Final Exam, Spring 2020 **Thursday April 30th, 2020** **Exam time: 09:00-12:00**

Student's name:..... **ID:** ..... **Section:** .....

Problem 1 (6 points)

For each of the question below, circle either T (for **True**) or F (for **False**). **No explanations** are needed. Incorrect answers or unanswered questions are worth zero points.

T F Consider a communication network of nodes where node v needs to broadcast a single message to all the other nodes efficiently. The message should be sent to the shortest paths tree from v.

T F Prim's algorithm is a greedy solution of the Minimum Spanning Tree problem.

T F Kruskal algorithm solves the Minimum Spanning Tree problem.

T F Let T be the minimum spanning tree of a connected, undirected, and weighted graph G = (V, E). If e is a new edge, then $T \cup \{e\}$ contains a cycle.

T F Let G be an edge-weighted directed graph with source vertex s and let T be a shortest path tree from s. Suppose we add a positive constant p to the cost of every edge in G. T remains a shortest path tree from s.

T F Let *G = (V, E)* be a weighted graph and let *M* be a minimum spanning tree of *G*. The path in *M* between any pair of vertices v_1 and v_2 must be a shortest path in *G*

T F Bellman-Ford algorithm works on all graphs with negative-cost edges.

T F Dijkstra's algorithm works on graphs with negative-cost edges.

T F Let G be an edge-weighted directed graph with source vertex s, and let T be a shortest path tree from s. If we add a positive constant p to the cost of every edge incident on s in G. T remains a shortest path tree from s.

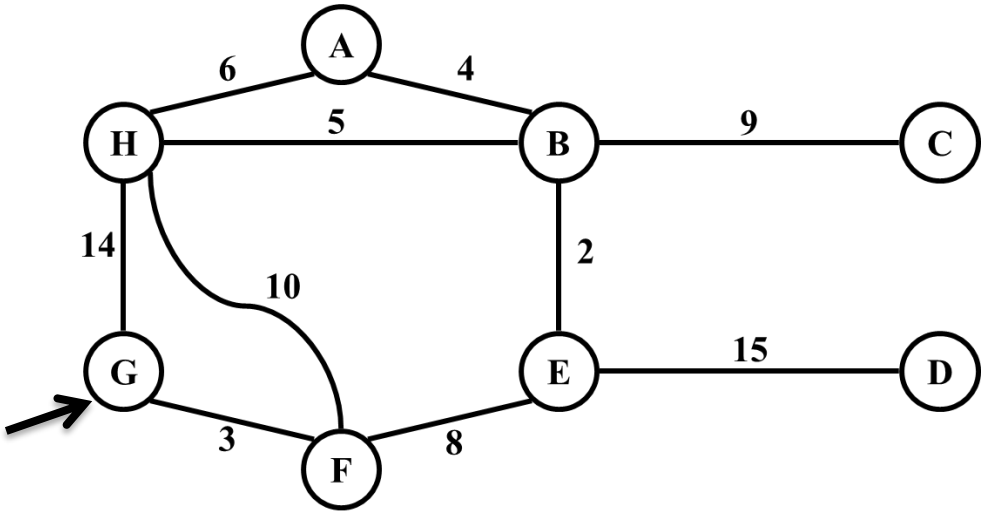
T F If someone was able to give an exponential time lower bound for a problem that is *NP-complete*. Then this would imply that *P* is not equal to *NP*.

T F There exist a polynomial time algorithm to determine whether an undirected graph contains a clique of size 3.

T F Suppose problem P_1 can be reduced to problem P_2 in linear time (i.e.,$P_1 \propto O(n)$ P_2). Then, if there exists a polynomial time algorithm for P_1, there exists a polynomial time algorithm for P_2.

Problem 2 (7 points)

For each of the algorithm below, list the edges of the Minimum Spanning Tree for the graph in the order selected by the algorithm. Then, draw the obtained MST.

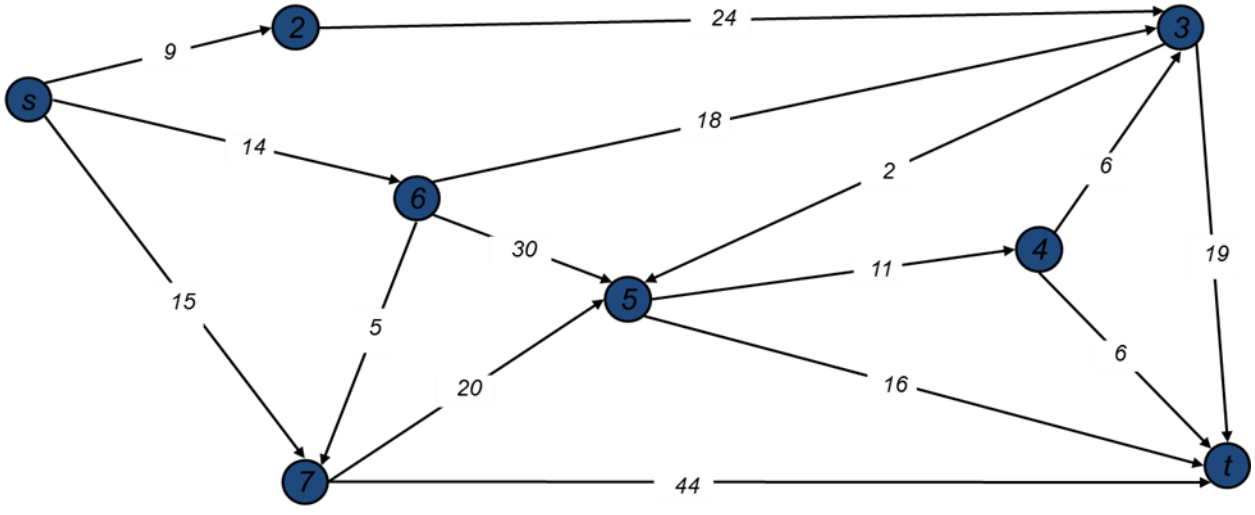


a- Prim's algorithm

b- Kruskal algorithm

Problem 3 (3 points)

Apply Bellman Ford algorithm on the graph below to solve the single source shortest path starting from vertex S. You need to show the results of each step with all details (hint: use a table as we did in class). Also, you need to show the final results.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Problem 4 (4 points)

A tow truck is moving through a straight one-way highway from location **A** to location **B**. The driver has received many requests from customers who need to tow their cars along this highway. Each customer **C** is defined by two numbers **C.from** and **C.to**. The customer C is located along the highway at position **C.from** units away from **A**, and wants to tow his car along the highway to the location that is **C.to** units away from **A** (**C.from** < **C.to**). Customers are charged a fixed amount (100 SR) regardless of the distance. The tow truck cannot carry more than one car at the same time. Also, the driver cannot drive back.

Assume that the customer information are available in an array **C**, i.e., **C[i].from** and **C[i].to** are the pickup and the destination of the i^{th} customer, respectively.

a- Describe an algorithm that assists the tow truck driver to maximize his profit. The algorithm should print the **.from** and **.to** properties of customer cars that should be towed.

b- What is the time complexity of your algorithm? Which programming design technique did you use?