An Overview on Deep Neural Networks (DNNs)

Hamed Araab

Common Tasks in Deep Learning

Binary Classification

- The primary objective is to assign each item to either Class 1 or Class 0.
- Consider the task of classifying images as either "cat" (Class 1) or "not cat" (Class 0):

🐈 → [1] 🐎 → [0] 🐶 → [0]

Multi-Class Classification

- The primary objective is to categorize items across multiple classes rather than just two.
- Consider a scenario where we aim to classify images into distinct categories such as "cat," "dog," and "neither"

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Multi-Label Classification

- In this scenario, the goal is to identify and assign multiple labels to each input.
- For instance, we may seek to find out whether each image contains a "cat," "dog," or "elephant":

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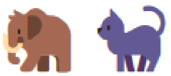
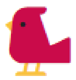
 $\rightarrow \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$
 $\rightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$
 $\rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
 $\rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Regression

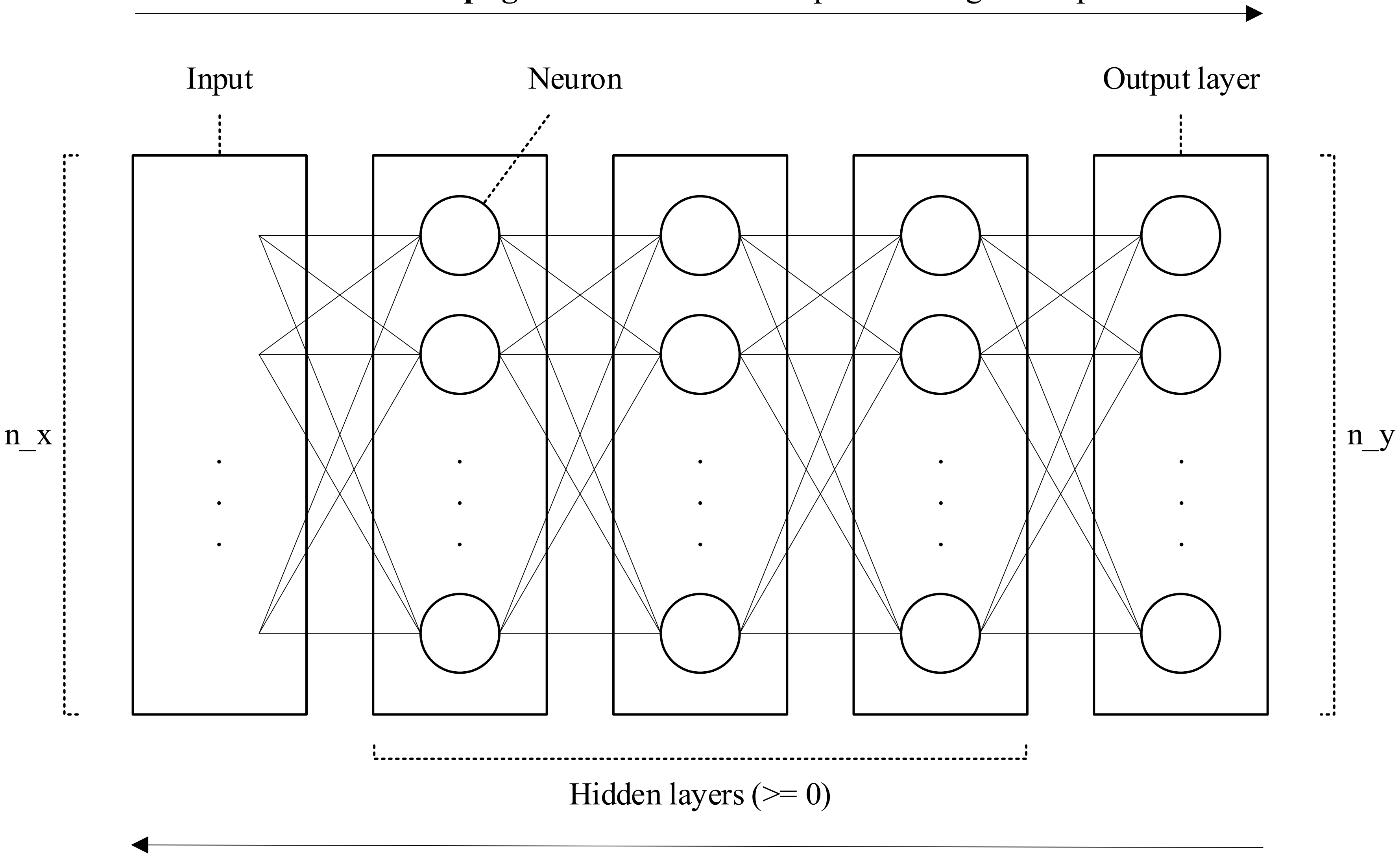
- In this task, the goal is to predict a continuous numerical value rather than a class label.
- It involves learning a mapping function from input features to a continuous output variable.
- For example, we can predict house prices based on features such as square footage, number of bedrooms, and location.

DNNs

What is a DNN?

- An artificial neural network (ANN) with multiple layers.
- Can model complex relationships due to its multi-layer structure.
- In each layer, there are neurons connected to each other.
- These connections have weights and each neuron has a bias.

Forward Propagation: Predict the output for the given input.



Backward Propagation: Updating the network's parameters by comparing the predicted output with the actual output

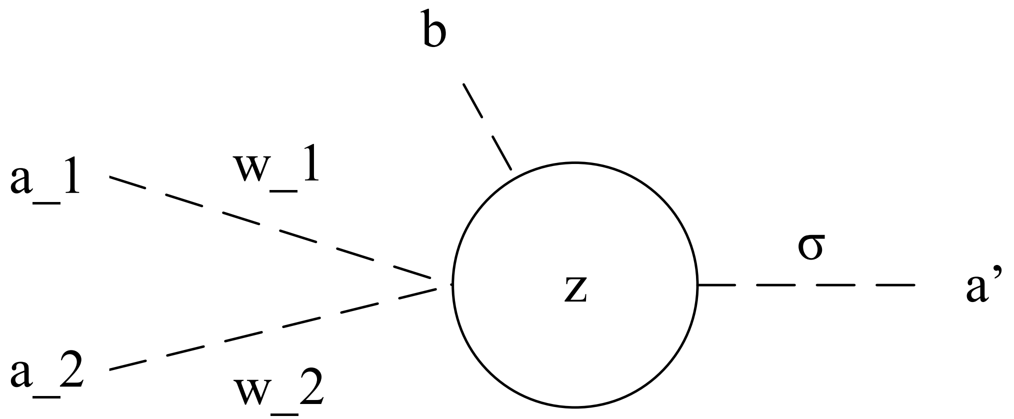
Notes

- The input is not usually considered to be a “layer”.
- Commonly, total layers = hidden layers + output layer

Forward Propagation

Forward Propagation: A Neuron

- a_i: the activation value of the i^{th} neuron in the previous layer
- w_j: weight of the connection to the j^{th} neuron in the previous layer
- b: bias
- z: pre-activation value (logit)
- σ: activation function
- a': activation value



Forward Propagation: A Neuron

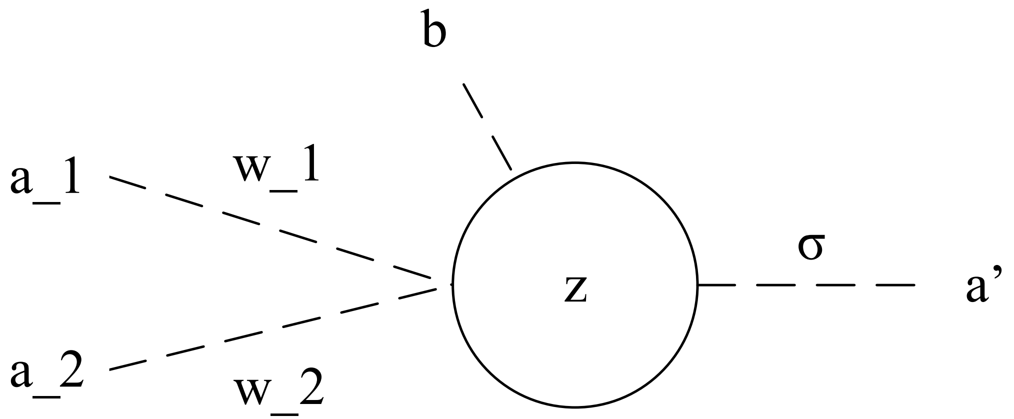
$$z = w_1 a_1 + w_2 a_2 + b$$

This can be vectorized as:

$$\vec{w} = \begin{bmatrix} w_1 & w_2 \end{bmatrix} \qquad \vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

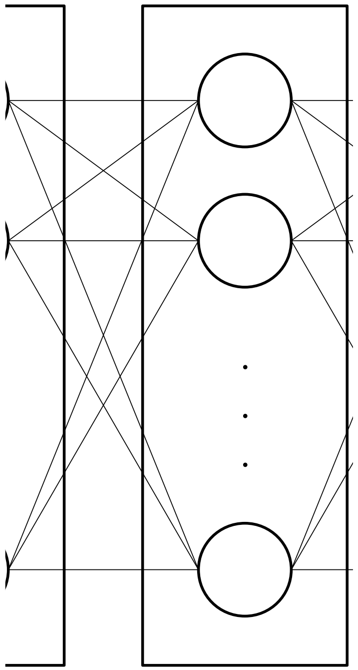
$$z = \vec{w}.\vec{a} + b$$

$$a' = \sigma(z)$$



Forward Propagation: A Layer

- $\vec{a}^{[l-1]}$: The activation vector of the previous layer
- $\vec{w}_i^{[l]}$: The weight vector of the i^{th} neuron in the current layer
- b_i: The bias of the i^{th} neuron in the current layer
- z_i: The pre-activation value of the i^{th} neuron in the current layer
- $\sigma^{[l]}$: The activation function of the l^{th} layer
- $\vec{a}^{[l]}$: The activation vector of the current layer



Forward Propagation: A Layer

$$z_1^{[l]} = \vec{w}_1^{[l]} . \vec{a}^{[l-1]} + b_1^{[l]}$$

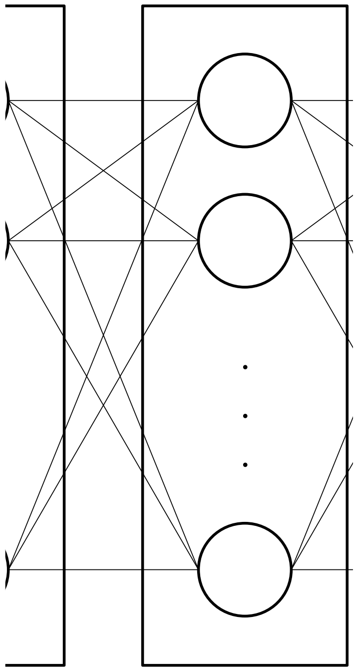
$$z_2^{[l]} = \vec{w}_2^{[l]} . \vec{a}^{[l-1]} + b_2^{[l]}$$

$\ldots$

These equations can be vectorized as:

$$W^{[l]} = \begin{bmatrix} - \ \vec{w}_1^{[l]} \ - \\ - \ \vec{w}_2^{[l]} \ - \\ - \ \ldots \ - \end{bmatrix} \qquad \vec{b}^{[l]} = \begin{bmatrix} b_1^{[l]} \\ b_2^{[l]} \\ \ldots \end{bmatrix}$$

$$\vec{z}^{[l]} = W^{[l]} \times \vec{a}^{[l-1]} + \vec{b}^{[l]} \qquad \vec{a}^{[l]} = \sigma^{[l]}(\vec{z}^{[l]})$$



Input and Output in Forward Propagation

- $\vec{x}_j$: The input vector of the j^{th} item in the dataset
- $\vec{y}_j$: The actual output vector of the j^{th} item in the dataset
- $\vec{\hat{y}}_j$: The predicted output vector for the j^{th} item in the dataset

Input and Output in Forward Propagation

$$\vec{a}_j^{[0]} = \vec{x}_j \qquad \forall j = 1,2,\dots,m$$

$$\vec{z}_j^{[l]} = W^{[l]} \times \vec{a}_j^{[l-1]} + \vec{b}^{[l]} \qquad \forall l = 1,2,\dots,L \qquad \forall j = 1,2,\dots,m$$

$$\vec{a}_j^{[l]} = \sigma^{[l]}\left(\vec{z}_j^{[l]}\right) \qquad \forall l = 1,2,\dots,L \qquad \forall j = 1,2,\dots,m$$

$$\vec{y}_j = \vec{a}_j^{[L]} \qquad \forall j = 1,2,\dots,m$$

Final Form of Forward Propagation

$$X = \begin{bmatrix} | & | & | \\ \vec{x}_1^{[l]} & \vec{x}_2^{[l]} & \vdots \\ | & | & | \end{bmatrix} \qquad Y = \begin{bmatrix} | & | & | \\ \vec{y}_1^{[l]} & \vec{y}_2^{[l]} & \vdots \\ | & | & | \end{bmatrix} \qquad \hat{Y} = \begin{bmatrix} | & | & | \\ \vec{\hat{y}}_1 & \vec{\hat{y}}_2 & \vdots \\ | & | & | \end{bmatrix}$$

$$Z^{[l]} = \begin{bmatrix} | & | & | \\ \vec{z}_1^{[l]} & \vec{z}_2^{[l]} & \vdots \\ | & | & | \end{bmatrix} \qquad A^{[l]} = \begin{bmatrix} | & | & | \\ \vec{a}_1^{[l]} & \vec{a}_2^{[l]} & \vdots \\ | & | & | \end{bmatrix}$$

Final Form of Forward Propagation

$$A^{[0]} = X$$

$$Z^{[l]} = W^{[l]}A^{[l-1]} + b^{[l]} \qquad \forall l = 1,2,\ldots,L$$

$$A^{[l]} = \sigma^{[l]}\left(Z^{[l]}\right) \qquad \forall l = 1,2,\ldots,L$$

$$\hat{Y} = A^{[L]}$$