KING SAUD UNIVERSITY

COLLEGE OF COMPUTER & INFORMATION SCIENCES
DEPT OF COMPUTER SCIENCE

CSC281 Discrete Mathematics	
Second Semester 1441 AH	(SPRING 2020)
Final Examination:	Thursday 7.05.2020 C.E. (duration = 3 hours)
Instructor:	Prof. Aqil Azmi

Instructions:

- Write your name, id and class serial number (if you remember it).
- Type your final answer in the designated space. Try showing your computation as much as possible.
- This is an open notes, open book final exam.
- Rename this file: ID-Firstname-Lastname.docx
- Upload your solution to Dropbox, https://www.dropbox.com/request/uBshVF61kKiJW1PX78Cy

S/N: **Name:** **ID:**

1. [Marks 2 each part carries equal weight]
Answer True or False. No need to state the reason.

	a.	For any positive integers a, b if $a^2 \nmid b \Rightarrow a \nmid b$
	b.	If $d \mid a$ and $a \mid b$ then $d \mid \gcd(a,b)$, where a, b, and d are all positive integers.
	c.	Let $f : A \to A$ be a 1-1 corresponding function. If $\lvert A \rvert = n$, then there can be n^n different functions f.
	d.	$\exists x \forall y (x + y = 5)$, where the universe of discourse is $\mathbb{R}$.

2. [Marks 2]

Calculate the following summation (show all details),

$$\sum_{k=1}^{100} (-1)^{k+1} \left(\frac{5 \times 5!}{4!} \right)^{1/2}$$

3. **[Marks 4]**
Suppose you have 10 boys, and 10 men. Calculate the following:

a. Count the number of ways to make a group of 10 people.

b. Count the number of ways to make a group of 10 people where a group cannot be all boys, or all men.

c. Count the number of ways to make a group of 10 people such that each group must have 5 boys and 5 men.

d. Count the number of ways to make a group of 10 people such that each group must have at least 3 boys.

4. [Marks 2]

Suppose we denote $(f \circ f)(x)$ by $f^{(2)}(x)$. Similarly, denote $(f \circ f \circ f)(x)$ by $f^{(3)}(x)$. Express the general formula for $f^{(n)}(x)$ if $f(x) = 3x + 4$, where $n \in \mathbb{Z}^+$.

5. **[Marks 2]**
A bag has 25 balls of four different colors: 10 red, 8 blue, 4 white, and 3 black. What is the minimum number of balls a blindfolded boy must pick up (with no replacement) so to guarantee that he has at least one ball of each color?

6. [Marks 3]

Suppose p and q are distinct primes. Find the general solution to the set of equations:

$$x \equiv -1 \bmod p$$

$$x \equiv -1 \bmod q.$$

Show all the steps/details.

7. [Marks 2]

Use the *extended binomial* expansion to expand (into 4 terms) and calculate the square root of 1.7, *i.e.* $(1 + 0.7)^{1/2}$.

8. [Marks 3]

Use mathematical induction to show that,

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$