سوال ۱

کد:

```
nums = [
    *[0.594, 0.928, 0.515, 0.055, 0.262, 0.797],
    *[0.788, 0.442, 0.262, 0.797, 0.788, 0.442],
    *[0.507, 0.351, 0.097, 0.798, 0.227, 0.127],
    *[0.474, 0.825, 0.007, 0.182, 0.929, 0.852],
]

D = framework.Tests.ks(nums)
D_alpha = 0.27

print(f"D = {D}")
print(f"D_alpha = {D_alpha}\n")

print(
    "Null hypothesis cannot be rejected. [Is uniform]"
    if D <= D_alpha
    else "Null hypothesis is rejected. [Is not uniform]"
)

z_0025 = 1.96

for start in range(len(nums)):
    for lag in range(1, len(nums) - start):
        z = framework.Tests.autoCorr(nums, start, lag)

        if abs(z) > 1.96:
            print("\nAutocorrelation detected with these values:")
            print(f"  start  = {start}")
            print(f"  lag    = {lag}")
            print(f"  z      = {z}")
            print(f"  z_0025 = {z_0025}")
```

رفرنس کد در فریمورک:

```
class Tests:
    @staticmethod
    def ks(nums: List[float | int]):
        nums = sorted(nums)
        n = len(nums)

        D_plus = max((i + 1) / n - num for i, num in enumerate(nums))
        D_minus = max(num - i / n for i, num in enumerate(nums))
```

```
        D = max(D_plus, D_minus)

        return D

    @staticmethod
    def autoCorr(nums: List[float | int], start: int, lag: int):
        pairs = (len(nums) - (start + 1)) // lag - 1

        nominator = (
            sum(
                nums[start + k * lag] * nums[start + (k + 1) * lag]
                for k in range(pairs + 1)
            )
            / (pairs + 1)
            - 0.25
        )

        denominator = math.sqrt(13 * pairs + 7) / (12 * (pairs + 1))

        return nominator / denominator
```

خروجی:

```
D = 0.16300000000000003
D_alpha = 0.27

Null hypothesis cannot be rejected. [Is uniform]

Autocorrelation detected with these values:
  start  = 1
  lag    = 9
  z      = 2.3649477803053496
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 1
  lag    = 14
  z      = 2.2248984533740606
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 1
  lag    = 18
  z      = 2.3385417874026904
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 1
  lag    = 21
  z      = 2.7762790740314895
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 1
  lag    = 22
  z      = 2.452185121431321
  z_0025 = 1.96
```

```
Autocorrelation detected with these values:
  start  = 5
  lag    = 17
  z      = 2.2243042932224903
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 6
  lag    = 16
  z      = 2.1863823617165283
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 9
  lag    = 13
  z      = 2.2243042932224903
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 10
  lag    = 12
  z      = 2.1863823617165283
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 11
  lag    = 4
  z      = 2.0136593004118826
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 15
  lag    = 4
  z      = 2.310976254746033
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 15
  lag    = 7
  z      = 2.228517841167597
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 19
  lag    = 3
  z      = 2.342283635685482
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 19
  lag    = 4
  z      = 2.054161317910548
  z_0025 = 1.96

Autocorrelation detected with these values:
  start  = 22
  lag    = 1
  z      = 2.4560494302033673
  z_0025 = 1.96
```

سوال ۲

کد:

```
for i in range(4):
    serviceTime = framework.RandomVariate.empirical(
        nums=[14, 30, 45, 60, 90, 120, 180, 300],
        frequencies=[10, 20, 25, 35, 30, 20, 10],
    )

    print(f"Customer #{i+1} service time: {serviceTime}")
```

رفرنس کد در فریمورک:

```
class Rng:
    @staticmethod
    def lcg(seed: int, modulus: int, multiplier: int, increment=0, returnInts=False):
        randomInt = seed

        while True:
            yield randomInt if returnInts else randomInt / modulus

            randomInt = (multiplier * randomInt + increment) % modulus


class RandomVariate:
    generator = Rng.lcg(seed=123_457, modulus=2**31 - 1, multiplier=7**5)

    @staticmethod
    def empirical(nums, frequencies):
        randomValue = next(RandomVariate.generator)
        total = sum(frequencies)

        index = -1
        cumulativeFrequency = 0

        for frequency in frequencies:
            relativeFrequency = frequency / total
            if cumulativeFrequency + relativeFrequency <= randomValue:
                cumulativeFrequency += relativeFrequency
                index += 1
            else:
                break

        x0 = nums[index]
        x1 = nums[index + 1]
        relativeFrequency = frequencies[index + 1] / total

        return x0 + (x1 - x0) / relativeFrequency * (randomValue - cumulativeFrequency)
```

خروجی:

```
Customer #1 service time: 40.10740459622229
Customer #2 service time: 112.87262176995753
Customer #3 service time: 22.082180094012145
Customer #4 service time: 49.92902146183374
```

سوال ۳

کد:

```python
def formatTime(time: float) -> str:
    hour = math.floor(currentTime)
    minutes = math.floor(60 * (currentTime - hour))

    return f"{hour}:{"0" if minutes < 10 else ""}{minutes}"

customersCount = framework.RandomVariate.poisson(mean=100)

print(f"Randomly generated value for customers' count: {customersCount}")

currentTime = 8
arrivalTimes = []

for _ in range(customersCount):
    interval = framework.RandomVariate.exponential(mean=8 / 100)
    currentTime += interval

    arrivalTimes.append(formatTime(currentTime))

print("\nArrival times:")
pp(arrivalTimes)
```

رفرنس کد در فریمورک:

```python
class Rng:
    @staticmethod
    def lcg(seed: int, modulus: int, multiplier: int, increment=0, returnInts=False):
        randomInt = seed

        while True:
            yield randomInt if returnInts else randomInt / modulus

            randomInt = (multiplier * randomInt + increment) % modulus


class RandomVariate:
```

```
    generator = Rng.lcg(seed=123_457, modulus=2**31 - 1, multiplier=7**5)

    @staticmethod
    def exponential(mean: float):
        randomValue = next(RandomVariate.generator)

        return -mean * math.log(randomValue)

    @staticmethod
    def poisson(mean: int):
        if mean >= 15:
            z = RandomVariate.normal(0, 1)

            return math.ceil(math.sqrt(mean) * z + mean - 0.5)

        n = 0
        p = 1

        while (p := p * next(RandomVariate.generator)) >= math.exp(-mean):
            n += 1

        return n
```

خروجی:

```
Randomly generated value for cusftomers' count: 87

Arrival times:
['8:02',
 '8:14',
 '8:19',
 '8:29',
 '8:33',
 '8:33',
 '8:42',
 '8:49',
 '8:53',
 '8:57',
 '9:00',
 '9:00',
 '9:05',
 '9:08',
 '9:25',
 '9:27',
 '9:36',
 '9:39',
 '9:40',
 '9:46',
 '9:58',
 '10:01',
 '10:01',
 '10:02',
 '10:09',
 '10:15',
 '10:24',
 '10:25',
 '10:32',
 '10:45',
```

'10:46',
'10:46',
'11:02',
'11:04',
'11:13',
'11:20',
'11:26',
'11:43',
'11:43',
'11:44',
'11:46',
'11:46',
'11:50',
'11:50',
'11:55',
'12:04',
'12:04',
'12:06',
'12:14',
'12:15',
'12:15',
'12:17',
'12:18',
'12:22',
'12:35',
'12:43',
'12:45',
'12:56',
'12:57',
'13:00',
'13:00',
'13:05',
'13:09',
'13:09',
'13:09',
'13:13',
'13:14',
'13:14',
'13:28',
'13:29',
'13:31',
'13:36',
'13:39',
'13:41',
'13:43',
'13:47',
'13:47',
'13:49',
'13:53',
'13:57',
'14:00',
'14:07',
'14:09',
'14:13',
'14:22',
'14:24',
'14:30']

سوال ۴

سوال ۵

زیرا اگر بر روی یک خط قرار بگیرد، به این معناست که روند (Trend) توزیع انباشته داده‌ها، مشابه تابع توزیع حدس زده شده است. اگر شیب آن ۱ باشد، به این معناست که نه تنها روندشان مشابه است، بلکه به تقریب خوبی یکسان هستند. این یکسانی به معنای انتخاب پارامترهای درست برای تابع توزیع است.

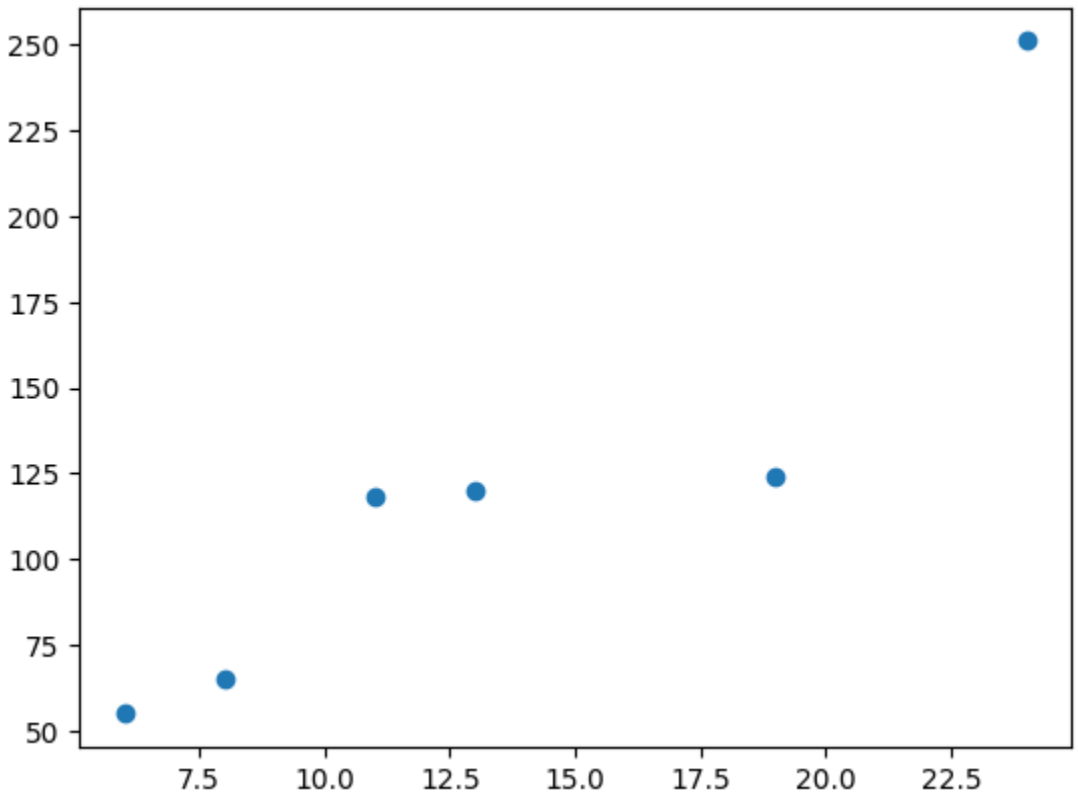
کد:

```
a = sorted([19, 8, 6, 11, 24, 13])
b = sorted([118, 55, 65, 251, 124, 120])

plt.scatter(a, b)
```

خروجی:

از یک توزیع پیروی نمی‌کنند.



سوال ۶

کد:

```
trucksCount = framework.RandomVariate.discreteUniform(3, 9)

print(f"Randomly generated value for trucks' count: {trucksCount}")

loadingTimes = []

for _ in range(trucksCount):
    loadingTime = framework.RandomVariate.normal(mean=34, std=5)

    loadingTimes.append(round(loadingTime, 2))

print("\nLoading times:")
print(loadingTimes)
```

رفرنس کد در فریمورک:

```
class Rng:
    @staticmethod
    def lcg(seed: int, modulus: int, multiplier: int, increment=0, returnInts=False):
        randomInt = seed

        while True:
            yield randomInt if returnInts else randomInt / modulus

            randomInt = (multiplier * randomInt + increment) % modulus


class RandomVariate:
    generator = Rng.lcg(seed=123_457, modulus=2**31 - 1, multiplier=7**5)

    @staticmethod
    def discreteUniform(low: int, high: int):
        k = high - low + 1
        randomValue = next(RandomVariate.generator)

        return math.floor(k * randomValue) + low

    @staticmethod
    def normal(mean: float, std: float):
        randomValue1 = next(RandomVariate.generator)
        randomValue2 = next(RandomVariate.generator)

        radius = math.sqrt(-2 * math.log(randomValue1))
```

```
        theta = 2 * math.pi * randomValue2

        randomVariate1 = radius * math.cos(theta)
        randomVariate2 = radius * math.sin(theta)

        randomVariate1 = std * randomVariate1 + mean
        randomVariate2 = std * randomVariate2 + mean

        return randomVariate1 if next(RandomVariate.generator) < 0.5 else randomVariate2
```

خروجی:

```
Randomly generated value for trucks' count: 6

Loading times:
[38.04, 33.82, 31.3, 39.11, 25.37, 33.93]
```