King Saud University

College of Sciences

Department of Mathematics

106 Math Exercises

(1)

Anti-Derivatives & Indefinite Integrals

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ANTI-DERIVATIVES & INDEFINITE INTEGRALS

Definition(1):

$$F(x)\ is\ anti-derivative\ of\ f(x)\ on\ the\ interval\ I,$$
$$if\ F'(x) = f(x) : \forall x \in I\ .$$

$$If\ F(x) and\ G(x) are\ both\ antiderivative\ of\ f(x) \Rightarrow$$
$$F(x) - G(x) = c : c\ is\ a\ constant \in \mathbb{R}$$

Definition(1):

$$If\ F(x)\ is\ anti-derivative\ of\ f(x)\ on\ the\ interval\ I \Rightarrow$$
$$\int f(x)\,dx = F(x) + c : F'(x) = f(x) : \forall x \in I$$

Basic Rules of Integration:

$$\int 1\,dx = x + c$$

$$\int k\,dx = kx + c : k \in \mathbb{R}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c : n \neq -1, n \in \mathbb{Q}$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int \cos x\,dx = \sin x + c$$

$$\int sec^2 x\,dx = \tan x + c$$

$$\int csc^2 x\,dx = -\cot x + c$$

$$\int \sec x \tan x\,dx = \sec x + c$$

$$\int \csc x \cot x\,dx = -\csc x + c$$

$$\sec x = \frac{1}{\cos x} \quad , 1 + tan^2 x = sec^2 x$$

$$\csc x = \frac{1}{\sin x} \quad , 1 + cot^2 x = csc^2 x$$

Properties :

i)

$$\int [\, f(x) \pm g(x) \,]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

ii)

$$\int kf(x)\, dx = k \int f(x) dx : \; k \in \mathbb{R}$$

iii)

$$\frac{d}{dx} \int f(x)\, dx = f(x)$$

iv)

$$\int \frac{d}{dx} f(x) dx = f(x) + c$$

Q. Evaluate the following integrals :

1)

$$\int (3sinx + \sqrt{x}) \, dx$$

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2)

$$\int (tan^2 x + 5x) \, dx$$

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3)

$$\int (\frac{1}{x^{2/3}} + \frac{1}{\sqrt{x}}) \, dx$$

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4)

$$\int (\cos x + \sqrt[3]{x^2}) \, dx$$

5)

$$\int secx(tanx + secx)dx$$

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6)

$$\int (\, s^2 - 2s + 1\,)dx$$

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7)

$$\int (\, x^2 + \, sec^2x + 1\,)dx$$

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8)

$$\int (tan^2x - sec^2x)dx$$

9)

$$\int \left(\frac{3}{t^5} - 4t\right) dt$$

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10)

$$\int \frac{3x^2 + x}{\sqrt{x}}\, dx$$

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11)

$$\int \left(\frac{1}{\sec x} - \frac{1}{\csc x}\right) dx$$