Q Determine whether each of these functions from $\{a, b, c, d\}$ to itself is one-to-one.

a) $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$

b) $f(a) = b$, $f(b) = b$, $f(c) = d$, $f(d) = c$

c) $f(a) = d$, $f(b) = b$, $f(c) = c$, $f(d) = d$

Sol

a) Yes

b) No

c) No

Q Which functions in the above exercise are onto?

Sol

a) Yes

b) No

c) No

Q Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one.

a) $f(n) = n - 1$

b) $f(n) = n^2 + 1$

c) $f(n) = n^3$

d) $f(n) = \lfloor \frac{n}{2} \rfloor$

Sol:

a) Yes

b) No

c) Yes

d) No

Q Which functions in above exercise are onto?

Sol:

a) Yes

b) No

c) No

d) Yes

Q Let $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c and d are constants. Determine necessary and sufficient conditions on a, b, c, d so that

$$f \circ g = g \circ f.$$

Sol:

$$f \circ g = f(cx+d)$$
$$= a(cx+d)+b$$
$$= acx + \underline{ad+b}$$

$$g \circ f = g(ax+b)$$
$$= c(ax+b)+d$$
$$= acx + \underline{bc+d}$$

So, the necessary and sufficient condition is,

$$\boxed{ad+b = bc+d}$$

Q What are the terms a_0, a_1, a_2 and a_3 of the sequence $\{a_n\}$, where a_n equals

a) $(-2)^n$

b) $7+4^n$

c) $2^n+(-2)^n$

Sol

a) $1, -2, 4, -8$

b) $8, 11, 23, 71$

c) $2, 0, 8, 0$

Q For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

a) 3, 6, 12, 24 ...

b) 15, 8, 1, −6, −13 ...

c) 3, 5, 8, 12, 17, 23 ...

d) 2, 16, 54, 128, 250 ...

e) 2, 3, 7, 25, 121, 721 ...

Sol:

a) $3 \cdot 2^{n-1}$ → This is a geometric series.

b) $22 - 7n$ → Arithmetic series.

c) $(n^2+n+4)/2$

d) $2n^3$

e) $n! + 1$