جامعة الملك سعود
King Saud University

Theory of Computation
CSC 339 – Spring 2021

Chapter-7: part1
Time Complexity

King Saud University

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Introduction

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- **What’s the difference between computability and complexity theories?**

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- **In this part, we will study time complexity.**

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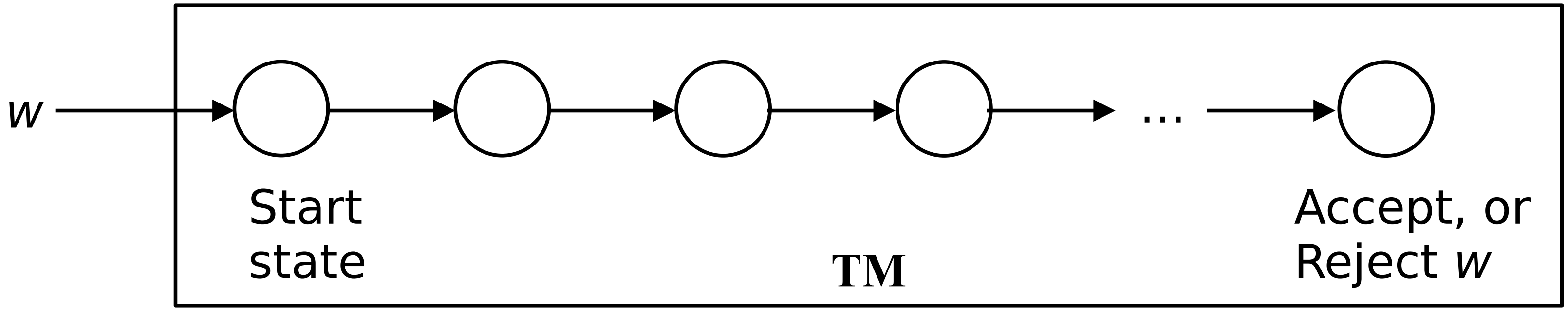

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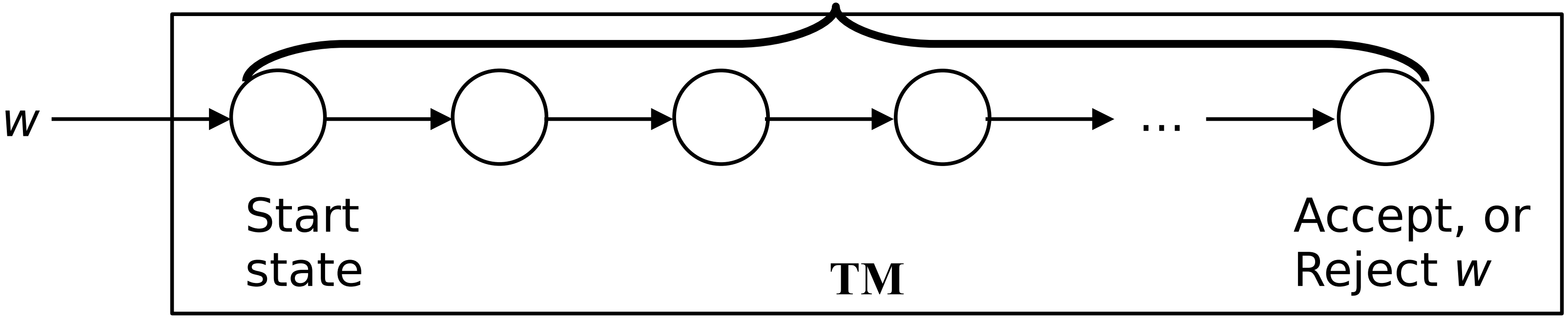
Running Time = number of transitions



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- **We compute the running time of an algorithm (or a TM) as a function of the length of the string representing the input.**

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- **Worst-case analysis**
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- **Average-case analysis**
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Complexity Theory: Time Complexity

Definition 7.1

Let M be a deterministic Turing machine that halts on all inputs. The ***running time*** or ***time complexity*** of M is the function f: $N \rightarrow N$, where $f(n)$ is the maximum number of steps that M uses on any input of length n. If $f(n)$ is the running time of M , we say that M runs in time $f(n)$ and that M is an $f(n)$ time Turing machine. Customarily, we use n to represent the length of the input.

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- ***Asymptotic notation* is also known as *big-O notation*.**

Definition 7.2

Let f and g be functions $f, g : N \rightarrow R+$. Say that $f(n) = O(g(n))$ if positive integers c and n_0 exist such that for every integer $n \geq n_0$,

$$f(n) \leq cg(n).$$

When $f(n) = O(g(n))$, we say that $g(n)$ is an <u>upper bound</u> for $f(n)$, or more precisely, that $g(n)$ is an asymptotic upper bound for $f(n)$, to emphasize that we are suppressing constant factors.

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Intuitively, $f(n) = O(g(n))$ means that f is less than or equal to g if we disregard differences up to a constant factor.

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> *i* Is $f(n) = O(n^4)$?

- ***Asymptotic notation* is also known as *big-O notation*.**

Complexity Theory: Asymptotic Analysis

- **Examples**
  - $\sqrt{n} = O(n)$.
  - $n = O(n \log \log n)$.
  - $n \log \log n = O(n \log n)$.
  - $n \log n = O(n^2)$.
  - $n^2 = O(n^3)$.