King Saud University

College of Sciences

Department of Mathematics

106 Math Exercises

(4)

Numerical Integration

Malek Zein AL-Abidin

1) Approximate the integral

$$\int_1^2 \ln(x+1)\,dx$$

using regular partition with n = 4 , using the <u>Trapezoidal rule</u> .

<u>Solution:</u>

i- $[a,b] = [1,2]$, $f(x) = \ln(x+1)$, $\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4} = 0.25$

$$x_i = a + i\,\Delta x = 1 + 0.25i \;:\; i = 0,1,2,3,4 = n$$

i	x_i	$f(x_i) = \ln(x_i+1)$	m_i	$m_i f(x_i)$
0	$x_0 = a = 1$	0.6930	1	0.6930
1	$x_1 = 1.25$	0.8109	2	1.6218
2	$x_2 = 1.5$	0,9163	2	1.8326
3	$x_3 = 1.75$	1.0116	2	2.0232
4	$x_4 = b = 2$	1.0986	1	1.0986
				$\sum_{i=0}^{4} \boldsymbol{m_i f(x_i)} = \mathbf{7.2692}$

$$T_n = \frac{b-a}{2n}\sum_{i=0}^{n} m_i f(x_i) \Rightarrow T_4 = \frac{2-1}{2(4)}\sum_{i=0}^{4} m_i f(x_i) = \frac{7.2692}{8}$$

$$\therefore \quad T_4 = 0.90865$$

$$\therefore \int_1^2 \ln(x+1)dx \approx 0.90865$$

2) Approximate the integral

$$\int_1^5 \frac{lnx}{x}dx$$

using regular partition with n = 4 , using the Trapezoidal rule .

Solution:

$$[a,b] = [1,5] \ , \ f(x) = \frac{lnx}{x} \quad , \quad \Delta x = \frac{b-a}{n} = \frac{5-1}{4} = 1$$

$$x_i = a + i\,\Delta x \ = \ 1 + i \ : \ \ i = 0,1,2,3,4 = n$$

i	x_i	$f(x_i) = \frac{lnx_i}{x_i}$	m_i	$m_i f(x_i)$
0	$x_0 = a = 1$	0.0000	1	0.0000
1	$x_1 = 2$	0.3466	2	0.6932
2	$x_2 = 3$	0.3662	2	0.7324
3	$x_3 = 4$	0.3466	2	0.6932
4	$x_4 = b = 5$	0.3218	1	0.3218
				$\sum_{i=0}^{4} m_i f(x_i) = 2.4406$

$$T_n = \frac{b-a}{2n}\sum_{i=0}^{n} m_i f(x_i) \ \Rightarrow \ T_4 = \frac{5-1}{2(4)}\sum_{i=0}^{4} m_i f(x_i) = \frac{2.4406}{2}$$

$$\therefore \quad T_4 = 1.2203$$

$$\int_1^5 \frac{lnx}{x}dx \approx 1.2203$$

3) Approximate the integral

$$\int_2^3 \sqrt{1+x^3}\,dx$$

, using regular partition with n=5 , using the <u>Trapezoidal rule</u> .

<u>Solution:</u>

$$[a,b]=[2,3] \;\; , f(x)=\sqrt{1+x^3} \quad , \; \Delta x=\frac{b-a}{n}=\frac{3-2}{5}=\frac{1}{5}=0.2$$

$$x_i = a + i\,\Delta x \;=\; 2+0.2\,i \;:\; \quad i=0,1,2,3,4,5=n$$

i	x_i	$f(x_i)=\sqrt{1+{x_i}^3}$	m_i	$m_i f(x_i)$
0	$x_0=a=2$	3.0000	1	3.0000
1	$x_1=2.2$	3.4129	2	6.8259
2	$x_2=2.4$	3.8520	2	7.7040
3	$x_3=2.6$	4.3099	2	8.6198
4	$x_4=2.8$	4.7915	2	9.5816
5	$x_5=b=3$	5.2915	1	5.2915
				$\sum_{i=0}^{5} m_i f(x_i)=41.0228$

$$T_n=\frac{b-a}{2n}\sum_{i=0}^{n} m_i f(x_i) \;\Rightarrow\; T_5=\frac{3-2}{2(5)}\sum_{i=0}^{5} m_i f(x_i)=\frac{41.0228}{10}$$

$$\therefore \quad T_5 = 4.10228$$

$$\int_2^3 \sqrt{1+x^3}\,dx \;\approx\; 4.10228$$

4) Approximate the integral

$$\int_{0}^{1} \sqrt{1 - x^2}\ dx$$

, using regular partition with n=5 , using the <u>Trapezoidal rule</u>

5) Approximate the integral

$$\int_0^1 e^{4x} dx$$

using regular partition with n = 4 , using the Simpson's rule .

Solution:

i- $[a, b] = [0,1]$, $f(x) = e^{4x}$, $\Delta x = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} = 0.25$

$$x_i = a + i\,\Delta x = 0 + 0.25i = 0.25i \quad : \quad i = 0,1,2,3,4 = n$$

i	x_i	$f(x_i) = e^{4x_i}$	δ_i	$\delta_i f(x_i)$
0	$x_0 = a = 0$	1.0000	1	1.0000
1	$x_1 = 0.25$	$e = 2.7183$	4	10.8732
2	$x_2 = 0.5$	$e^2 = 7.3891$	2	14.7782
3	$x_3 = 0.75$	$e^3 = 20.0855$	4	80.3420
4	$x_4 = b = 1$	$e^4 = 54.5982$	1	54.5982
				$\sum_{i=0}^{4} \delta_i f(x_i) = 161.5916$

$$S_n = \frac{b-a}{3n} \sum_{i=0}^{n} \delta_i f(x_i) \Rightarrow S_4 = \frac{1-0}{3(4)} \sum_{i=0}^{4} \delta_i f(x_i) = \frac{161.5916}{12}$$

$S_4 = 13.466$

$$\int_0^1 e^{4x} dx \approx 13.466$$

6) Approximate the integral

$$\int_1^5 \sqrt{1+x^2}\, dx$$

, using regular partition with n=4 , using the <u>Simpson's rule</u> .

<u>Solution:</u> $[a,b]=[1,5]$, $f(x)=\sqrt{1+x^2}$, $\Delta x=\frac{b-a}{n}=\frac{5-1}{4}=1$

$x_i = a + i\,\Delta x = 1 + i$: $i = 0,1,2,3,4 = n$

i	x_i	$f(x_i)=\sqrt{1+{x_i}^2}$	δ_i	$\delta_i f(x_i)$
0	$x_0 = a = 1$	1.4142	1	1.4142
1	$x_1 = 2$	2.2360	4	8.9440
2	$x_2 = 3$	3.1622	2	6.3244
3	$x_3 = 4$	4.1231	4	16.4924
4	$x_4 = b = 5$	5.0990	1	5.0990
				$\sum_{i=0}^{4}\delta_i f(x_i) = 38.274$

$$S_n = \frac{b-a}{3n}\sum_{i=0}^{n}\delta_i f(x_i) \Rightarrow S_4 = \frac{5-1}{3(4)}\sum_{i=0}^{4}\delta_i f(x_i) = \frac{38.274}{3}$$

$\therefore \quad S_4 = 12.758 \Rightarrow$

$$\int_1^5 \sqrt{1+x^2}\, dx \approx 12.758$$

7) Approximate the integral

$$\int_{0}^{\pi} \cos(sinx)\,dx$$

, using regular partition with n=6 , using the Simpson's rule .

<u>Solution:</u> $[a,b]=[0,\pi]$, $f(x)=\cos(\sin x)$, $\Delta x=\frac{b-a}{n}=\frac{\pi-0}{6}=\frac{\pi}{6}$

$$x_i=a+i\,\Delta x = 0+i\,\frac{\pi}{6}: \quad i=0,1,2,3,4,5,6=n$$

i	x_i	$f(x_i)=\cos(sinx_i)$	δ_i	$\delta_i f(x_i)$
0	$x_0=a=0$	1.000 00	1	1.000 00
1	$x_1=\frac{\pi}{6}$	0.999 96	4	3.999 84
2	$x_2=\frac{\pi}{3}$	0.999 88	2	1.999 76
3	$x_3=\frac{\pi}{2}$	0.999 84	4	3.999 36
4	$x_4=\frac{3\pi}{2}$	0.999 88	2	1.999 76
5	$x_5=\frac{5\pi}{6}$	0.999 96	4	3.999 84
6	$x_6=b=\pi$	1.000 00	1	1.000 00
				$\sum_{i=0}^{6}\delta_i f(x_i)=17.99856$

$$S_n=\frac{b-a}{3n}\sum_{i=0}^{n}\delta_i f(x_i) \Rightarrow S_6=\frac{\pi-0}{3(6)}\sum_{i=0}^{4}\delta_i f(x_i)=\frac{\pi}{18}\ 17.99856$$

$$\therefore\quad S_6=3.14134$$

$$\therefore \int_{0}^{\pi}\cos(sinx)\,dx \approx 3.14134$$

8) Approximate

$$\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx$$

, using regular partition with n=4 , using the Simpson's rule .