King Saud University
College of Science
Department of Mathematics

106 Math Exercises

Exponential & Logarithmic Functions

(5)

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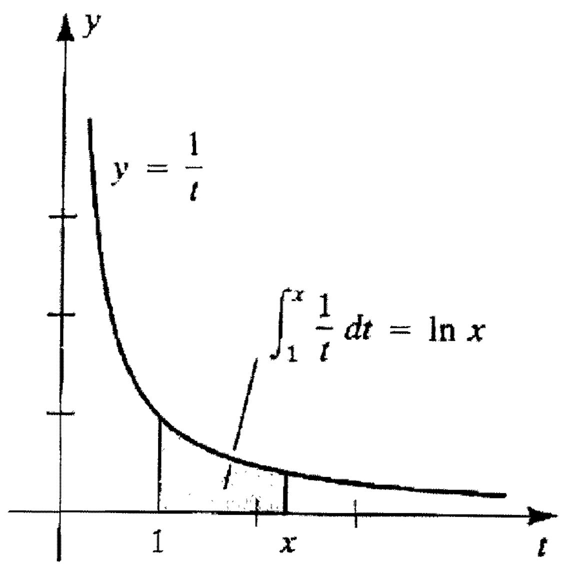
Natural Logarithm Function

Definition:

$$\ln x = \int_1^x \frac{1}{t}\,dt \quad : \; for\ every\ \ x > 0$$

$$\ln 1 = \int_1^1 \frac{1}{t}\,dt = 0 \qquad \frac{d}{dx}\int_1^x \frac{1}{t}\,dt = \frac{1}{x} \;\; : x > 0$$

$$\lim_{x\to\infty}(\ln x) = \infty \quad , \quad \lim_{x\to 0^-}(\ln x) = -\infty$$



Theorem: i) $\frac{d}{dx}(\ln x) = \frac{1}{x}$

ii) $If\ u = g(x) and\ g\ is\ differentiable\ then$:

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx} \quad if\ u > 0$$

$$\boxed{\frac{d}{dx}(\ln|u|) = \frac{1}{u}\frac{du}{dx} \quad if\ \ u \neq 0 \ , \qquad \int \frac{1}{u}\,du = \ln|u| + C}$$

Law of Natural Logarithms: $If\ p > 0\ and\ q > 0\ , then$

i) $\ln pq = lnp + lnq$

ii) $\ln \frac{p}{q} = lnp - lnq$

iii) $\ln p^r = r\ lnp \quad , \forall r \in \mathbb{Q}$

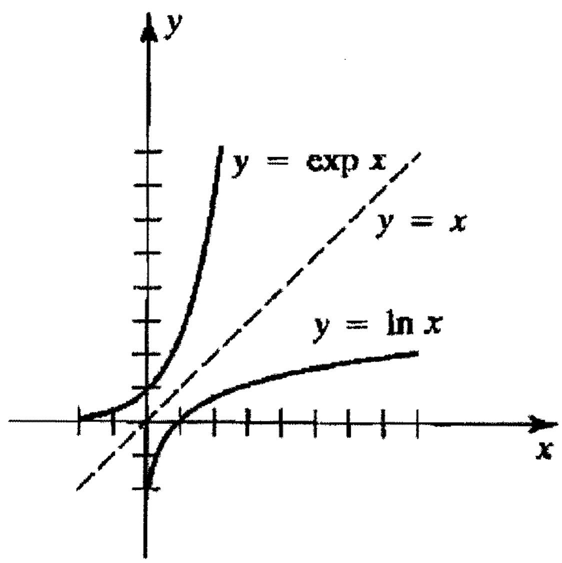
The Exponential Function

Theorem: To every real number x there corresponds exactly one Positive real number y such that $\ln y = x$

Definition: The **natural exponential function** , denoted **exp ,** is the inverse of the natural logarithm function .

$$\boxed{y = \exp x \quad \Leftrightarrow \quad x = ln y}$$

$$\lim_{x \to \infty} \exp x = \infty \quad and \quad \lim_{x \to -\infty} \exp x = 0$$



Definition: The letter **e** denotes the positive real number that

$$\ln e = 1 \quad , \quad e = 2.7182818284590 \ldots \ldots \ldots \ldots$$

Definition of e^x : $\boxed{\text{If } x \in \mathbb{R}, \quad e^x = y \iff \ln y = x}$

Theorem: i) $\boxed{\ln e^x = x \quad , \forall\ x \in \mathbb{R}}$

ii) $\boxed{e^{\ln x} = x \quad , \forall\, x > 0}$

Theorem: If p and q $\in \mathbb{R}$ and $r \in \mathbb{Q}$, then :

i) $e^p e^q = e^{p+q}$

ii) $\dfrac{e^p}{e^q} = e^{p-q}$

iii) $(e^p)^r = e^{pr}$

Theorem: i) $\dfrac{d}{dx}(e^x) = e^x$

ii) *If $u = g(x)$ and g is differentiable, then*

$$\boxed{\frac{d}{dx}(e^u) = e^u \frac{du}{dx}} \qquad \boxed{\int e^u\, du = e^u + C}$$

Theorem:

i) $\int \tan u\ du = -\ln|\cos u| + C = \ln|\sec u| + C$

ii) $\int \cot u\ du = \ln|\sin u| + C$

iii) $\int \sec u\, du = \ln|\sec u + \tan u| + C$

iv) $\int \csc u\, du = \ln|\csc u - \cot u) + C$

Exercieses

1) Find the value of x that satisfies the equation : $ln\frac{1}{x} = 2$

2) Find the value of x that satisfies the equation :$e^{5x+3} = 4$

3) Find the value of x that satisfies the equation :$e^{2x-4} = 1$

4) Find the value of x that satisfies the equation:

$2lnx = \ln(x+2)$

5) Simplify $\ln(e^x)^4$

6) Simplify $3[\frac{1}{2}\ln|x+2| + \ln|x| - \ln|5+x^2|]$

7) Find $g(x)$ such that:

i- $\int [\ln|x|]^2 g(x)dx = \frac{2}{3}[\ln\ |x|]^3 + c$

ii- $\int e^{3x^2} g(x)dx = -\ e^{3x^2} + c$

8) Find the domain of the following functions :

i- $f(x) = \ln\left(\frac{2}{x-2}\right)$

ii- $f(x) = \ln(1 - x)$

iii- $f(x) = \ln\left(\frac{1}{2-x}\right)$

iv- $f(x) = \ln(x^2 + x)$

9) Find $f'(x)$ *of the following* :

i- $f(x) = e^{3x} + \frac{2}{e^x}$

ii- $f(x) = ln\frac{2}{x} + \frac{2}{lnx}$

iii- $f(x) = ln|5x^2 - 1|^3$

iv- $f(x) = \cos(ln5x) + e^{cos5x}$

v- $f(x) = ln\frac{(6x-5)^2}{\sqrt{x^2+1}}$

vi- $f(x) = \tan(e^{2x}) + e^{sin5x}$

vii- $f(x) = \frac{x}{e^{x^2}} + e^{-3x} cos3x$

viii- $f(x) = \ln\,(csce^{3x})$

ix- $f(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

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10) Use logarithmic differentiation to find y':

i- $y = \sqrt{4x+7}\,(x-5)^3\,(x+3)^4$

ii- $y = \sqrt[3]{\frac{(x^2+3)^5\,(7x\_2)^2}{\sqrt{3x+4}}}$

11) Evaluate the following integrals :

i- $\int e^{(x^2+lnx)}\, dx=$

ii- $\int e^{2x} sec^2(e^{2x}) dx$

iii- $\int \frac{e^{-x}}{(1-e^{-x})^2} dx$

iv- $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

v- $\int \frac{e^{\frac{3}{x}}}{x^2} dx$

vi- $\int \frac{e^{sinx}}{secx} dx$

vii- $\int_1^e \frac{\sqrt[3]{lnx}}{x}dx$

viii- $\int \frac{cot\sqrt[3]{x}}{\sqrt[3]{x^2}}dx$

ix- $\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$

x- $\int \frac{1}{e^{-x}+1}dx$

xi- $\int_0^1 \frac{9x^2+12x}{x^3+2x^2+1}dx$

xii- $\int \frac{lnx^2}{x}dx$

xiii- $\int \frac{e^x}{cos^2(e^x-2)} dx$

xiv- $\int \frac{sinx-cosx}{sinx+cosx} dx$

xv- $\int \frac{x-1}{x+1} dx$

xvi- $\int_1^e \frac{1}{x\sqrt{lnx}} dx$

General Exponential & Logaritmic Functions

1- General Exponential

Definition of a^x : $\boxed{a^x = e^{x \ln a} \quad , \forall\, a > 0 \;\; and \;\; \forall\, x \in \mathbb{R}}$

Laws of Exponents: *Let* $a > 0$ *and* $b > 0$, *If* u *and* $v \in \mathbb{R}$, *then*

i) $a^u a^v = a^{u+v}$

ii) $(a^u)^v = a^{uv}$

iii) $(ab)^u = a^u b^u$

iv) $\dfrac{a^u}{a^v} = a^{u-v}$

v) $\left(\dfrac{a}{b}\right)^u = \dfrac{a^u}{b^u}$

Theorem:

i) $\dfrac{d}{dx}(a^x) = a^x \ln a \quad , \quad \int a^x\, dx = \left(\dfrac{1}{\ln a}\right) a^x + C$

ii) $\boxed{\dfrac{d}{dx}(a^u) = (a^u \ln a)\dfrac{du}{dx}} \qquad \boxed{\int a^u\, du = \left(\dfrac{1}{\ln a}\right) a^u + C}$

2- Logaritmic Functions

Definition: $\boxed{If\ a > 0\ and\ a \neq 1\ , \quad y = \log_a x \quad \Leftrightarrow \quad x = a^y}$

$$\log_e x = \ln x \qquad , \ \log_{10} x = \log x$$

$$\log_a x = \frac{\ln x}{\ln a}$$

Theorem:

i) $$\frac{d}{dx}(\log_a x) = \frac{1}{\ln a}\frac{1}{x}$$

ii) $$\boxed{\frac{d}{dx}(\log_a u) = \frac{1}{\ln a}\frac{1}{u}\frac{du}{dx}}$$

iii) $$\boxed{\lim_{h \to 0} (1+h)^{1/h} = e}$$

iv) $$\boxed{\lim_{h \to \infty} (1+\frac{1}{h})^h = e}$$

Exercises

1) Find the value of x if $\log_2 x = 3$?

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2) Find the value of a if $\log_a 125 = 3$?

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3) Find the value of x if $2\log|x| = \log 2 + \log|3x - 4|$

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4) Find the value of x if $\log\left(\frac{x}{x-1}\right) = 1$?

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5) Find the value of x if $\log_4 x^2 = 1$?

6) Find the value of x if $\log_2(\frac{x-1}{x}) = 1$?

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7) Find y' if

i- $2x = 4^y$

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ii- $y = (\sin x)^x$

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iii- $y = (1 + x^2)^{2x+1}$

iv- $y = (x^4 + x^2 + 1)^{\ln(2x+1)}$

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v- $y = (x^2 + x + 1)^{\sin(2x)}$

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8) Find $f'(x)$ if $f(x) = 7^{\sqrt[3]{x}} + \pi^{3x}$

9) Find $f'(e)$ if $f(x) = x^{\frac{1}{x}}$

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10) Find $f'(x)$ if $f(x) = x^{(e^x)}$

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11) Evaluate the following integrals :

i- $\int x^2 7^{x^3}\, dx =$

ii- $\int \frac{2^x}{2^x+1}\, dx =$

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iii- $\int \frac{5^{cotx}}{sin^2 x}\, dx =$

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iv- $\int 2^{x\ln x}\,(1+\ln x)\, dx =$

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v- $\int 4^x\ 5^{(4^x)}\, dx =$

vi- $\int 3^x \left[1 + \sin(3^x) \right] dx$

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vii- $\int \frac{3^{\sqrt{x}}}{\sqrt{x}} \, dx$

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viii- $\int 3^x (3^x + 3^{-x})^2 dx$

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ix- $\int \frac{5^{\cos x}}{\csc x} \, dx =$

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x- $\int_0^1 (7x) \, 7^{x^2} \, dx =$

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
22. $\frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$
23. $\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2-1}}$
24. $\frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2-1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\text{sech}^2 u)u'$
28. $\frac{d}{dx}[\coth u] = -(\text{csch}^2 u)u'$
29. $\frac{d}{dx}[\text{sech } u] = -(\text{sech } u \tanh u)u'$
30. $\frac{d}{dx}[\text{csch } u] = -(\text{csch } u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2+1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2-1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1-u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1-u^2}$
35. $\frac{d}{dx}[\text{sech}^{-1} u] = \frac{-u'}{u\sqrt{1-u^2}}$
36. $\frac{d}{dx}[\text{csch}^{-1} u] = \frac{-u'}{|u|\sqrt{1+u^2}}$

Basic Integration Formulas

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
5. $\int e^u\,du = e^u + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \tan u\,du = -\ln|\cos u| + C$
9. $\int \cot u\,du = \ln|\sin u| + C$
10. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
11. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
12. $\int \sec^2 u\,du = \tan u + C$
13. $\int \csc^2 u\,du = -\cot u + C$
14. $\int \sec u \tan u\,du = \sec u + C$
15. $\int \csc u \cot u\,du = -\csc u + C$
16. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$
17. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
18. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C$

Integration and Differentiation Formula

1. $\int u^n du = \frac{u^{n+1}}{n+1} + c \quad n \neq 1$

2. $\int x dx = \frac{x^2}{2} + c$

3. $\int a^u du = \frac{a^u}{\ln a} + c$

4. $\int \frac{du}{u} = \ln|u| + c$

5. $\int e^u du = e^u + c$

6. $\int \sin u du = -\cos u + c$

7. $\int \csc^2 u du = -\cot u + c$

8. $\int \tan^2 u du = \tan u - u + c$

9. $\int \ln u du = u \ln u - u + c$

10. $\int \tan u du = \ln|\sec u| + c$

10. $\int \cot u du = \ln|\sin u| + c$

11. $\int \sec u du = \ln|\sec u + \tan u| + c$

12. $\int u e^u du = u e^u - e^u + c$

13. $\int \sin^2 u du = \frac{1}{2}u - \frac{1}{4}\sin 2u + c$

14. $\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\sin 2u + c$

15. $\int u \sin u du = \sin u - u \cos u + c$

16. $\int u \cos u du = \cos u + u \sin u + c$

17. $\int \sinh u du = \cosh u + c$

18. $\int \cosh u du = \sinh u + c$

19. $\int \tanh u du = Ln \cosh u + c$

20. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + c$

21. $\int \frac{-du}{\sqrt{a^2 - u^2}} = \cos^{-1}\frac{u}{a} + c$

22. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$

23. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + c$.

24. $\int e^{au}\cos bu du = \frac{e^{au}}{a^2 + b^2}(a\cos bu + b\sin bu) + c$

25. $\int e^{au}\sin bu du = \frac{e^{au}}{a^2 + b^2}(a\sin bu - b\cos bu) + c$

26. $\int \sin^{-1} u du = u\sin^{-1} u + \sqrt{1 - u^2} + c$

27. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln\left|u + \sqrt{u^2 - a^2}\right| = \cosh^{-1}\frac{u}{a}$

28. $\int \frac{du}{\sqrt{u^2 + a^2}} = \ln\left|u + \sqrt{u^2 + a^2}\right| = \sinh^{-1}\frac{u}{a}$

Derivative Formula

1. $\frac{d}{dx}x^n = nx^{n-1}$

2. $\frac{d}{dx}e^x = e^x$

3. $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

4. $\frac{d}{dx}\ln x = \frac{1}{x}$

5. $\frac{d}{dx}\sin x = \cos x$

6. $\frac{d}{dx}\cos x = -\sin x$

7. $\frac{d}{dx}\sec x = \sec x \tan x$

8. $\frac{d}{dx}\tan x = \sec^2 x$

9. $\frac{d}{dx}\cot x = -\mathrm{cosec}^2 x$

10. $\frac{d}{dx}\mathrm{cosec}\, x = -\mathrm{cosec}\, x \cot x$

11. $\frac{d}{dx}\sinh x = \cosh x$

12. $\frac{d}{dx}\cosh x = \sinh x$

13. $\frac{d}{dx}\tanh x = \mathrm{sech}^2 x$

14. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$

15. $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1 - x^2}}$

16. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1 + x^2}$

17. $\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{1 + x^2}}$

18. $\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2 - 1}}$

Implicit Differentiation

$$\frac{d}{dx}y^n = ny^{n-1}\frac{dy}{dx}$$

$$\frac{d}{dx}(x^2y^2) = 2x^2y\frac{dy}{dx} + 2xy$$

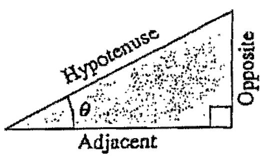
$$\frac{d}{dx}(x^2 + y^2) = 2x + 2y\frac{dy}{dx}$$

$$\frac{d}{dx}x\sin y = \sin y + x\cos y.\frac{dy}{dx}$$

TRIGONOMETRY

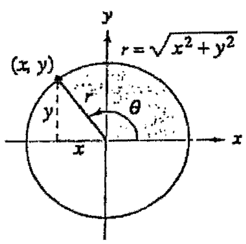
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

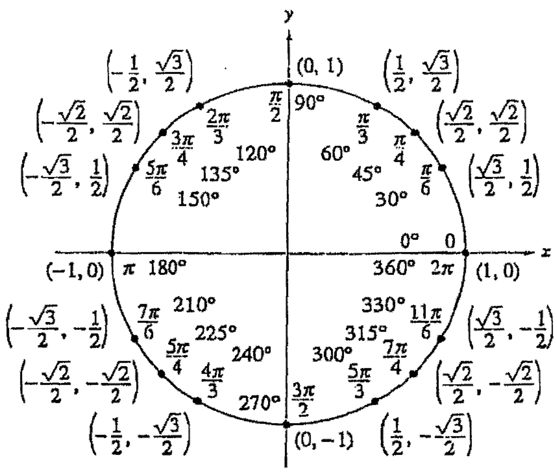


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$
$$\cos\theta = \frac{x}{r} \quad \sec\theta = \frac{r}{x}$$
$$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \sec x = \frac{1}{\cos x} \quad \tan x = \frac{1}{\cot x}$$
$$\csc x = \frac{1}{\sin x} \quad \cos x = \frac{1}{\sec x} \quad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$
$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$
$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x$$
$$\csc(-x) = -\csc x \quad \tan(-x) = -\tan x$$
$$\sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$
$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$
$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$
$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \quad u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$
26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$
27. $\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u'$
28. $\frac{d}{dx}[\coth u] = -(\operatorname{csch}^2 u)u'$
29. $\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u'$
30. $\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}}$
32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}}$
33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1 - u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1 - u^2}$
35. $\frac{d}{dx}[\operatorname{sech}^{-1} u] = \frac{-u'}{u\sqrt{1 - u^2}}$
36. $\frac{d}{dx}[\operatorname{csch}^{-1} u] = \frac{-u'}{|u|\sqrt{1 + u^2}}$

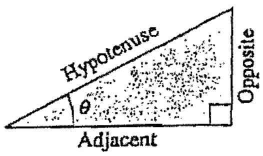
Basic Integration Formulas

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
5. $\int e^u\,du = e^u + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \tan u\,du = -\ln|\cos u| + C$
9. $\int \cot u\,du = \ln|\sin u| + C$
10. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
11. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
12. $\int \sec^2 u\,du = \tan u + C$
13. $\int \csc^2 u\,du = -\cot u + C$
14. $\int \sec u \tan u\,du = \sec u + C$
15. $\int \csc u \cot u\,du = -\csc u + C$
16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$
17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

TRIGONOMETRY

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

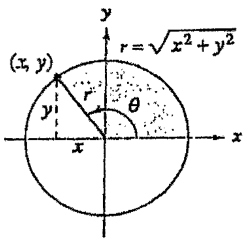


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

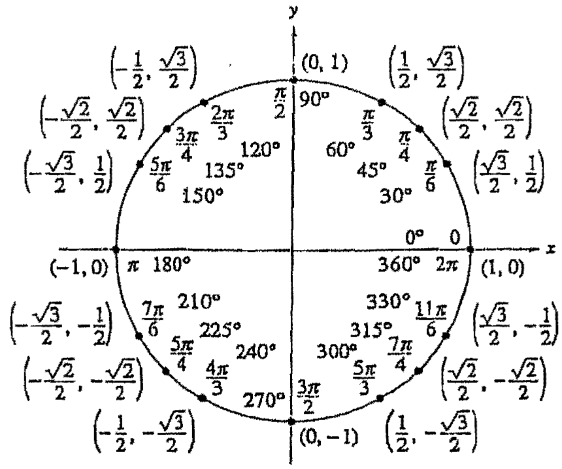
Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \sec x = \frac{1}{\cos x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \cos x = \frac{1}{\sec x} \qquad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x$$

$$\csc(-x) = -\csc x \qquad \tan(-x) = -\tan x$$

$$\sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$