King Saud University

College of Sciences

Department of Mathematics

106 Math Exercises

(16 & 17)

Area & Volume

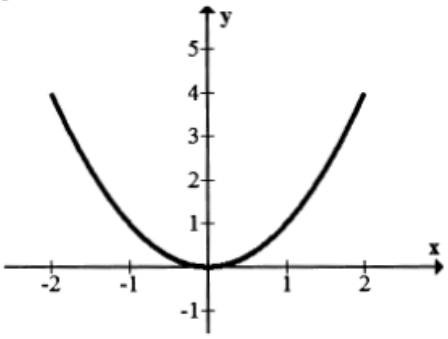
By

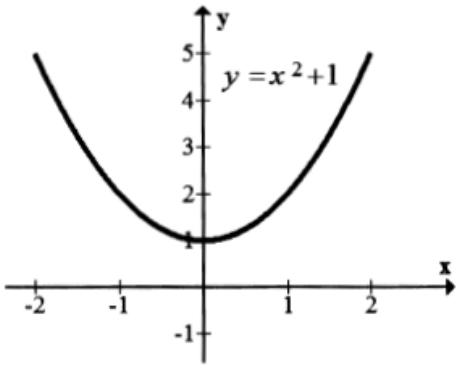
MALEK ZEIN AL-ABIDIN

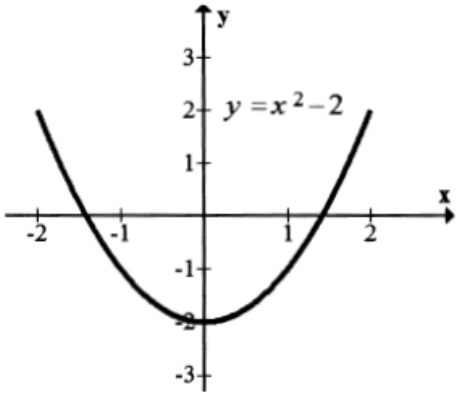
$$\frac{1440\ H}{2019\ G}$$

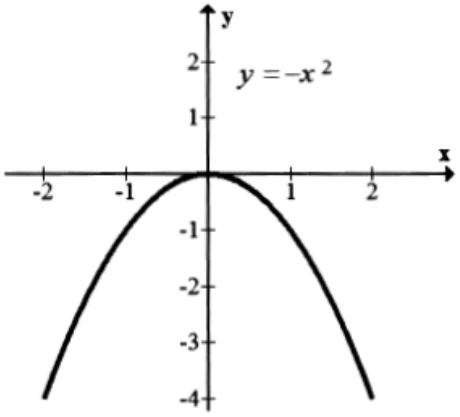
List of Famous Curves

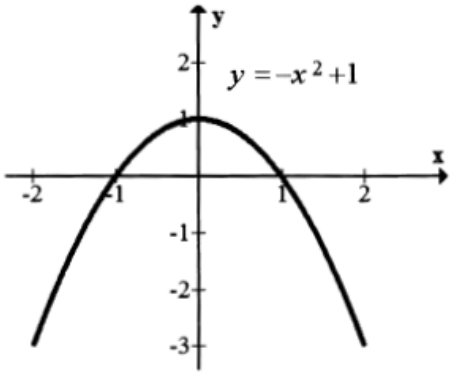
$y = x^2$

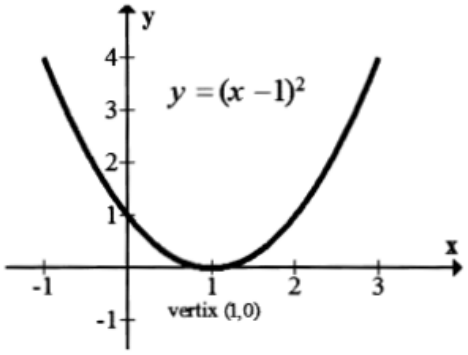


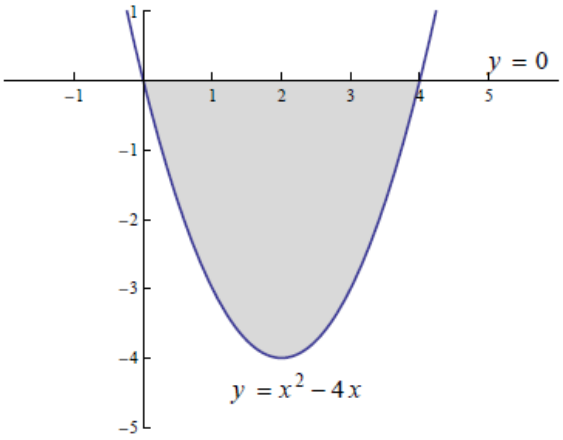


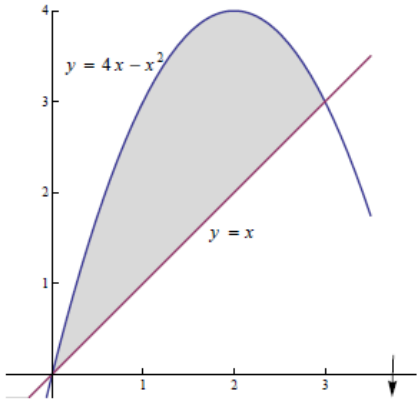


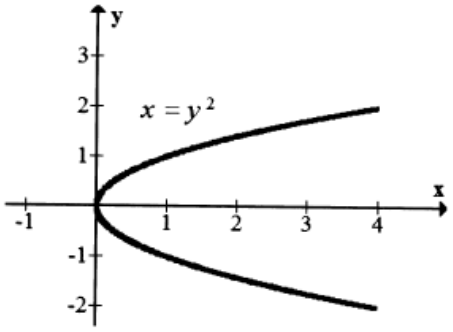


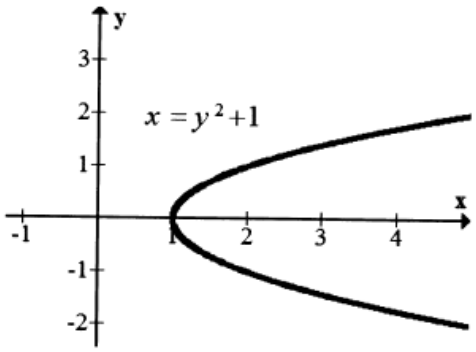


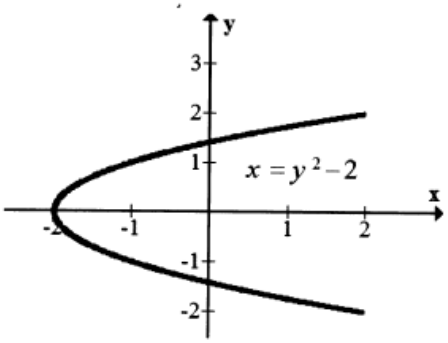


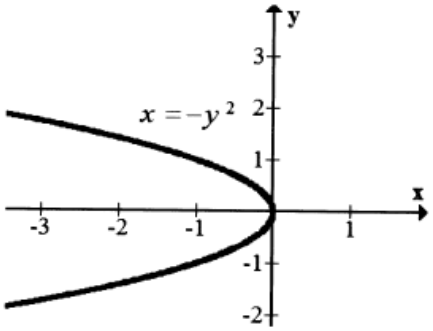


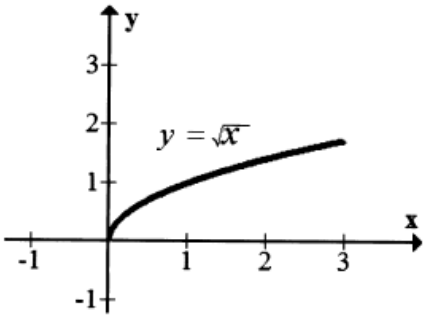


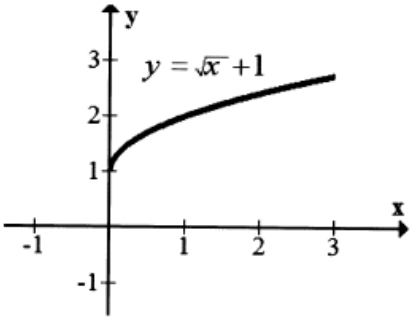

y
x
$x = y^2$

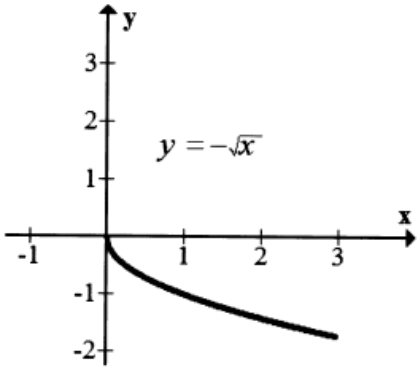

y
x
$x = y^2 + 1$

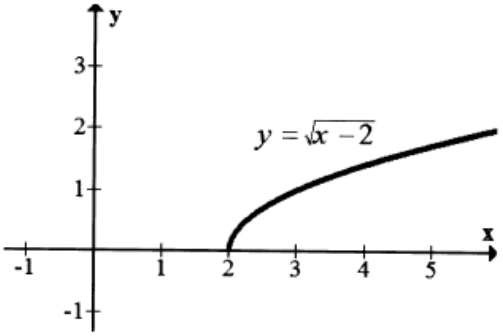

y
x
$x = y^2 - 2$

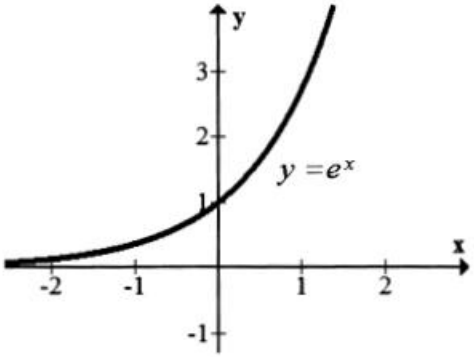

y
x
$x = -y^2$

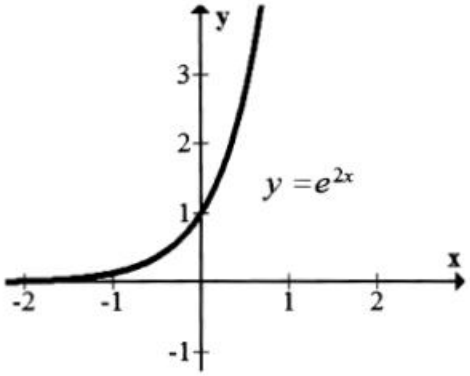

y
x
$y = \sqrt{x}$

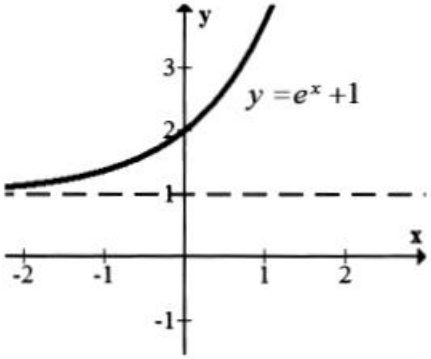

y
x
$y = \sqrt{x} + 1$

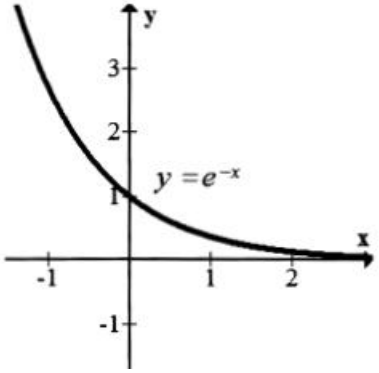

y
x
$y = -\sqrt{x}$

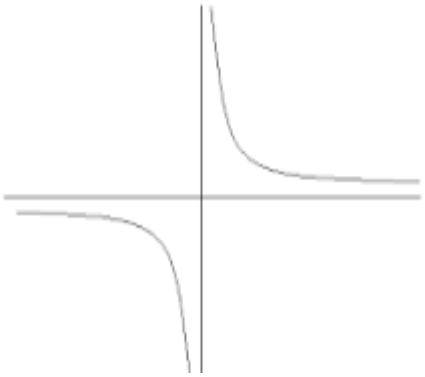

y
x
$y = \sqrt{x - 2}$



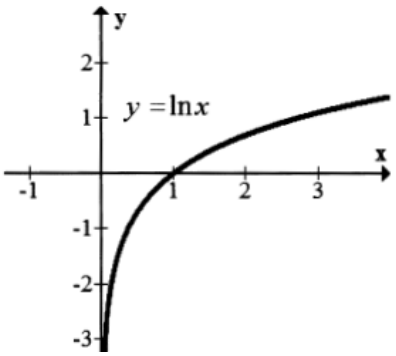


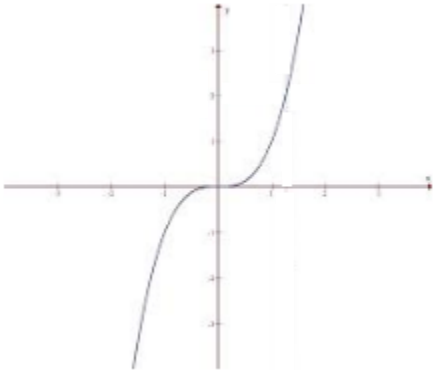




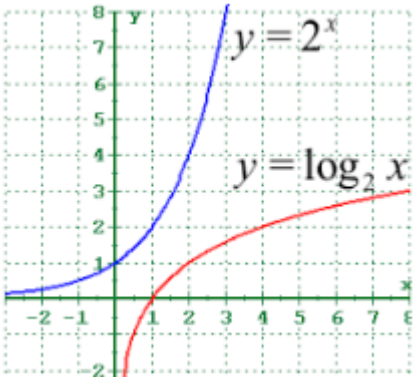


$y = \frac{1}{x}$

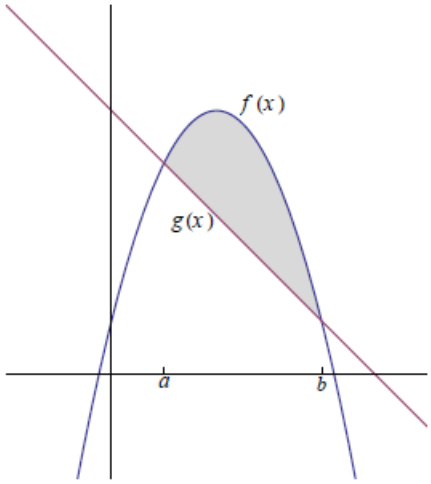




$y = x^3$



AREA BETWEEN CURVES

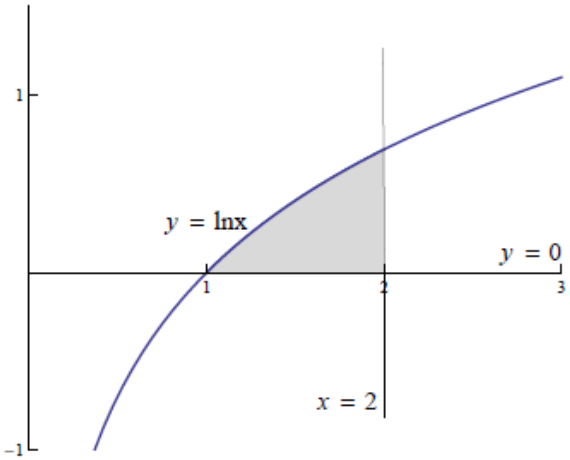


In the above figure the graphs of $f(x)$ and $g(x)$ intersect at the points $x = a$ and $x = b$.
The area bounded by the graphs of the curves of $f(x)$ and $g(x)$ equals

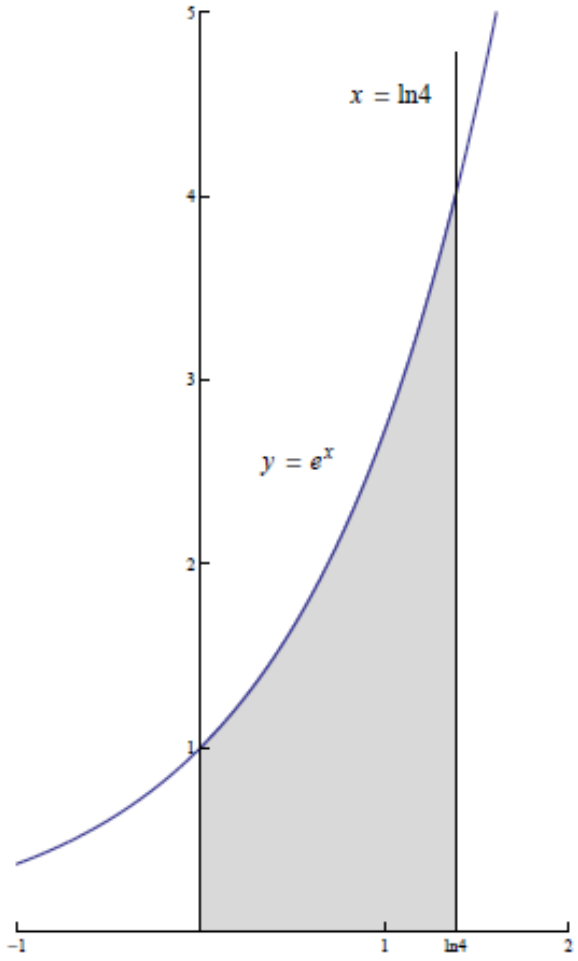
$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b [f(x) - g(x)]\,dx$$

Examples : Set up integrals to evaluate the areas bounded by the graphs of the curves of :

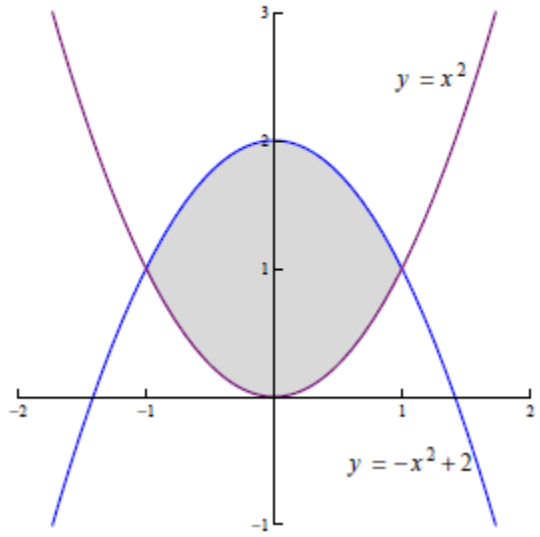
1. $y = \ln x$, $y = 0$ and $x = 2$.



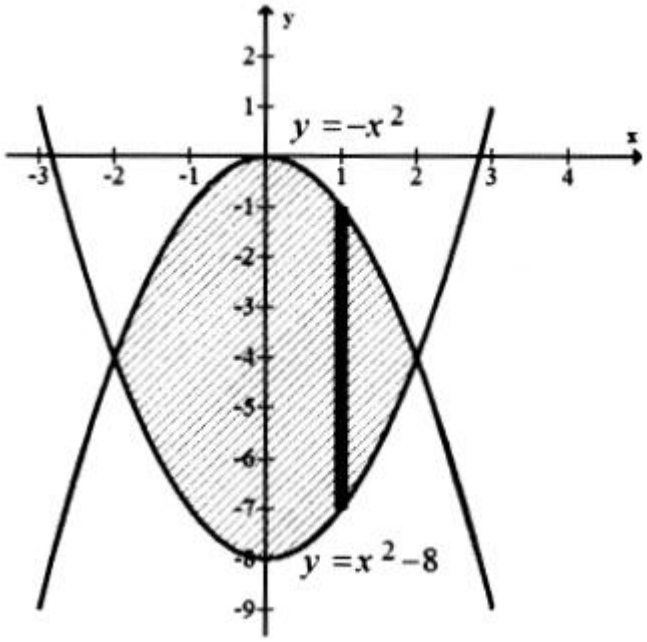
2. $y = e^x$, $x = \ln 4$, $x = 0$ and $y = 0$.



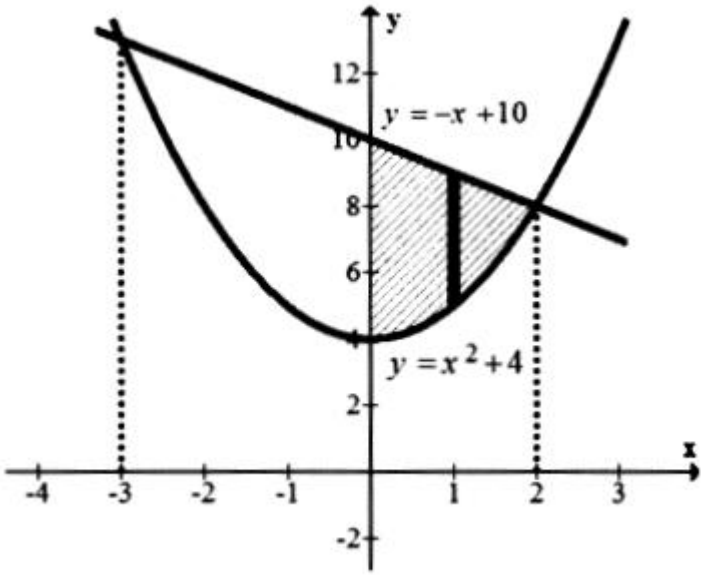
3. $y = x^2$ and $y = -x^2 + 2$

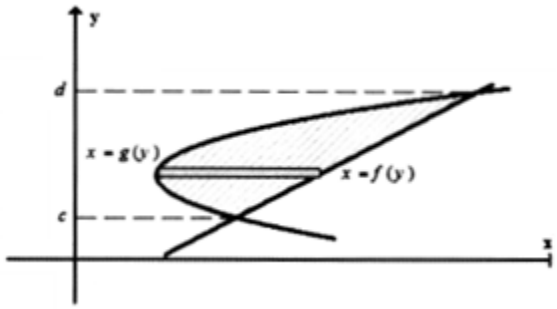


4. Sketch the region R bounded by the graphs $y = x^2 - 8$, $y = -x^2$.
Find its area .



5. Sketch the region R bounded by the graphs $y = x^2 + 4$, $y = -x + 10$ Find the area of the region in the first quadrant .





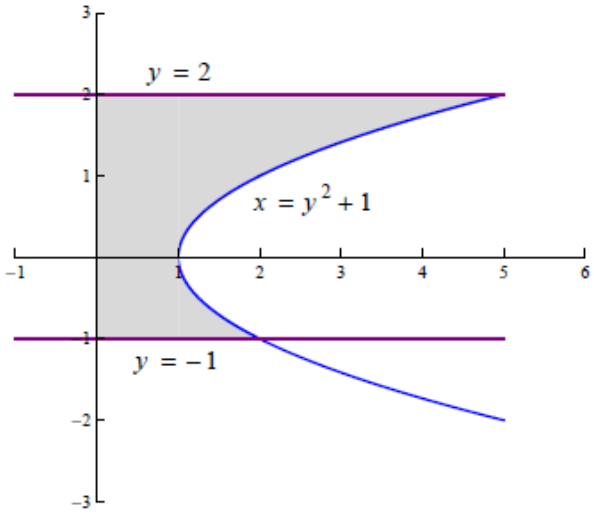
In the above figure the graphs of $x = f(y)$ *and* $x = g(y)$ intersect at the points $y = c$ *and* $y = d$.

The area bounded by the graphs of the curves of $x = f(y)$ *and* $x = g(y)$ equals

$$A = \int_c^d [\, f(y) - \, g(y)]dy$$

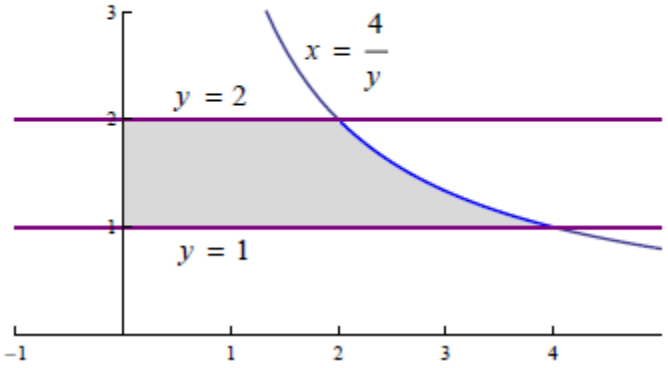
$$A = \int_c^d [\, Right\ bound - \ Left\ bound]dy$$

6. Find the area bounded by the graphs of the curves of $x = y^2 + 1$, $x = 0$, $y = -1$ and $y = 2$.



7. Sketch the region R bounded by the curves , and find its area:

$y = \frac{4}{x}$, $x = 0$, $y = 1$ and $y = 2$.



8. Sketch the region R bounded by the graphs of :

$x = y^2$ *and* $x = 4 - y^2$, then, find its area.

9. Sketch the region R bounded by the graphs of :

$x = y^2 - 1 \quad and \quad x = y + 1$, then, find its area.

.

10. Sketch the region R bounded by the graphs of :

$y = x^2 \quad and \quad y = 2x - 1 \quad , \quad x - axis.$ then, find its area.

11. Sketch the region R bounded by the graphs of :

$x = y^2 - 1 \quad and \quad x = y + 1$ then, find its area.

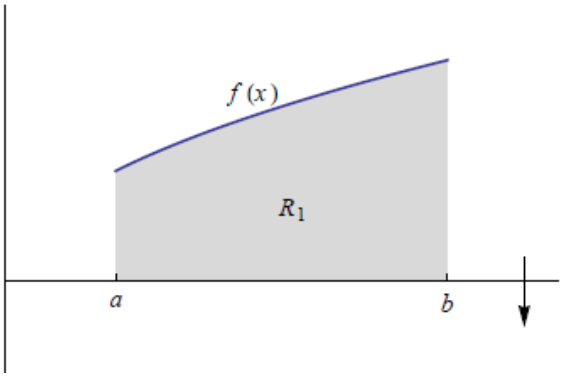
12. Sketch the region R bounded by the graphs of :

$x = -y^2$, $x = y + 4$ and $y = -1$, $y = 1$ then, find its area.

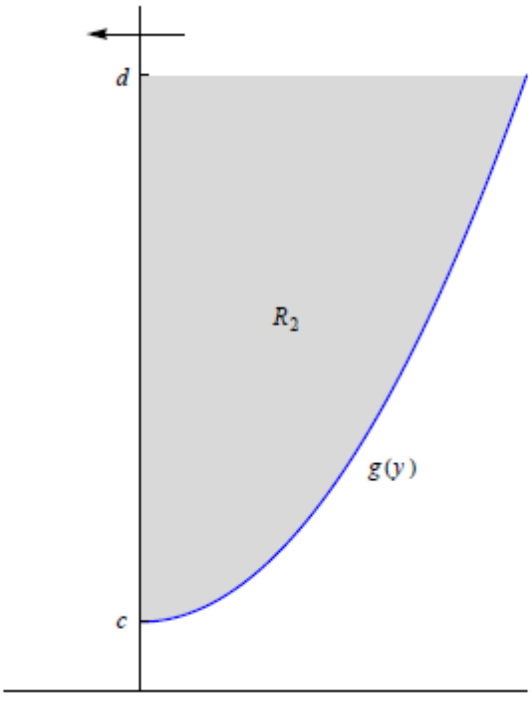
VOLUME OF A SOLID OF REVOLUTION
Disk or Washer method

1. Disk Method

Recall that the volume of a right circular cylinder equals $\pi r^2 h$ where r is the radius of the base (which is a circle) and h is the hight of the cylinder .



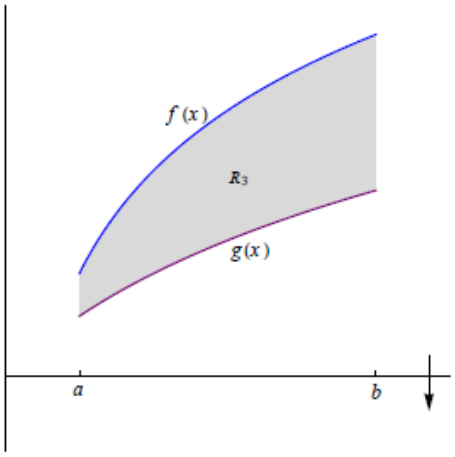
In the above figure R_1 is the region bounded by the graphs of the curves of $f(x)$, $x = a$, $x = b$ and the x-axis.
Using disk method , the volume of the solid of revolution generated by revolving the region R_1 around the x-axis is $V = \pi \int_a^b [f(x)]^2 \, dx$



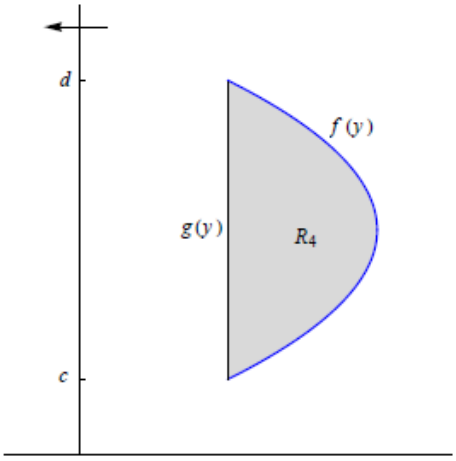
In the above figure R_2 is the region bounded by the graphs of the curves of $g(y)$, $y = d$ and the y-axis.
Using disk method , the volume of the solid of revolution generated by revolving the region R_2 around the y-axis is $V = \pi \int_c^d [g(y)]^2 \, dy$

2. Washer Method

$Volume\ of\ a\ washer\ = \pi\left[(outer\ radius)^2 - (inner\ radius)^2\right]\ (thickness)$



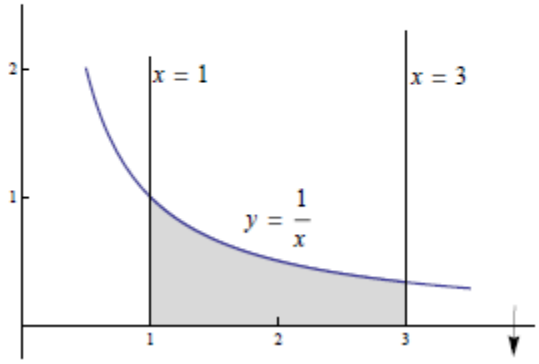
In the above figure R_3 is the region bounded by the graphs of the curves of $f(x)$, $g(x)$, $x = a$ and $x = b$.
Using washer method , the volume of the solid of revolution generated by revolving the region R_3 around the x-axis is $V = \pi \int_a^b \left[(f(x))^2 - (g(x))^2\right]\ dx$



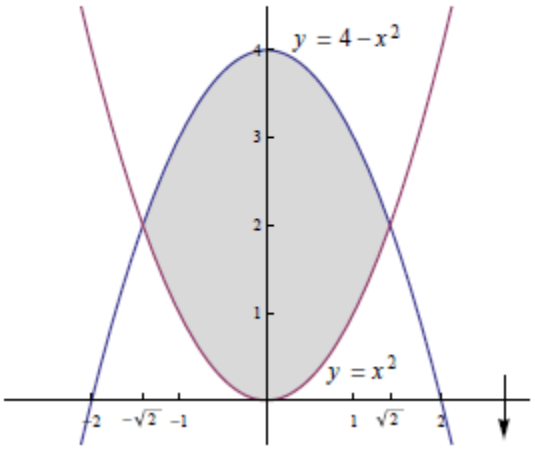
In the above figure R_4 is the region bounded by the graphs of the curves of $f(y)$ and $g(y)$, where $f(y)$ and $g(y)$ intersect at the points $y = c$ and $y = d$.
Using washer method , the volume of the solid of revolution generated by revolving the region R_4 around the y-axis is $V = \pi \int_c^d \left[(f(y))^2 - (g(y))^2\right]\ dy$

Examples : Use disk or washer method to find the volume of the solid of revolution generated by revolving the region bounded by the graphs of the curves of :

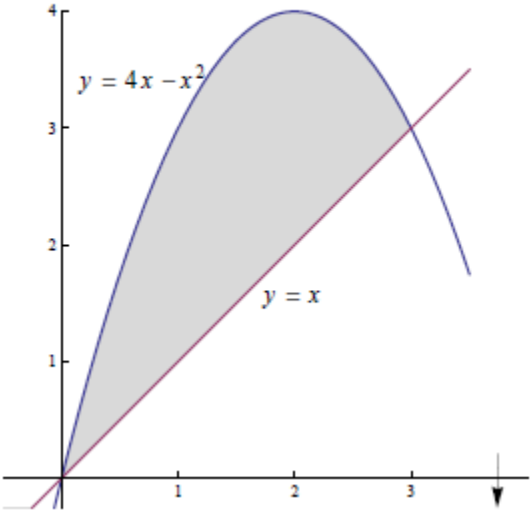
1. $y = \frac{1}{x}$, $x = 1$, $x = 3$ and $y = 0$, around the x-axis.



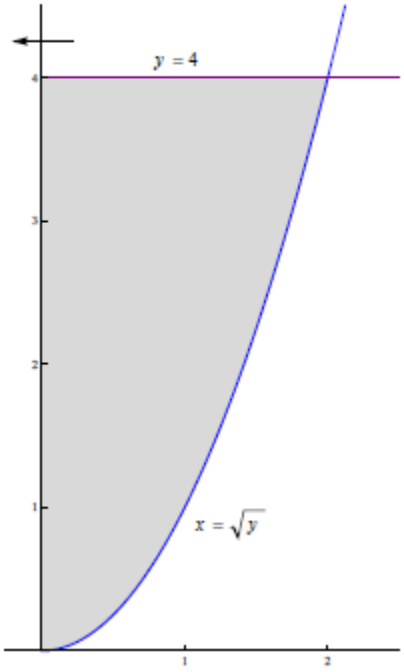
2. $y = x^2$ and $y = 4 - x^2$, around the x-axis .



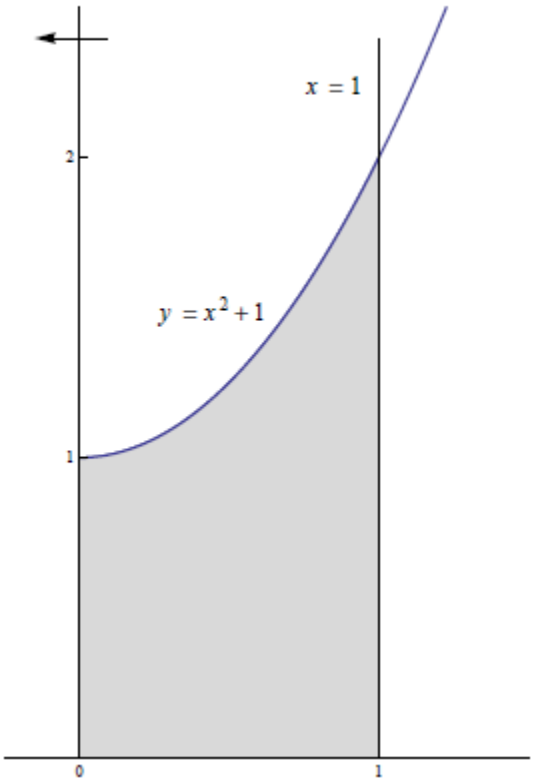
3. $y = 4x - x^2$ and $y = x$, around the x-axis .



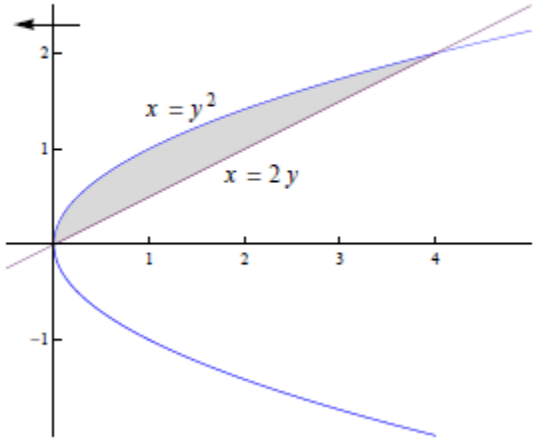
4. $x = \sqrt{y}$, $x = 0$ and $y = 4$, around the y-axis



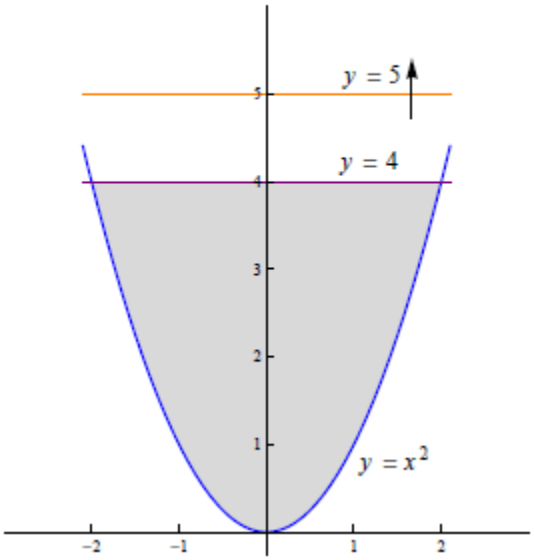
5. $y = x^2 + 1$, $y = 0$, $x = 0$ and $x = 1$, around the y-axis .



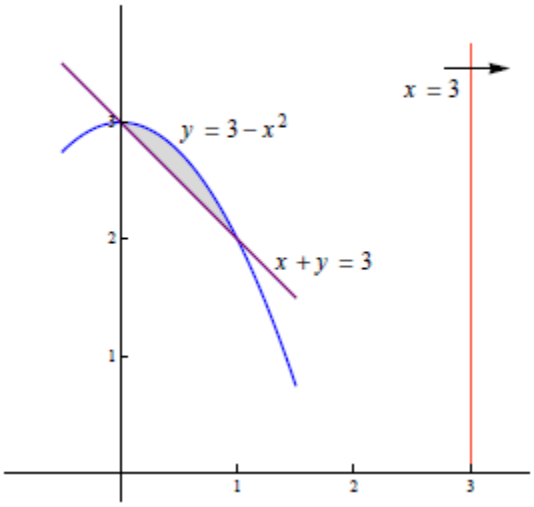
6. $x = y^2$ and $x = 2y$, around the y-axis .



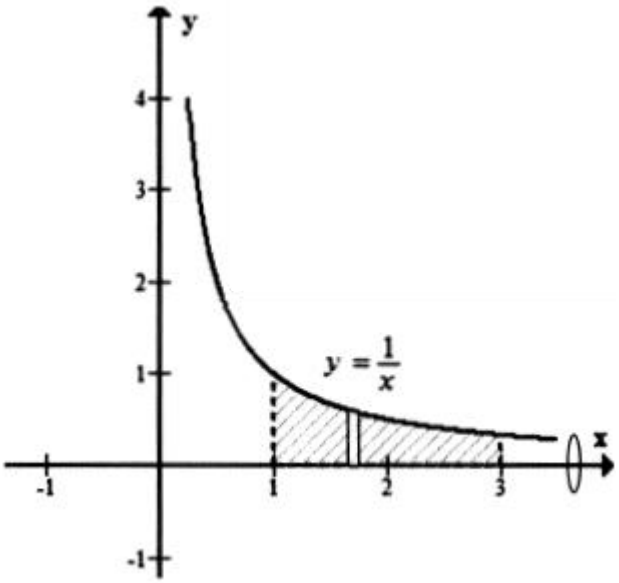
7. $y = x^2$ and $y = 4$, around the line $y = 5$.



8. $y + x^2 = 3$ and $y + x = 3$, around the line $x = 3$



9. Sketch the region R and find the volume of solid generated by revolving R around $x-axis$: $y=\frac{1}{x}$, $x=1$, $x=3$, $y=0$

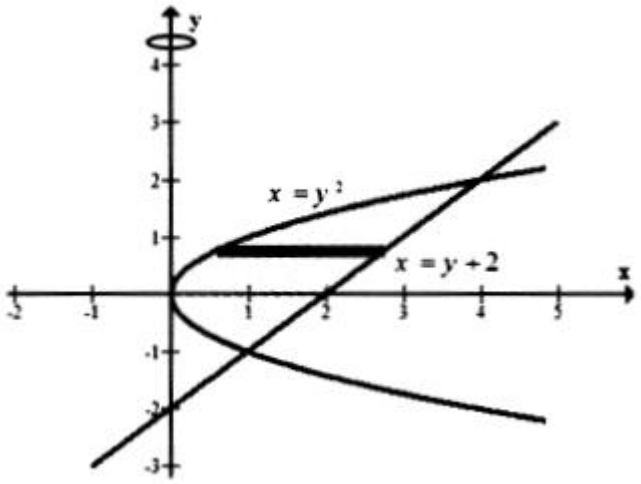


10. Sketch the region R bounded by the graphs $x = y^2$, $x - y = 2$

and find

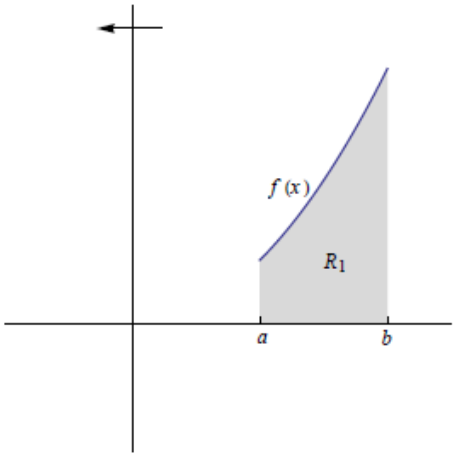
(i) The area of the region R .

(ii) the volume of the solid generated by revolving R around $y - axis$

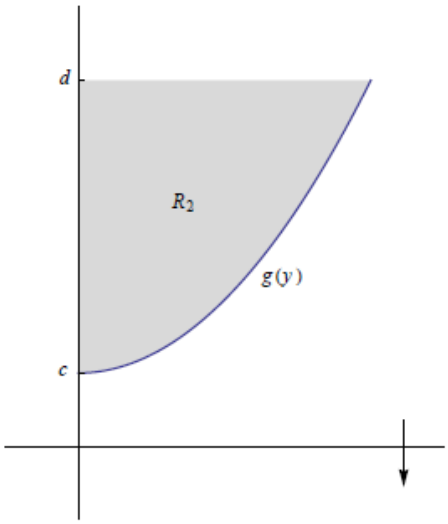


VOLUME OF A SOLID OF REVOLUTION
Cylindrical shells method

Volume of a shell $=$ 2π *(average radius) (altitude) (thickness)*



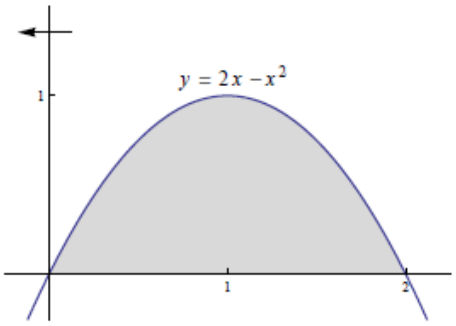
In the above figure R_1 is the region bounded by the graphs of the curves of $f(x)$, $x = a$, $x = b$ and the x-axis.
Using cylindrical shells method , the volume of the solid of revolution generated by revolving the region R_1 around the y-axis is $V = 2\pi \int_a^b x\ f(x)\ dx$



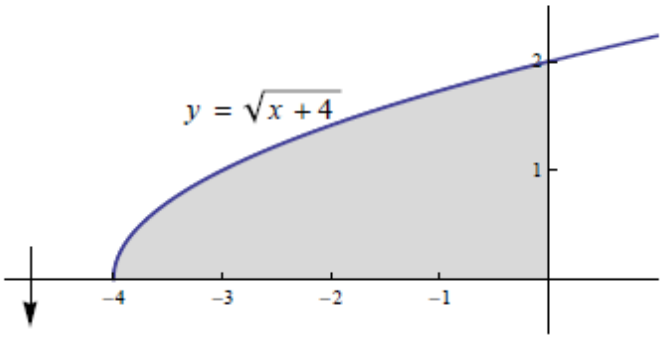
In the above figure R_2 is the region bounded by the graphs of the curves of $g(y)$, $y = d$ and the y-axis.
Using cylindrical shells method , the volume of the solid of revolution generated by revolving the region R_2 around the x-axis is $V = 2\pi \int_c^d y\ g(y)\ dy$

Examples : Use cylindrical shells method to find the volume of the solid of revolution generated by revolving the region bounded by the graphs of the curves of :

1. $y = 2x - x^2$ and $y = 0$, around the y-axis .

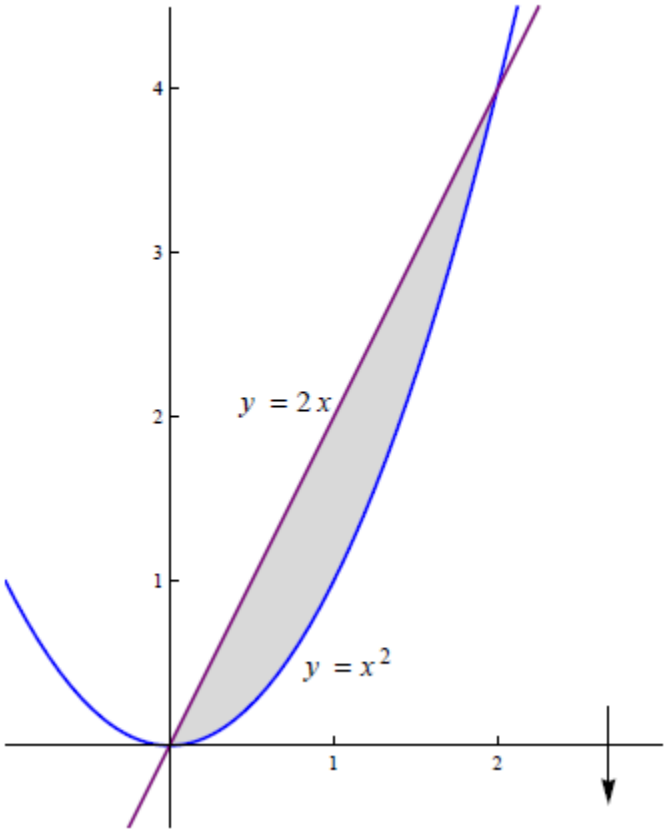


2. $y = \sqrt{x+4}$, $y = 0$ and $x = 0$, around the x-axis .



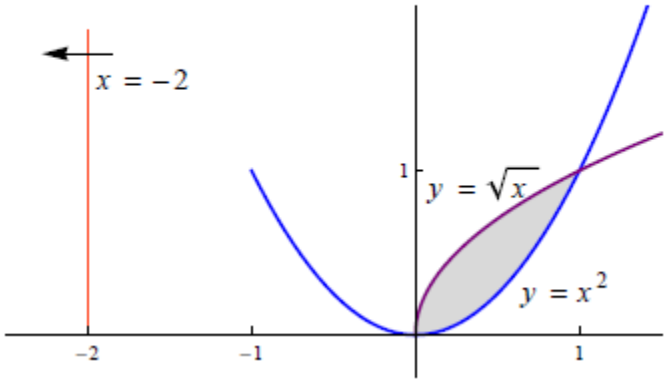
3.

$y = x^2$ and $y = 2x$, around the x-axis .



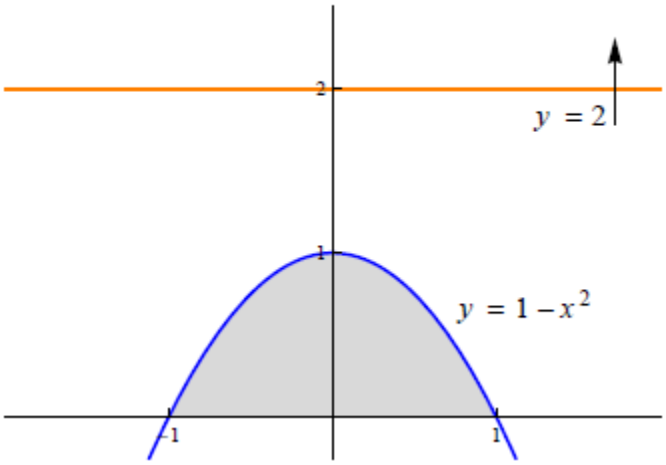
4.

$y = \sqrt{x}$ and $y = x^2$, around the line $x = -2$.



5.

$y = 1 - x^2$ and $y = 0$, around the line $y = 2$.



6. Sketch the region R bounded by the graphs of

$$y = x^2 - 4 \ \ and \ \ y = 2x - 4$$, then find :

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

$a)\ x - axis$ b) $y - axis$ c) $x = 3$

d) $x = -1$ e) $y = 2$ f) $y = -5$

7. Sketch the region R bounded by the graphs of

$$y = x^2 - 1 \ \ and \ \ y = x - 1 \ ,$$ then find :

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

$a)$ $x - axis$ b) $y - axis$ c) $x = 3$

d) $x = -1$ e) $y = 2$ f) $y = -2$

8. Sketch the region R bounded by the graphs of

$y = \sqrt{x}$ and $x = 4$, $y = 0$, then find :

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

$a)$ $x - axis$ b) $y - axis$ c) $x = 4$

d) $x = -1$ e) $y = 2$ f) $y = -1$

9. Sketch the region R bounded by the graphs of

$$y = \sqrt{x} \quad and \quad y = x^2$$, then find :

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

(a) $x - axis$ (b) $y - axis$ (c) $x = 2$

(d) $x = -1$ (e) $y = -2$ (f) $y = 3$

10. Sketch the region R bounded by the graphs of

$y = 2x - x^2$ and $y = x$, then find :

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

(a) $x - axis$ (b) $y - axis$ (c) $x = 1$

(d) $x = -1$ (e) $y = -2$ (f) $y = 3$

11. Sketch the region R bounded by the graphs of

$y = 3 - x \quad and \quad y = 1 + \sqrt{x}$, then find :

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

$(a)\ \ x - axis \qquad (b)\ \ y - axis \qquad (c)\ \ x = 1$

$(d)\ \ x = -1 \qquad (e)\ \ y = -2 \qquad (f)\ \ y = 4$

12. Sketch the region R bounded by the graphs of

$x = y^2$, $x = 2 - y^2$ *and the* $x - axis$ ($y = 0$) ,then find

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

(a) $x - axis$ (b) $y - axis$ (c) $x = 2$

(d) $x = -1$ (e) $y = -2$ (f) $y = 2$

13. Sketch the region R bounded by the graphs of $y = x^2 \ and \ y = 2x$ then find :

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

$(a) \ x - axis$ $(b) \ y - axis$ $(c) \ x = 2$

$(d) \ x = -1$ $(e) \ y = -2$ $(f) \ y = 4$

14. Sketch the region R bounded by the graphs of

$y = 2x - x^2$ and $y = x$, then find :

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

(a) $x - axis$ (b) $y - axis$ (c) $x = 2$

(d) $x = -1$ (e) $y = -1$ (f) $y = 2$

15. Sketch the region R bounded by the graphs of

$y = x^2$, $x + y = 2$ and $x = 2$, then find :

(i) The area of the region R .

(ii) The volume of solid generated by revolving R around :

(a) $x - axis$ (b) $y - axis$ (c) $x = 3$

(d) $x = 1$ (e) $x = -1$ (f) $y = -1$ (g) $y = 4$