Q Let $P(x)$ denote the statement "$x \leq 4$". What are truth values of

i, $P(0)$

ii, $P(4)$

iii, $P(6)$

Sol: i, True

ii, True

iii, False

Q Let $P(x)$ be "x spends more than 5 hours every weekday in class", where domain of x is all students. Express these quantifications in English.

(a) $\exists x\, P(x)$

(b) $\forall x\, P(x)$

(c) $\exists x\, \neg P(x)$

(d) $\forall x\, \neg P(x)$

Sol: (a) There exists a student that spends more than 5 hours every weekday in class.

(b) Every student spends ...

(c) There exists a student that does not spend ...

(d) No student spends ...

Q Let $P(x)$ be the statement "x speaks Russian" and $Q(x)$ be the statement "x knows C++". Express each of these sentences in terms of $P(x)$, $Q(x)$ quantifiers and logical connectives. The domain for quantifiers consists of all students at your school.

(a) There is a student at your school who can speak Russian and knows C++.

(b) There is a student who can speak Russian but does not know C++.

(c) Every student either can speak Russian or knows C++.

(d) No student knows Russian or C++.

Sol:

(a) $\exists x [P(x) \wedge Q(x)]$

(b) $\exists x [P(x) \wedge \neg Q(x)]$

(c) $\forall x (P(x) \vee Q(x)]$

(d) $\forall x [\neg P(x) \wedge \neg Q(x)]$.

The last one mean the student doesn't understand anything, that why it is and .

Q Let $P(x,y)$ be the statement "Student x has taken class y", where the domain for x consists of all students in your class and for y consists of all computer science courses at your school. Express each of these quantifications in English.

a) $\exists x \exists y P(x,y)$ b) $\exists x \forall y P(x,y)$

c) $\forall x \exists y P(x,y)$ d) $\exists y \forall x P(x,y)$

e) $\forall y \exists x P(x,y)$ f) $\forall x \forall y P(x,y)$

Sol:

a) There exists a student x who has taken course y.

b) There exists a student x who has taken every course y.

c) Every student x has taken atleast one course y.

d) There is a course y that every student has taken.

e) Every course has been taken by atleast one student.

f) Every student has taken every course.

Q Let $A = \{a, b, c\}$, $B = \{x, y\}$ and $C = \{0, 1\}$.

Find a) $A \times B \times C$

b) $C \times B \times A$

c) $C \times A \times B$

Sol

a) $\{(a, x, 0), (a, y, 0), (a, x, 1), (a, y, 1),$
$(b, x, 0), (b, y, 0), (b, x, 1), (b, y, 1),$
$(c, x, 0), (c, y, 0), (c, x, 1), (c, y, 1)\}$

Q Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$.

Find

a) $A \cup B$

b) $A \cap B$

c) $A - B$

d) $B - A$

Sol

a) $\{0, 1, 2, 3, 4, 5, 6\}$

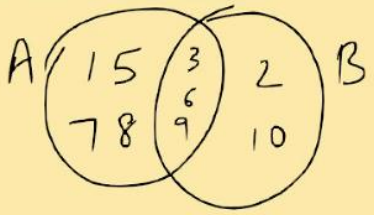
b) $\{3\}$

c) $\{1, 2, 4, 5\}$

d) $\{0, 6\}$

Q Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$.

Sol



$A = \{1, 3, 5, 6, 7, 8, 9\}$

$B = \{2, 3, 6, 9, 10\}$