جامعة الملك سعود
King Saud University

Theory of Computation
CSC 339 – Spring 2021

Chapter-3: part1
Turing Machines

King Saud University

Department of Computer Science

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Outline

- Recap
- Introduction
- Turing machines

Recap

- **Finite automata (DFA/NFA) are good computational memory when memory is limited.**

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What if we could have unlimited memory

and access it in any way we wish?

Introduction

- **Turing machines**
  - **Turing machines are more accurate than automata in modeling modern computers.**

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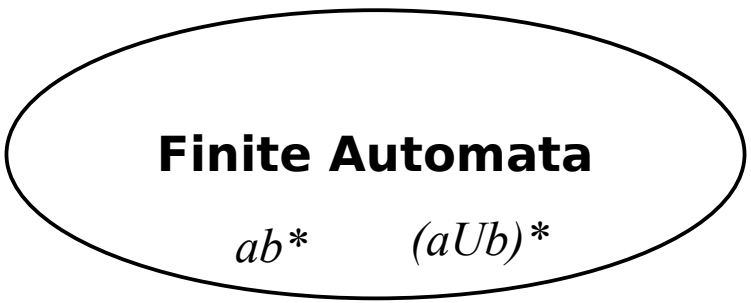

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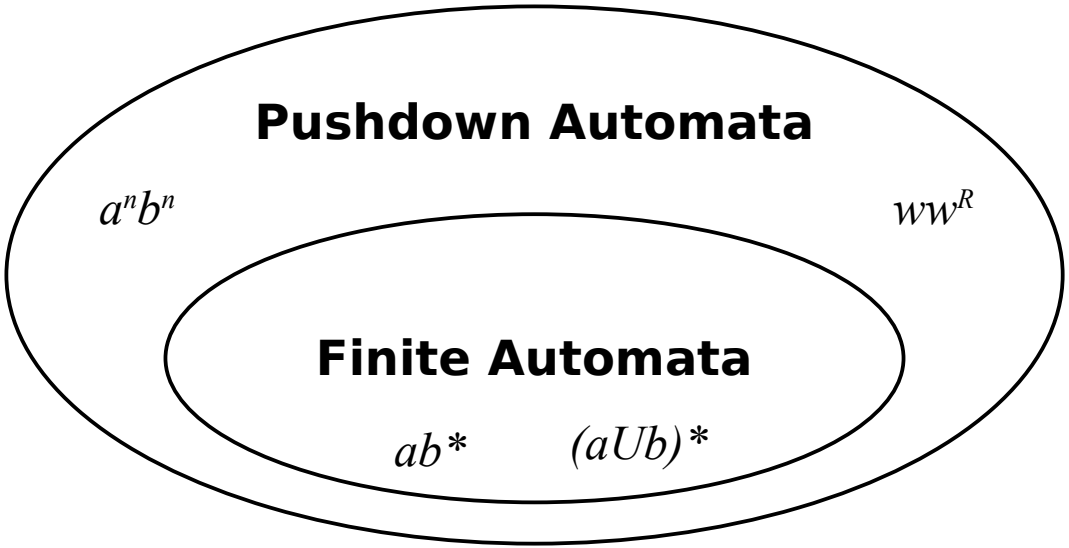

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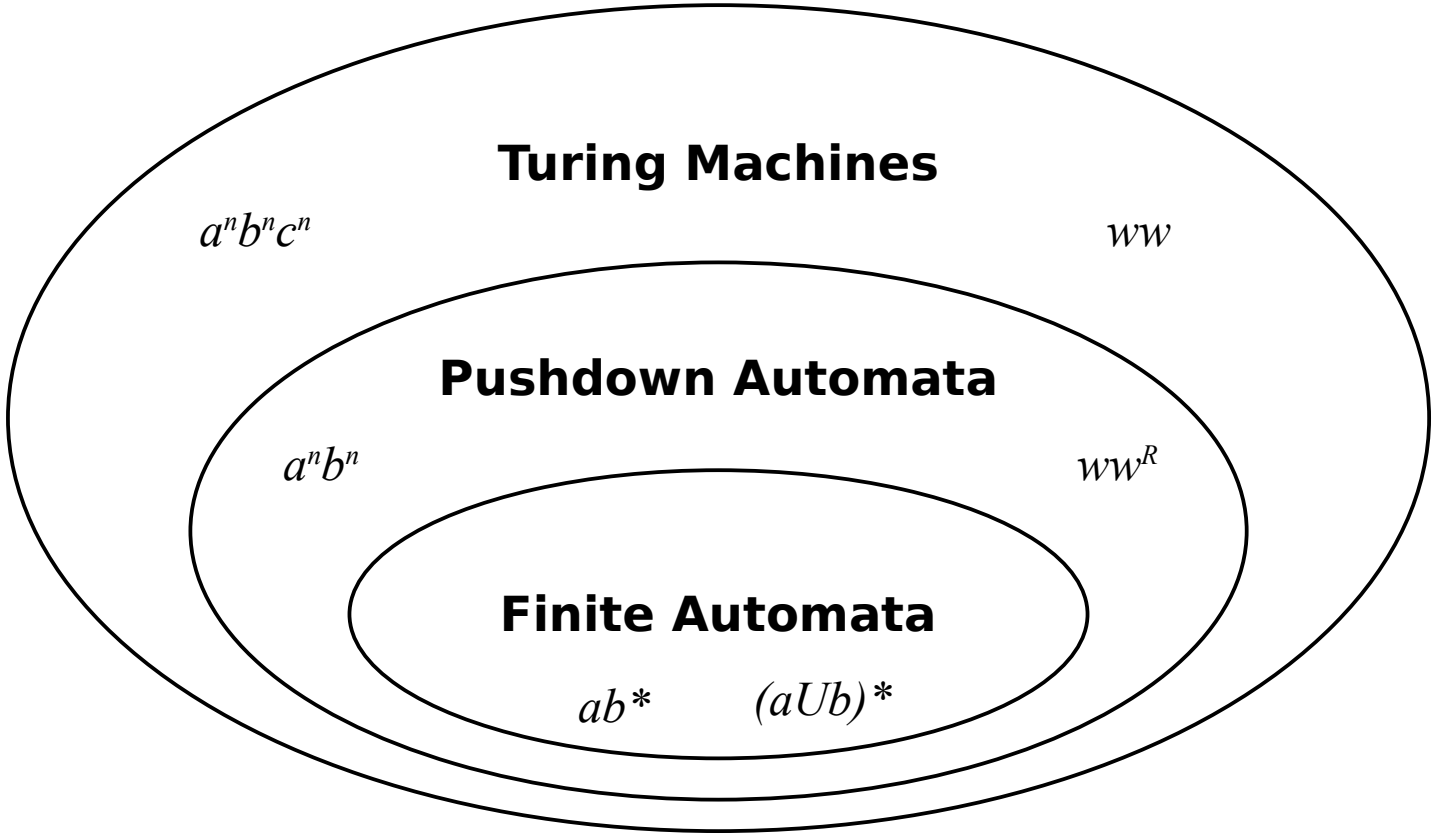

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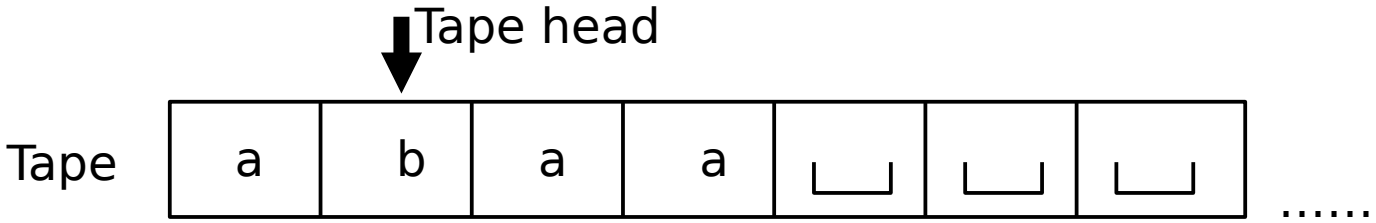

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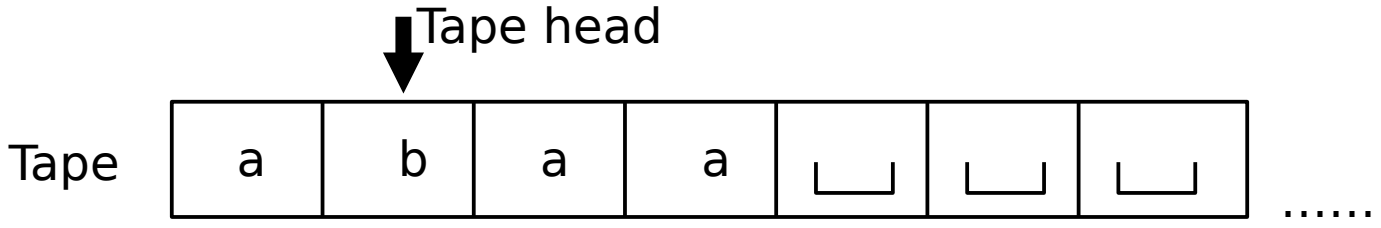
Unlimited & unrestricted memory



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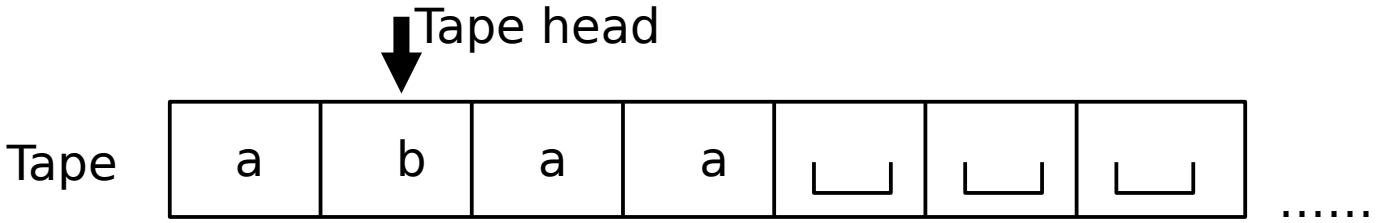
  - **Turing machines have a “tape” that can be used to read and store information.**
  - **The tape initially contains the input string, followed by the blank symbol**



Turing Machines vs. Finite Automata

- Turing machines can read from the tape and write to it.
- The tape “head” can move left and right.
- The tape is infinite.
- Separate accept and reject states.
- The accept and reject states take effect immediately.

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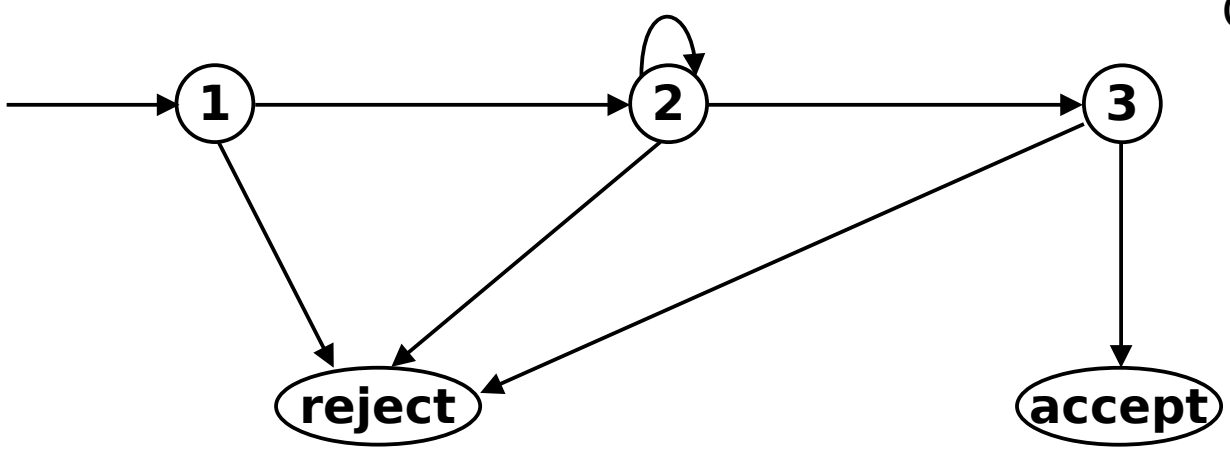

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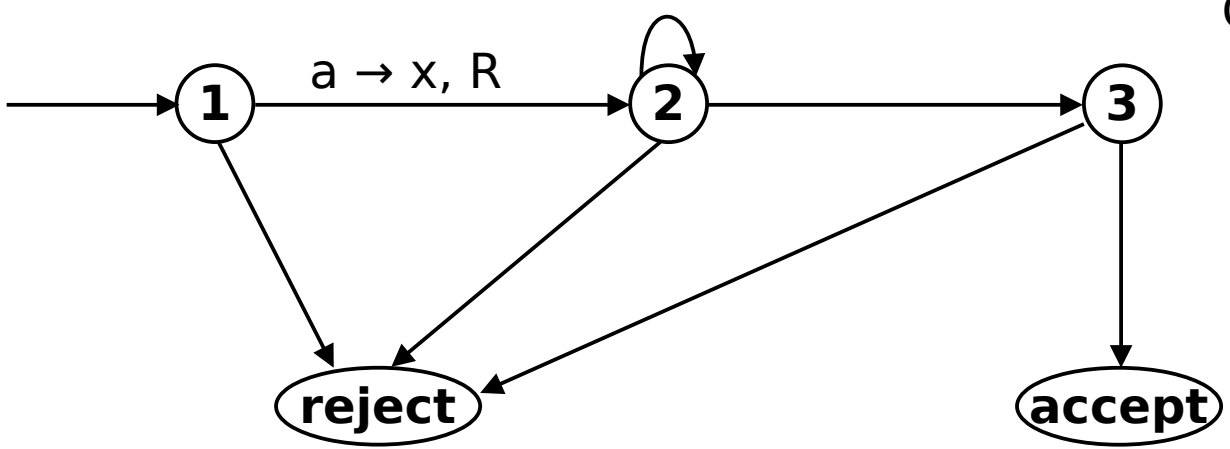

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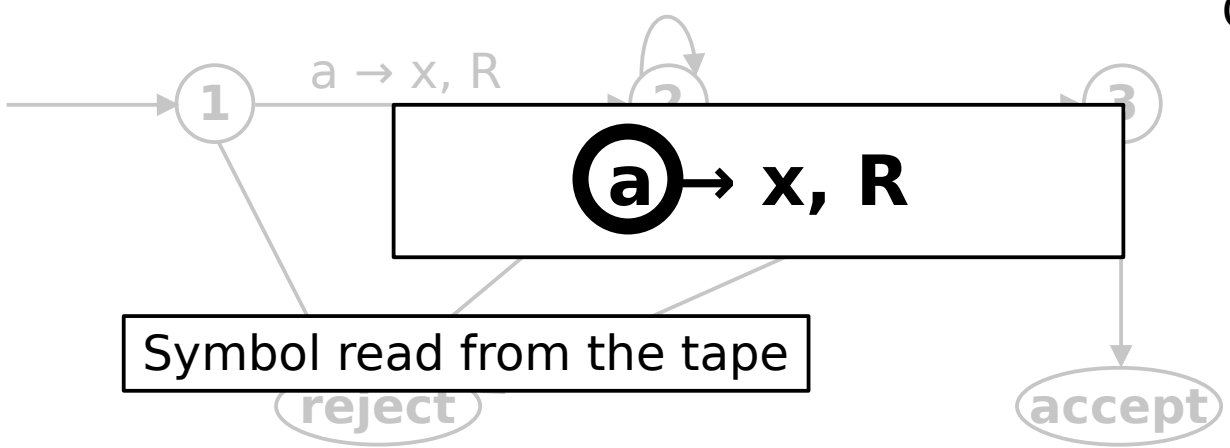

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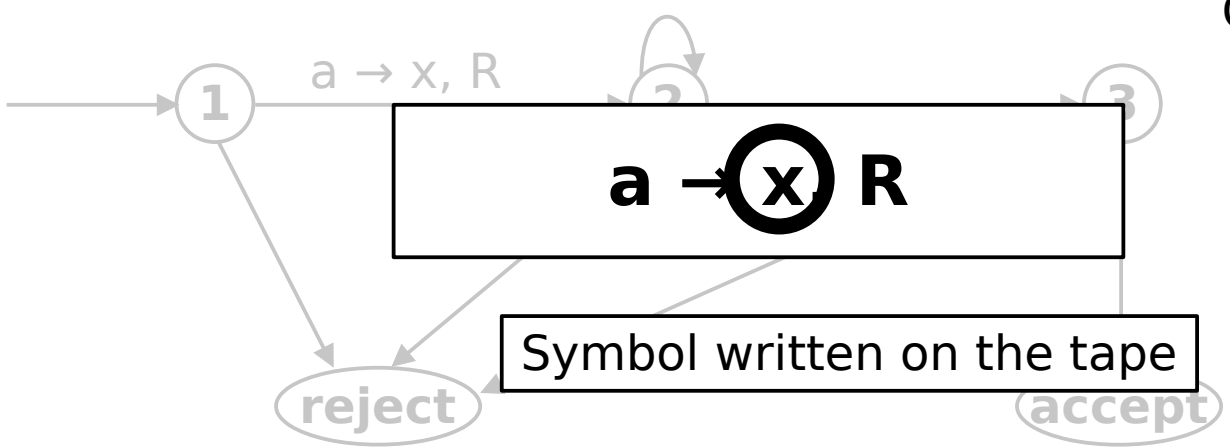

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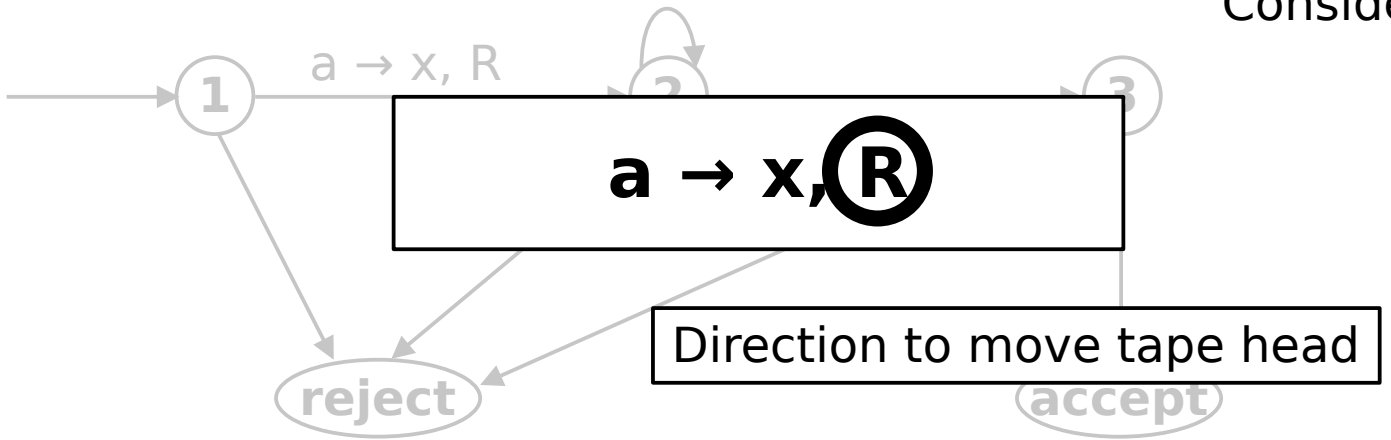

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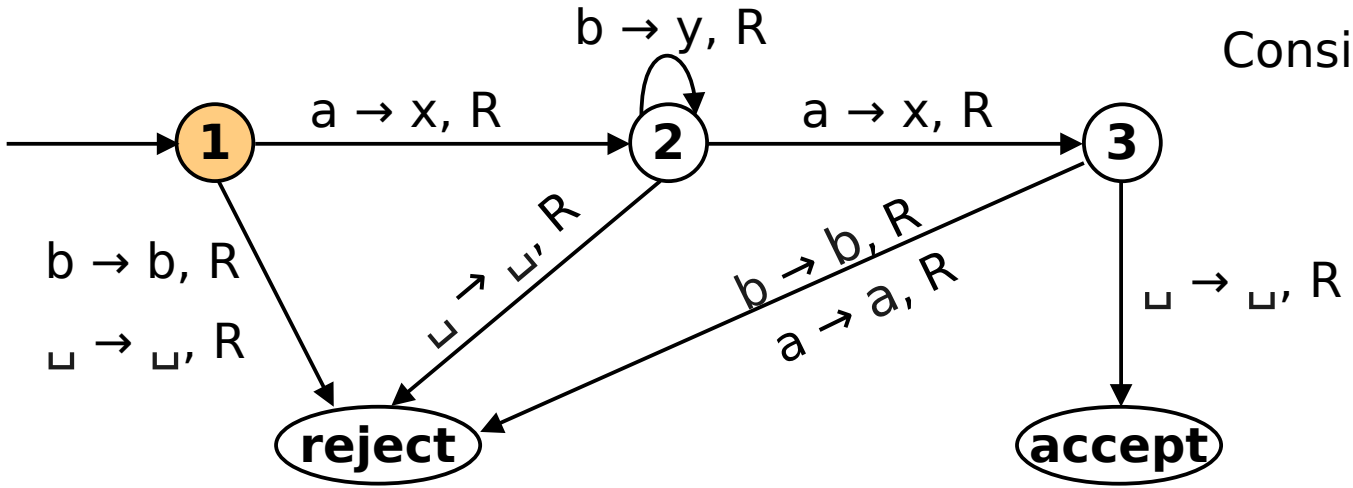

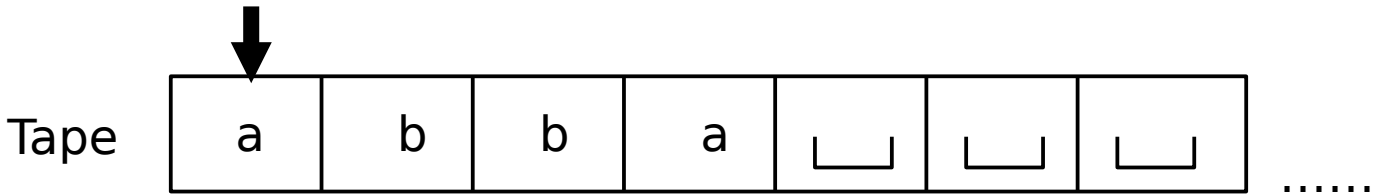

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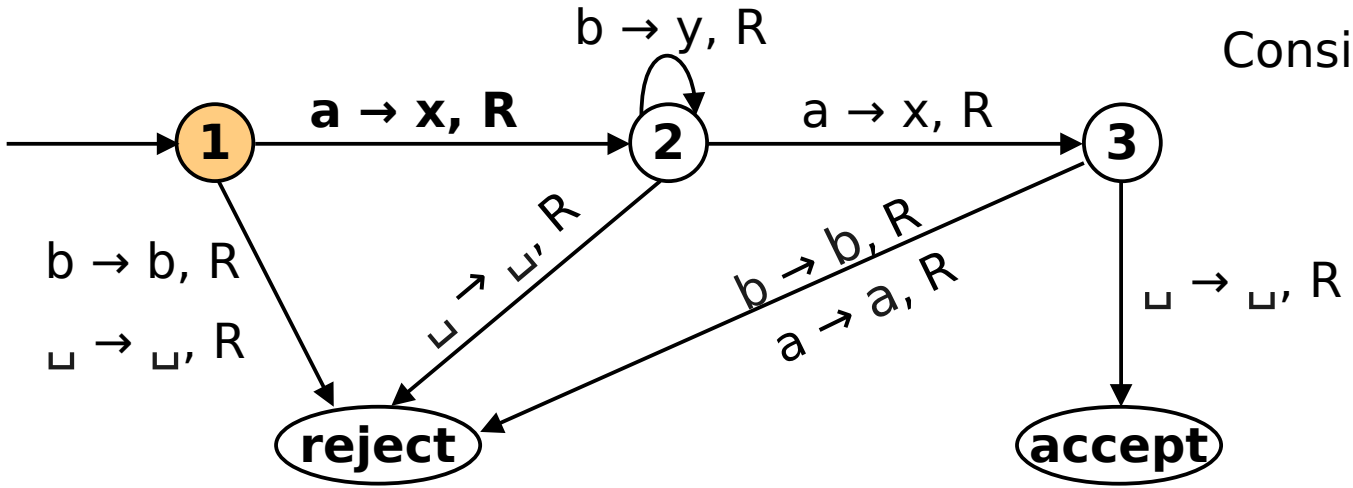

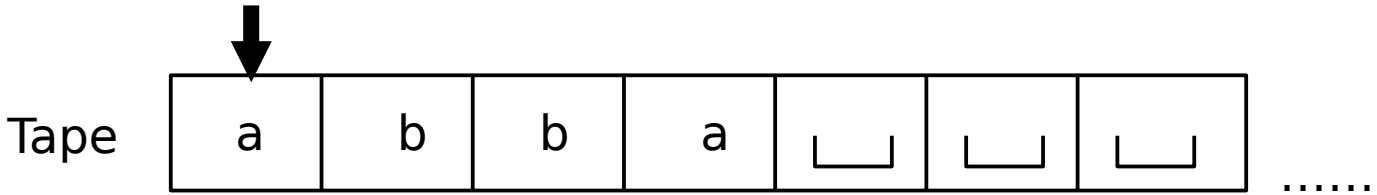

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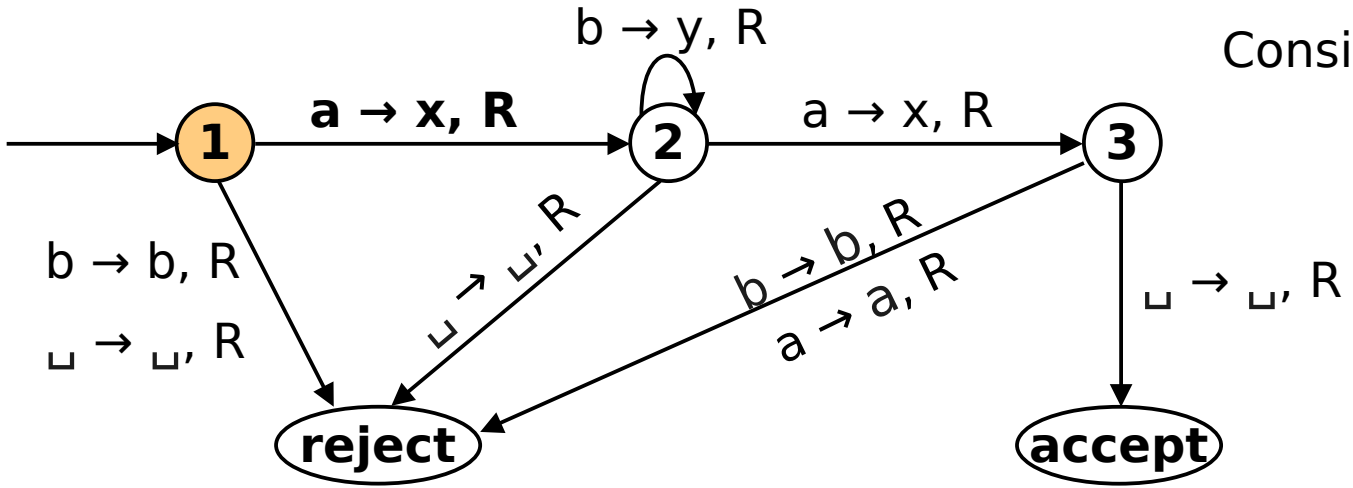

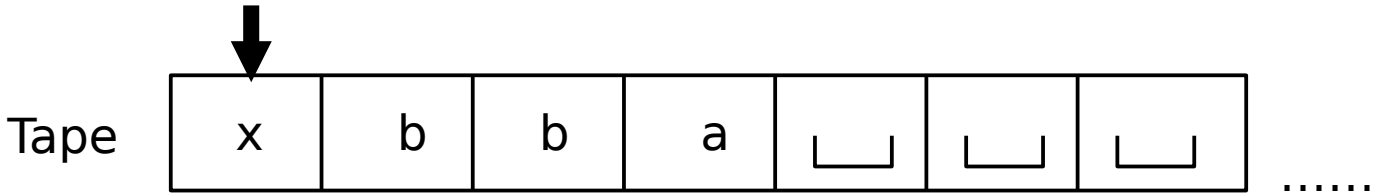

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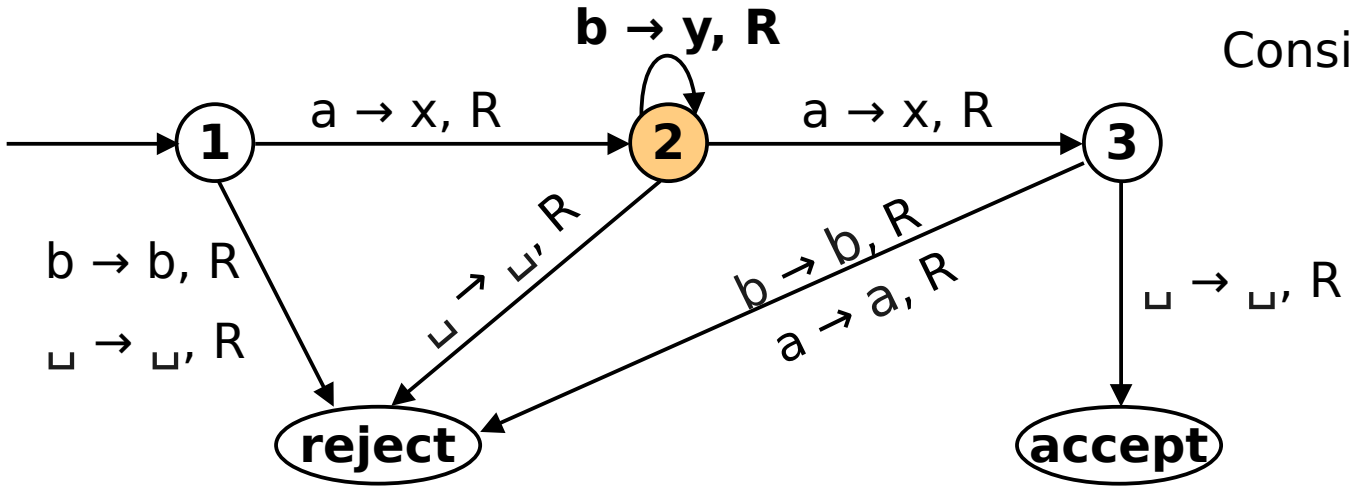

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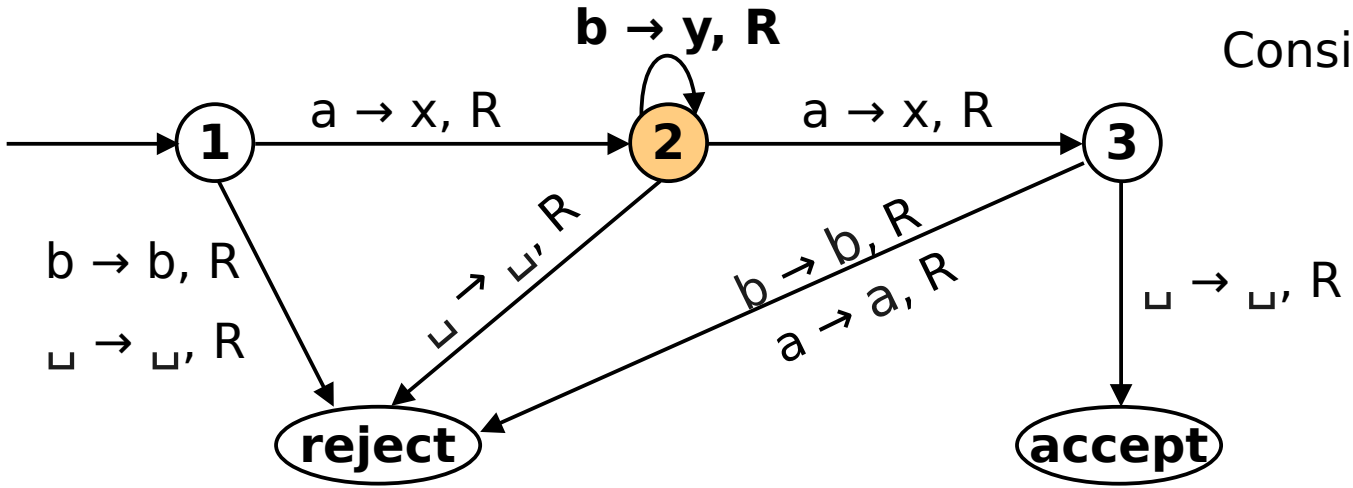

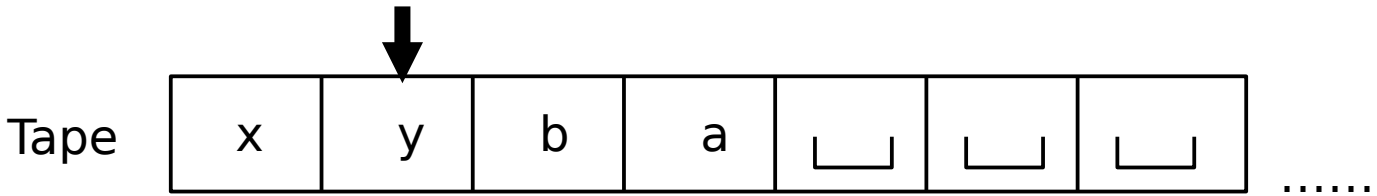

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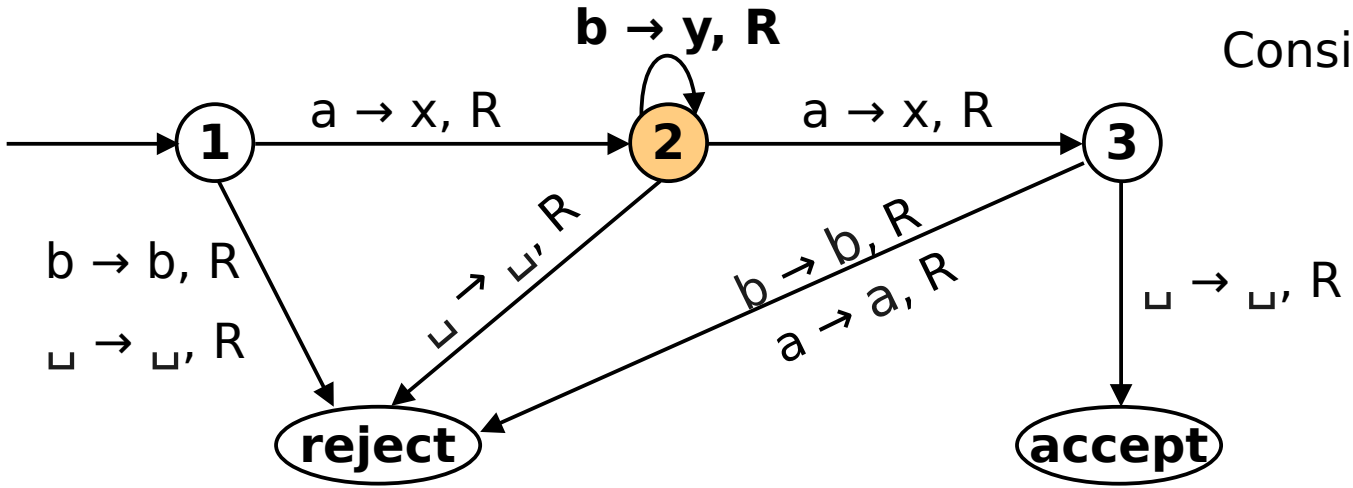

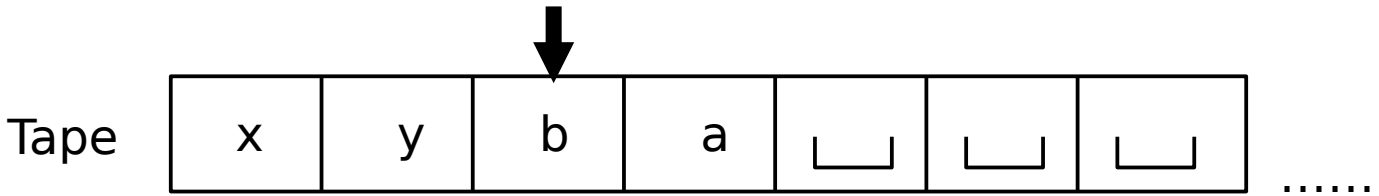

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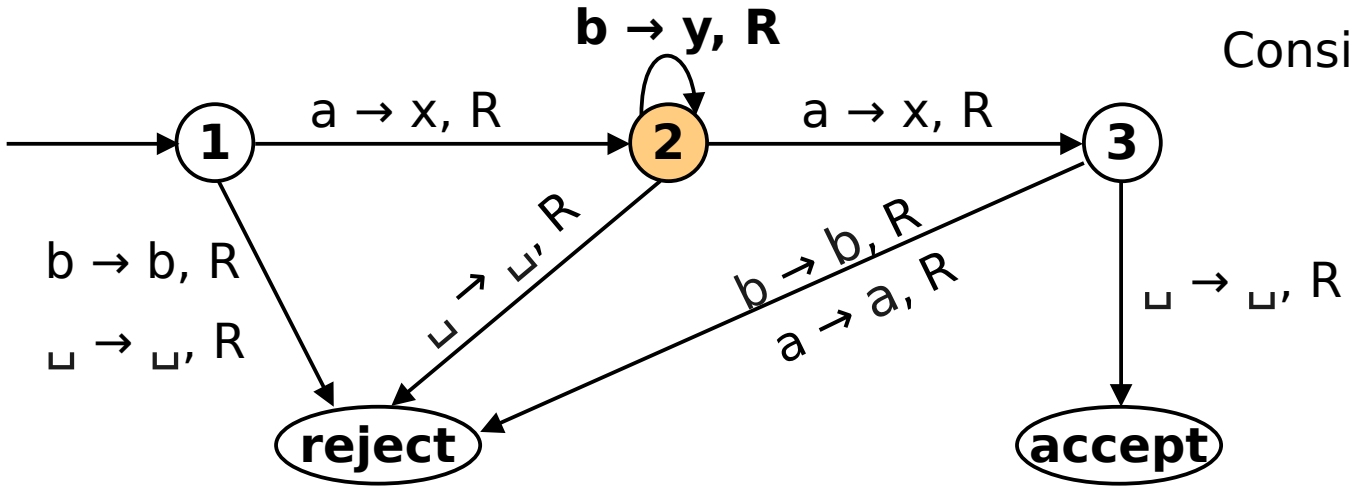

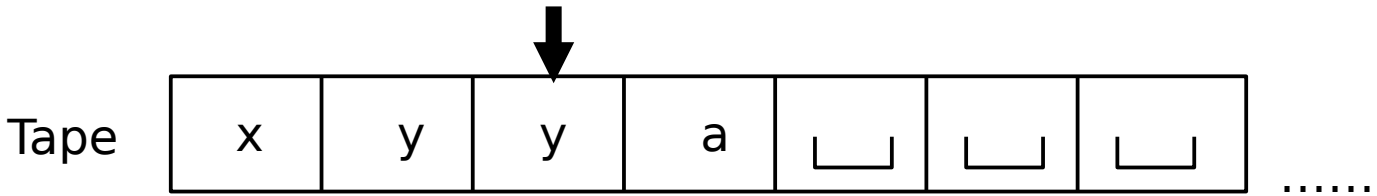

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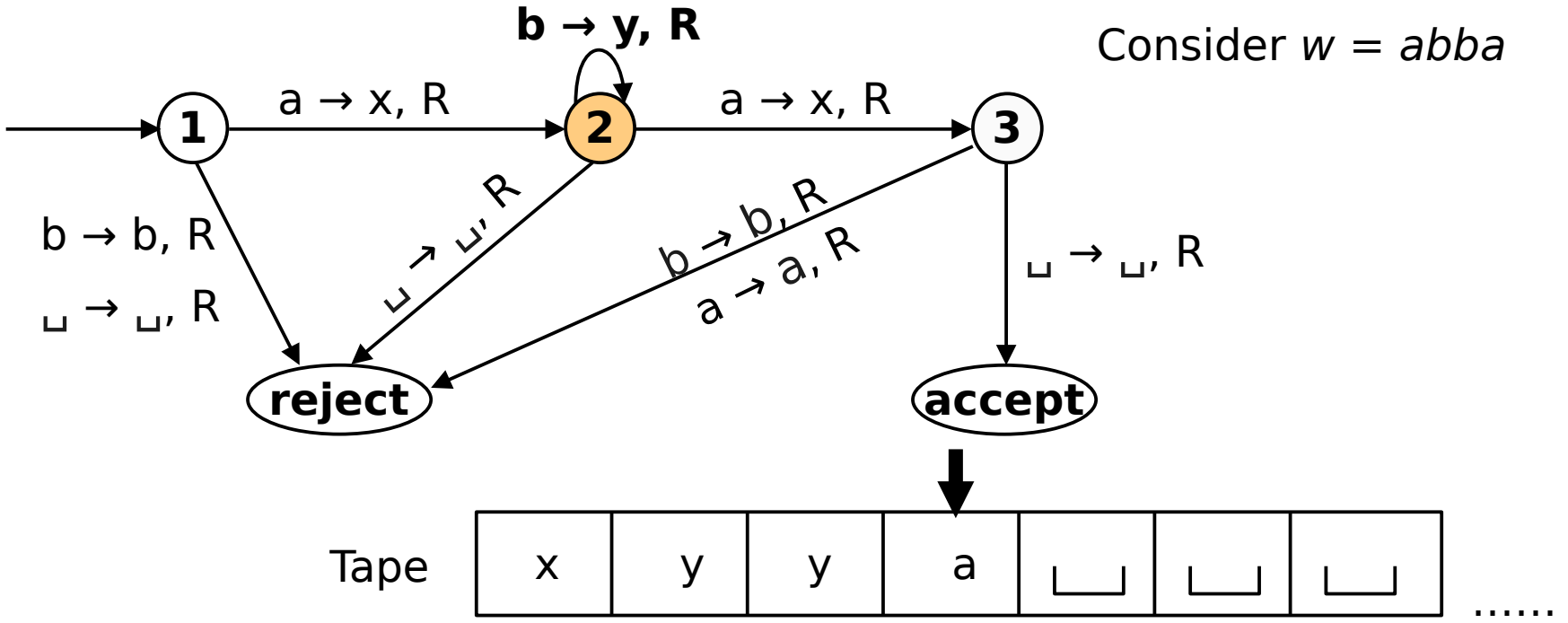

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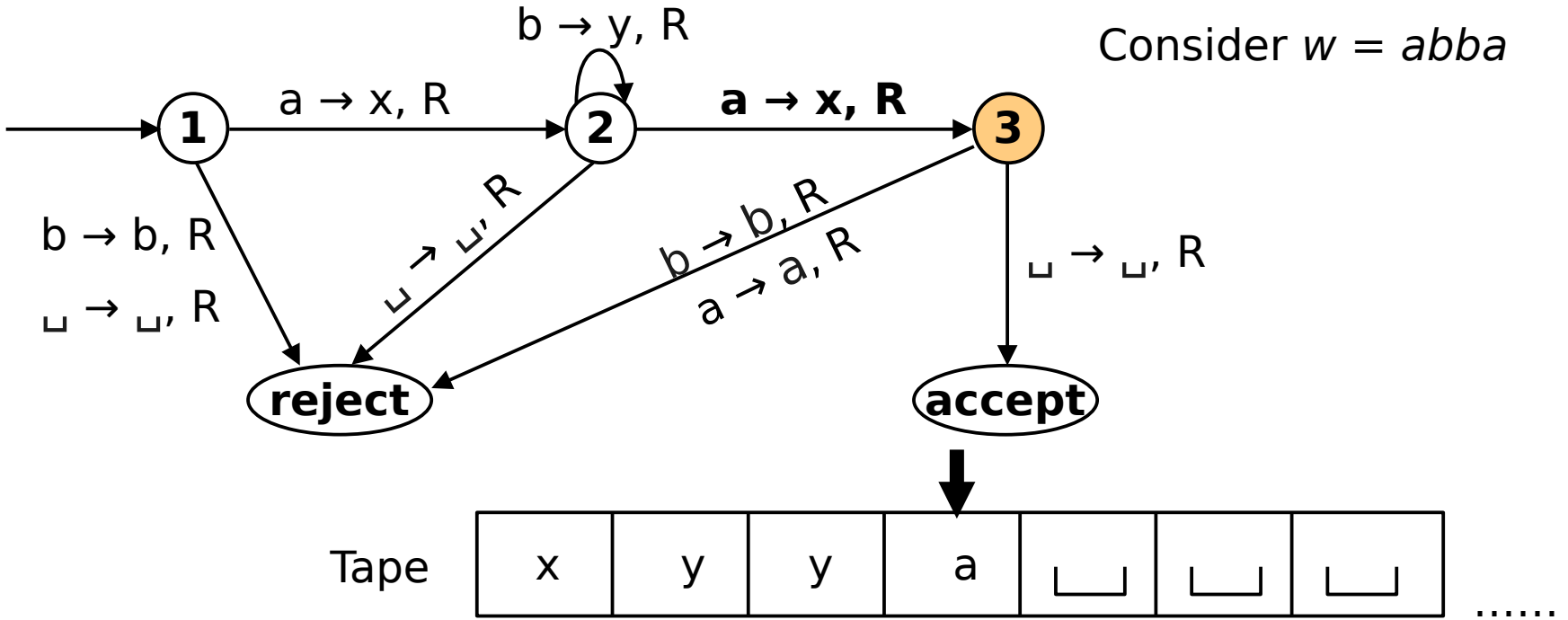

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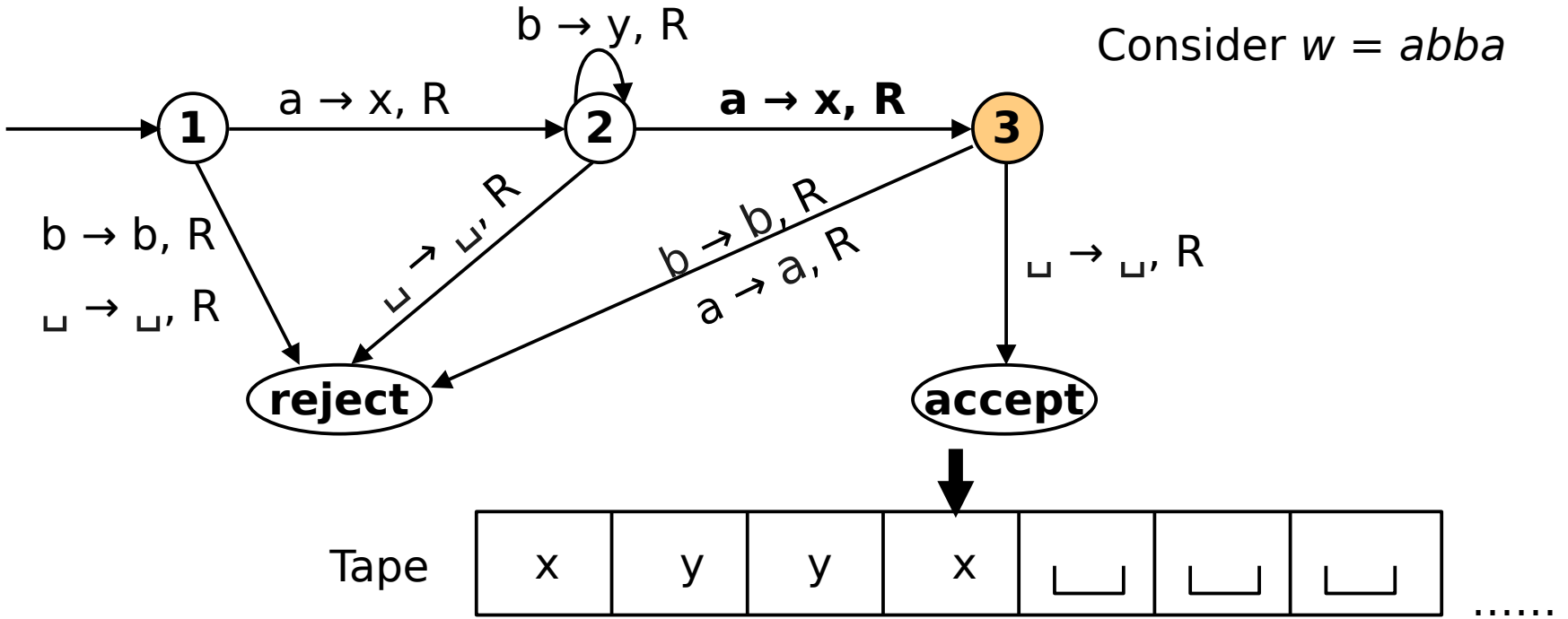

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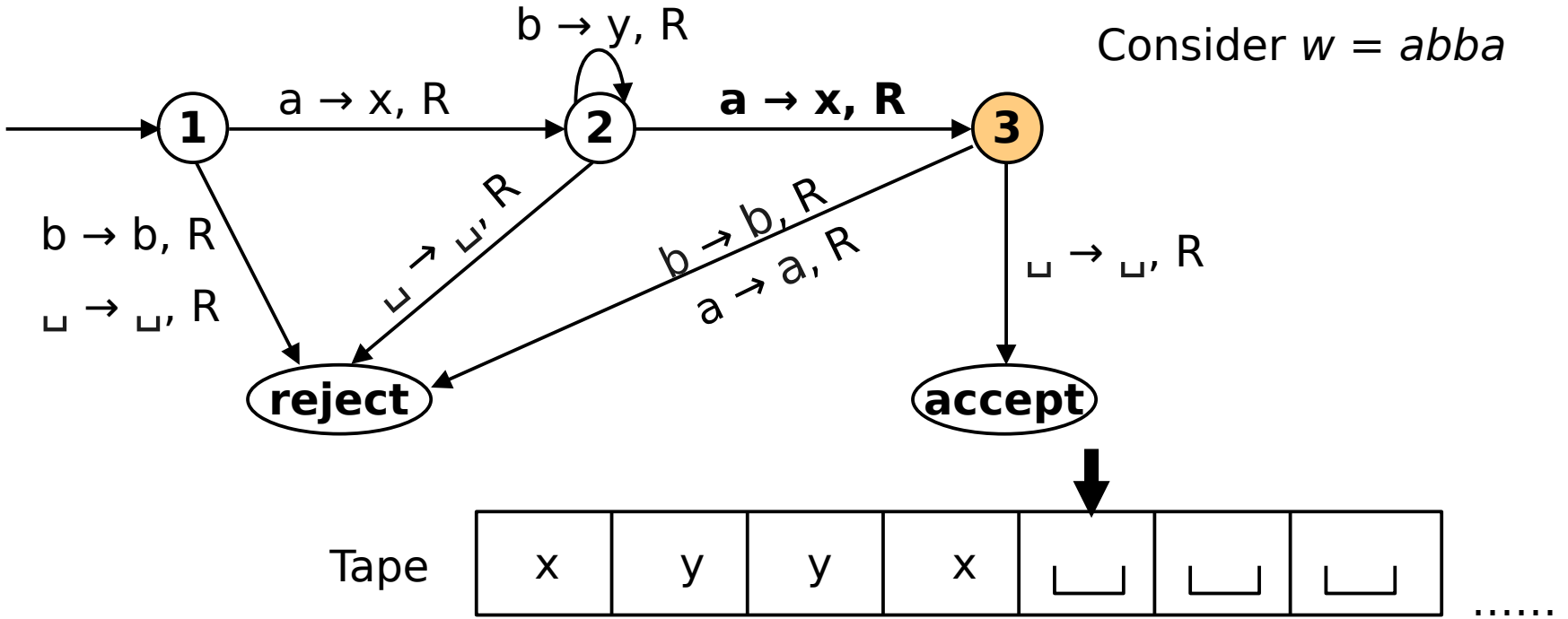

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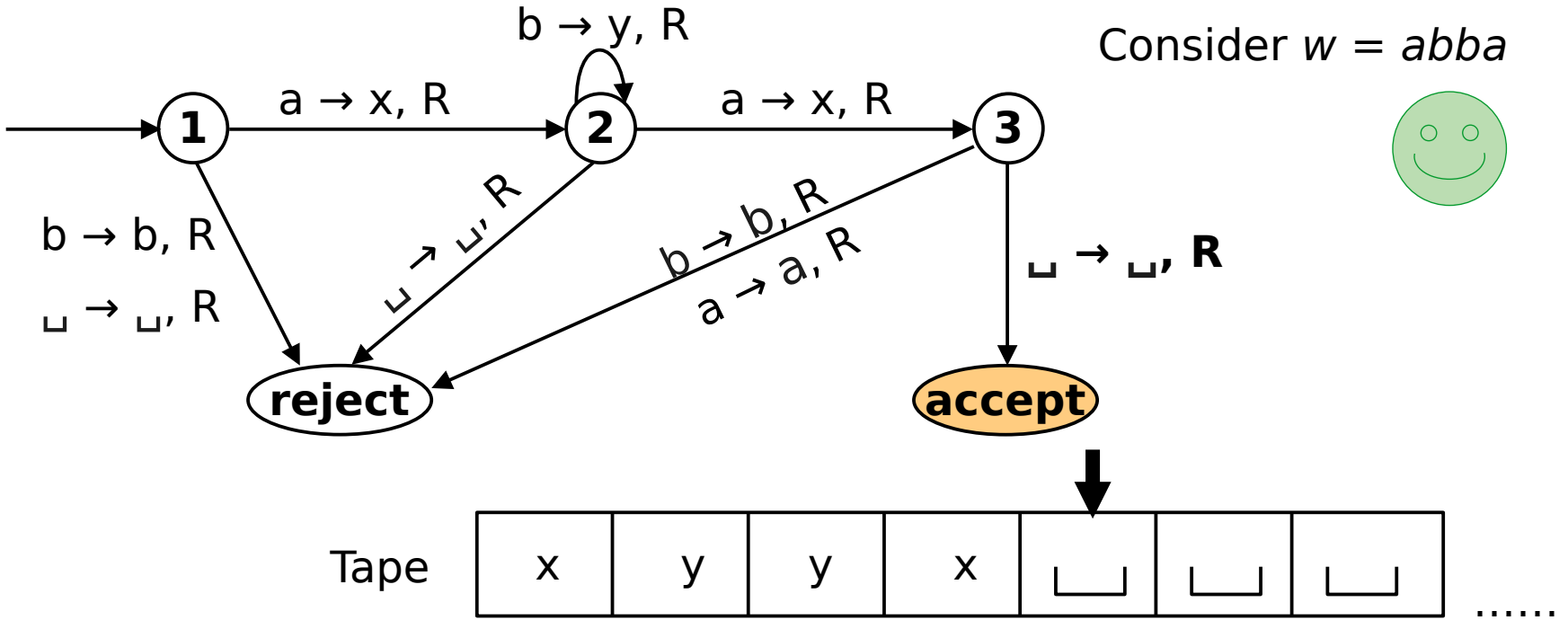

Turing Machines: Formal Definition

- A turing machine is a 7-tuple
  - Q is the set of states
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Current tape content

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$$u\ q(v)$$

Tape head location is the first symbol of v

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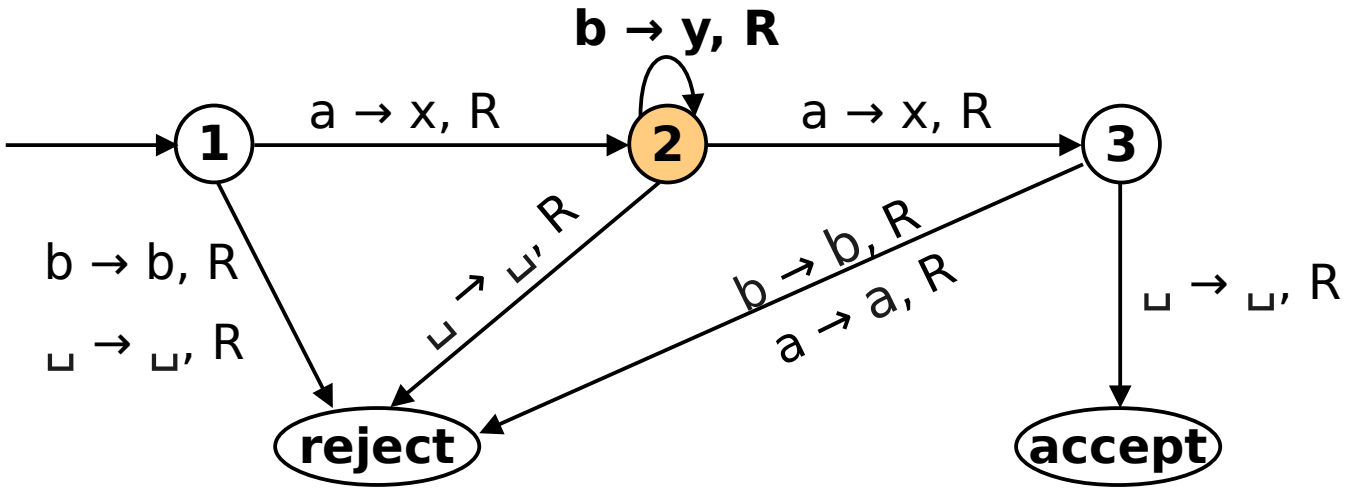

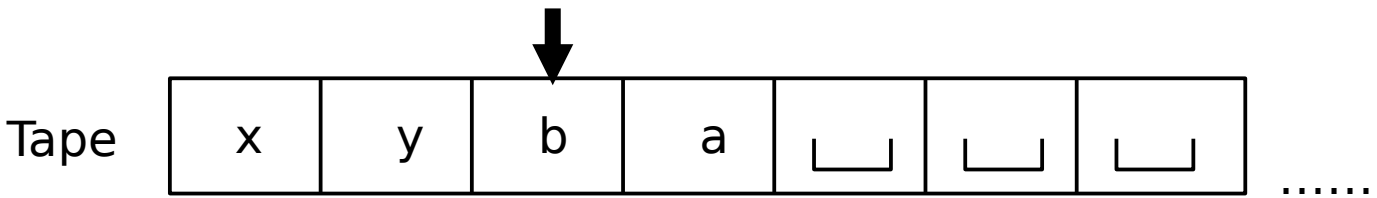

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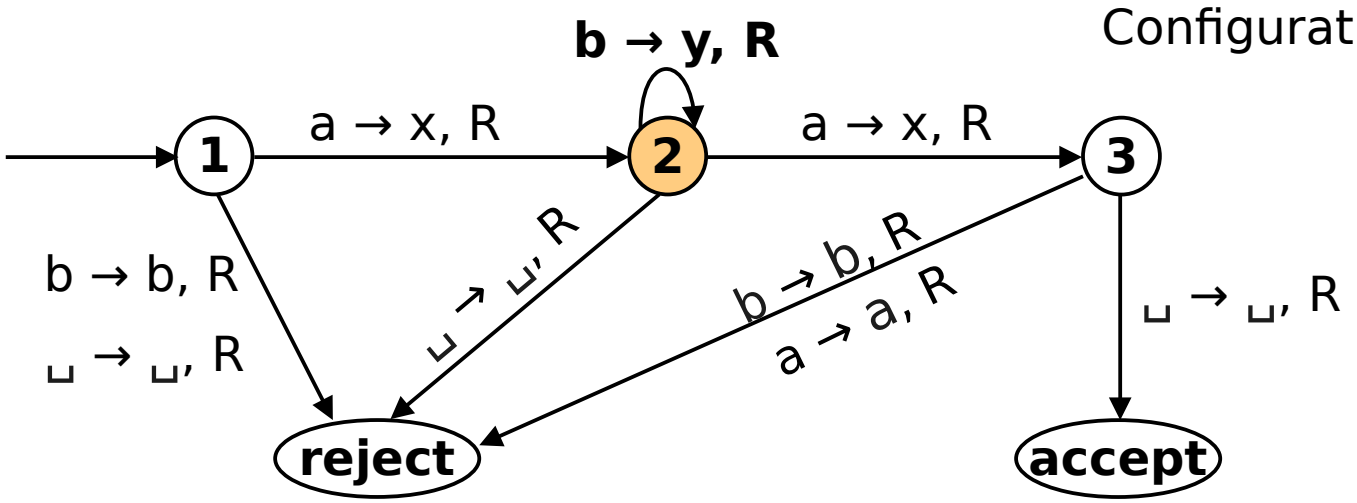
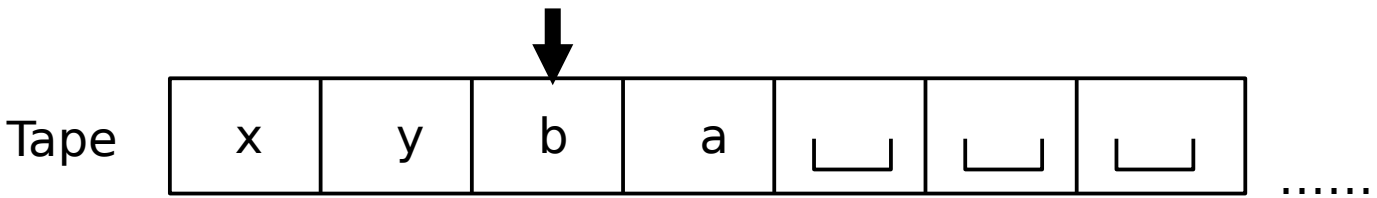


Configuration is **xy2ba**



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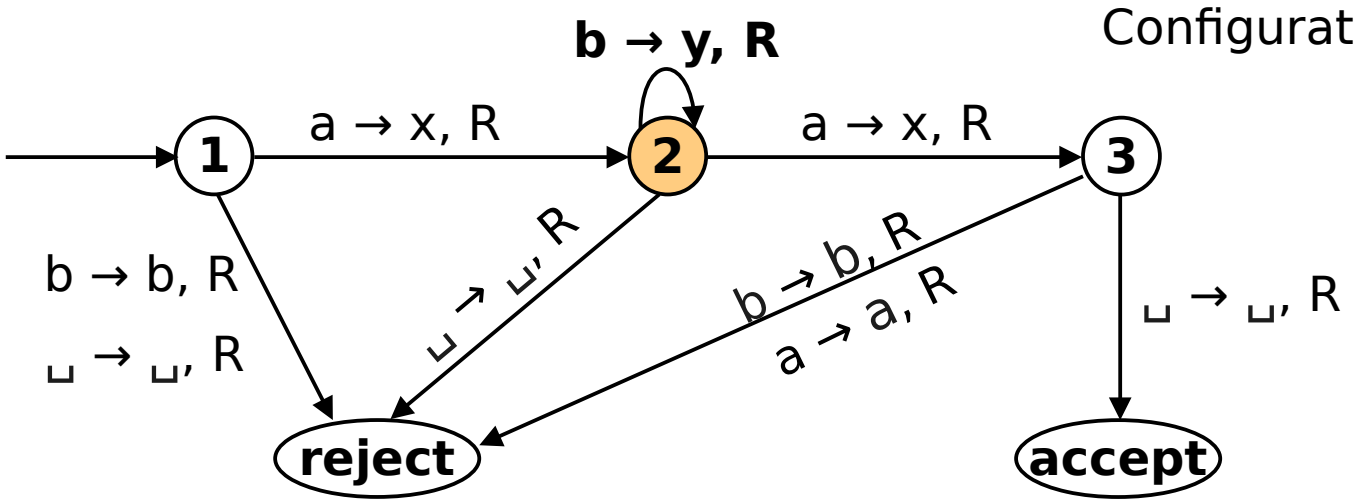

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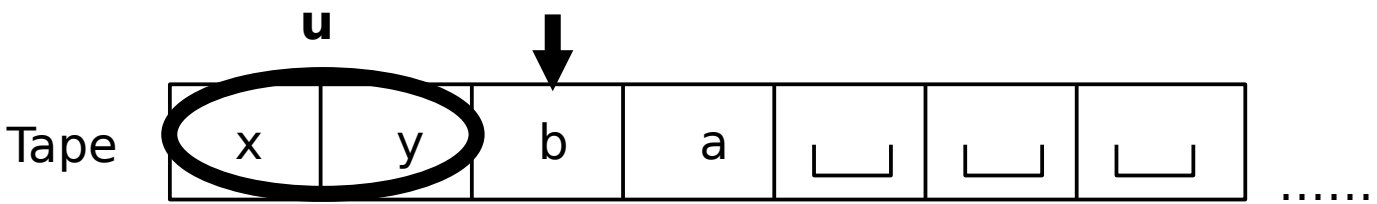

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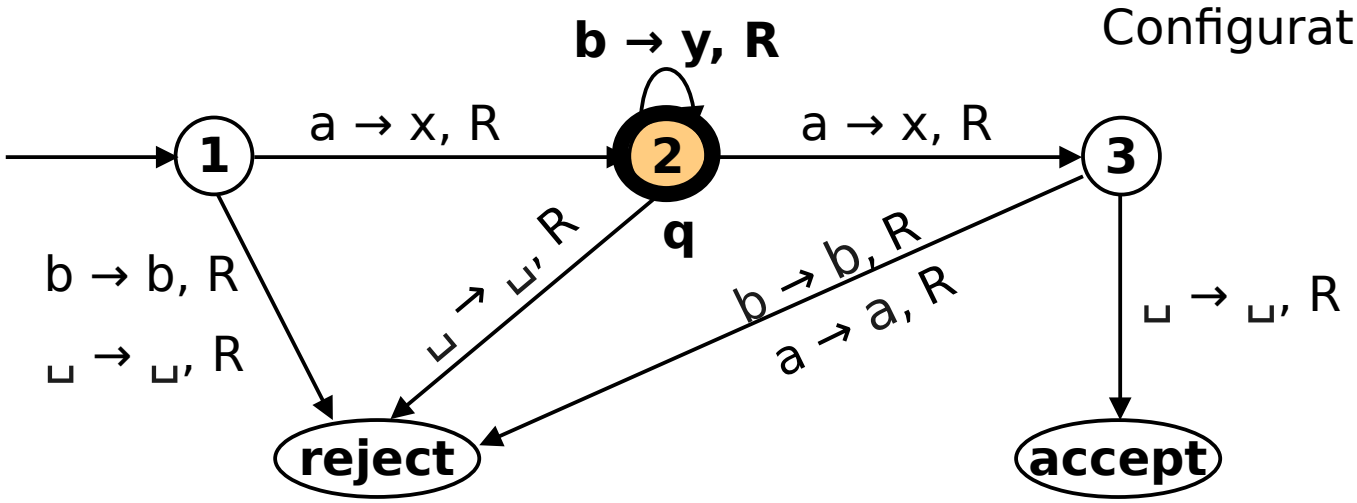

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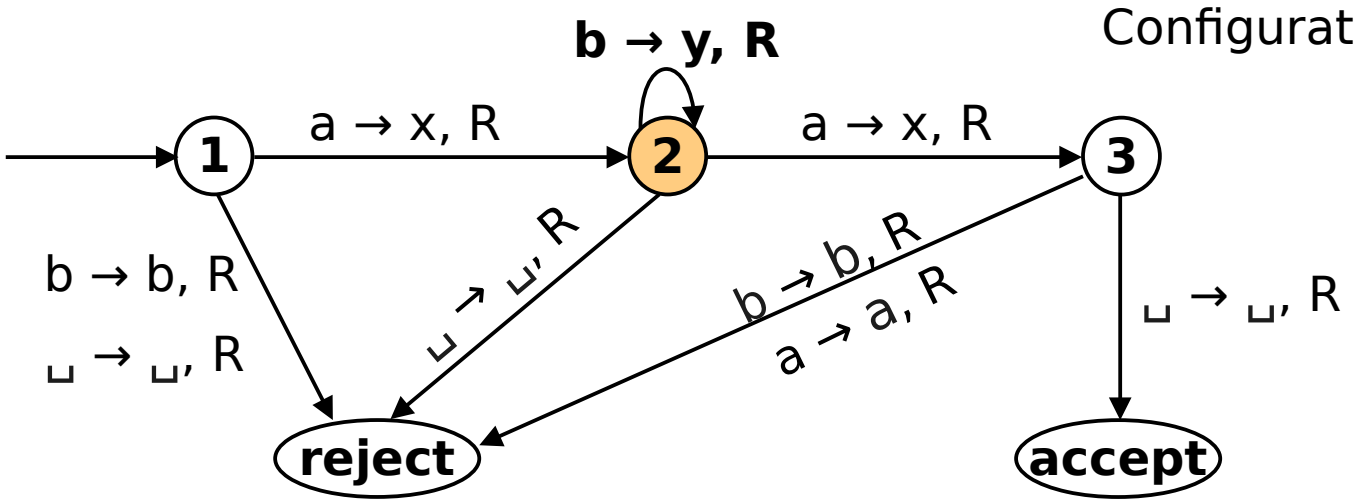

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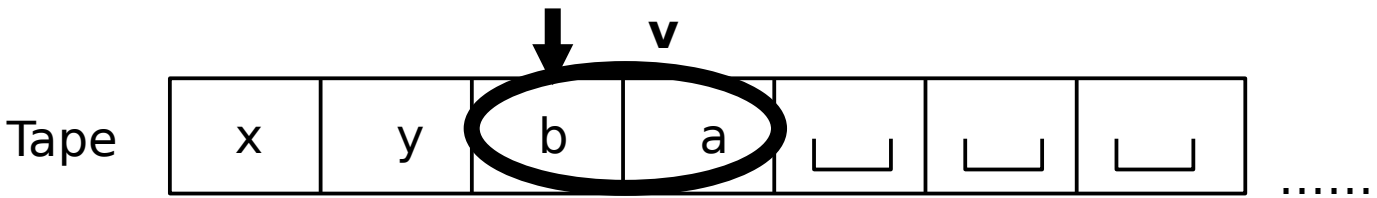


Turing Machines: Configuration

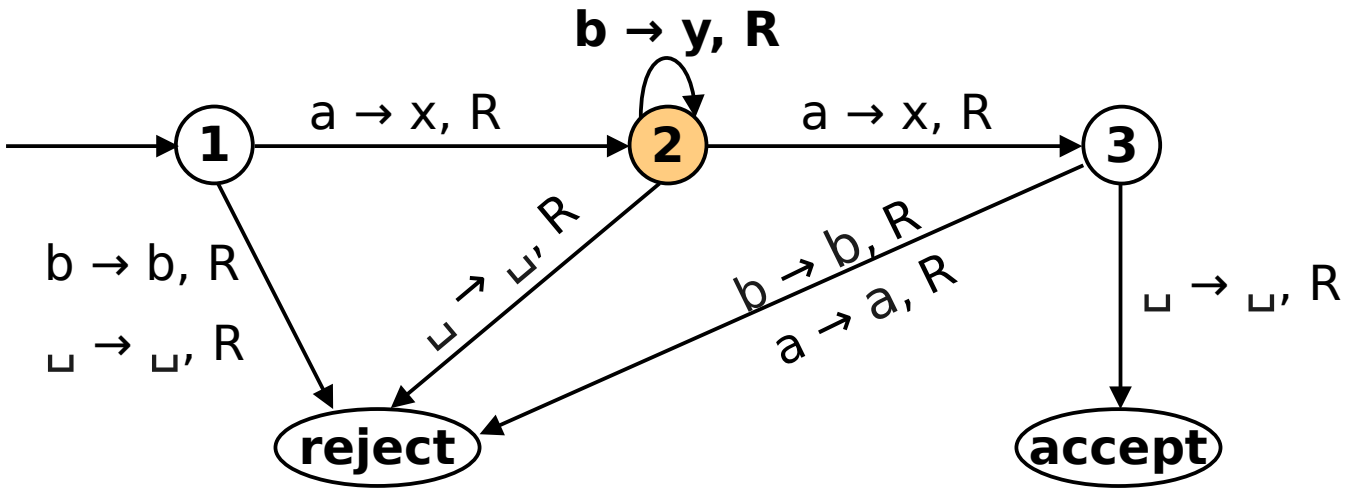
- **Configuration** C_1 **yields** C_2 **if the TM can move from** C_1 **to** C_2 **in a single step.**

Turing Machines: Configuration

C_1 = **xy2ba**

C_2 = **xyy2a**

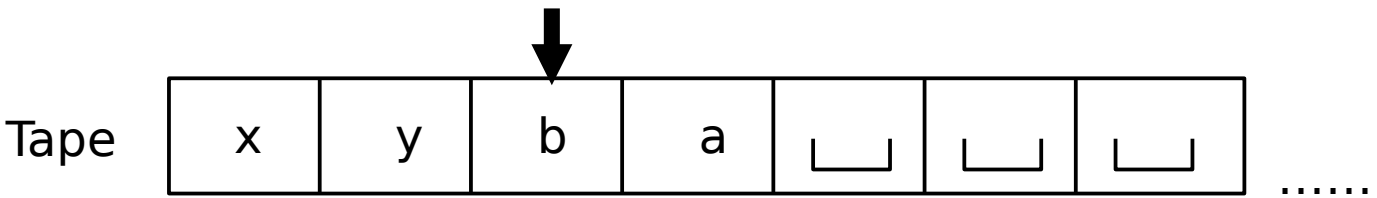
C_3 = **xyyx3**



C_1 yields C_2

C_2 yields C_3

C_1 doesn't yield C_3



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Halting configurations

Turing Machines: Configuration

- **A TM M accepts its input w if there is a sequence of configurations $C_1, C_2, \ldots, C_k$ exists, where:**
  - **C_1 is the start configuration of M on input w.**
  - **each C_i yields C_{i+1}, and**
  - **C_k is an accepting configuration.**

Turing Machines: Turing-recognizable

- **The collection of strings that *M* accepts is the language of *M*, or language recognized by *M*, *L(M)*.**
  - *A language is called **Turing-recognizable** if some TM recognizes it.*

Turing Machines: Turing-decidable

- When a TM starts on an input, there are three possible outcomes. The machine may:
  - Accept,
  - Reject, or
  - Loop

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- **Machines that loop forever never halt, and thus, never decide whether the input is accepted or rejected.**

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> *A language **Turing-decidable** or simply decidable if some Turing machine decides it*

- **Every Turing-decidable language is also Turing-recognizable.**
  - **But, not the opposite!**

Turing Machines: Example

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How can we design a Turing machine?

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What are the instructions that need to be executed in order to recognize a certain string?

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How can we design a Turing machine?

Think of it as an algorithm

Use the algorithm as a guide to design the TM

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Algorithm for $\mathbf{M_1}$

1) Scan the first leftmost input symbol on the tape, and cross it off (replace with x).

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3) Repeat steps 1 and 2 until all symbols left to # are crossed off.
4) Scan symbols right to #, and reject if any symbols remain. Else, accept.

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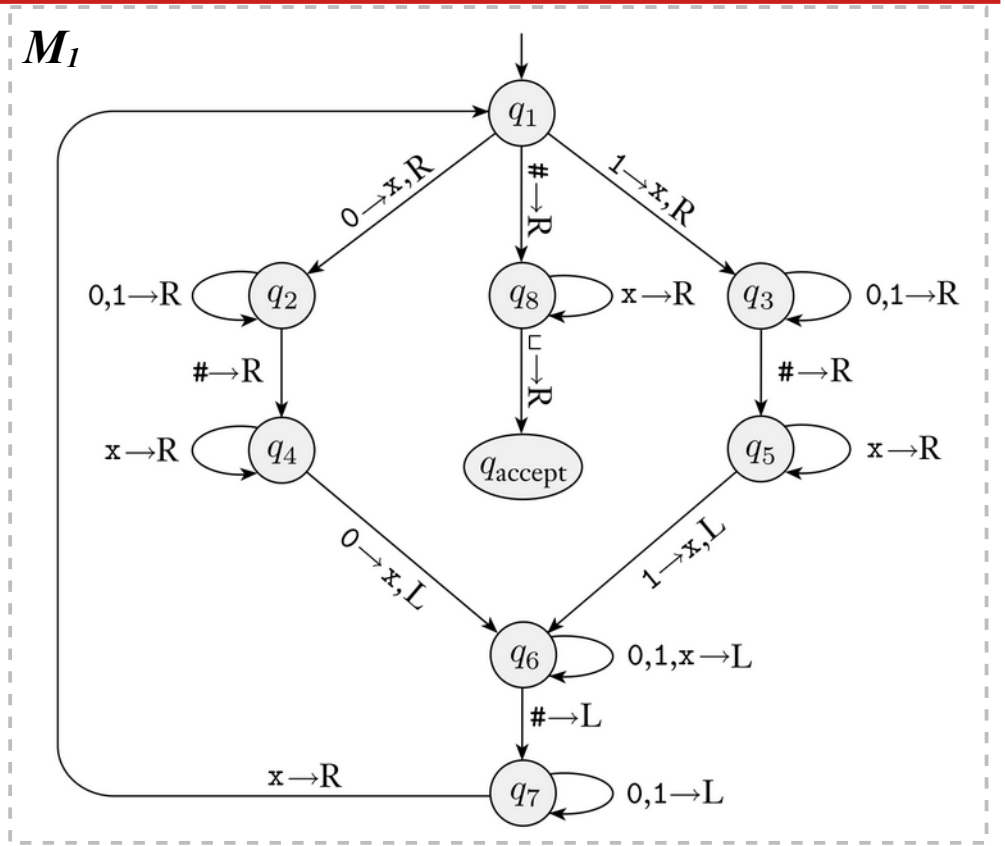

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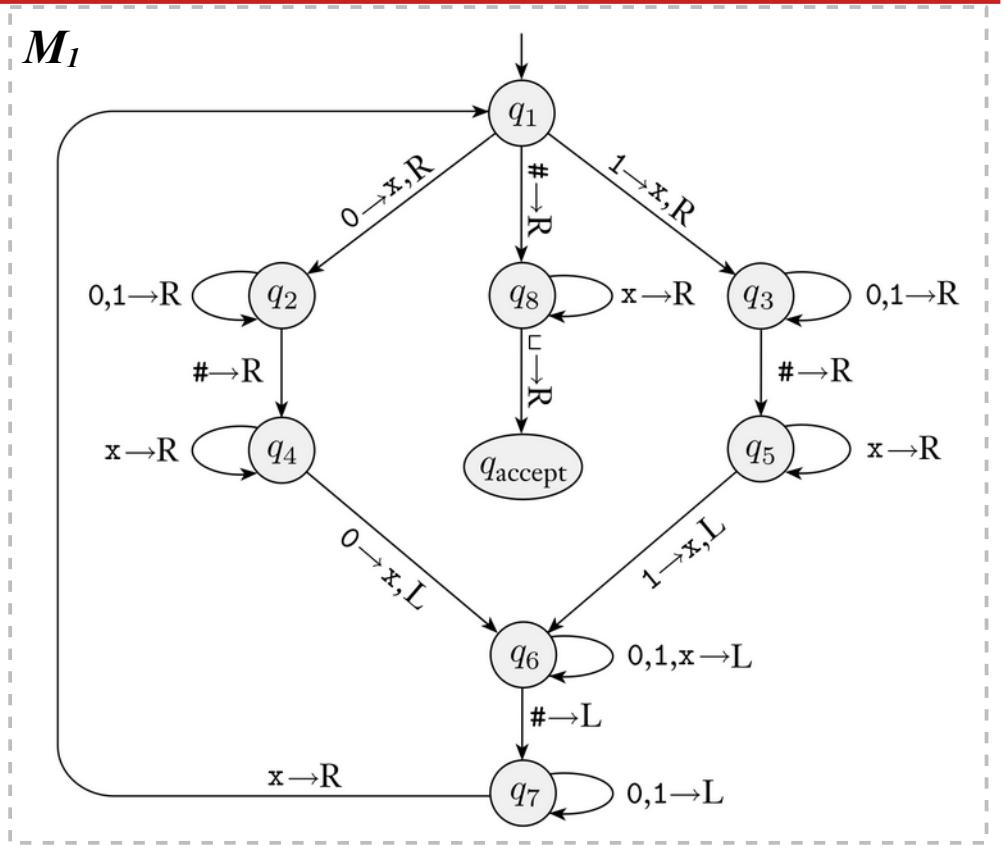
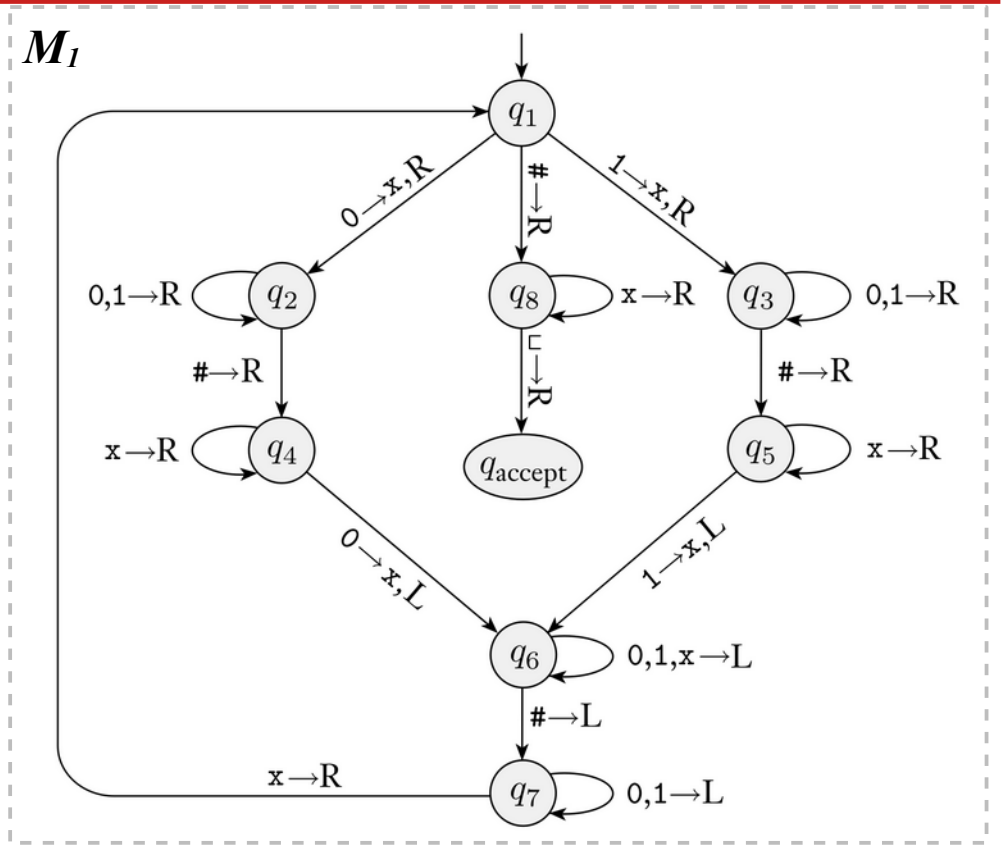
Definition of M_1

- $Q = \{q_1, \ldots, q_8, q_{accept}, q_{reject}\}$
- $\Sigma = \{0,1,\#\}$, and
- $\Gamma = \{0,1,\#,x,\sqcup\}$
- δ is the state diagram
- Start state is q_1
- Accept state is q_{accept}
- Reject state is q_{reject} (not shown)



Turing Machines: Example

Consider $w = 001\#001$



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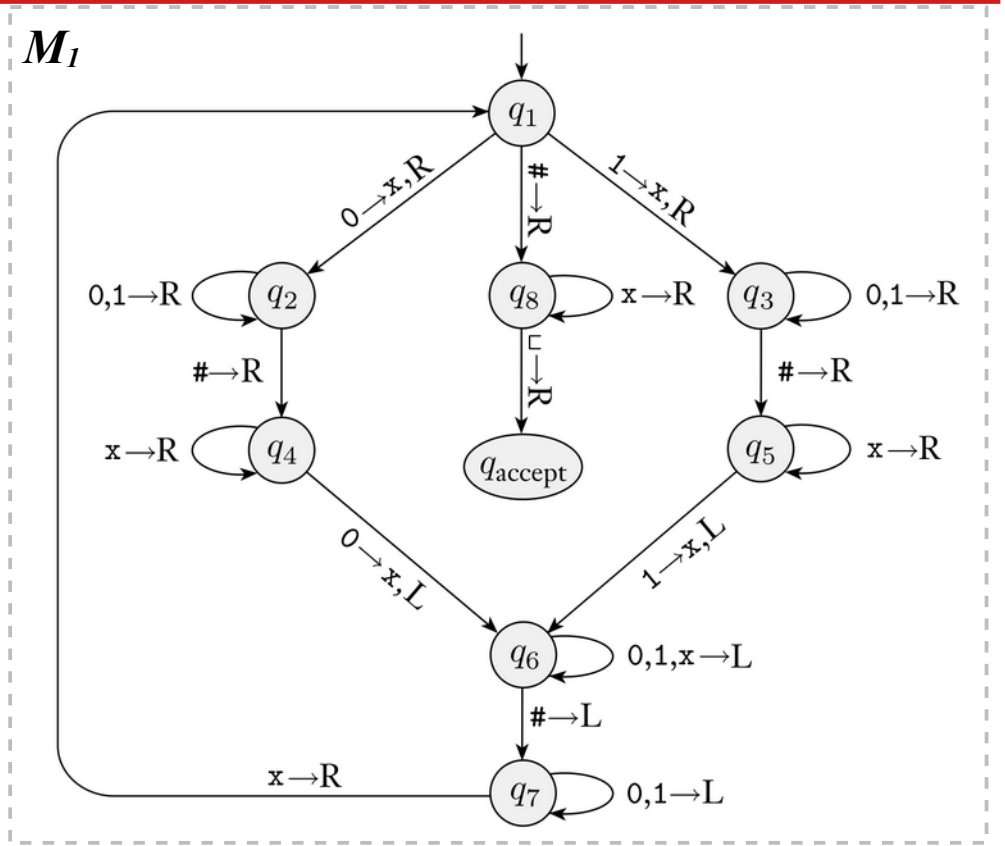
$q_1001\#001\sqcup$

Start configuration



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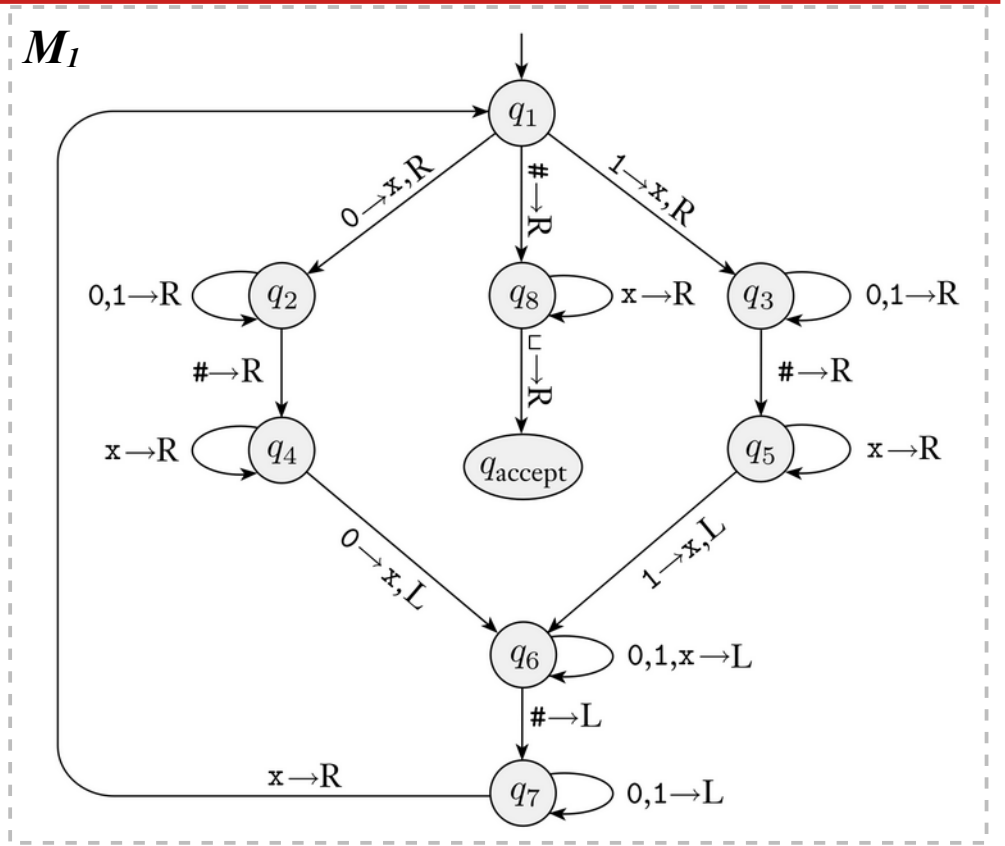
$q_1001\#001\sqcup \Rightarrow xq_201\#001\sqcup$



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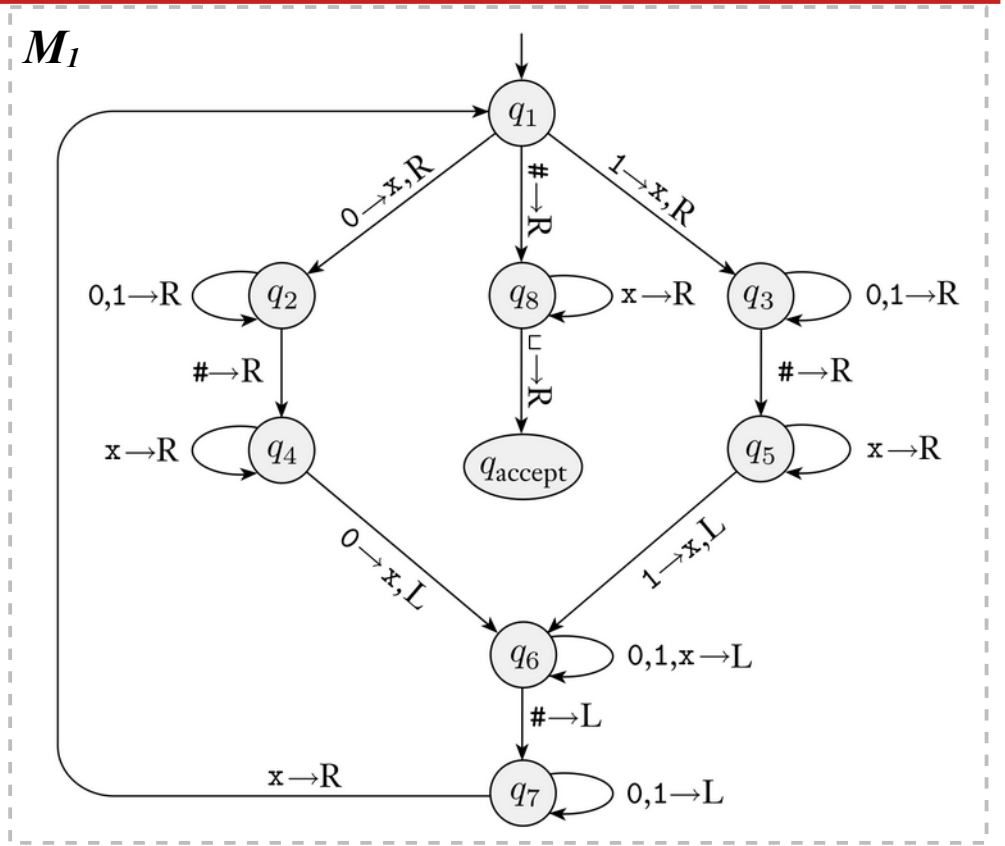

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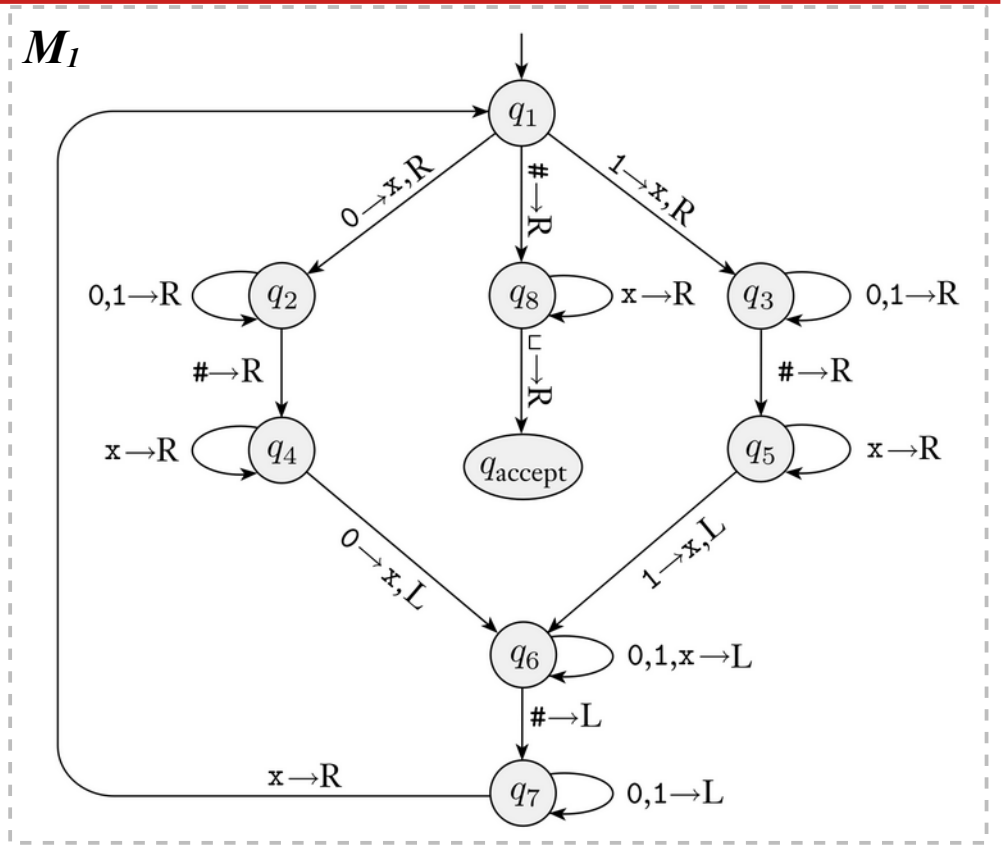

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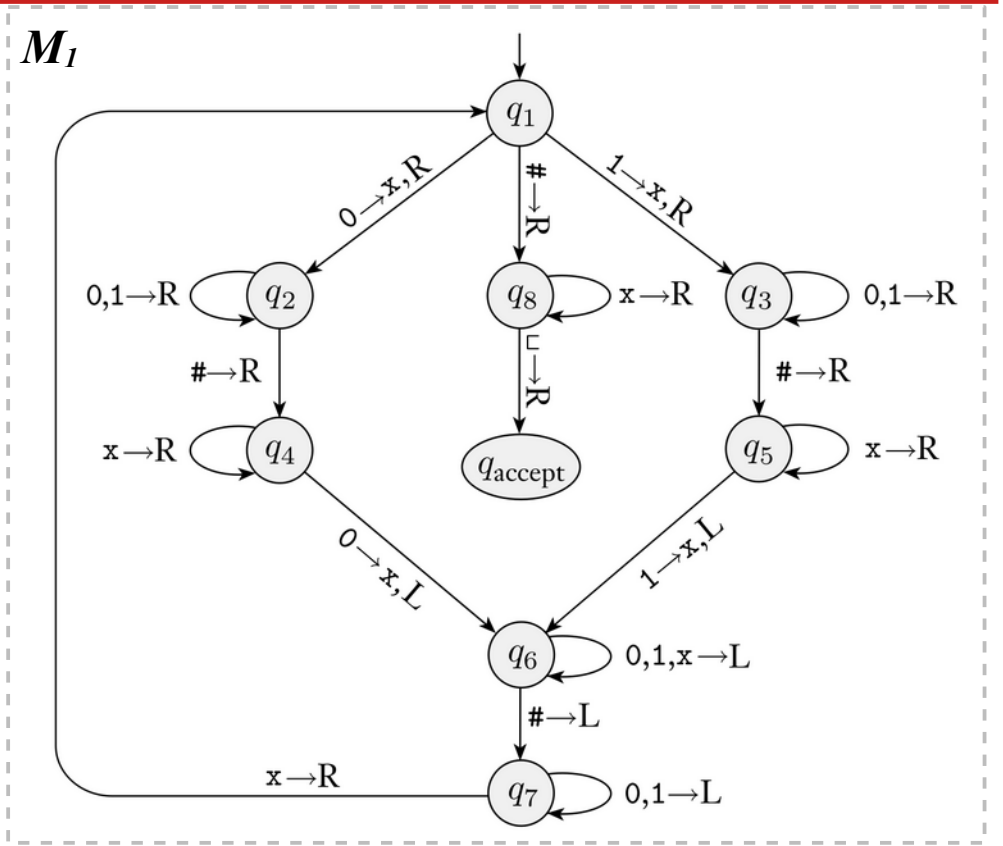

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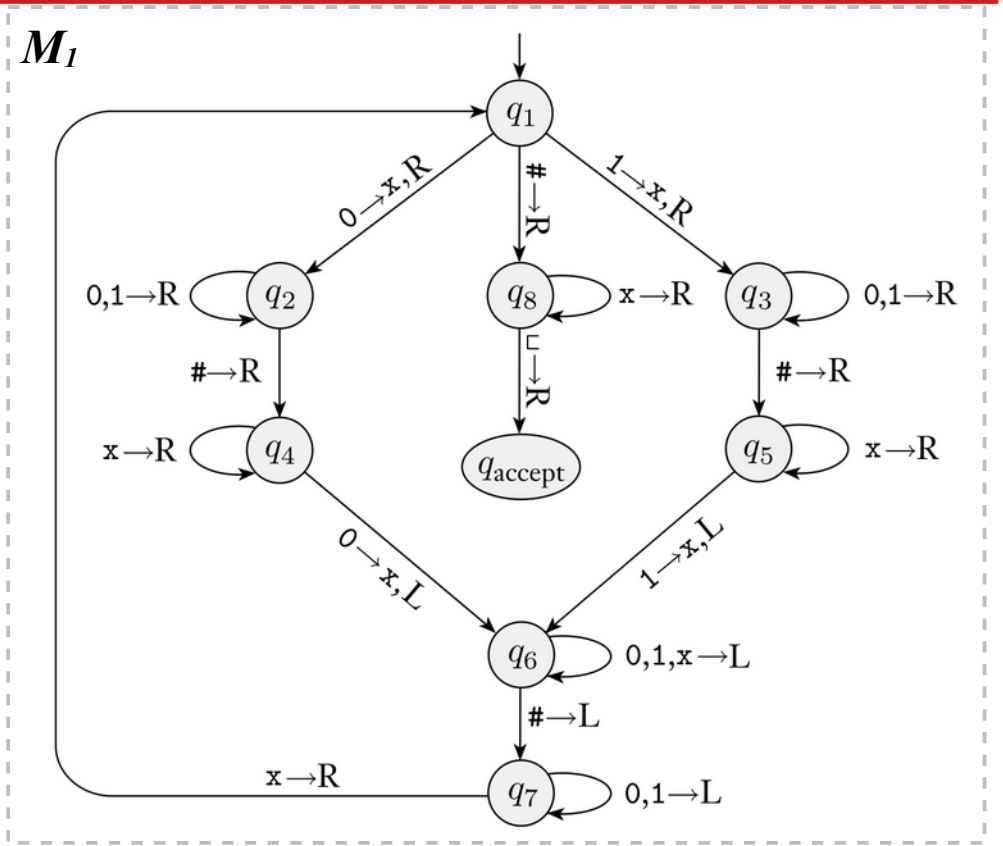

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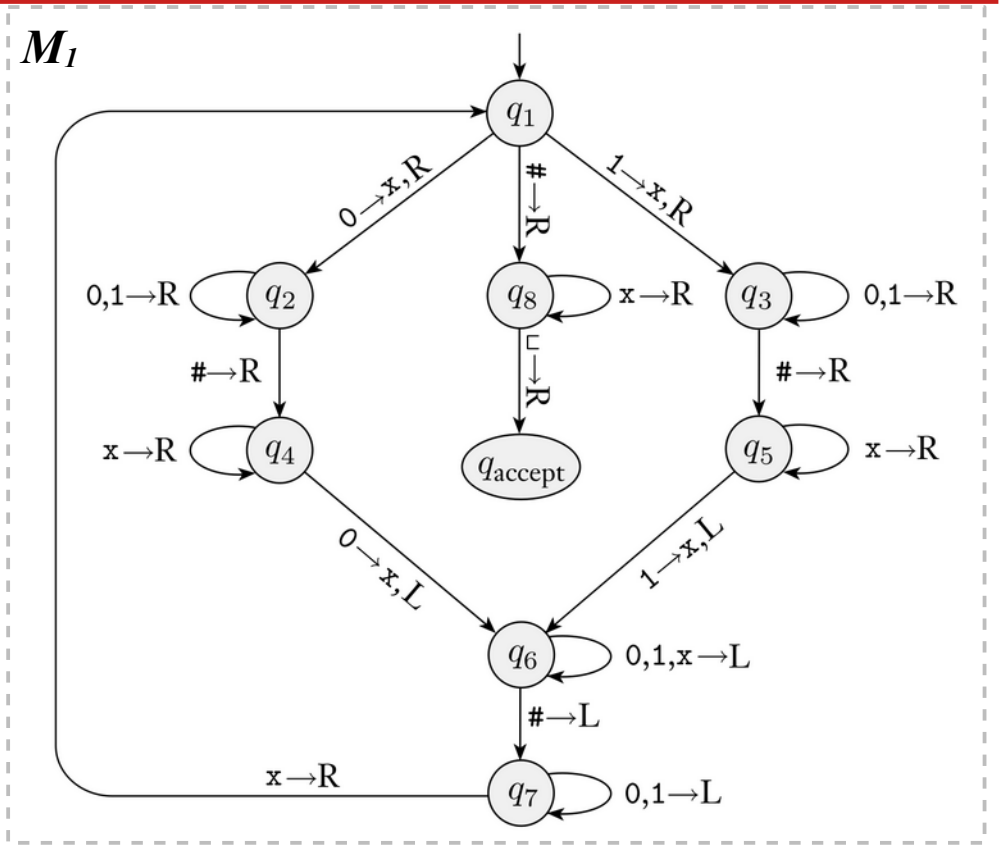

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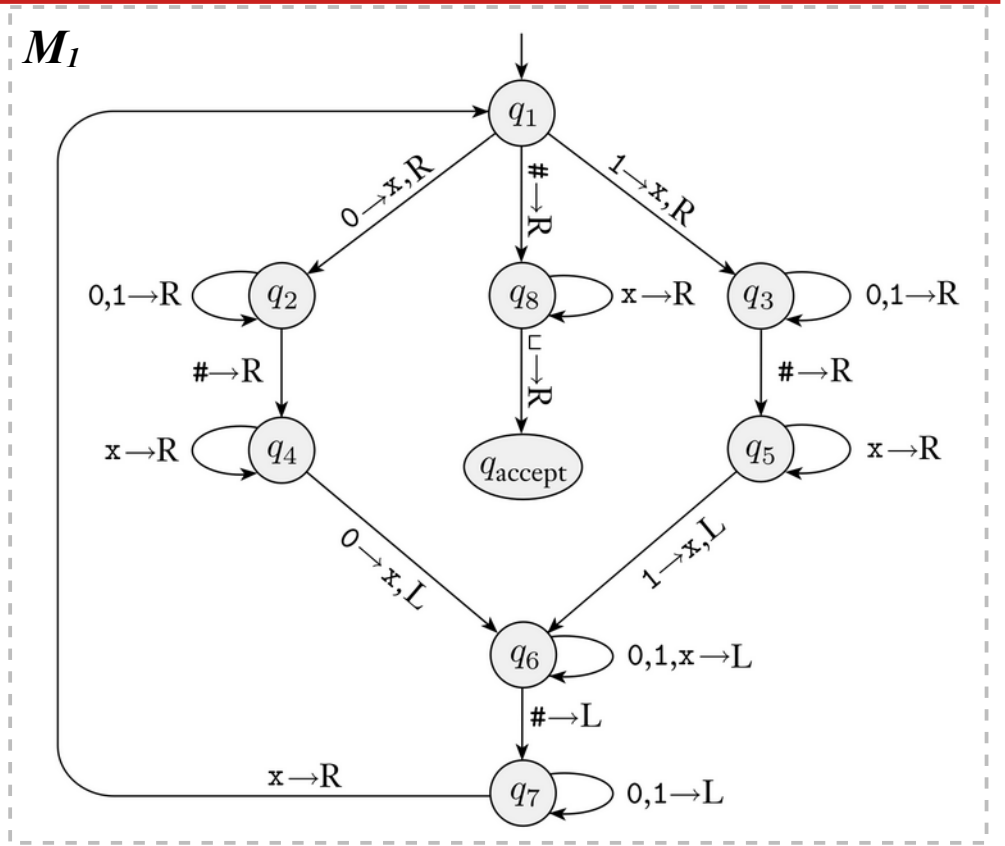

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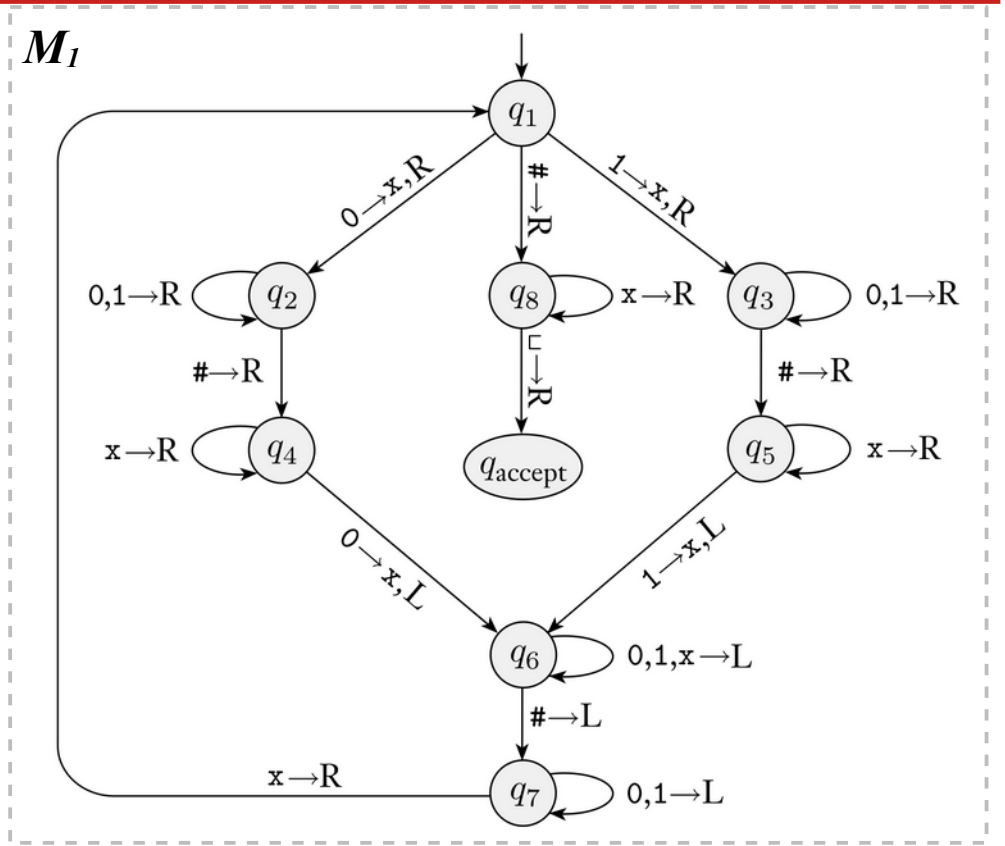

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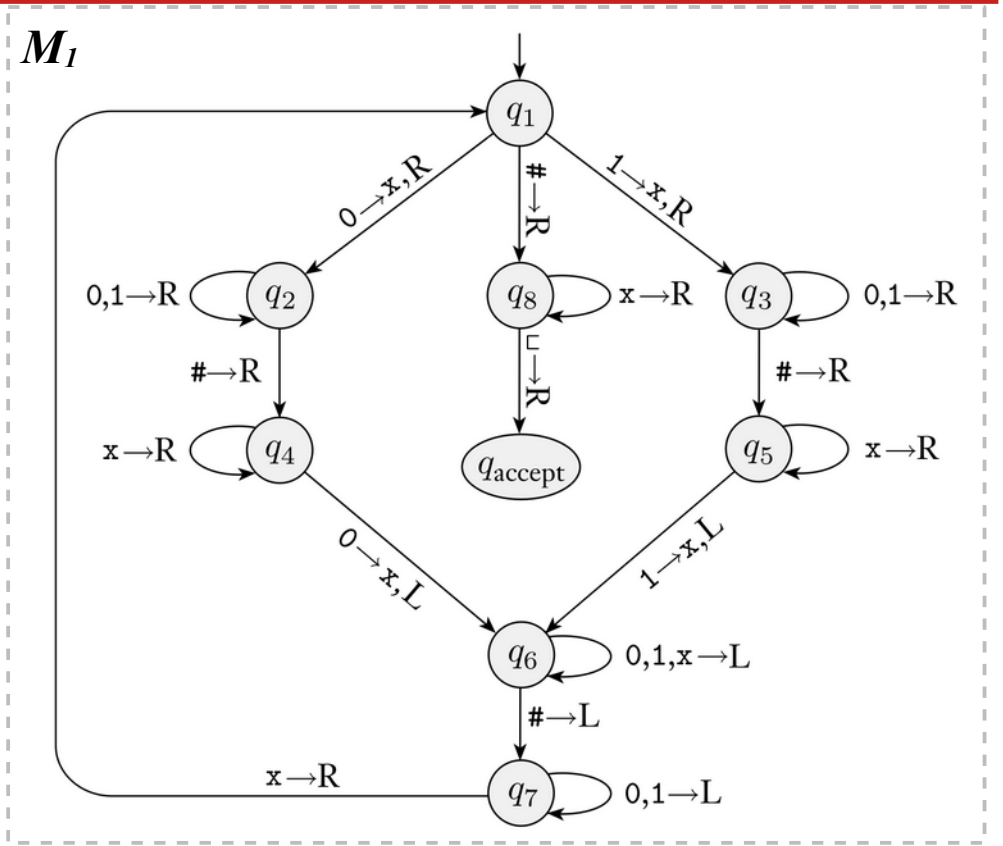

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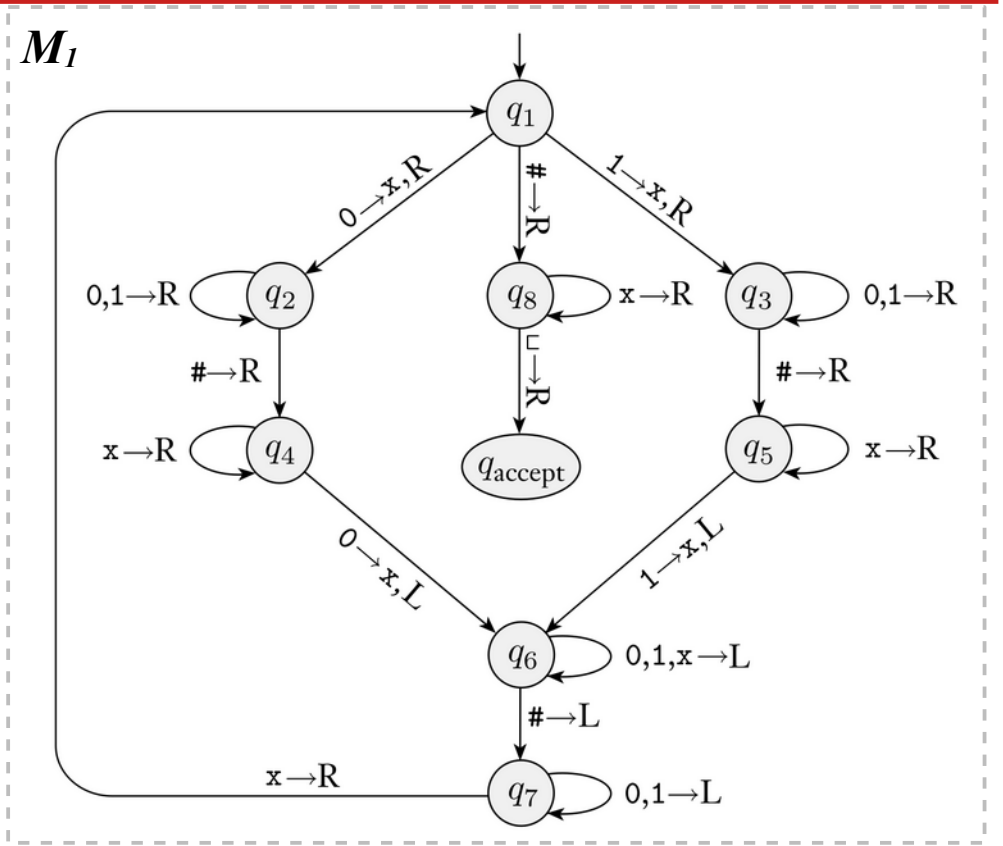

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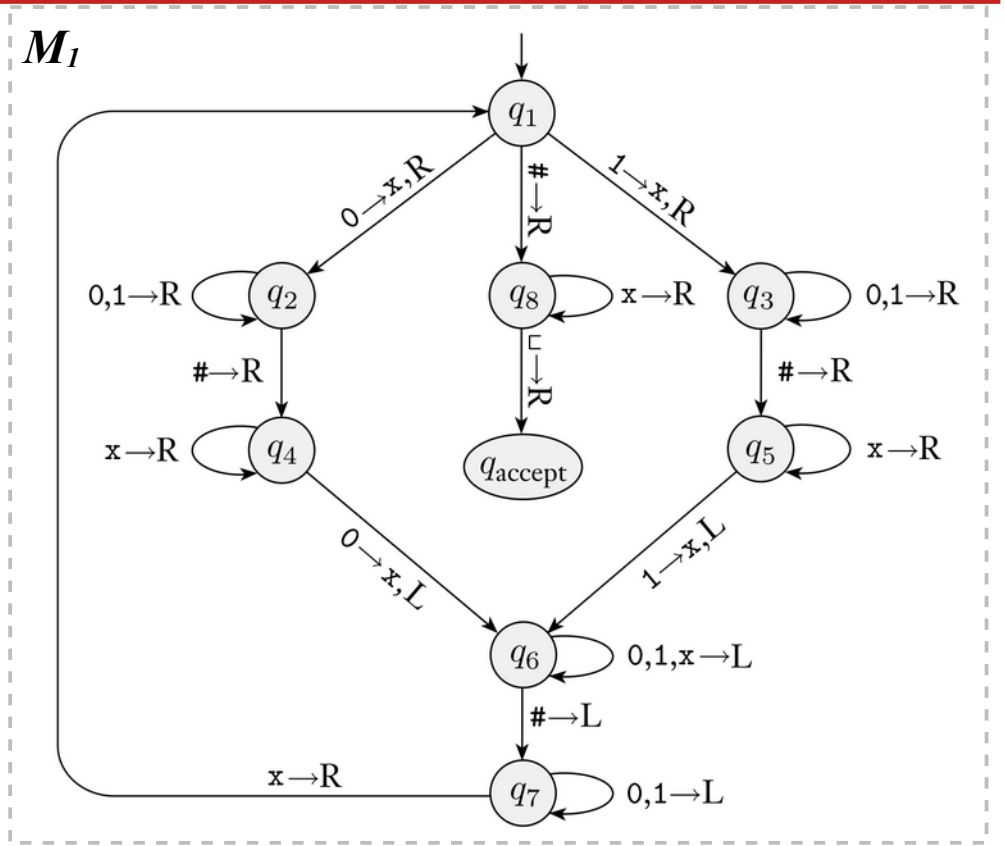

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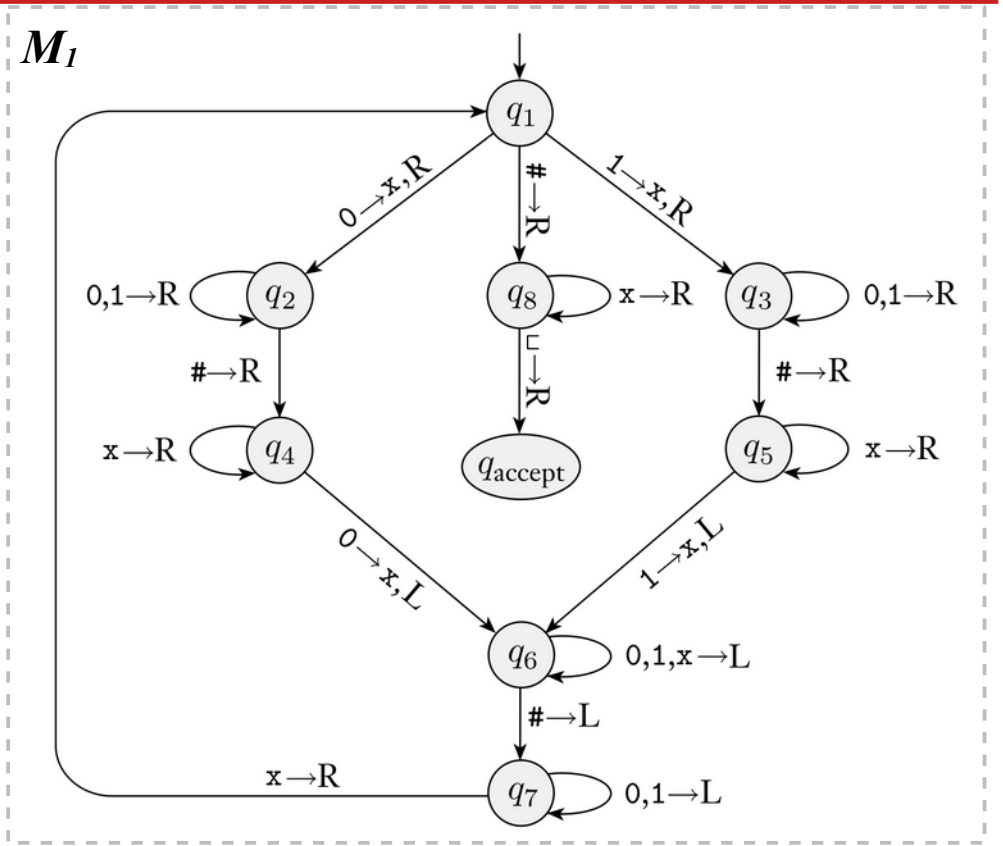

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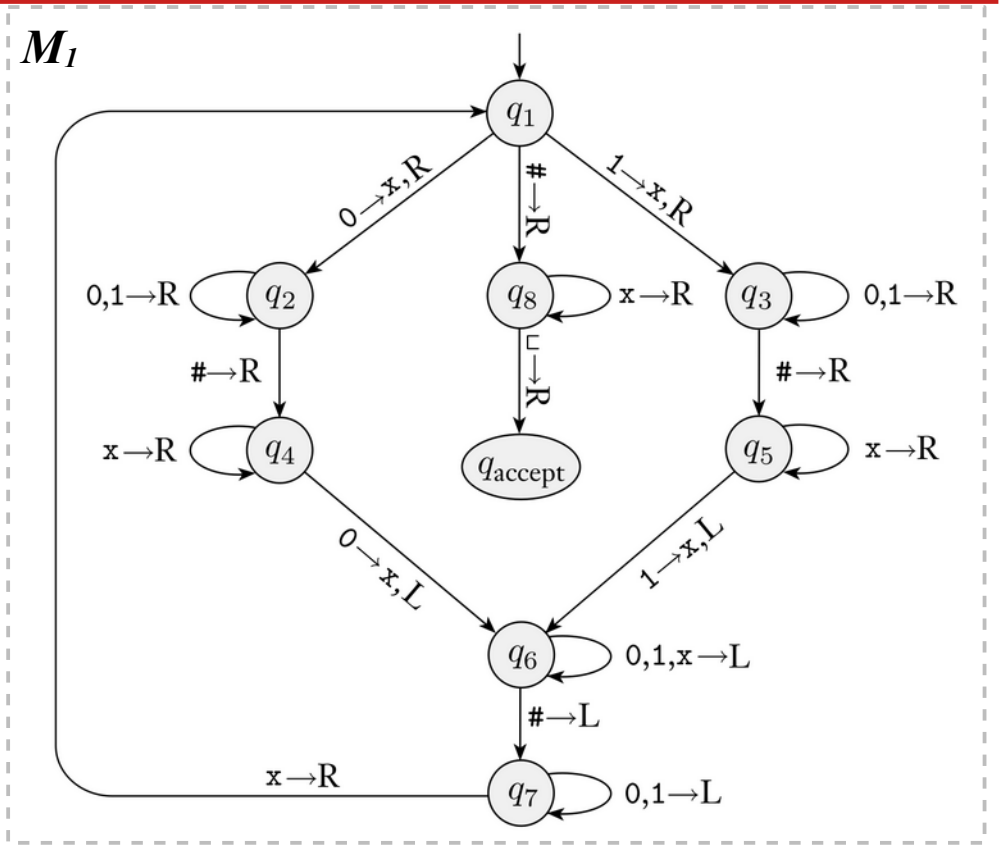

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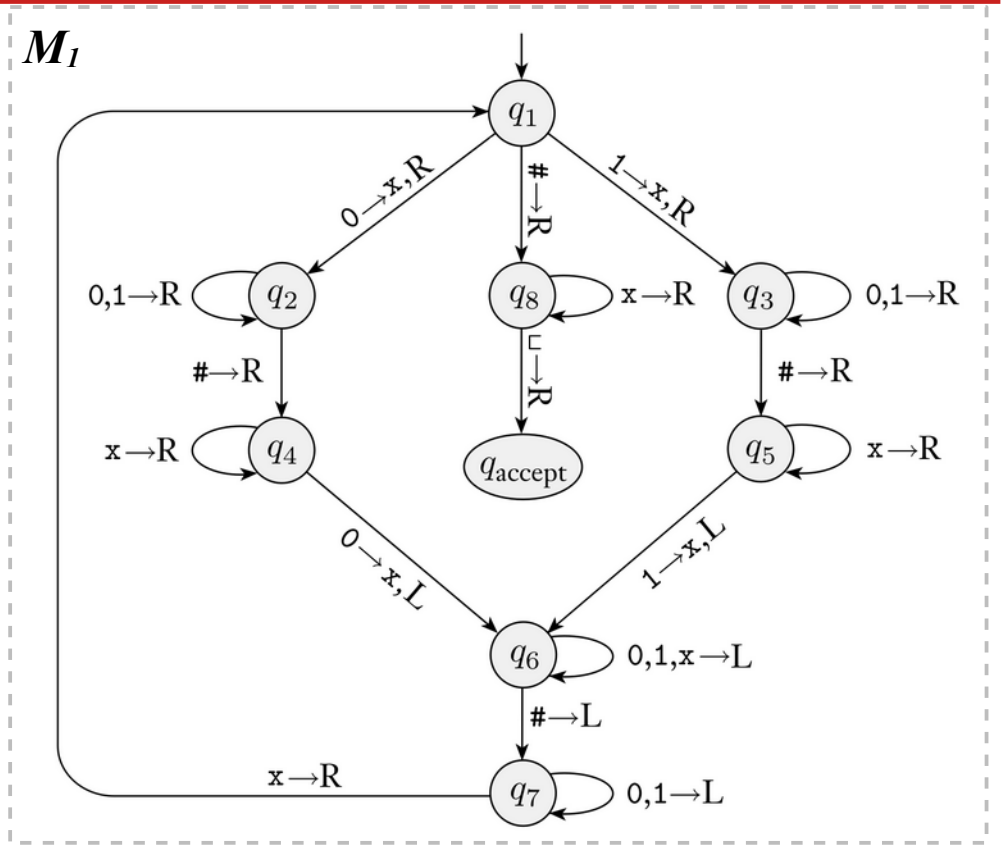

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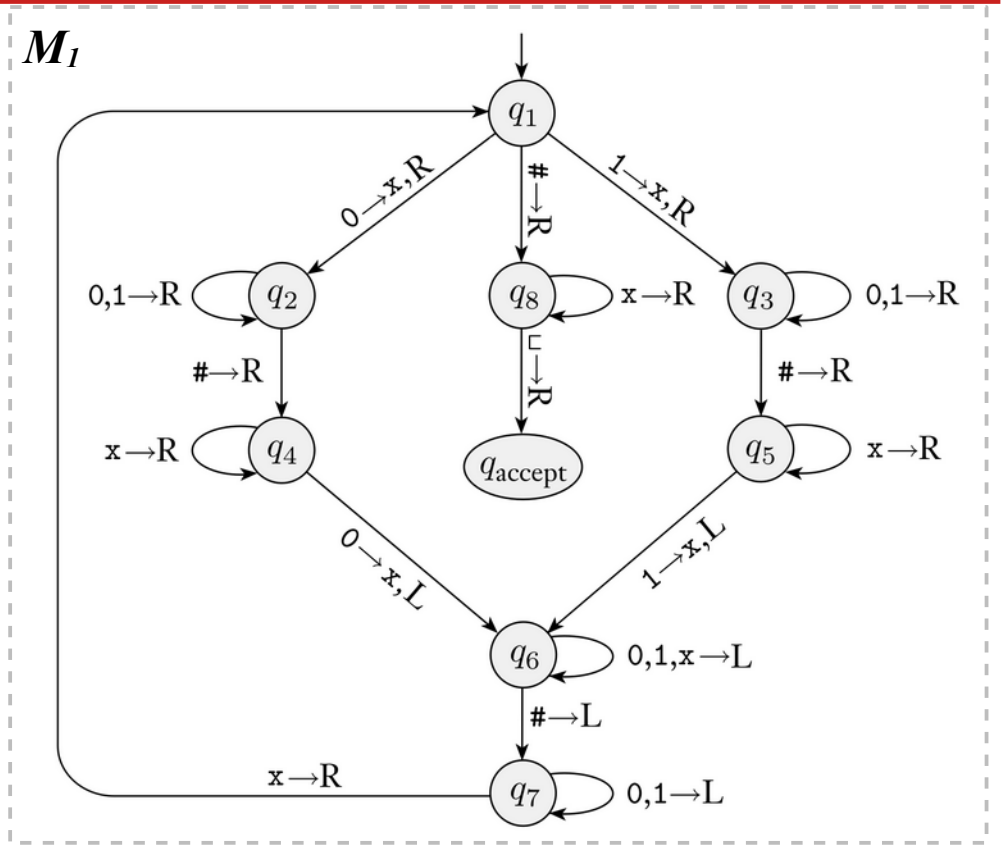

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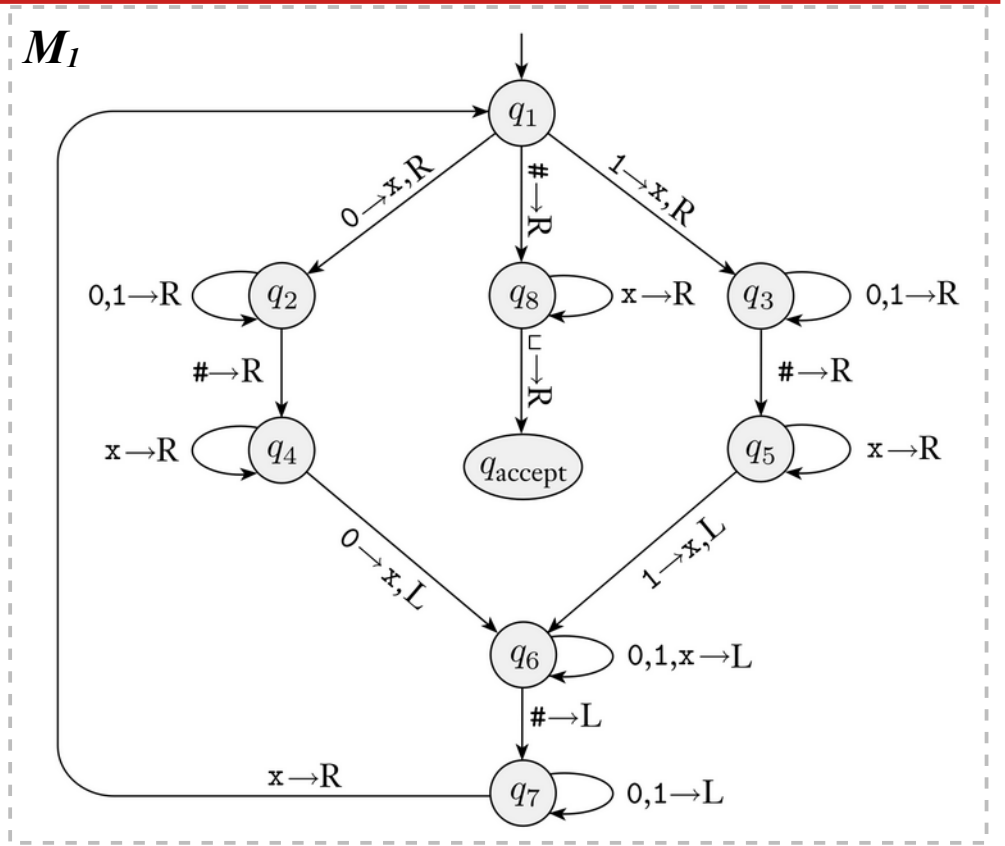

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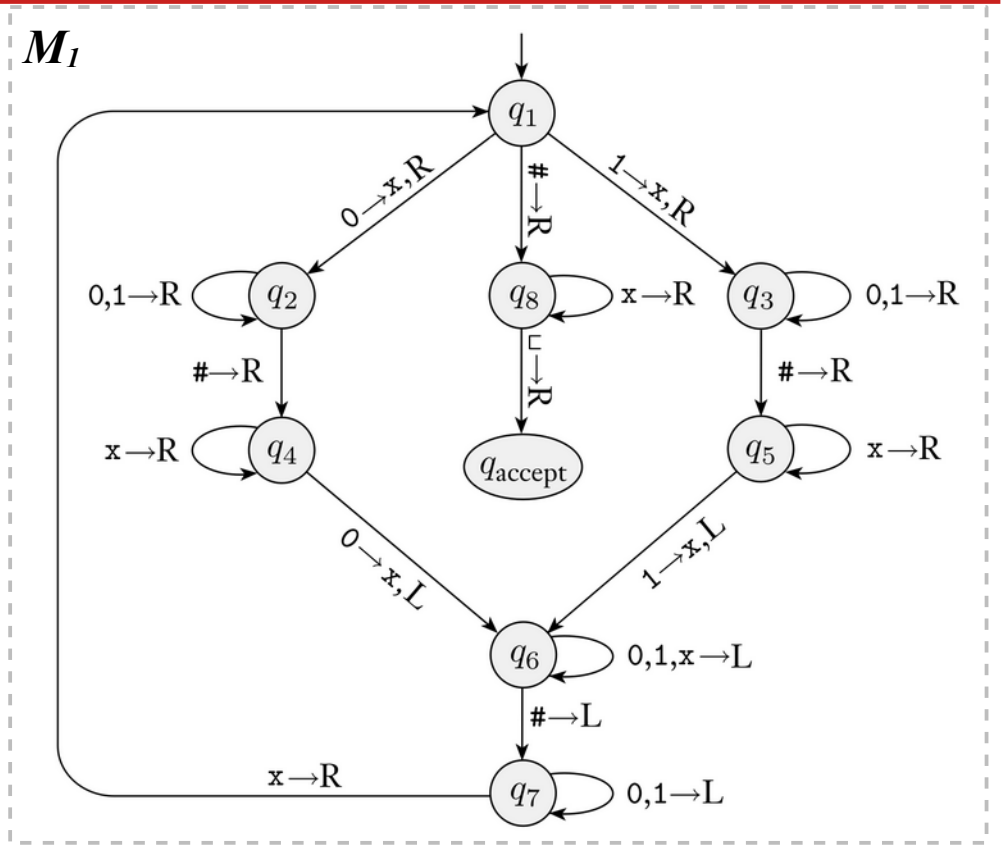

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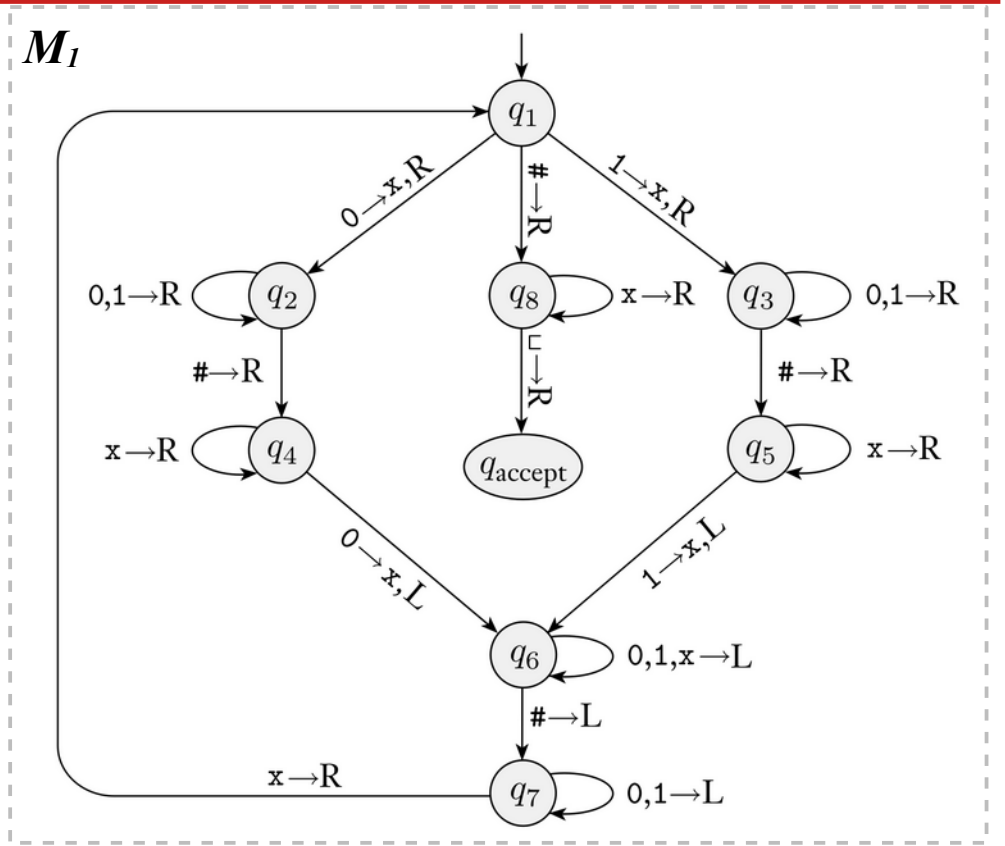
$xxxq_3\#xx1_\sqcup \Rightarrow$



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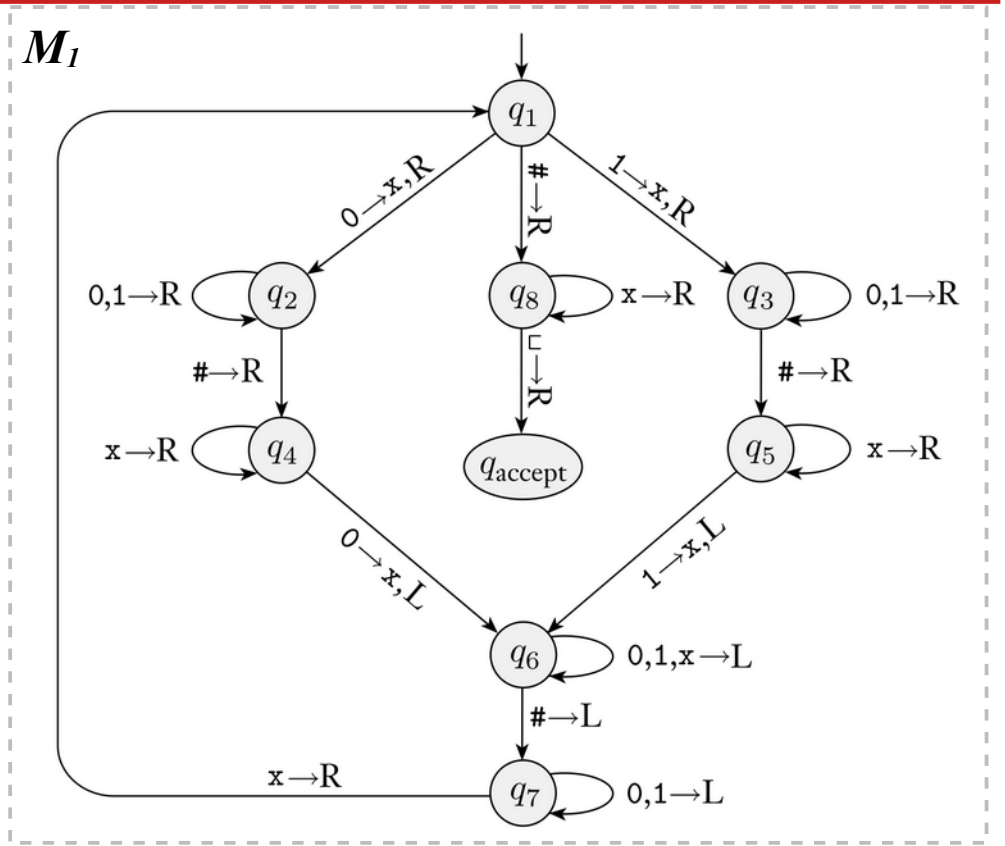

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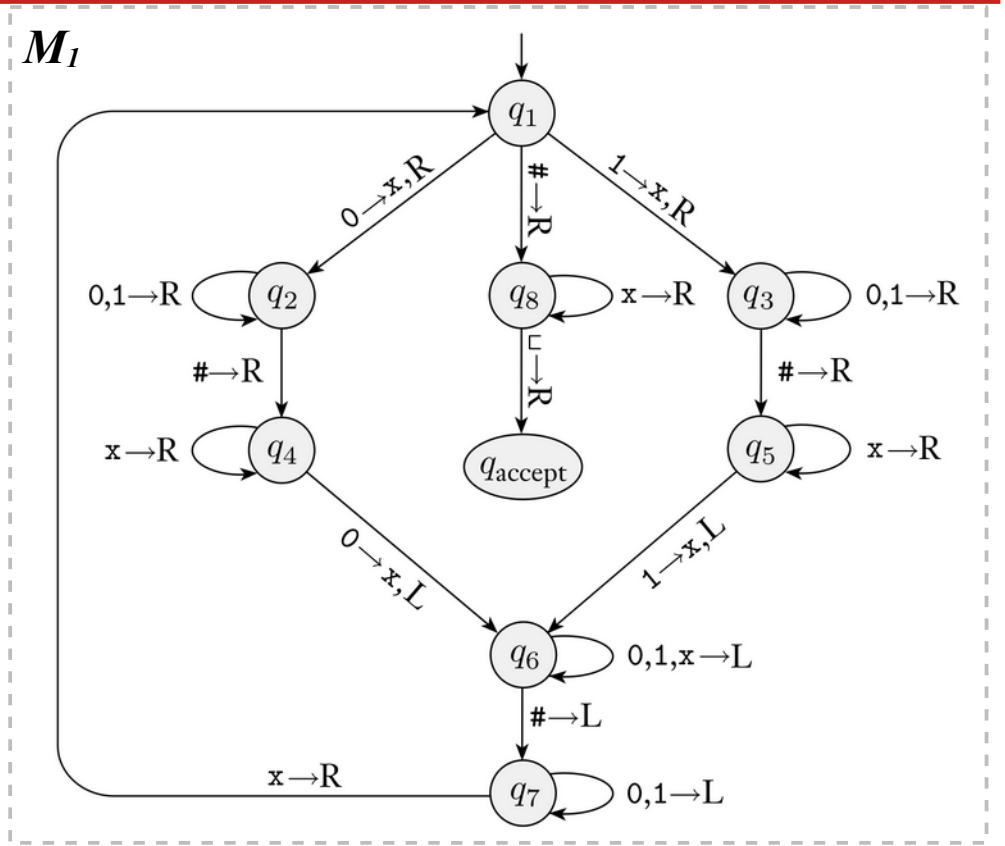

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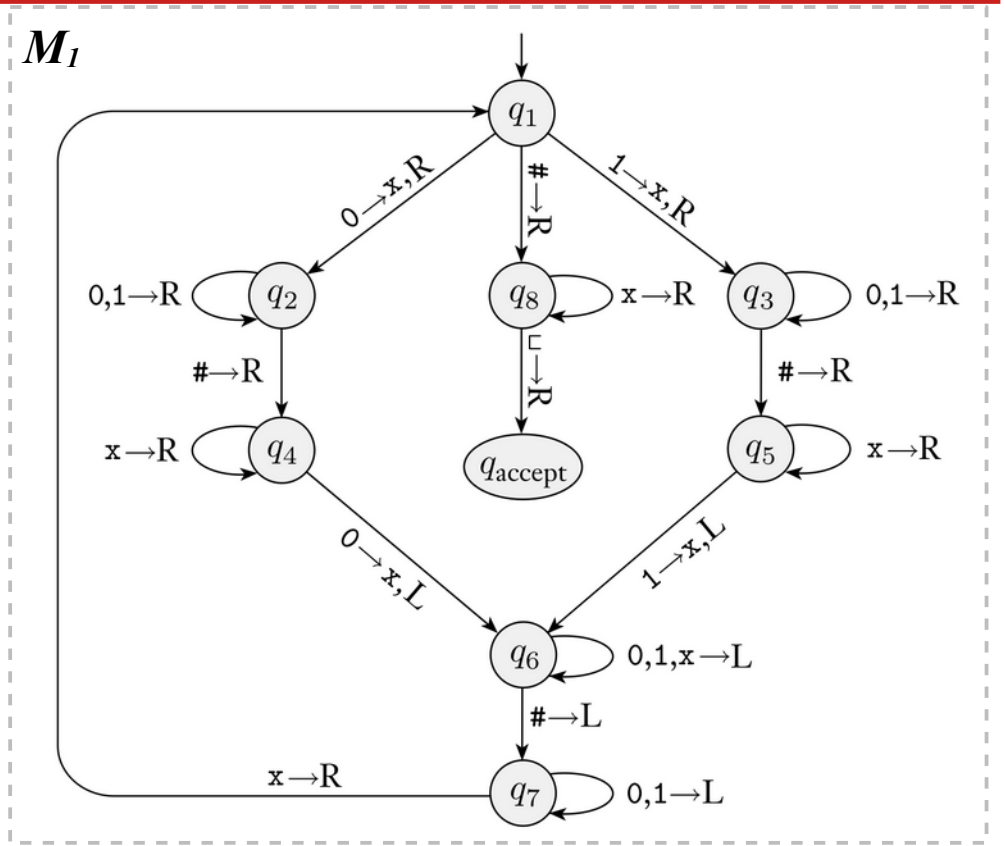

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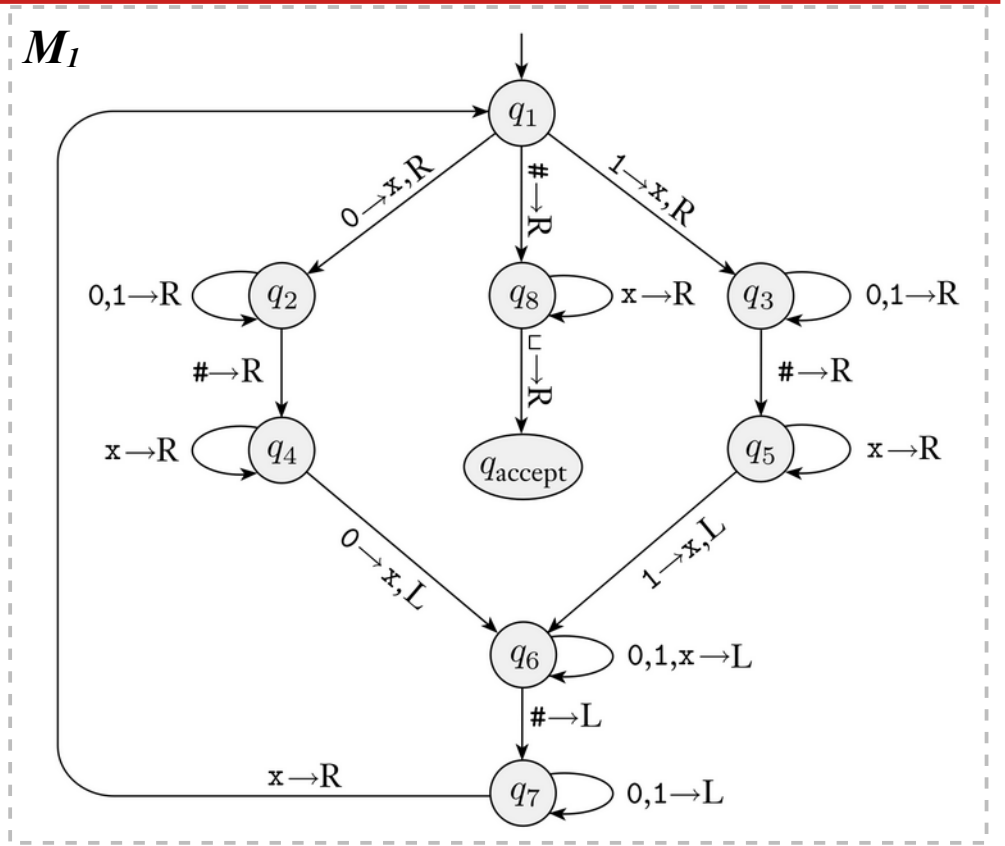

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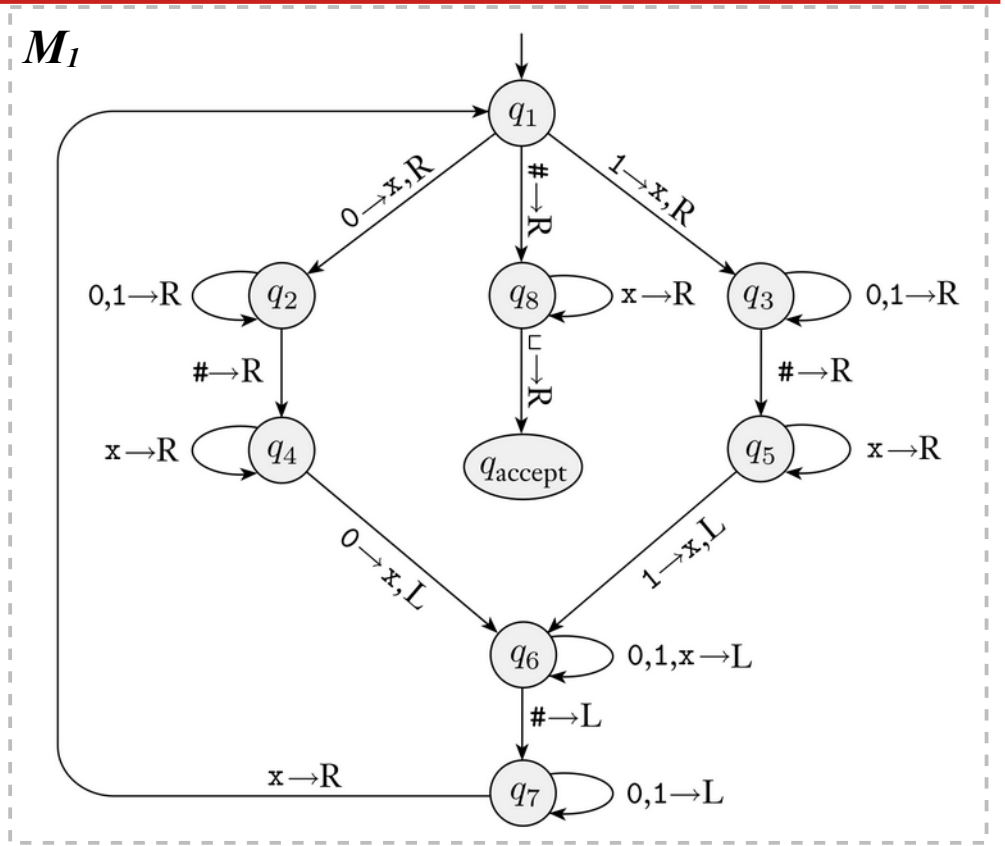

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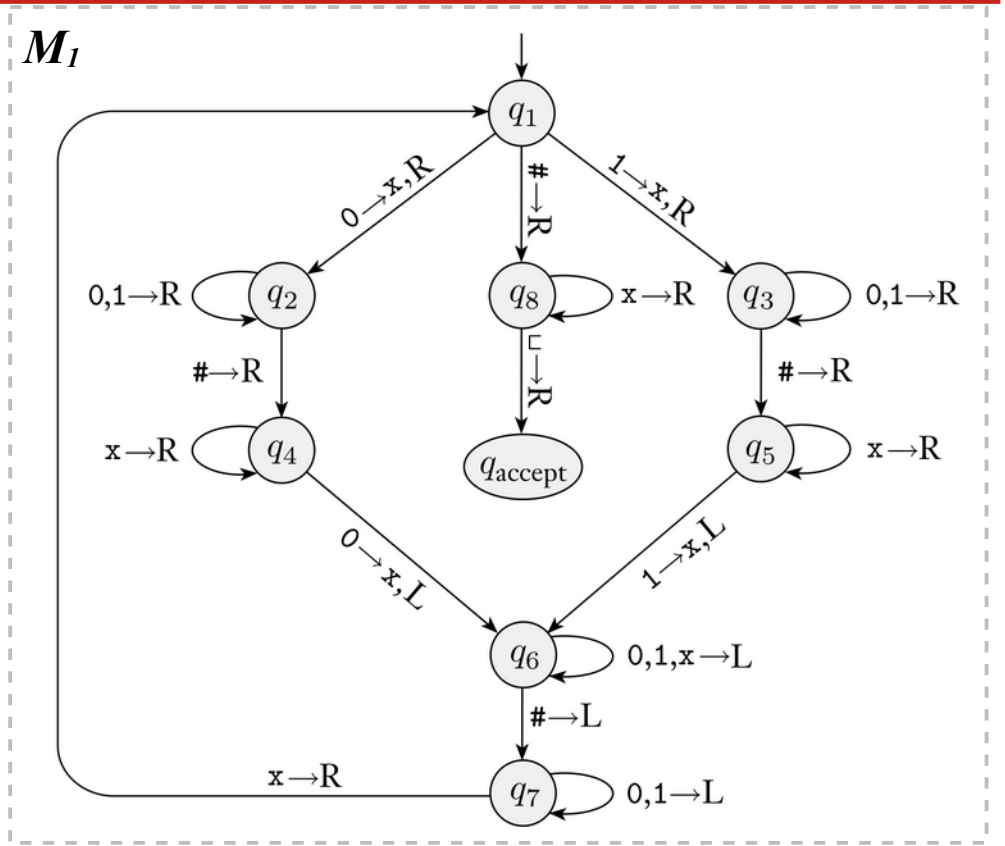

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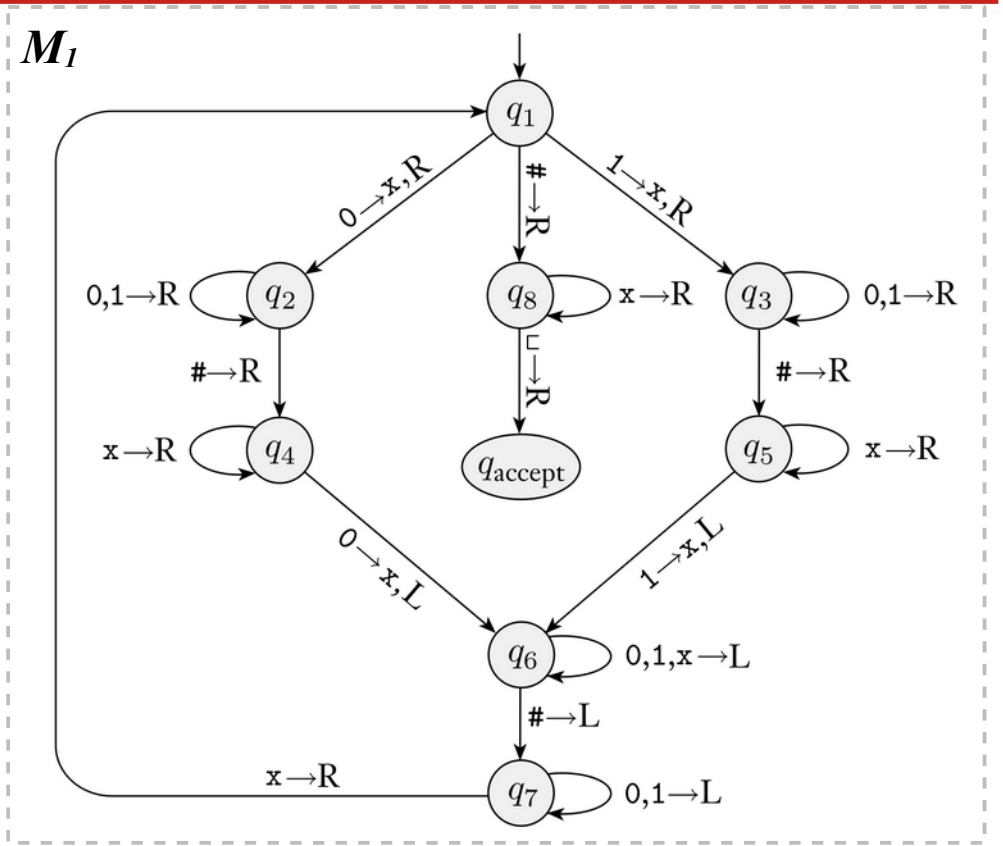

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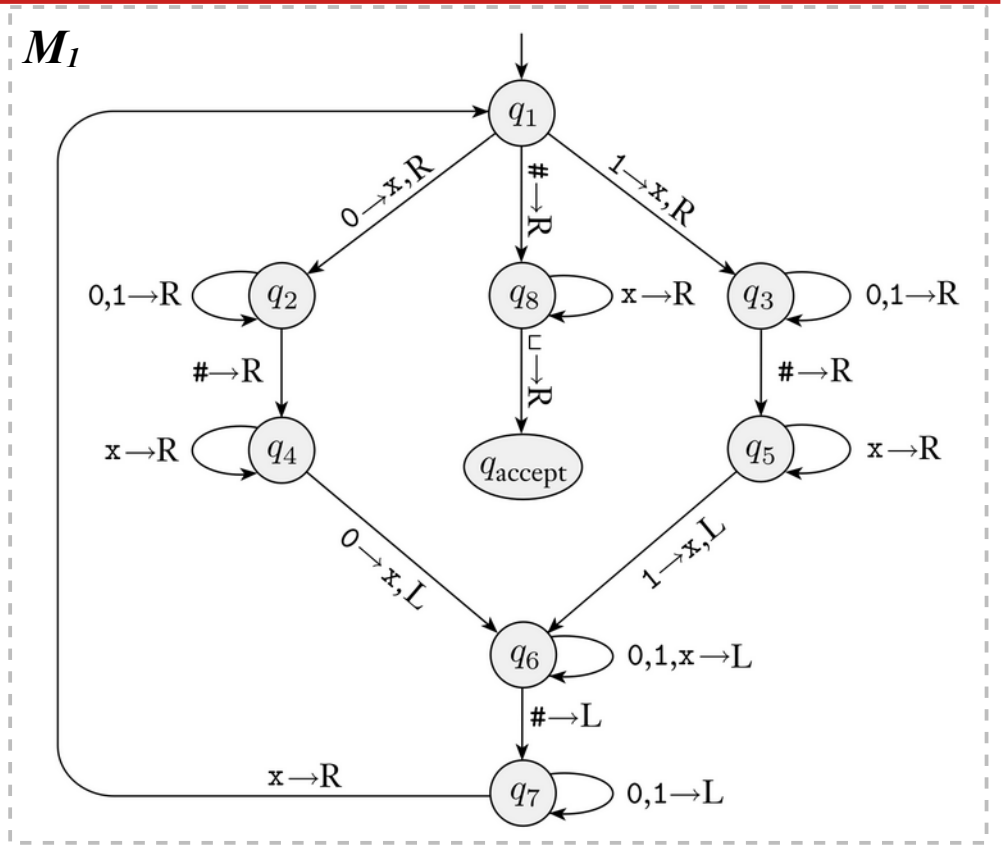

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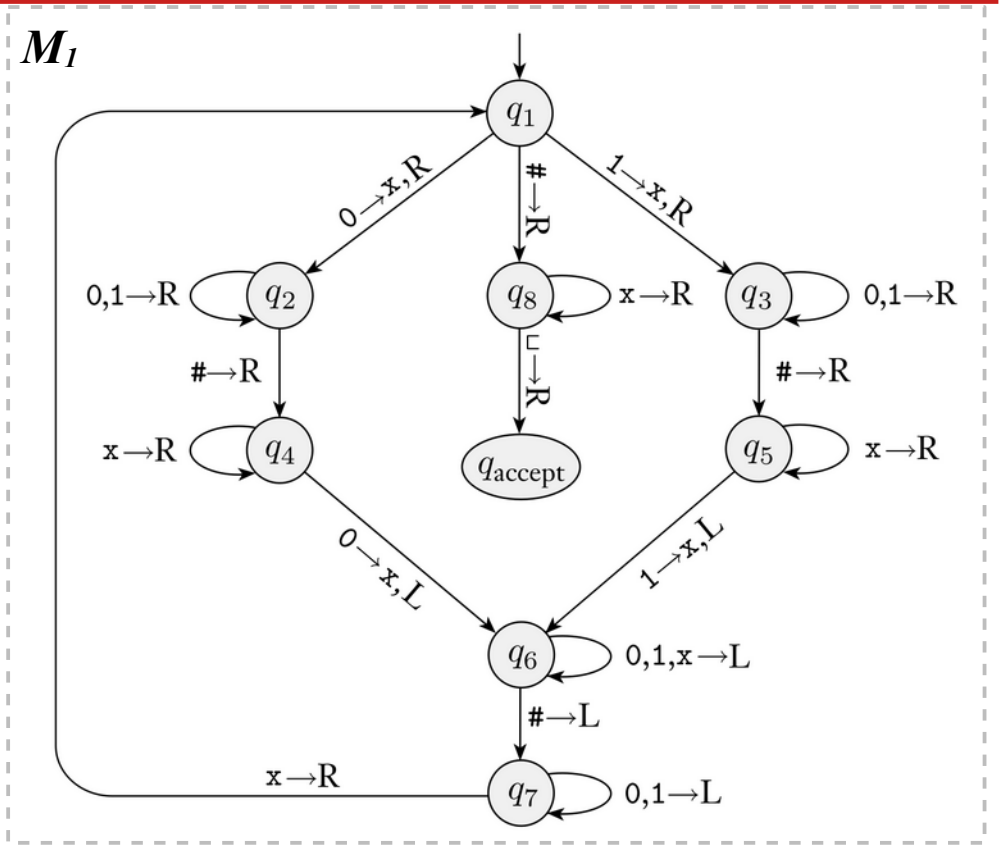
$xxx\#q_8xxx \Rightarrow$



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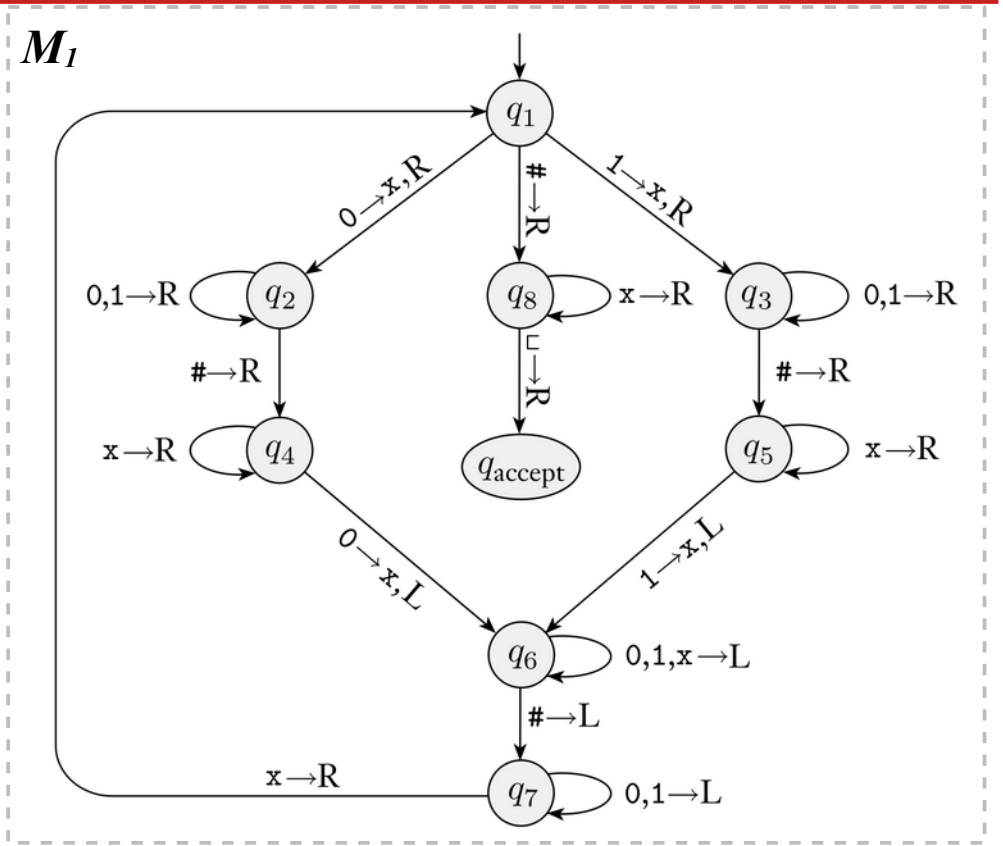

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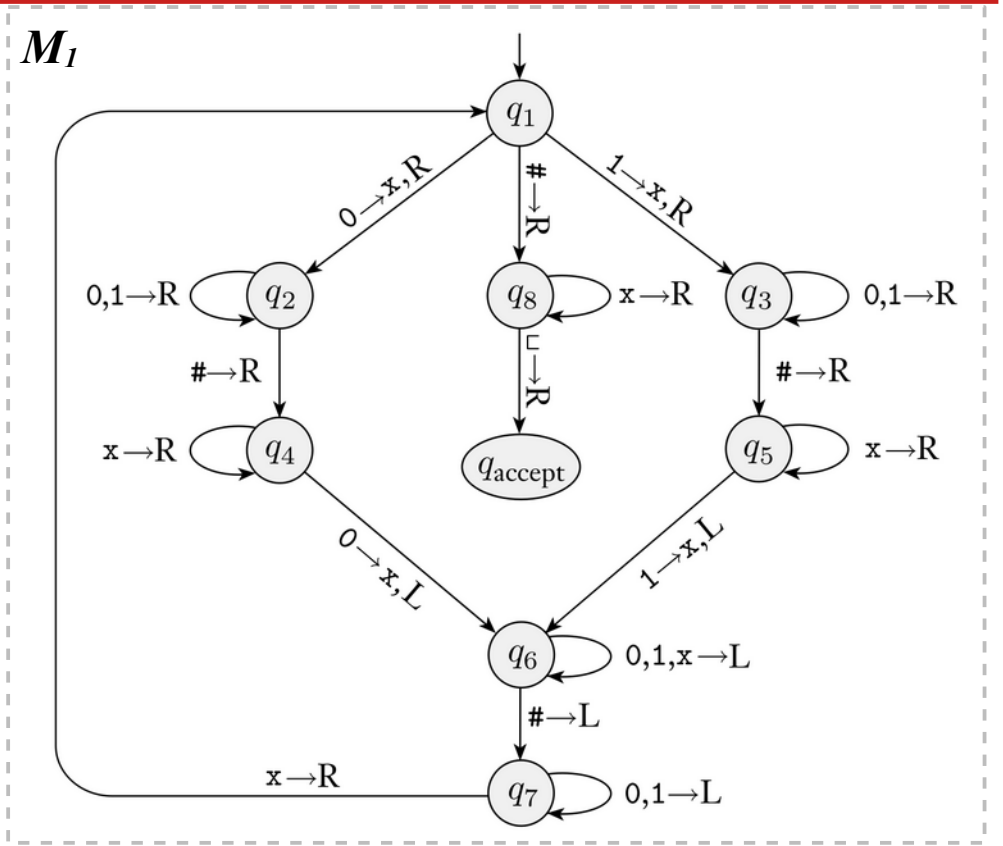

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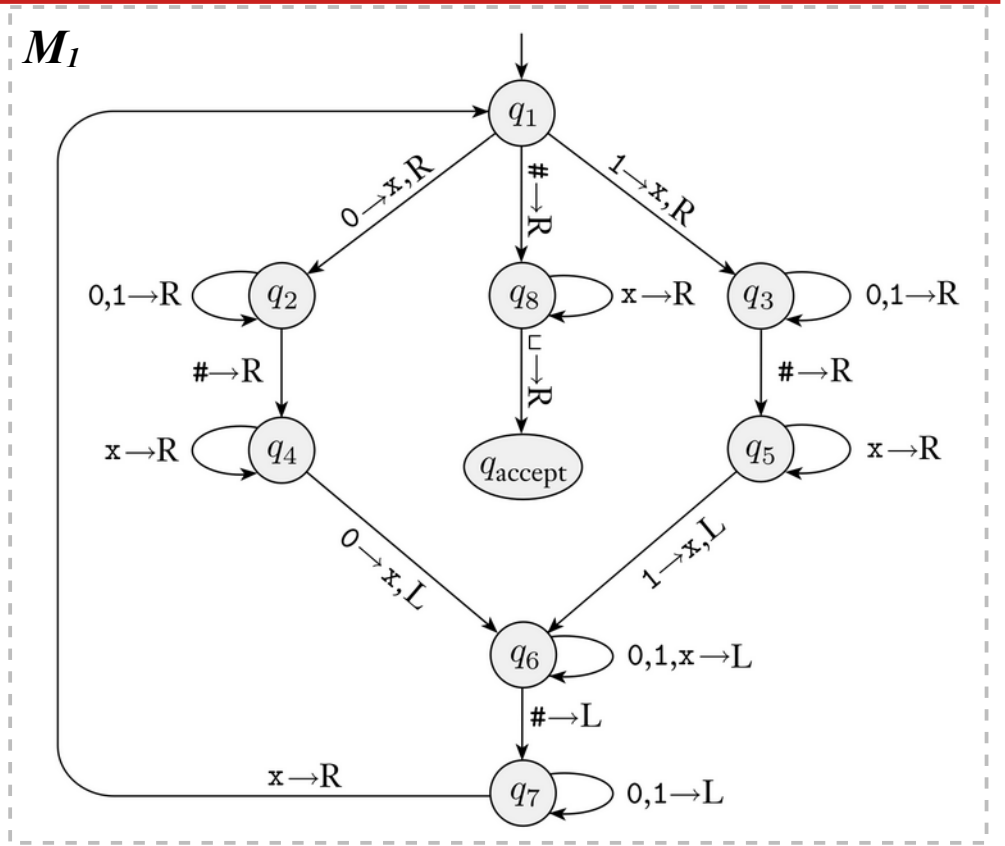

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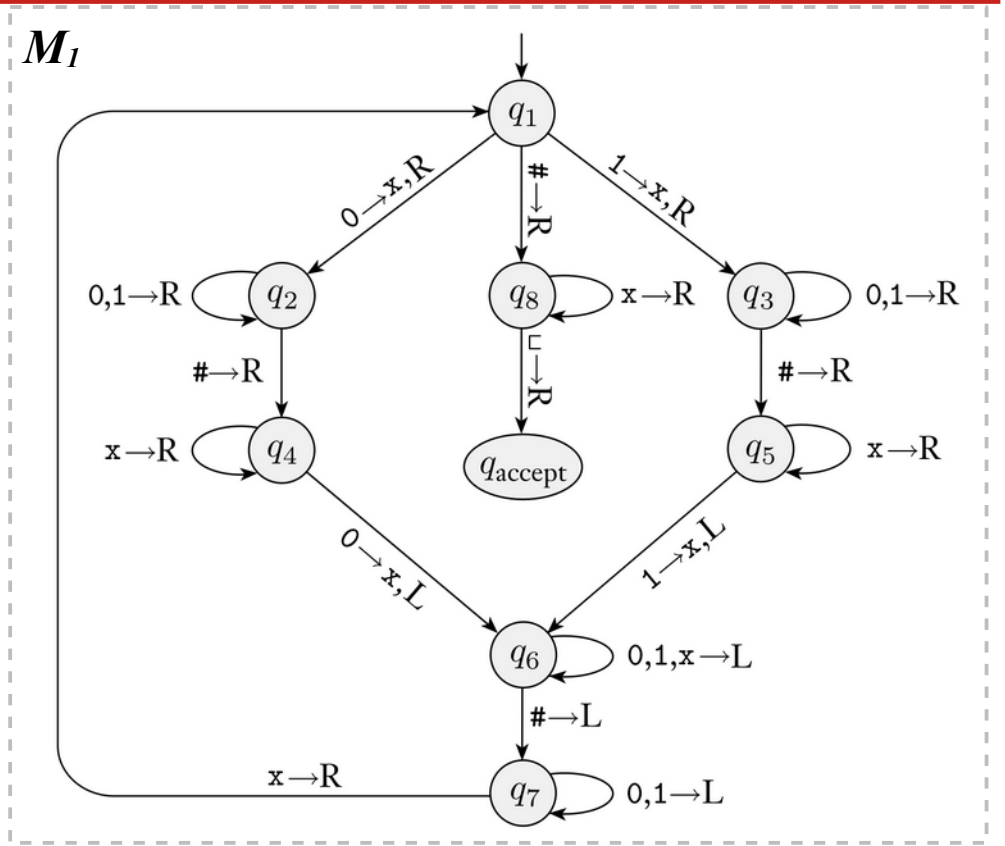

Homework

- **Exercises:**
  - **3.1, 3.2, 3.5, and 3.8a**
- **Read 3.2 and 3.3**