King Saud University

College of Sciences

Department of Mathematics

106 Math Exercises

(6)

Inverse Trigonometric Functions

Malek Zein AL-Abidin

Inverse Trigonometric Functions

$' = \frac{d}{dx}$

$(sin^{-1}x)' = \frac{1}{\sqrt{1-x^2}}$, $(cos^{-1}x)' = \frac{-1}{\sqrt{1-x^2}}$: $|x| < 1$

$(tan^{-1}x)' = \frac{1}{1+x^2}$, $(cot^{-1}x)' = \frac{-1}{1+x^2}$

$(sec^{-1}x)' = \frac{1}{x\sqrt{x^2-1}}$, $(csc^{-1}x)' = \frac{-1}{x\sqrt{x^2-1}}$: $|x| > 1$

$u = u(x)$

$(sin^{-1}u)' = \frac{u'}{\sqrt{1-u^2}}$ $(cos^{-1}u)' = \frac{-u'}{\sqrt{1-u^2}}$: $|u| < 1$

$(tan^{-1}u)' = \frac{u'}{1+u^2}$, $(cot^{-1}x)' = \frac{-u'}{1+u^2}$

$(sec^{-1}u)' = \frac{u'}{u\sqrt{u^2-1}}$ $(csc^{-1}u)' = \frac{-u'}{u\sqrt{u^2-1}}$: $|u| > 1$

$$\int \frac{1}{\sqrt{a^2-u^2}}du = sin^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\,tan^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}du = \frac{1}{a}sec^{-1}\left(\frac{u}{a}\right) + c$$

Exercises

Q(1) Find $f'(x)$ for the following :

1) $f(x) = 2^{sin^{-1}x} + tan^{-1}(e^x)$

---

2) $f(x) = sec^{-1}(2^x) + \tan(sin^{-1}x)$

---

3) $f(x) = e^{tan^{-1}x} + cos^{-1}(e^x)$

4) $f(x) = tan^{-1}(lnx) + 2^{sec^{-1}x}$

---

5) $f(x) = 5^{\arcsin(x^2)} + cot^{-1}(4x^3 + 1)$

---

6) $f(x) = \frac{7^{x^2}}{sin^{-1}x} - \arctan(e^x) +$

7) $f(x) = x^{sec^{-1}(x)}$

Q(2) Evaluate the integrals:

1)

$$\int \frac{1}{\sqrt{e^{2x}-1}}\,dx$$

---

2)

$$\int \frac{1}{x\sqrt{x^4-1}}\,dx$$

---

3)

$$\int \frac{6x}{16+x^4}\,dx$$

4)

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{4-(sin^{-1}x)^2}}\,dx$$

---

5)

$$\int \frac{1}{x\sqrt{x-1}}\,dx$$

---

6)

$$\int \frac{1}{x\sqrt{1-(lnx)^2}}\,dx$$

7)

$$\int \frac{e^x}{e^{2x}+7}\,dx$$

---

8)

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx$$

---

9)

$$\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$$

10)

$$\int \frac{x+9}{x^2+9}\,dx$$

---

11)

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx$$

---

12)

$$\int \frac{x+2}{\sqrt{4-x^2}}\,dx$$

13)

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$$

---

14)

$$\int_0^{1/\sqrt{2}} \frac{arc\ sinx}{\sqrt{1-x^2}}\,dx$$

---

15)

$$\int_0^1 \frac{x^3}{4+x^8}\,dx$$

16)

$$\int \frac{cotx}{\sqrt{sin^2x - 1}}\,dx$$

---

17)

$$\int \frac{cosx}{\sqrt{9 - sin^2x}}\,dx$$

---

18)

$$\int \frac{1}{1 + 3x^2}\,dx$$

19)

$$\int \frac{x + tan^{-1}x}{1 + x^2}\, dx$$

---

20)

$$\int \frac{1 + x}{1 + x^2}\, dx$$

---

21)Find the value of $\sin\left(arc\ tan\frac{1}{2} + arc\ cos\frac{4}{5}\right)$