CSC 311 HW 1

439101298 [illegible]

Q1:

Prove $0.5n^3 - 4n^2 + 2 \in \Theta(n^3)$

Step 1: Prove $0.5n^3 - 4n^2 + 2 \in O(n^3)$

$$\leq 0.5n^3 + 2$$

$$\leq 2.5n^3 \quad ; \; n \geq 1$$

Choose $c = 2.5$ & $n_0 = 1$

Step 2: Prove $0.5n^3 - 4n^2 + 2 \in \Omega(n^3)$

$$\geq 0.5n^3 - 4n^2$$

$$\geq 0.25n^3$$

[illegible] [illegible]

$0.25n^3 \geq 4n^2$

$n \geq 16$

$n_0 = 16$

Choose $C = 0.25$ & $n_0 = 16$

Step 3: Since we proved big oh and big omega $\longrightarrow 0.5n^3 - 4n^2 + 2 \in \Theta(n^3)$ is True

choose $c_1 = 2.5$, $c_2 = 0.25$ and $n_0 = \max(1, 16) = 16$

Q2:

Since $f(n) \in O(g_1(n)) \iff c_1 g_1(n) \ge f(n)$; $n \ge n_1$

$h(n) \in O(g_2(n)) \iff c_2 g_2(n) \ge h(n)$; $n \ge n_2$

then Let $c_m = \max(c_1, c_2)$, $n_m = \max(n_1, n_2)$

$\rightarrow f(n) + h(n)$

$\le c_1 g_1(n) + c_2 g_2(n)$; $n \ge n_m$

$\le c_m g_1(n) + c_m g_2(n)$; $n \ge n_m$ (Since $c_m = \max(c_1, c_2)$)

$\le c_m (g_1(n) + g_2(n))$; $n \ge n_m$

$\le c_m \cdot 2 \max(g_1(n), g_2(n))$

Choose $c = 2\max(c_1, c_2)$ $n_0 = \max(n_1, n_2)$

$\rightarrow f(n) + h(n) \in O(\max(g_1(n), g_2(n)))$

Q3: Lowest $\rightarrow$ Highest

$\ln^2 n$

$5\log^{10}(n+100)$

$\sqrt[3]{n}$

$3n^2+1$

$0.001n^4$

3^n

2^{2n}

$(n-2)!$

Q4:

Line No.	Count
1	$\lfloor \frac{n}{2} \rfloor + 1 \approx \frac{n}{2} + 1$
2	$\cancel{\frac{3n^2}{8} + \frac{3n}{4} + 1}$ $\frac{3n^2+6n}{8}$
3	$\frac{3n^2+2n}{8}$

Big Oh: $O(n^2)$

$$\text{operation\_count}(n) = \frac{n}{2} + 1 + \frac{3n^2+6n}{8} + \frac{3n^2+2n}{8}$$

$$= \frac{6n^2+12n+8}{8}$$

$$\sum_{0}^{n/2-1} \left((n-1) - i + 1 + 1\right)$$

$$= \sum_{0}^{n/2-1} (n+1-i) =$$

$$(n/2) (n+1) - \frac{n(n-2)}{8}$$

$$= \frac{n^2}{2} + \frac{n}{2} - \frac{n^2}{8} + \frac{n}{4} = \frac{3n^2}{8} + \frac{3n}{4}$$

$$= \frac{3n^2+6n}{8}$$

$$\sum_{0}^{n/2-1} (n-i) = \frac{3n^2}{8} + \frac{n}{4} = \frac{3n^2+2n}{8}$$