Q Show that each of these conditional statements is a TAUTOLOGY using truth tables.

i. $p \wedge q \rightarrow p$

ii. $p \rightarrow p \vee q$

iii. $\neg p \rightarrow (p \rightarrow q)$

iv. $(p \wedge q) \rightarrow (p \rightarrow q)$

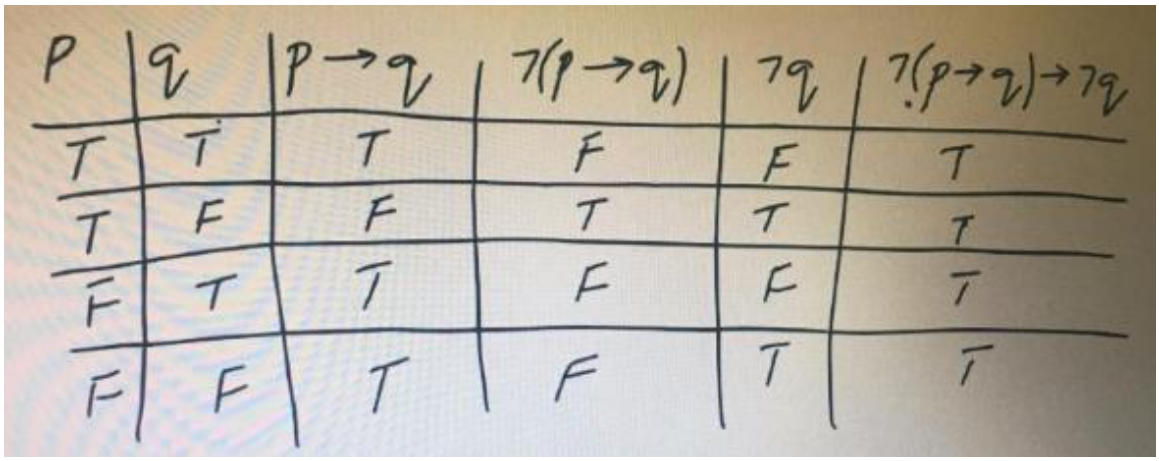
v. $\neg(p \rightarrow q) \rightarrow p$

vi. $\neg(p \rightarrow q) \rightarrow \neg p$

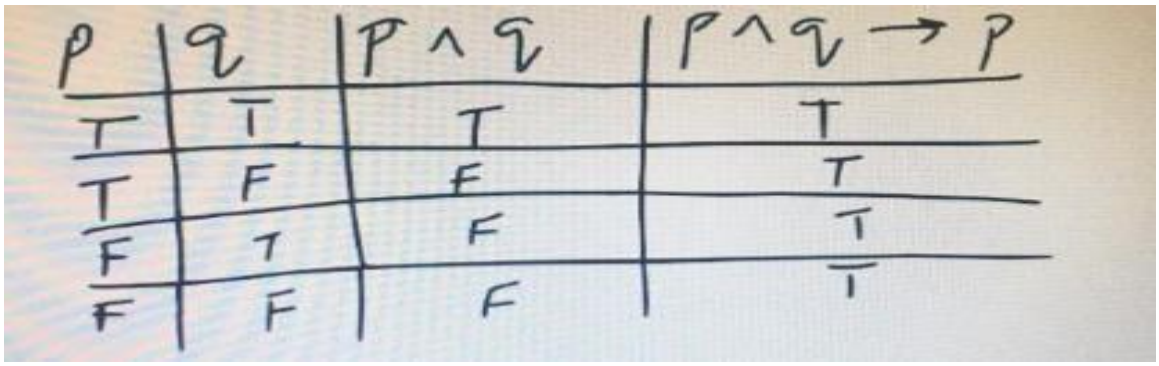
p	q	$\neg p$	$p \rightarrow q$	$\neg p \rightarrow (p \rightarrow q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	T	T

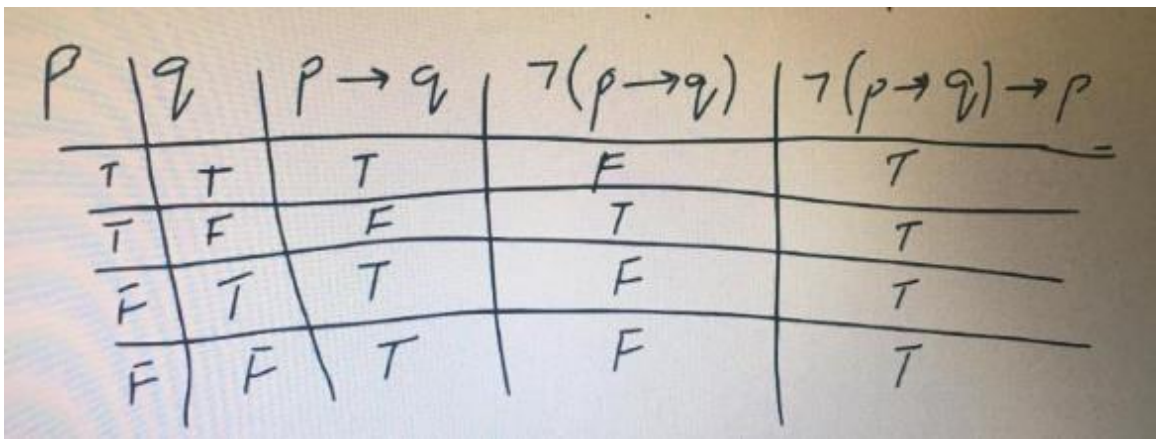
p	q	$p \vee q$	$p \rightarrow p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T



p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T



p	q	$p \wedge q$	$p \wedge q \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T



p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

Q: Show each conditional statement in the above question without using truth tables. (Use logical equivalences)

Sol: i, $(p \wedge q) \rightarrow p$

(Use logical equ

$\equiv \neg(p \wedge q) \vee p$

$\equiv (\neg p \vee \neg q) \vee p$

$\equiv (\neg p \vee p) \vee \neg q$

$\equiv T \vee \neg q$

$\equiv T$

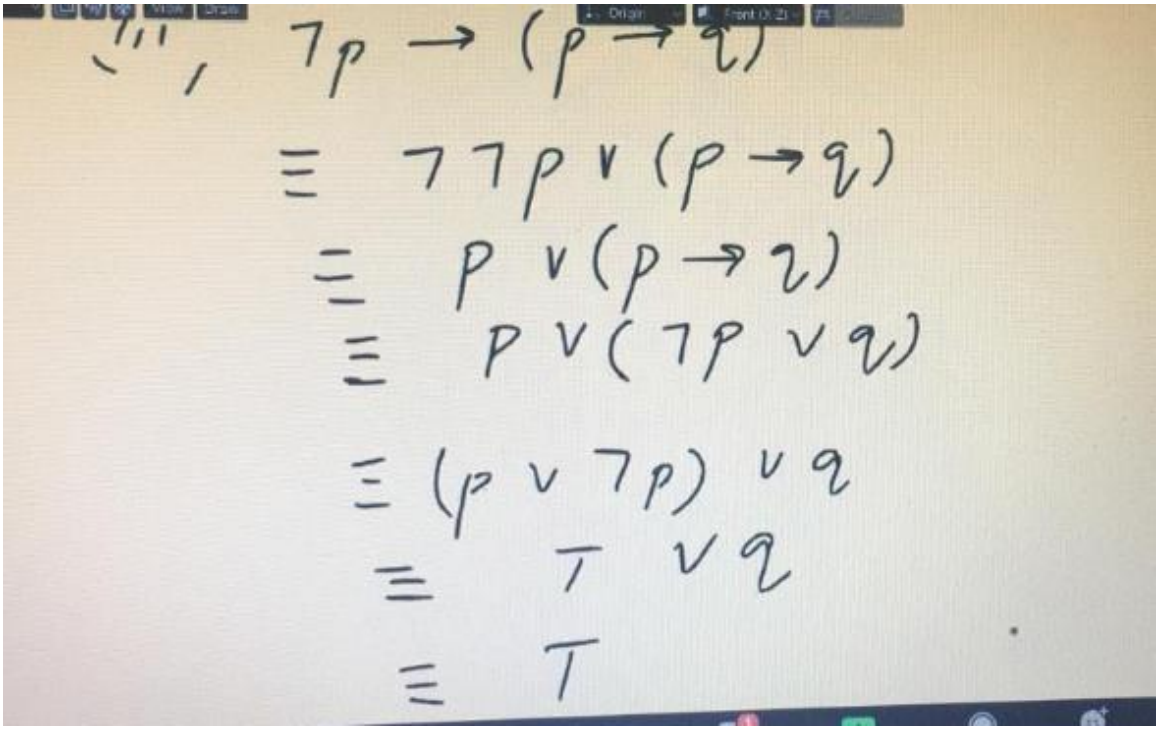
ii, $p \rightarrow (p \vee q)$

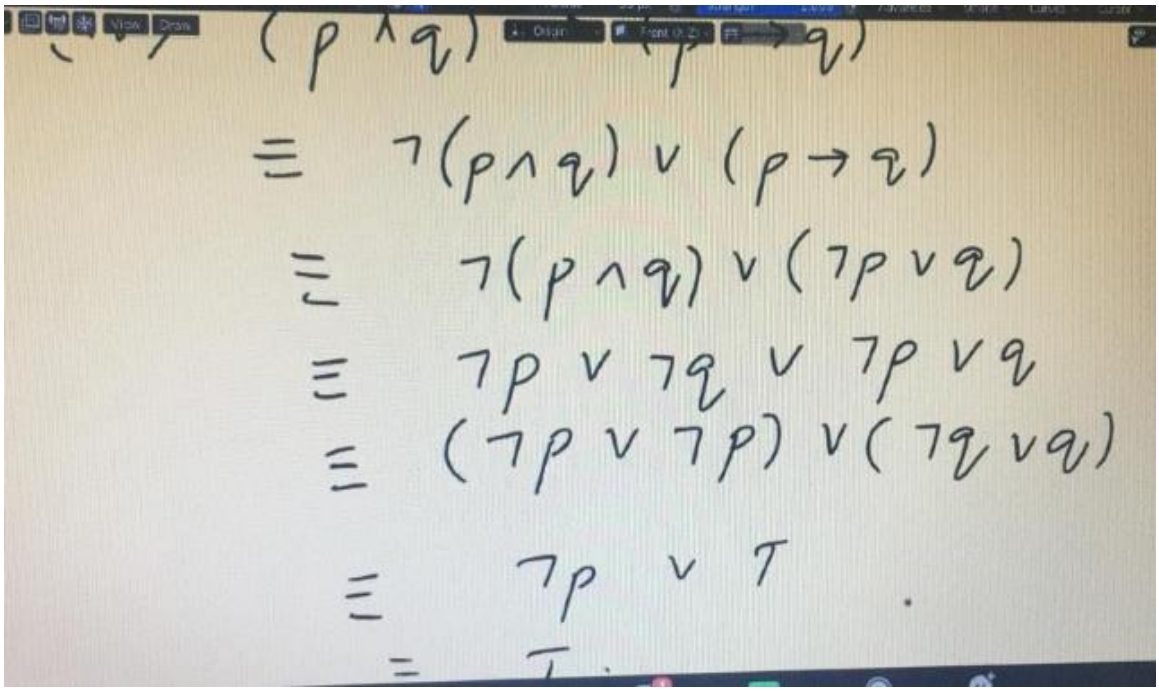
$\equiv \neg p \vee (p \vee q)$

$\equiv (\neg p \vee p) \vee q$

$\equiv T \vee q$

$\equiv T$

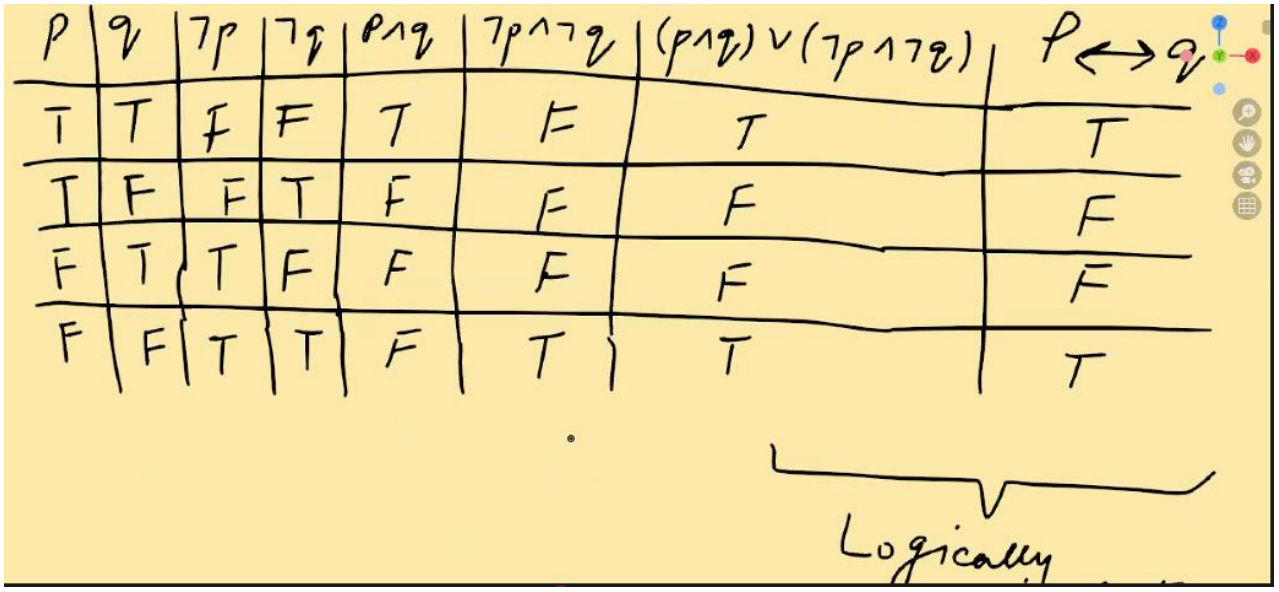

$\neg p \rightarrow (p \rightarrow q)$
$\equiv \neg\neg p \vee (p \rightarrow q)$
$\equiv p \vee (p \rightarrow q)$
$\equiv p \vee (\neg p \vee q)$
$\equiv (p \vee \neg p) \vee q$
$\equiv T \vee q$
$\equiv T$


$(p \wedge q) \rightarrow (p \rightarrow q)$
$\equiv \neg(p \wedge q) \vee (p \rightarrow q)$
$\equiv \neg(p \wedge q) \vee (\neg p \vee q)$
$\equiv \neg p \vee \neg q \vee \neg p \vee q$
$\equiv (\neg p \vee \neg p) \vee (\neg q \vee q)$
$\equiv \neg p \vee T$
$\equiv T$

$$\equiv \neg(\neg p \vee q) \rightarrow p$$
$$\equiv (\neg\neg p \wedge \neg q) \rightarrow p$$
$$\equiv (p \wedge \neg q) \rightarrow p$$
$$\equiv \neg(p \wedge \neg q) \vee p$$
$$\equiv (\neg p \vee q) \vee p$$
$$\equiv (\neg p \vee p) \vee q$$
$$\equiv T \vee q$$

(vi) $\neg(p \rightarrow q) \rightarrow \neg q$
$$\equiv \neg(\neg p \vee q) \rightarrow \neg q$$
$$\equiv (\neg\neg p \wedge \neg q) \rightarrow \neg q$$
$$\equiv (p \wedge \neg q) \rightarrow \neg q$$
$$\equiv \neg(p \wedge \neg q) \vee \neg q$$
$$\equiv (\neg p \vee \neg\neg q) \vee \neg q$$
$$\equiv \neg p \vee (q \vee \neg q)$$

Q Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent. [Use truth tables]



p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$p \leftrightarrow q$
T	T	F	F	T	F	T	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	F	T	T	T

Logically