جامعة الملك سعود
King Saud University

Theory of Computation
CSC 339 – Spring 2021

Chapter-3: part2
Turing Machines

King Saud University

Department of Computer Science

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Note on Completeness

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It also doesn't say whether the empty string is part of ***L***.

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  - **e.g., $L = \{0^n 1^n \mid n \geq 0\}$**
  - **Incomplete description of L may say something like: *L* consists of strings of equal number of 0's and 1's.**
  - **A more complete description, however, should say something like: *L* consists of strings that have a number of 0's *followed by* the same number of 1's, and L includes the empty string.**

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- **On the other hand, a language *L* is said to be <u>*Turing-decidable*</u> if and only if there is a TM that recognizes L, and always halts.**
  - **For strings ∈ L, the TM always accepts.**
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Variants of Turing Machines

- **Turing machines have a number of variants that are equivalent in power with the original model.**
  - **TM with an additional tape head action (S: stay)**
  - **Multi-tape TM**
  - **Nondeterministic TM**
  - **Enumerators**

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$$\Downarrow$$

$$a \rightarrow x, R$$

$$y \rightarrow L \qquad \text{for some } y \in \Gamma$$

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$$\delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k ,$$

where k is the number of tapes

Multi-tape Turing Machines

$$\delta(q_i, a_1 , \ldots, a_k) = (q_j , b_1 , . . . , b_k , L, R, . . . , L)$$

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If the machine is in state q_i and reads symbols a_1 through a_k from tapes *1* through k. Then, the machine goes to state q_j, writes symbols b_1 through b_k, and directs each tape head to move *left*, *right*, or *stay put*.

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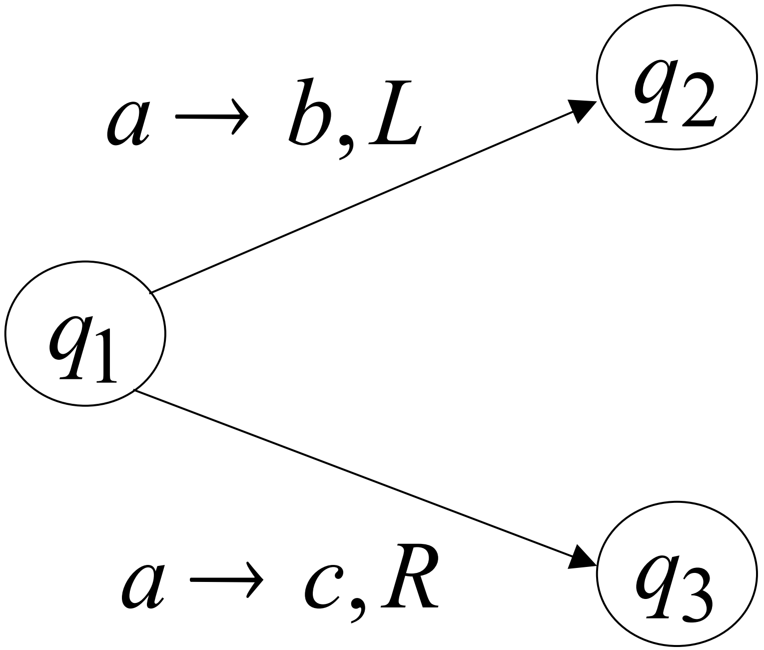

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- **The concept behind Turing machines is a great embodiment of computability theory.**

Algorithms

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- What's an algorithm?
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- Turing machines capture all algorithms: we can design a TM that simulates a given algorithm.

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- **Finding the integral root of a polynomial.**
  - $4x^3 - 2x^2 + x - 7$
- **The integral root is an assignment of an integer value to the variables of the polynomial.**
- **Can we design an algorithm that tells us whether a given polynomial has an integral root?**
  - **Short answer: NO.**

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- **Though, we can convert the TM to be a decider if we provide <u>bounds</u> within which the integral root should fall within.**

- **For instance, the integral root should be within the range (x, y)**

Homework

- **Read page 193 and section 4.1**