] Zhao S, Zhang T, Ma S, Wang M. Sea-horse optimizer: a novel nature-inspired meta-heuristic for global optimization problems. Appl Intelligence 2023;53:11833–60.

[13] Aydemir SB. Enhanced marine predator algorithm for global optimization and engineering design problems. Adv Engineer Software 2023;184:103517.

[14] Gharehchopogh FS, Abdollahzadeh B, Barshandeh S, Arasteh B. A multi-objective mutation-based dynamic Harris Hawks optimization for botnet detection in IoT. Int Things 2023;24:100952.

[15] Özbay E. An active deep learning method for diabetic retinopathy detection in segmented fundus images using artificial bee colony algorithm. Artif Intell Rev 2023;56:3291–318.

[16] Gharehchopogh FS. An improved harris hawks optimization algorithm with multi-strategy for community detection in social network. J Bionic Eng 2023;20:1175–97.

[17] Özbay E, Özbay FA, Gharehchopogh FS. Peripheral blood smear images classification for acute lymphoblastic leukemia diagnosis with an improved convolutional neural network. J Bionic Eng 2023.

[18] Gharehchopogh FS, Ibrikci T. An improved African vultures optimization algorithm using different fitness functions for multi-level thresholding image segmentation. Multimed Tools Appl 2024;83:16929–75.

[19] Wolpert DH, Macready WG. No free lunch theorems for optimization. IEEE Transac Evolution Computat 1997;1:67–82.

[20] Holland JH. Genetic Algorithms. Sci Am 1992;267:66–73.

[21] Storn R, Price K. Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces. J Glob Optimizat 1997;11:341–59.

[22] Beyer H-G, Schwefel H-P. Evolution strategies – A comprehensive introduction. Nat Comput 2002;1:3–52.

[23] Simon D. Biogeography-Based Optimization. IEEE Transac Evolution Computat 2008;12:702–13.

[24] Zhang J-H, Xu X-H. An efficient evolutionary programming algorithm. Comput Oper Res 1999;26:645–63.

[25] Faramarzi A, Heidarinejad M, Stephens B, Mirjalili S. Equilibrium optimizer: a novel optimization algorithm. Knowl Based Syst 2020;191:105190.

[26] Hashim FA, Hussain K, Houssein EH, Mabrouk MS, Al-Atabany W. Archimedes optimization algorithm: a new metaheuristic algorithm for solving optimization problems. Appl Intelligence 2021;51:1531–51.

[27] Abualigah L, Diabat A, Mirjalili S, Abd Elaziz M, Gandomi AH. The arithmetic optimization algorithm. Comput Methods Appl Mech Eng 2021;376:113609.

[28] Eskandar H, Sadollah A, Bahreininejad A, Hamdi M. Water cycle algorithm – A novel metaheuristic optimization method for solving constrained engineering optimization problems. Comput Struct 2012;110-111:151–66.

[29] Abdel-Basset M, Mohamed R, Azeem SAA, Jameel M, Abouhawwash M. Kepler optimization algorithm: a new metaheuristic algorithm inspired by Kepler's laws of planetary motion. Knowl Based Syst 2023;268:110454.
[30] Hatamlou A. Black hole: a new heuristic optimization approach for data clustering. Info Sci 2013;222:175–84.
[31] Rashedi E, Nezamabadi-pour H, Saryazdi SGSA. A gravitational search algorithm. Info Sci 2009;179:2232–48.
[32] Abdel-Basset M, El-Shahat D, Jameel M, Abouhawwash M. Young's double-slit experiment optimizer: a novel metaheuristic optimization algorithm for global and constraint optimization problems. Comput Methods Appl Mech Eng 2023;403:115652.
[33] Zong Woo G, Joong Hoon K, Loganathan GV. A new heuristic optimization algorithm: harmony search. Simulation 2001;76:60–8.
[34] Shi Y. Brain Storm Optimization Algorithm. In: Tan Y, Shi Y, Chai Y, Wang G, editors. Advances in Swarm Intelligence Berlin. Heidelberg: Springer Berlin Heidelberg; 2011. p. 303–9.
[35] Das B, Mukherjee V, Das D. Student psychology based optimization algorithm: a new population based optimization algorithm for solving optimization problems. Adv Engineer Software 2020:146.
[36] Rao RV, Savsani VJ, Vakharia DP. Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems. Comp-Aid Des 2011;43:303–15.
[37] Askari Q, Younas I, Saeed M. Political Optimizer: a novel socio-inspired meta-heuristic for global optimization. Knowl Based Syst 2020;195:105709.
[38] Atashpaz-Gargari E, Lucas C. Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition. In: 2007 IEEE Congress on Evolutionary Computation; 2007. p. 4661–7.
[39] Kennedy J, Eberhart R. Particle swarm optimization. In: Proceedings of ICNN'95 - International Conference on Neural Networks. 4; 1995. p. 1942–8. vol.
[40] Dorigo M, Birattari M, Stutzle T. Ant colony optimization. IEEE Comput Intell Mag 2006;1:28–39.
[41] Mirjalili S, Lewis A. The whale optimization algorithm. Adv Engineer Software 2016;95:51–67.
[42] Heidari AA, Mirjalili S, Faris H, Aljarah I, Mafarja M, Chen H. Harris hawks optimization: algorithm and applications. Future Generat Comp Sys 2019;97:849–72.
[43] Zhong C, Li G, Meng Z. Beluga whale optimization: a novel nature-inspired metaheuristic algorithm. Knowl Based Syst 2022;251:109215.
[44] Abdel-Basset M, Mohamed R, Zidan M, Jameel M, Abouhawwash M. Mantis search algorithm: a novel bio-inspired algorithm for global optimization and engineering design problems. Comput Methods Appl Mech Eng 2023;415:116200.
[45] Wang L, Cao Q, Zhang Z, Mirjalili S, Zhao W. Artificial rabbits optimization: a new bio-inspired meta-heuristic algorithm for solving engineering optimization problems. Eng Appl Artif Intell 2022;114:105082.
[46] Mohammadi-Balani A, Nayeri MD, Azar A, Taghizadeh-Yazdi M. Golden eagle optimizer: a nature-inspired metaheuristic algorithm. Comput Ind Eng 2021:152.
[47] Li S, Chen H, Wang M, Heidari AA, Mirjalili S. Slime mould algorithm: a new method for stochastic optimization. Future Generat Comp Sys 2020;111:300–23.
[48] Abdollahzadeh B, Khodadadi N, Barshandeh S, Trojovský P, Gharehchopogh FS, El-kenawy E-SM, et al. Puma optimizer (PO): a novel metaheuristic optimization algorithm and its application in machine learning. Cluster Comput 2024.
[49] Özbay FA. A modified seahorse optimization algorithm based on chaotic maps for solving global optimization and engineering problems. Engineer Sci Technol, an Int J 2023;41:101408.
[50] Gharehchopogh FS, Nadimi-Shahraki MH, Barshandeh S, Abdollahzadeh B, Zamani H. CQFFA: a chaotic quasi-oppositional farmland fertility algorithm for solving engineering optimization problems. J Bionic Eng 2023;20:158–83.
[51] Shen B, Khishe M, Mirjalili S. Evolving Marine Predators Algorithm by dynamic foraging strategy for real-world engineering optimization problems. Eng Appl Artif Intell 2023;123:106207.
[52] Khishe M. Greedy opposition-based learning for chimp optimization algorithm. Artif Intell Rev 2023;56:7633–63.
[53] Saffari A, Zahiri SH, Khishe M. Fuzzy whale optimisation algorithm: a new hybrid approach for automatic sonar target recognition. J Experim Theoret Artificial Intelligence 2023;35:309–25.
[54] Bo Q, Cheng W, Khishe M. Evolving chimp optimization algorithm by weighted opposition-based technique and greedy search for multimodal engineering problems. Appl Soft Comput 2023;132:109869.
[55] Gharehchopogh FS, Ghafouri S, Namazi M, Arasteh B. Advances in manta ray foraging optimization: a comprehensive survey. J Bionic Eng 2024;21:953–90.
[56] Zamani H, Nadimi-Shahraki MH, Mirjalili S, Soleimanian Gharehchopogh F, Oliva D. A critical review of moth-flame optimization algorithm and its variants: structural reviewing, performance evaluation, and statistical analysis. Arch Computat Methods Engineer 2024.
[57] Khishe M, Orouji N, Mosavi MR. Multi-objective chimp optimizer: an innovative algorithm for multi-objective problems. Exp Syst Appl 2023;211:118734.
[58] Wang B, Liu L, Li Y, Khishe M. Robust grey wolf optimizer for multimodal optimizations: a cross-dimensional coordination approach. J Sci Comput 2022;92:110.
[59] Liu L, Khishe M, Mohammadi M, Hussein Mohammed A. Optimization of constraint engineering problems using robust universal learning chimp optimization. Adv Engineer Inform 2022;53:101636.
[60] Gong S-P, Khishe M, Mohammadi M. Niching chimp optimization for constraint multimodal engineering optimization problems. Exp Syst Appl 2022;198:116887.
[61] Kaidi W, Khishe M, Mohammadi M. Dynamic levy flight chimp optimization. Knowl Based Syst 2022;235:107625.
[62] Khishe M, Nezhadshahbodaghi M, Mosavi MR, Martín D. A weighted chimp optimization algorithm. IEEE Access 2021;9:158508–39.
[63] Wang J, Khishe M, Kaveh M, Mohammadi H. Binary chimp optimization algorithm (BChOA): a new binary meta-heuristic for solving optimization problems. Cognit Comput 2021;13:1297–316.
[64] Qian L, Khishe M, Huang Y, Mirjalili S. SEB-ChOA: an improved chimp optimization algorithm using spiral exploitation behavior. Neural Comput Applicat 2024;36:4763–86.
[65] Burnham KK, Burnham JL, Johnson JA, Huffman A. Migratory movements of Atlantic puffins Fratercula arctica naumanni from high Arctic Greenland. PLoS One 2021;16:e0252055.
[66] Guilford T, Freeman R, Boyle D, Dean B, Kirk H, Phillips R, et al. A dispersive migration in the atlantic puffin and its implications for migratory navigation. PLoS One 2011;6:e21336.
[67] Fayet AL, Clucas GV, Anker-Nilssen T, Syposz M, Hansen ES. Local prey shortages drive foraging costs and breeding success in a declining seabird, the Atlantic puffin. J Animal Ecol 2021;90:1152–64.
[68] Zhong C, Li G, Meng Z, He W. Opposition-based learning equilibrium optimizer with Levy flight and evolutionary population dynamics for high-dimensional global optimization problems. Exp Syst Appl 2023;215:119303.
[69] He Q, Liu H, Ding G, Tu L. A modified Lévy flight distribution for solving high-dimensional numerical optimization problems. Math Comput Simul 2023;204:376–400.
[70] Wang Z, Chen Y, Ding S, Liang D, He H. A novel particle swarm optimization algorithm with Lévy flight and orthogonal learning. Swarm Evol Comput 2022;75:101207.
[71] Wang Y, Ran S, Wang G-G. Role-oriented binary grey wolf optimizer using foraging-following and Lévy flight for feature selection. Appl Math Modelling 2024;126:310–26.
[72] Awad N.H., Ali M.Z., Suganthan P.N., Liang J.J., Qu B.Y. Problem definitions and evaluation criteria for the CEC 2017 competition and special session on constrained single objective real-parameter optimization. 2016.
[73] Kumar A., Price K.V., Mohamed A.W., Hadi A.A., Suganthan P.N. Problem definitions and evaluation criteria for the CEC 2022 special session and competition on single objective bound constrained numerical optimization. 2021.
[74] Abdollahzadeh B, Gharehchopogh FS, Mirjalili S. African vultures optimization algorithm: a new nature-inspired metaheuristic algorithm for global optimization problems. Comput Ind Eng 2021;158:107408.
[75] Abdollahzadeh B, Soleimanian Gharehchopogh F, Mirjalili S. Artificial gorilla troops optimizer: a new nature-inspired metaheuristic algorithm for global optimization problems. Int J Intell Syst 2021;36:5887–958.
[76] Abualigah L, Elaziz MA, Sumari P, Geem ZW, Gandomi AH. Reptile Search Algorithm (RSA): a nature-inspired meta-heuristic optimizer. Exp Syst Appl 2022;191:116158.
[77] Ahmadianfar I, Heidari AA, Gandomi AH, Chu X, Chen H. RUN beyond the metaphor: an efficient optimization algorithm based on Runge Kutta method. Expert Syst Appl 2021;181:115079.
[78] Hansen N, Müller SD, Koumoutsakos P. Reducing the time complexity of the derandomized evolution strategy with covariance matrix adaptation (CMA-ES). Evol Comput 2003;11:1–18.
[79] Martin L, Leblanc R, Toan NK. Tables for the Friedman rank test. Canadian J Stat 1993;21:39–43.
[80] Kumar A, Wu G, Ali MZ, Mallipeddi R, Suganthan PN, Das S. A test-suite of non-convex constrained optimization problems from the real-world and some baseline results. Swarm Evol Comput 2020;56:100693.
[81] Abdel-Basset M, Mohamed R, Jameel M, Abouhawwash M. Nutcracker optimizer: a novel nature-inspired metaheuristic algorithm for global optimization and engineering design problems. Knowl Based Syst 2023;262:110248.
[82] Zamani H, Nadimi-Shahraki MH, Gandomi AH. Starling murmuration optimizer: a novel bio-inspired algorithm for global and engineering optimization. Comput Methods Appl Mech Eng 2022;392:114616.
[83] Faramarzi A, Heidarinejad M, Mirjalili S, Gandomi AH. Marine Predators Algorithm: a nature-inspired metaheuristic. Expert Syst Appl 2020;152:113377.
[84] Li K, Huang H, Fu S, Ma C, Fan Q, Zhu Y. A multi-strategy enhanced northern goshawk optimization algorithm for global optimization and engineering design problems. Comput Methods Appl Mech Eng 2023;415:116199.
[85] McCarthy GT. The unit hydrograph and flood routing. Proc Confer North Atlantic Division, US Army Corps Engineers 1938;1938:608–9.
[86] Wang W, Tian W, Xu L, Liu C, Xu D. Mε-OIDE algorithm for solving constrained optimization problems and its application in flood control operation of reservoir group. Shuili Xuebao/J Hydraulic Engineer 2023;54:148–58.