

Theory of Computation

CSC 339 – Spring 2021

Introduction

Week-1&2

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Outline

- **Complexity Theory**
- **Computability Theory**
- **Automata Theory**
- **Mathematical and terminology review**

Theories ..

- **Complexity Theory**
- **Computability Theory**
- **Automata Theory**

Theories ..

- **Complexity Theory**

- What makes some problems computationally hard and others easy?

- **Computability Theory**

- **Automata Theory**

Theories ..

‣ **Complexity Theory**

- What makes some problems computationally hard and others easy?

‣ **Computability Theory**

- Can we determine if a problem is solvable or not?

‣ **Automata Theory**

Theories ..

‣ **Complexity Theory**

- What makes some problems computationally hard and others easy?

‣ **Computability Theory**

- Can we determine if a problem is solvable or not?

‣ **Automata Theory**

- Can we design models of computations to solve certain problems?

Complexity Theory

- **The million dollar question**

- *“What makes some problems computationally hard and others easy?”*

- **This question has been studied for over 40 years**

Complexity Theory

- **The million dollar question**
 - *“What makes some problems computationally hard and others easy?”*
- **This question has been studied for over 40 years**
- **We cannot prove that a given problem is computationally hard**
- **But we can point to some evidence that certain problems are computationally hard..**

Complexity Theory

› **What to do when you encounter a computationally hard problem?**

Complexity Theory

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 - **Identify the root of the difficulty, and alter it to make the problem more tractable.**

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- **What to do when you encounter a computationally hard problem?**
 - **Identify the root of the difficulty, and alter it to make the problem more tractable.**
 - **It might be fine to settle for a less than perfect solution.**
 - **Some problems are hard only in worst case scenario, so design solutions that are optimized for the common case.**

Complexity Theory

➤ **What to do when you encounter a computationally hard problem?**

- **Identify the root of the difficulty, and alter it to make the problem more tractable.**
- **It might be fine to settle for a less than perfect solution.**
- **Some problems are hard only in worst case scenario, so design solutions that are optimized for the common case.**
- **May consider alternate types (e.g., randomized) of computation to speed up tasks.**

Computability Theory

- **Some “basic” problems cannot be solved by computers**
 - **Determining whether a mathematical statement is true or false.**

Computability Theory

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Computability Theory

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Computability Theory

- **Complexity and computability theories are closely related**
 - **In complexity theory, the objective is to classify problems as hard or easy ones.**
 - **In computability theory, the classification of problems is by those that are solvable and those that are not.**

Automata Theory

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Automata Theory

- **Automata theory deals with definitions and properties of mathematical models of computation.**
- **Such models comprise a significant portion of applied areas in computer science.**
 - **e.g., finite automata is used in text processing, compilers, and HW design**
 - **Context-free grammar (CFG) is used in PL and AI.**

Mathematical and Terminology Review

- **Sets**
- **Sequences & Tuples**
- **Functions**
- **Graphs**
- **Strings and Languages**

Mathematical and Terminology Review

➤ Sets

- A set is a group of objects (aka members, elements) represented as a unit. Order of elements does not matter.
- It contains only 1 occurrence of each element
- $S1 = \{1,2,3\}$ is equivalent to $S2 = \{2,1,3\}$
- Is $S3 = \{1,\{2,3\}\}$ equivalent to $S1$?
- Is $S4 = \{1,2,2\}$ equivalent to $S1$?
- Unions and intersections

Mathematical and Terminology Review

➤ Sequences and tuples

➤ Order of elements does matter.

➤ $(1,2,3)$ is not the same as $(3,2,1)$.. whereas $\{1,2,3\}$ is the same as $\{3,2,1\}$

➤ **K-tuple:** $(1,2,3)$ is a 3-tuple

Mathematical and Terminology Review

➤ Functions

➤ “a *function* is an object that sets up an input-output relationship.”

➤ Example..

➤ *Increment(x)*

➤ Takes x as input, adds 1, and outputs result

➤ Can take multiple arguments

➤ Unary functions take single argument

➤ Binary functions take two

➤ 3-ary functions take three, and so on.

➤ Predicates are special type of functions (returns True or False)

Mathematical and Terminology Review

➤ Graphs

- nodes and edges
- Path: a sequence of nodes connected by edges
- A graph is connected if every two nodes have a path between them
- Graphs can be undirected or directed
 - Edges in directed graphs have arrows pointing to their direction
 - Outdegree (# of outgoing edges) & indegree (# of incoming edges)

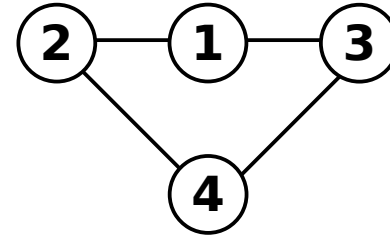
Mathematical and Terminology Review

➤ Graphs

➤ $G = (V, E)$

➤ $V = \{1, 2, 3, 4\}$

➤ $E = \{\{1, 2\}, \{1, 3\}, \{2, 4\}, \{3, 4\}\}$



Mathematical and Terminology Review

➤ Strings and languages

- Alphabet is a non-empty finite set
- A string over an alphabet is a finite sequence of symbols from that alphabet.
- $\{0,1\}$
- $\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z\}$
- $\{0,1,x,y,z\}$

Mathematical and Terminology Review

➤ Strings and languages

➤ Prefix

- $xz = y$.. string x is a prefix of string y
- x is a proper prefix of y if $x \neq y$

Mathematical and Terminology Review

➤ Boolean logic

➤ True or False $\{0,1\}$

➤ Conjunction (AND)

➤ $0 \wedge 1 = 0$

➤ $1 \wedge 1 = 1$

➤ Disjunction (OR)

➤ $0 \vee 1 = 1$

➤ $0 \vee 0 = 0$

➤ Exclusive or (XOR)

➤ $0 \text{ xor } 0 = 0$

➤ $1 \text{ xor } 0 = 1$

➤ $1 \text{ xor } 1 = 0$

Homework

➤ Exercise

➤ 0.3, 0.8

➤ Reading

➤ 1.1 (p31 - p47)