KING SAUD UNIVERSITY

COLLEGE OF COMPUTER & INFORMATION SCIENCES
DEPT OF COMPUTER SCIENCE

CSC281 Discrete Mathematics
Second Semester 1441 AH (SPRING 2020)
Final Examination: Thursday 7.05.2020 C.E. (duration = 3 hours)
Instructor: Prof. Aqil Azmi

Instructions:

- Write your name, id and class serial number (if you remember it).
- Type your final answer in the designated space. Try showing your computation as much as possible.
- This is an open notes, open book final exam.
- Rename this file: ID-Firstname-Lastname.docx
- Upload your solution to Dropbox, https://www.dropbox.com/request/uBshVF61kKiJW1PX78Cy

S/N: **Name:** **ID:**

1. [Marks 2 each part carries equal weight]

Answer True or False. No need to state the reason.

F	**a.**	For any positive integers a, b if $a^2 \nmid b$ ق $a \nmid b$
T	**b.**	If $d\|a$ and $a\|b$ then $d\|\gcd(a,b)$, where a, b, and d are all positive integers.
F	**c.**	Let $f : A$ ® A be a 1-1 corresponding function. If $\|A\| = n$, then there can be n^n different functions f.
F	**d.**	$x" y(x + y = 5), where the universe of discourse is ‹ .

2. [Marks 2]

Calculate the following summation (show all details),

$$\sum_{k=1}^{100} (-1)^{k+1} \left(\frac{5 \times 5!}{4!}\right)^{1/2}$$

$$\sum_{k=1}^{100} -1^{k}(-1)\left(\frac{5\times 5!}{4!}\right)^{\frac{1}{2}}$$

$$=\sum_{k=1}^{100} -1^{k}(-1)(5) = -5\sum_{k=1}^{100} -1^{k}$$

$$-5\left(\frac{-1^{100+1} - -1^{1}}{-1-1}\right) = -5(0) = 0$$

3. **[Marks 4]**
Suppose you have 10 boys, and 10 men. Calculate the following:
   a. Count the number of ways to make a group of 10 people.

$$20C10 = \frac{20!}{(20-10)!\times 10!} = 184756$$

   b. Count the number of ways to make a group of 10 people where a group cannot be all boys, or all men.

$$20C10 - (10C10 + 10C10) =$$
$$184756 - (1 + 1) = 184754$$

   c. Count the number of ways to make a group of 10 people such that each group must have 5 boys and 5 men.

$$10C5(10C5) = 252^2 = 63504$$

   d. Count the number of ways to make a group of 10 people such that each group must have at least 3 boys.

$$\sum_{n=3}^{10} \mathbf{10}\boldsymbol{Cn} \times (\mathbf{10})\boldsymbol{C}(\mathbf{10} - \boldsymbol{n}) = \mathbf{5019807}$$

4. [Marks 2]

Suppose we denote $(f \circ f)(x)$ by $f^{(2)}(x)$. Similarly, denote $(f \circ f \circ f)(x)$ by $f^{(3)}(x)$. Express the general formula for $f^{(n)}(x)$ if $f(x) = 3x + 4$, where n ż ¢$^{+}$.

$$\boldsymbol{f^{n}(x) = 3^{n}x + \sum_{k=0}^{n-1} 4(3)^{k}}$$

5. **[Marks 2]**
A bag has 25 balls of four different colors: 10 red, 8 blue, 4 white, and 3 black. What is the minimum number of balls a blindfolded boy must pick up (with no replacement) so to guarantee that he has at least one ball of each color?

The worst case scenario the boy will pick 10 red then 8 blue then 4 white then we just need one of 3 black balls to have a ball of each color so we will need :

10 + 8 + 4 + 1 = 23 balls

6. **[Marks 3]**
Suppose p and q are distinct primes. Find the general solution to the set of equations:

$$x - \quad 1 \bmod p$$

$$x - \quad 1 \bmod q.$$

Show all the steps/details.

Since p and q are distinct primes then we can apply the theorem

$$\boldsymbol{that\ if\ a\ \equiv b\ mod\ p\ and\ a\ \equiv b\ mod\ q\ then\ a\ \equiv b\ mod\ pq}$$

Then $\boldsymbol{x\ \equiv -1\ mod\ pq}$ so the general formula is

$$\boldsymbol{x = -1 + pqK\ ; K \in Z}$$

7. **[Marks 2]**
Use the *extended binomial* expansion to expand (into 4 terms) and calculate the square root of 1.7, *i.e.* $(1 + 0.7)^{1/2}$.

$$\mathbf{1.7^{\frac{1}{2}} = (1 + 0.7)^{\frac{1}{2}} = \sum_{k=0}^{\infty} \frac{1}{2} CK \times 0.8^{k} =}$$

$$\mathbf{1 + \frac{1}{2}(0.7) + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2!} 0.7^{2} + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right)}{3!} 0.7^{3}}$$

$$\mathbf{= 1 + 0.35 - 0.06125 + 0.0214375 = 1.3101875}$$

8. [Marks 3]

Use mathematical induction to show that,

$$1 \times 2 + 2 \times 3 + 3 \times 4 + L + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

$$let\ p(n) = n(n+1) = \frac{n(n+1)(n+2)}{3}$$

1- Base case (n = 1):

LHS: 1(1+1) = 2

RHS: $\frac{1(1+1)(1+2)}{3}$ = 2

Since RHS = LHS then p(n) is true

2- Inductive case:

Assume p(n) is true then p(n+1) , we show that p(n+1) is also true

LHS of p(n+1) = n+1(n+1+1) = (n+1)(n+2)

= n(n+1) + (n+1)(n+2) = $\frac{n(n+1)(n+2)}{3} + (n+1)(n\ +2) =$

$\frac{n(n+1)(n+2)+3(n+1)(n+2)}{3} = \frac{(n+1)(n+2)[n+3]}{3} = RHS\ of\ p(n+1)$