	King Saud University	College of Computer and Information Sciences Department of Computer Science
	Data Structures CSC 212	**Final Exam - Fall 2017**
	Date: 30/12/2017	Duration: 3 hours

Guidelines

- No calculators or any other electronic devices are allowed in this exam.
- Use a pencil in choice questions.

Student ID:	Name:
Section:	Instructor:

1	2.1	2.2	3.1	3.2	4	5	6	7	8	Total

Question 1 ..... 16 points

(a) Choose the correct frequency for every line as well as the total O of the following code:

```
1  sum = 1;
2  for (i = 1; i <= n; i++) {
3    sum+= i;
4    for (j = i; j >= 2; j--)
5      sum--;}
```

1. Line 1: (A) 1 (B) 2 (C) 3 (D) n (E) $2n$
2. Line 2: (A) n (B) $n+1$ (C) $n-1$ (D) $n+2$ (E) $n-2$
3. Line 3: (A) n (B) $n+1$ (C) $n-1$ (D) $n+2$ (E) $n-2$
4. Line 4: (A) n^2 (B) $n(n-1)/2$ (C) $(2n+1)/2$ (D) $(2n-1)/2$ (E) $n(n+1)/2$
5. Line 5: (A) n^2 (B) $n(n-1)/2$ (C) $(2n+1)/2$ (D) $(2n-1)/2$ (E) $n(n+1)/2$
6. Total O: (A) 1 (B) n (C) n^2 (D) $n\log(n)$ (E) n^3

(b) Choose the correct frequency for every line as well as the total O of the following code:

```
1  count = 0;
2  for (i = 1; i < n+1; i++)
3    count ++;
4  for (j = 0; j <= count; j++)
5    k = j+1;
```

1. Line 1: (A) 0 (B) 1 (C) 2 (D) n (E) n^2
2. Line 2: (A) n (B) $n+1$ (C) $n-1$ (D) $n+2$ (E) $n-2$
3. Line 3: (A) n (B) $n+1$ (C) $n-1$ (D) $n+2$ (E) $n-2$
4. Line 4: (A) $count+2$ (B) $n+1$ (C) $n-1$ (D) $n+2$ (E) $n(n+1)/2$
5. Line 5: (A) $count+1$ (B) $n+1$ (C) $n-1$ (D) $n+2$ (E) $n(n-1)/2$
6. Total O: (A) 1 (B) n (C) n^2 (D) $n\log(n)$ (E) n^3

(c) Choose the correct answer:

1. $n^2 + n \log n^4$ is : (A) $O(n)$ (B) $O(n^2)$ (C) $O(n \log(n))$ (D) $O(n^4)$ (E) None
2. $n^2 + 1000n$ is : (A) $O(n)$ (B) $O(n^2)$ (C) $O(n \log(n))$ (D) $O(nn^2)$ (E) None
3. $n^4 \log n + n!$ is : (A) $O(n!)$ (B) $O(n^4)$ (C) $O(n^5)$ (D) $O(\log(n))$ (E) None
4. Algorithm A is $O(n)$, and Algorithm B is $O(2n)$. Given the same input:
   (A) A always finishes before B. (B) B always finishes before A. (C) A and B finish at the same time. (D) B requires double the time taken by A. (E) None

Question 2 ..... 10 points

(a) Given a queue of time intervals represented as pairs of integers, write the method **public static** `Queue<Pair<Integer, Integer>> getIntervals(Queue<Pair<Integer, Integer>> q, int start, int end)`, which returns all intervals of `q` intersecting the interval [start, end]. The input `q` must not change. Assume that all intervals in `q` are valid (that is `first <= second`), non-overlapping and ordered in chronological order. The class `Pair` is given below.

```
public class Pair<U, V> {
  public U first;
  public V second;
  Pair(U first, V second) {
    this.first = first;
    this.second = second;
  }
}
```

```
Queue<Pair<Integer,Integer>> qq = new Queue<Pair<Integer,Integer>>();
if(q.length() == 0)
return qq;

for(int i = 0; i < q.length(); i++){
    int pair = q.serve();
    if(pair.first >= start && pair.second <= end)
        qq.enqueue(pair);
    q.enqueue(pair);
}

return qq;
```

(b) Consider a stack of decreasing time intervals, that is, starting from the top, each interval contains the next. Write the method **public static** `Pair<Integer, Integer> smallest(Stack<Pair<Integer, Integer>> st, int t)`, which returns the smallest interval containing `t` if it exists, **null** otherwise. Assume that all intervals in `st` are valid (that is `first <= second`).

Example 0.1. *If* `st :` $\{[0,8],[1,6],[1,5],[2,4]\}$*, then* `smallest(st, 1)` *returns* $[1,5]$*,* `smallest(st, 3)` *returns* $[2,4]$*,* `smallest(st, 9)` *returns* **null**.

Complete the code below by choosing the correct answer:

```
1  public static Pair<Integer, Integer>smallest(Stack<Pair<Integer, Integer>> st, int t){
2    ...
3    Pair<Integer, Integr> itm = null;
4    while (!st.empty()) {
5      Pair<Integer, Integr> it = st.pop();
6      ...
7      if (...)
8        itm = it;
9      else
10       ...
11   }
12   while (...) {
13     ...
14   }
15   return itm; }
```

1. Line 2:
   - (A) `Queue<Pair<Integer, Integer>> r = new LinkedQueue<Pair<Integer, Integer>>();`
   - (B) `Stack<Integer> r = new LinkedStack<Integer>();`
   - (C) `List<Pair<Integer, Integer>> r = new LinkedList<Pair<Integer, Integer>>();`
   - (D) `Stack<Pair<Integer, Integer>> r = new LinkedStack<Pair<Integer, Integer>>();`
   - (E) None
2. Line 6:
   - (A) `r.push(it.first);`
   - (B) `r.insert(it);`
   - (C) `r.enqueue(it);`
   - (D) `r.push(it);`
   - (E) None
3. Line 7:
   - (A) **if** `(it.first < t && t <= it.second)`
   - (B) **if** `(it.first <= t && t <= it.second)`
   - (C) **if** `(it.first < t || it.second > t)`
   - (D) **if** `(it.first <= t && it.second <= t)`
   - (E) None
4. Line 10:
   - (A) `r.serve();`
   - (B) **break**;
   - (C) `r.pop();`
   - (D) `r.findNext();`
   - (E) None
5. Line 12:
   - (A) **while** `(r.empty()){`
   - (B) **while** `(!r.empty()){`
   - (C) **while** `(r.pop()!= null){`
   - (D) **while** `(r.length()!= 0){`
   - (E) None
6. Line 13:
   - (A) `st.push(r.pop());`
   - (B) `st.push(r.serve());`
   - (C) `st.push(r.retrieve()); r.findNext();`
   - (D) `st.push(r.push());`
   - (E) None

Question 3 ..... 10 points

(a) The method **private** `BTNode<T> mirrorCopy(BTNode<T> t)` creates **recursively** a mirror copy of the subtree `t`. Choose the correct option to complete the code of this method:

```
1 private BTNode<T> mirrorCopy(BTNode<T> t) {
2   ...
3   ...
4   ...
5   ...
6   ...
7   ...
8 }
```

1. Line 2:
   - (A) **if** `(t.left == null || t.right == null)`
   - (B) **if** `(t.left == null && t.right == null)`
   - (C) **if** `(t == null)`
   - (D) **if** `(root != null)`
   - (E) None
2. Line 3:
   - (A) **return null**;
   - (B) **return** `root;`
   - (C) **return** `mirrorCopy(root);`
   - (D) **return** `mirrorCopy(t);`
   - (E) None
3. Line 4:
   - (A) `BTNode<T> p = new BTNode<T>(t.data);`
   - (B) `BTNode<T> p = new BTNode<T>(root);`
   - (C) `BTNode<T> p = new BTNode<T>(t);`
   - (D) `BTNode<T> p = new BTNode<T>(root.data);`
   - (E) None
4. Line 5:
   - (A) `p.right = mirrorCopy(t.left);`
   - (B) `t.left = mirrorCopy(t.left);`
   - (C) `p.right = mirrorCopy(t.right);`
   - (D) `t.left = mirrorCopy(t.right);`
   - (E) None
5. Line 6:
   - (A) `t.right = mirrorCopy(t.left);`
   - (B) `p.left = mirrorCopy(t.left);`
   - (C) `p.left = mirrorCopy(t.right);`
   - (D) `t.right = mirrorCopy(t.right);`
   - (E) None
6. Line 7:
   - (A) **return** `p;`
   - (B) **return** `mirrorCopy(t);`
   - (C) `mirrorCopy(t.left); mirrorCopy(t.right);`
   - (D) **return** `t;`
   - (E) None

(b) Consider the function `f` below, member of `DoubleLinkedList`:

```
public void f(int n) {
  Node<T> p = head; Node<T> q =  null;
  for (int i = 0; i < n; i++) {
    q = p;
    p = p.next;
  }
  if (p != null) {
    p.previous = null;
    while (p.next != null)
      p = p.next;
    p.next = head;
    head = q.next;
    q.next = null;
  }
}
```

Choose the correct result in each of the following cases:

1. The list `l`: A, B, C, D, E, after calling `l.f(2)`, `l` becomes:
   Ⓐ A, B Ⓑ D, E, A, B, C Ⓒ C, D, E, A, B Ⓓ A, D, E, B, C Ⓔ None
2. The list `l`: A, B, C, D, E, after calling `l.f(0)`, `l` becomes:
   Ⓐ *empty* Ⓑ A, B, C, D, E Ⓒ B, C, D, E, A Ⓓ B, C, D, E Ⓔ None
3. The list `l`: A, B, C, D, E, after calling `l.f(5)`, `l` becomes:
   Ⓐ *empty* Ⓑ A, B, C, D, E Ⓒ E, D, C, B, A Ⓓ A, D, E, B, C Ⓔ None
4. The list `l`: A, B, C, D, E, after calling `l.f(1)`, `l` becomes:
   Ⓐ A Ⓑ E, A, B, C, D Ⓒ C, D, E, A, B Ⓓ B, C, D, E, A Ⓔ None

Question 4 ..... 14 points

(a) Consider the following heap represented as an array: 3, 7, 9, 13, 8, 11. Choose the correct answer for every operation (all operations are done on the above heap).

1. Heap after inserting 5:
   Ⓐ 3,7,9,13,8,11,5 Ⓑ 3,5,7,13,8,11,9 Ⓒ 3,7,9,13,8,5,11 Ⓓ 5,7,3,13,8,11,9 Ⓔ None
2. Heap after inserting 10:
   Ⓐ 3,7,10,13,8,11,9 Ⓑ 3,7,9,13,8,10,11 Ⓒ 3,7,9,13,8,11,10 Ⓓ 3,7,9,10,8,11,13 Ⓔ None
3. Heap after inserting 2:
   Ⓐ 3,7,9,13,8,11,2 Ⓑ 3,7,2,13,8,11,9 Ⓒ 2,7,3,13,8,9,11 Ⓓ 2,7,3,13,8,11,9 Ⓔ None
4. Heap after deleting one key:
   Ⓐ 7,13,9,11,8 Ⓑ 7,8,9,13,11 Ⓒ 9,7,11,13,8 Ⓓ 7,9,8,11,13 Ⓔ None
5. Heap after deleting two keys:
   Ⓐ 7,13,9,11 Ⓑ 8,11,9,13 Ⓒ 7,8,9,13 Ⓓ 13,9,8,11 Ⓔ None

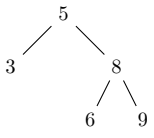
(b) What is the result of a bottom-up min-heap construction of the following array: 2,4,6,3,5,1?
Ⓐ 1,2,3,5,4,6 Ⓑ 2,1,3,4,5,6 Ⓒ 1,3,2,4,5,6 Ⓓ 1,3,2,5,6,4 Ⓔ None.

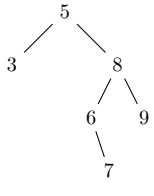
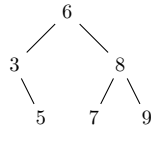
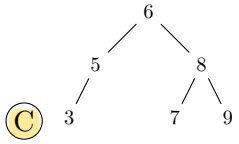
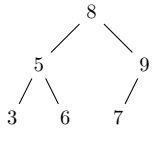
(c) Choose the correct answer:

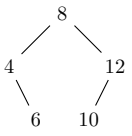
1. What is the height of a heap of size k?
   Ⓐ $\log\log k$ Ⓑ $k/2$ Ⓒ $k \log k$ Ⓓ $\log k$ Ⓔ None.
2. Bottom-up heap construction is:
   Ⓐ $O(n)$ Ⓑ $O(\log n)$ Ⓒ $O(n \log n)$ Ⓓ $O(n^2)$ Ⓔ None.
3. The enqueue operation in a heap priority queue is:
   Ⓐ $O(1)$ Ⓑ $O(\log n)$ Ⓒ $O(n)$ Ⓓ $O(n \log n)$ Ⓔ None.

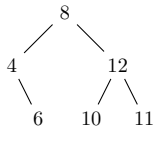
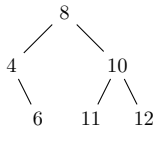
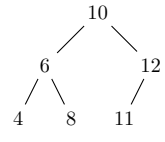
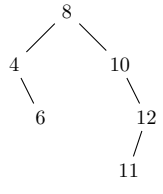
Question 5 . . . . . 14 points

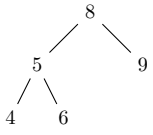
Choose the correct result in each of the following cases:

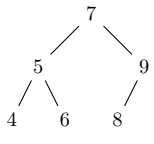
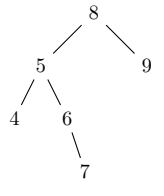
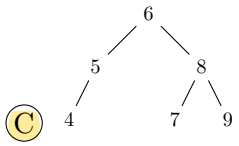
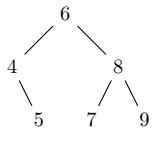
1. After inserting the key 7 in the AVL  , the tree becomes:

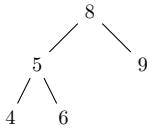
(A) 
(B) 
(C) 
(D) 
(E) None

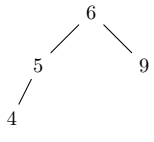
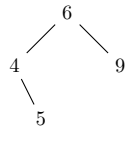
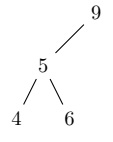
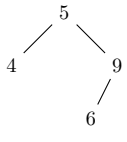
2. After inserting the key 11 in the AVL  , the tree becomes:

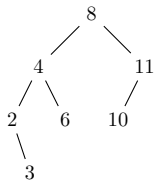
(A) 
(B) 
(C) 
(D) 
(E) None

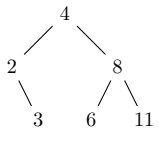
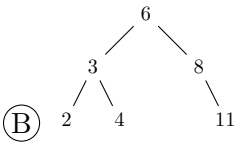
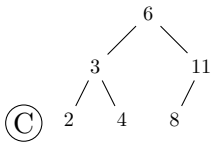
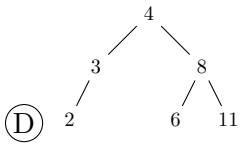
3. After inserting the key 7 in the AVL  , the tree becomes:

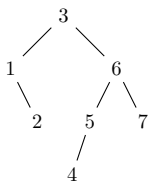
(A) 
(B) 
(C) 
(D) 
(E) None

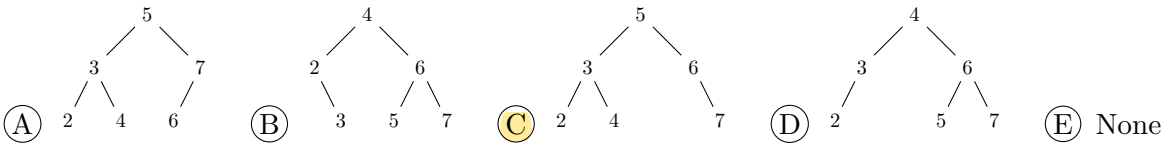
4. After deleting the key 8 from the AVL  , the tree becomes:

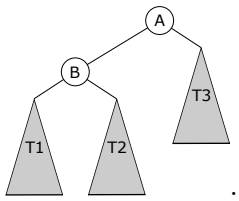
(A) 
(B) 
(C) 
(D) 
(E) None

5. After deleting the key 10 from the AVL  , the tree becomes:

(A) 
(B) 
(C) 
(D) 
(E) None

6. After deleting the key 1 from the AVL  , the tree becomes:



7. Consider the following tree 

. If the balance of A is -2 and that of B is 0, then after performing a single right rotation at A, then:

   (a) The balance of A becomes: -1

   (b) The balance of B becomes: 1

Question 6 ..... 14 points

Use the hash function $H(key) = key\%11$ to store the sequence of keys $16, 14, 27, 5, 21, 43, 10, 38, 19, 18, 20$ in the hash table. Use the following collision resolution strategies:

1. Linear rehashing (c=1). Fill in the following table:

Key	16	14	27	5	21	43	10	38	19	18	20
Position	5	3	6	7	10	11	0	8	9	1	2
Number of probes	1	1	2	3	1	2	3	4	2	7	6

2. External chaining. Fill in the following table:

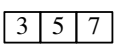
Key	16	14	27	5	21	43	10	38	19	18	20
List position	5	3	5	5	10	10	10	5	8	7	9

3. Coalesced chaining with cellar size 2 (do not change the hash function). Fill in the following table (put -1 if there is no next element):

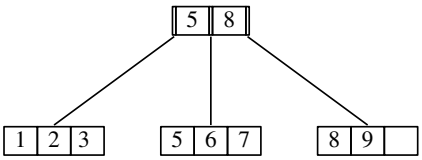
Key	16	14	27	5	21	43	10	38	19	18	20
Position	5	3	12	11	10	9	8	7	6	4	2
Next	12	-1	11	7	9	8	6	4	2	-1	-1

Question 7 ..... 14 points

Choose the correct result in each of the following cases:

1. After inserting the key 6 in the B+ tree 

, the **root** of tree becomes:

2. After inserting the key 4 in the B+ tree 

, the **root** of the tree becomes:

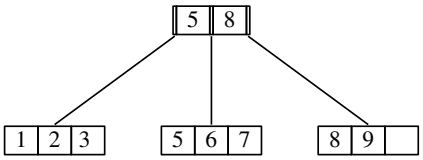
(A) 

(B) 

(C) 

(D) 

(E) None

3. After inserting the key 10 in the B+ tree 

, the **root** of the tree becomes:

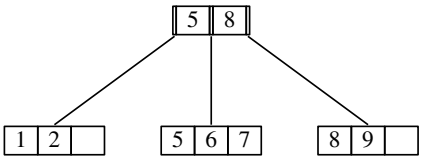
(A) 

(B) 

(C) 

(D) 

(E) None

4. After deleting the key 2 from the B+ tree 

, the **root** of the tree becomes:

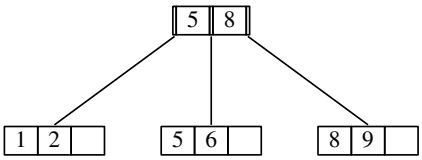
(A) 

(B) 

(C) 

(D) 

(E) None

5. After deleting the key 5 from the B+ tree 

, the **root** of the tree becomes:

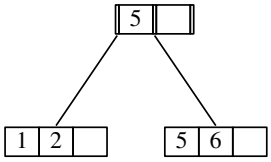
(A) 

(B) 

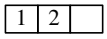
(C) 

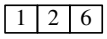
(D) 

(E) None

6. After deleting the key 5 from the B+ tree 

, the **root** of the tree becomes:

(A) 

(B) 

(C) 

(D) 

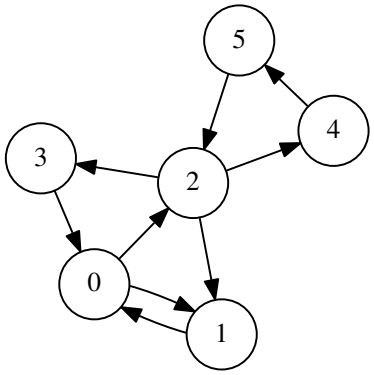
(E) None

7. A B+ tree of order 3 containing n keys has a height that is:

(A) $O(n)$ (B) $O(n/3)$ (C) $O(n^3)$ (D) $O(\log n)$ (E) None

Question 8 ..... 8 points

Consider the following graph.



1. Give the adjacency matrix of the graph.

	0	1	2	3	4	5
0	0	1	1	0	0	0
1	1	0	0	0	0	0
2	0	1	0	1	1	0
3	1	0	0	0	0	0
4	0	0	0	0	0	1
5	0	0	1	0	0	0

2. Give the adjacency list representation of the graph.

0-1-2
1-0
2-1-3-4
3-0
4-5
5-2

3. What is the number of edges in the subgraph containing the nodes $\{1, 2, 3\}$.

2

4. What is the maximum number of edges in a directed graph with n nodes (loops, edges from a node to itself, are not allowed)?

n(n-1)

ADT Queue Specification

- enqueue (Type e): **requires**: Queue Q is not full. **input**: Type e. **results**: Element e is added to the queue at its tail. **output**: none.
- serve (Type e): **requires**: Queue Q is not empty. **input**: none. **results**: the element at the head of Q is removed and its value assigned to e. **output**: Type e.
- length (int length): **requires**: none. **input**: none. **results**: The number of elements in the Queue Q is returned. **output**: length.
- full (boolean flag): **requires**: none. **input**: none. **results**: If Q is full then flag is set to true, otherwise flag is set to false. **output**: flag.

ADT Stack Specification

- push(Type e): **requires**: Stack S is not full. **input**: Type e. **results**: Element e is added to the stack as its most recently added elements. **output**: none.
- pop(Type e): **requires**: Stack S is not empty. **input**: **results**: the most recently arrived element in S is removed and its value assigned to e. **output**: Type e.
- empty(boolean flag): **requires**: none. **input**: none. **results**: If Stack S is empty then flag is true, otherwise false. **output**: flag.
- full(boolean flag): **requires**: none. **input**: none. **results**: If S is full then Full is true, otherwise Full is false. **output**: flag.