جامعة الملك سعود
King Saud University

# Theory of Computation
CSC 339 – Spring 2021

# Chapter-1: part4
Non-regular Languages

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# Outline

- Recap
- Introduction
- Non-regular Languages

# Recap

- **Regular expressions**
  - **A language is regular if and only if some regular expression describes it.**
    - **We saw how we could convert regular expressions into NFA.**
    - **We also saw how we could convert a finite automaton into regular expressions via state elimination.**

# Non-regular Languages: Introduction

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- **However..**

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  - **How can we prove that a language is not regular?**

# Non-regular Languages: Introduction

- Finite automata are useful to describe numerous <u>regular</u> languages.

- However..

  - They have some limitations, and some languages cannot be described/recognized by finite automata.
  - How can we prove that a language is not regular?
    - <u>“Pumping lemma”</u>

# Non-regular Languages: The Pumping Lemma

- **If A is a regular language, then there is a number p where if s is any string in A of length at least p, then we can divide s into three pieces, s = xyz, such that the following conditions are true:**

# Non-regular Languages: The Pumping Lemma

- **If A is a regular language, then there is a number <u>p</u> where if <u>s</u> is any string in <u>A</u> of length at least <u>p</u>, then we can divide <u>s</u> into three pieces, s = xyz, such that the following conditions are true:**

  1) **$xy^iz \in A$ for each $i \geq 0$,**
  2) **$|y| > 0$, and**
  3) **$|xy| \leq p$.**

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- **We cannot use the pumping lemma to prove that a given language is regular.**

# Non-regular Languages: The Pumping Lemma

- **If language A fails the pumping lemma –> A is definitely non-regular.**

- **If language B passes the pumping lemma –> we cannot decide whether B is regular or not.**

- **We cannot use the pumping lemma to prove that a given language is regular.**

    - **But.. we can use it to show that if a language violates the pumping lemma, then it must be a non-regular language.**

# Non-regular Languages: The Pumping Lemma

- **We will use the pumping lemma to prove that some languages are not regular.**

# Non-regular Languages: The Pumping Lemma

- **We will use the pumping lemma to prove that some languages are not regular.**
- **Proof by contradiction**
  - **Assume the language is regular.**
  - **Pick a string in that language.**
  - **Show that this string violates one of the conditions of the pumping lemma.. This means the language is not regular.**

# Non-regular Languages: The Pumping Lemma

- **Step by step**

  1) **Assume language *A* is regular.**

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# Non-regular Languages: The Pumping Lemma

- **Step by step**

  1) **Assume language *A* is regular.**
  2) **Let *p* be the pumping length.**
  3) **Pick a string $s \in A$, such that $s = xyz$, where $y \neq \varepsilon$ & $|xy| \leq p$**
  4) **Show that a "pumped" version of *s* <u>is not in</u> *A*.**

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  - **Let s = $0^p1^p$**

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  - **Assume A = $\{0^n1^n \mid n > 0\}$ is regular**
  - **Let $p$ be the pumping length.**
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  - **s = xyz where y $\neq \varepsilon$ & $|xy| \leq p$**
  - **Since $|xy| \leq p$, then xy must consist of only 0's**

# Non-regular Languages: The Pumping Lemma

- **Prove that A = $\{0^n1^n \mid n > 0\}$**
  - **Assume A = $\{0^n1^n \mid n > 0\}$ is regular**
  - **Let *p* be the pumping length.**
  - **Let s = $0^p1^p$**
  - **s = xyz where y ≠ ε & |xy| ≤ *p***
  - **Since |xy| ≤ *p*, then xy must consist of only 0's**
  - **Since y ≠ ε , then y is one or more 0's. Suppose |y| = k (y=$0^k$)**

# Non-regular Languages: The Pumping Lemma

- **Prove that $A = \{0^n1^n \mid n > 0\}$**
  - **Assume $A = \{0^n1^n \mid n > 0\}$ is regular**
  - **Let *p* be the pumping length.**
  - **Let $s = 0^p1^p$**
  - **$s = xyz$ where $y \neq \varepsilon$ & $|xy| \leq p$**
  - **Since $|xy| \leq p$, then xy must consist of only 0's**
  - **Since $y \neq \varepsilon$, then y is one or more 0's. Suppose $|y| = k$ ($y=0^k$)**
  - **By the pumping lemma, xyyz must be in A. But, it's not!**
    - **$xyyz = 0^{p+k}1^p$ is not in A**

# Homework

- Read examples 1.73-1.77