جـــامـــعـــة
الملك سعود
King Saud University

Theory of Computation
CSC 339 – Spring 2021

Chapter-4: part1
Decidability

King Saud University

Department of Computer Science

Dr. Azzam Alsudais

Introduction

- **Turing-recognizable languages**
  - **A language L is called Turing-recognizable if there is a TM that recognizes it.**

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  - **The TM always halts on every input.**
  - **Also called recursive languages**

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- **Corresponding language: PRIMES = {1, 2, 3, 5, 7, …}**
- **Can we decide (solve) this problem?**
  - **If we can devise a TM (or an algorithm) that decides whether a given number is prime, then the language is decidable.**

Decidability

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- **Suggested Algorithm:**

```
On input x:
    - Divide x by all possible numbers between 2 and $\sqrt{x}$
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- **Problem: is X a prime number?**
- **Suggested Algorithm:**

```
On input x:
    - Divide x by all possible numbers between 2 and $\sqrt{x}$
    - If any of those numbers can divide x, then reject
    - Else, accept
```

Decidability

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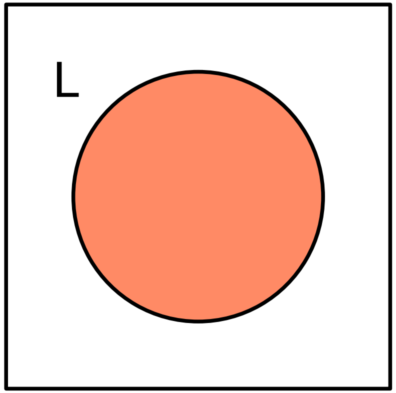
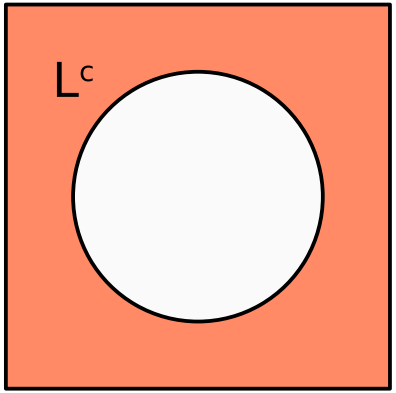




Decidability

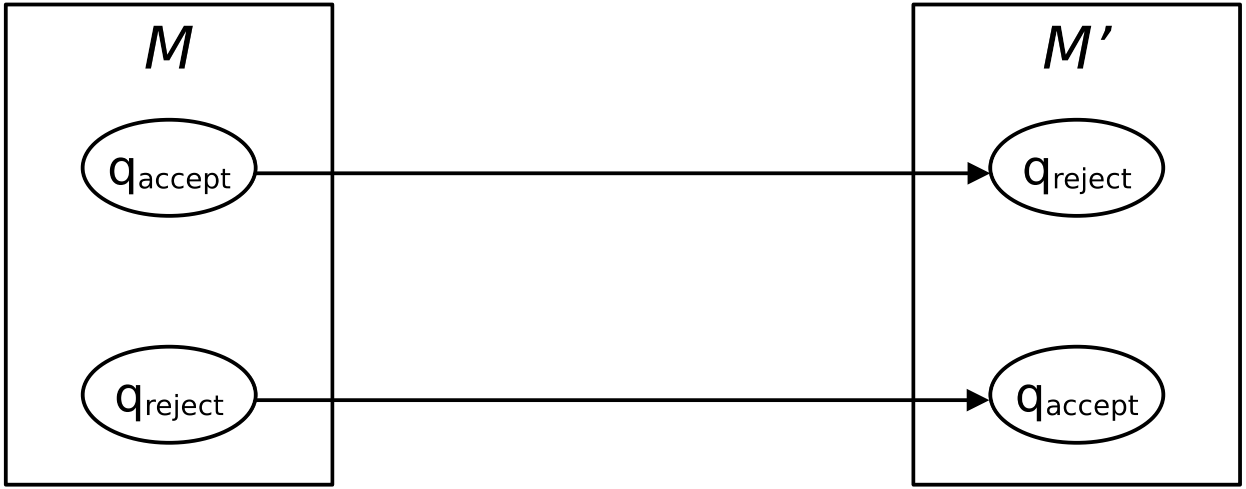
- **If L is decidable so is L^c.**

- **Given a TM M that decides L, how can we build TM M' that decides L^c?**

Decidability

- **If *L* is decidable so is *L*^{c}.**

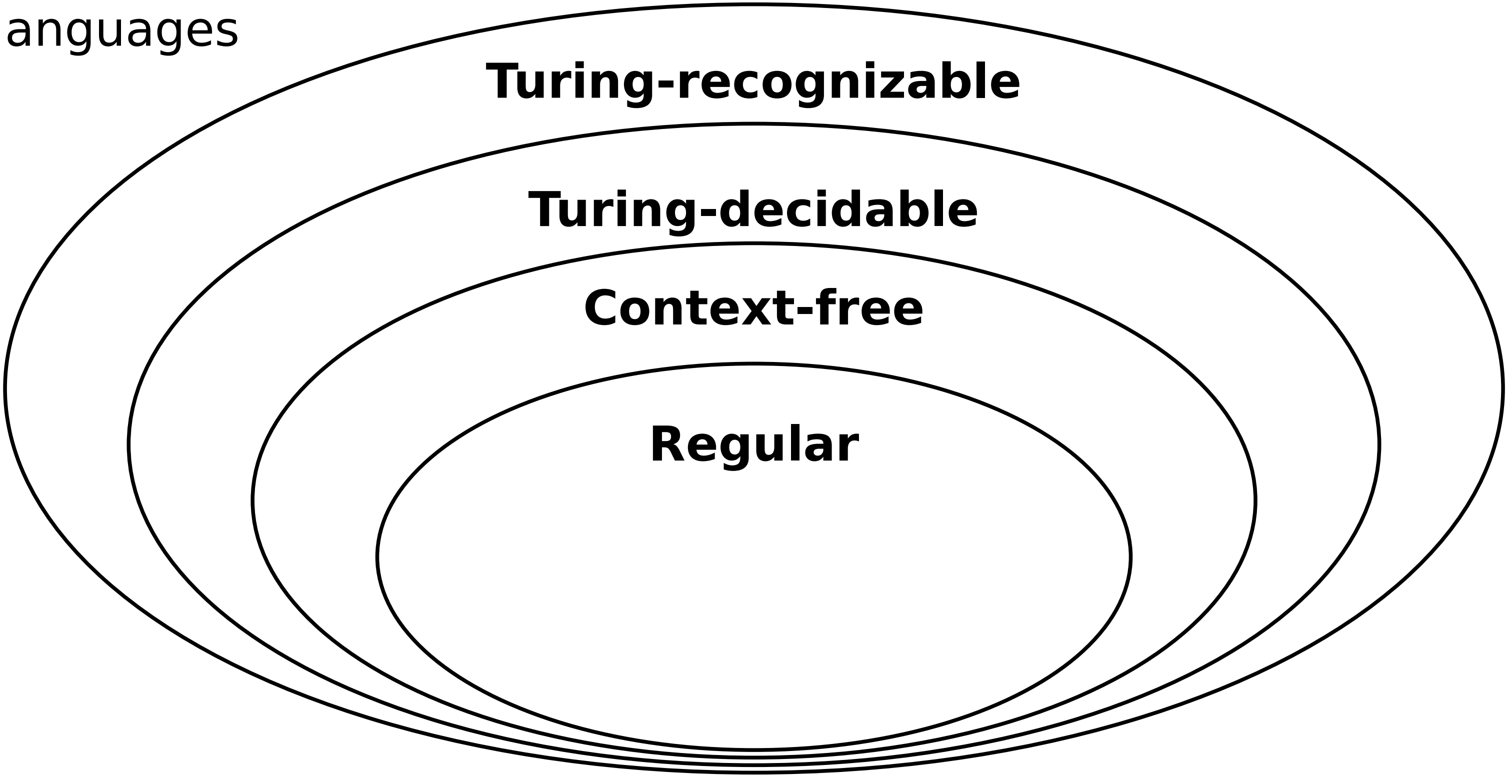
- **Given a TM *M* that decides *L*, how can we build TM *M'* that decides *L*^{c}?**



Replace every accept state with a reject state, and vice versa

Decidability

Classes of languages


Turing-recognizable
Turing-decidable
Context-free
Regular