King Saud University

College of Science

Department of Mathematics

106 Math Exercises

(3-1)

RIEMANN SUM

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Sums And Sigma Notation

Let a_k , b_k , $C \in \mathbb{R}$

$$\sum_{k=1}^{n}(a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$$

$$\sum_{k=1}^{n} C\, a_k = C\sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n} C = nC$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$(a+b)^2 = a^2 + 2ab + b^2$, $(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$, $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^2 - b^2 = (a-b)(a+b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Exercises

1) Find the value of α such that

$$\sum_{k=1}^{n}(k+\alpha) = \frac{n^2}{2} \quad : (n \geq 1)$$

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2) Find the value of α such that

$$\sum_{k=1}^{4}(\alpha k+1) = 14$$

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3) Find the sum

$$\sum_{k=1}^{n}\left(\frac{1}{2}-k\right) =$$

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4) Find the sum

$$\sum_{k=1}^{n}(k+3) =$$

5) Find the sum

$$\sum_{k=1}^{4} k(k^2 - 1) =$$

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6) Find the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2k}{n^2}$$

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7) Find the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{5k}{n^2}$$

8) Find the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{(6k+2)}{n^2}$$

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9) Find the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{(k-1)^2}{n^3}$$

Norm of Partition :

Let $P = \{x_0, x_1, x_2, \dots, x_n\}$ in the interval [a,b] $\Rightarrow$

$[x_0, x_1], [x_1, x_2], [x_2, x_3], \dots, [x_{k-1}, x_k]$ are sub-intervals of [a,b]

$$\Delta x_k = x_k - x_{k-1} \qquad : k = 1,2,3, \dots, n$$

$$\|P\| = max\,\{\Delta x_1, \Delta x_2, \Delta x_3, \dots, \Delta x_n\}$$

$\|P\|$ *called a norm of partition P of the interval*$[a, b]$.

Riemann Sum:

Let $f(x)$ be defined on the closed interval $[a, b]$,

Let $P = \{x_0, x_1, x_2, \dots, x_n\}$ be a partition of $[a, b]$,

Let $w_k \in [x_{k-1}, x_k]$, $k = 1,2,3, \dots, n$.

Riemann sum of $f(x)$ for *a partition P* is

$$R_p = f(w_1)\Delta x_1 + f(w_2)\Delta x_2 + f(w_3)\Delta x_3 + \cdots + f(w_n)\Delta x_n$$

$$R_p = \sum_{k=1}^{n} f(w_k)\,\Delta x_k$$

For a regular partition of the interval $[a, b]$, $\Delta x = \frac{b-a}{n} \Rightarrow$

$$R_p = \sum_{k=1}^{n} f(w_k)\Delta x$$

Area under the graph of a function:

If $f(x) \geq 0 : x \in [a, b]$ ⇒ Area under the graph of a function $f(x)$

$$Area = A = \lim_{\|P\| \to 0} R_p = \lim_{n \to \infty} \sum_{k=1}^{n} f(w_k)\, \Delta x$$

Definite Integral:

If $f(x)$ is a continuous function on [a,b]

$$\int_a^b f(x)dx = \lim_{\|P\| \to 0} R_p = \lim_{n \to \infty} \sum_{k=1}^{n} f(w_k)\, \Delta x$$

If $f(x) \geq 0 : x \in [a, b]$ ⇒

$\int_a^b f(x)dx = A$, *where A is the* Area under the graph of a function $f(x)$

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10)Find the norm of the partition $P = \{0, 0.3, 0.8, 1\}$ of the interval [0,1] ?

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11)Find the Riemann sum R_p for the function $f(x) = 2x + 1$ on the partition

$P=\{-1,0,2,3,4.5,5\}$ of the interval [-1,5] by choosing on each subinterval of P :

(a) The left-hand end point , $w_k = x_{k-1}$

(b) The right-hand end point , $w_k = x_k$

(c) The mid-point , $w_k = \frac{x_{k-1}+x_k}{2}$

12)Find the area under the curve of the function $f(x) = 3x + 1$, on the

interval [1,3] using Riemann sum R_p (regular partition) by choosing on each subinterval of P :

(a) The left-hand end point , $w_k = x_{k-1}$

(b) The right-hand end point , $w_k = x_k$

(c) The mid-point , $w_k = \frac{x_{k-1}+x_k}{2}$

13) Use Riemann sum to find the integral

$$\int_{0}^{6} (3 - x)dx$$

14)Use Riemann sum to find the integral

$$\int_{-1}^{2} (9 - x^2)dx$$

15) Find the definite integral representing $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{3+x}\, \Delta x$ using regular partition of the interval $[1,2]$