Q What are the values of these sums, where $S = \{1, 3, 5, 7\}$?

a) $\sum_{j \in S} j$

b) $\sum_{j \in S} j^2$

c) $\sum_{j \in S} (1/j)$

d) $\sum_{j \in S} 1$

Sol

a) 16

b) 84

c) $\frac{176}{105}$

d) 4

Q What is the value of each of these sums of terms of a geometric progression?

a) $\sum_{j=0}^{8} 3 \cdot 2^j$

b) $\sum_{j=1}^{8} 2^j$

c) $\sum_{j=2}^{8} (-3)^j$

d) $\sum_{j=0}^{8} 2 \cdot (-3)^j$

Sol: a) $r = 2, a = 3, n = 8$

$\Rightarrow 3(2^9 - 1)/(2-1) = \underline{1533}$

b) $r = 2, a = 1, n = 8$

From 0 – 8 the sum is

$1 \cdot (2^9 - 1)/(2-1)$

$\Rightarrow 511$

Now subtract term 2^0

$\Rightarrow 511 - 1 \Rightarrow \underline{510}$

c) $[((-3)^9 - 1)/(-3-1)] - (-3)^0 - (-3)^1$

$\Rightarrow 4921 - 1 + 3 \Rightarrow \underline{4923}$

d) $2((-3)^9 - 1)/(-3-1)$

$\Rightarrow 9843$

Q Show that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$, where $a_0, a_1, a_2, \ldots, a_n$ is a sequence of real numbers. This type of sum is called "telescoping".

Sol:

$$\cancel{a_1} - a_0 + \cancel{a_2} - \cancel{a_1} + \cancel{a_3} - \cancel{a_2} + \cdots + \cancel{a_{n-1}} - \cancel{a_{n-2}} + a_n - \cancel{a_{n-1}}$$

$$\Rightarrow \boxed{a_n - a_0}$$

Q Use the identity $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$ and "telescoping" from previous exercise to compute

$$\sum_{k=1}^{n} \frac{1}{k(k+1)}$$

Sol:

$$\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+1}\right)$$

$$\Rightarrow -\sum_{k=1}^{n}\left(\frac{1}{k+1} - \frac{1}{k}\right)$$

Let $a_k = \frac{1}{k+1}$

$$\Rightarrow -\sum_{k=1}^{n}\left(a_k - a_{k-1}\right)$$

Since $\sum_{i=1}(a_i - a_{i-1}) = a_n - a_0$

$$\Rightarrow -(a_n - a_0)$$

$$\Rightarrow a_0 - a_n$$

$$\Rightarrow \boxed{1 - \frac{1}{n+1}}$$

Q Compute each of these double sums

a) $\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j)$

b) $\sum_{i=0}^{2}\sum_{j=0}^{3}(2i+3j)$

c) $\sum_{i=1}^{3}\sum_{j=0}^{2} i$

d) $\sum_{i=0}^{2}\sum_{j=1}^{3} ij$

Sol

a) $\sum_{i=1}^{2}(i+1)+\sum_{i=1}^{2}(i+2)+\sum_{i=1}^{2}(i+3)$

$\Rightarrow 1+1+1+2+1+3+2+1+2+2+2+3$

$\Rightarrow \underline{21}$

b) $(0+0)+(0+3)+(0+6)+(0+9)+$
$(2+0)+(2+3)+(2+6)+(2+9)+$
$(4+0)+(4+3)+(4+6)+(4+9)$

$\Rightarrow 75$

c) $(1+1+1)+(2+2+2)+(3+3+3)$

$\Rightarrow \underline{18}$

d) $(0+0+0)+(1+2+3)+(2+4+6)$

$\Rightarrow \underline{18}$

Q What are the values of the following products?

a) $\prod_{i=0}^{10} i$ b) $\prod_{i=5}^{8} i$

c) $\prod_{i=1}^{100} (-1)^{i}$ d) $\prod_{i=1}^{10} 2$

Sol

a) 0 {0 times anything is 0}

b) $5 \cdot 6 \cdot 7 \cdot 8 = 1680$

c) $(-1)^{50} \times (1)^{50} = 1$

d) $2 \cdot 2 \cdot 2 \cdots 2 = 2^{10} = 1024$

Q Find a formula for $\sum_{k=1}^{m} \lfloor \sqrt{k} \rfloor$,

when m is a positive integer.

Sol: If we expand first few terms, we get

$1+1+1+2+2+2+2+2+3+3+3+3+3+3+3+$
$4+4+4+4+4+4+4+4+4+\ldots$

There are $2i+1$ copies of i, so we need to sum $i(2i+1)$ for an appropriate range of values for i

$$i(2i+1)$$

$$\Rightarrow 2i^2+i$$

$$\Rightarrow \sum_{i=1}^{n} 2i^2+i$$

$$\Rightarrow 2\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i$$

$$\Rightarrow \frac{2(n(n+1)(2n+1))}{6} + \frac{n(n+1)}{2}$$

$$\Rightarrow \frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}$$