King Saud University

College of Science

Department of Mathematics

106 Math Exercises

(3-1)

RIEMANN SUM

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Sums And Sigma Notation

Let
$$a_k$$
 , b_k , $C \in \mathbb{R}$

$$\sum_{k=1}^{n} (a_k \pm b_k) = \sum_{k=1}^{n} a_k \pm \sum_{k=1}^{n} b_k$$

$$\sum_{k=1}^{n} C a_k = C \sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n} C = nC$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$
, $(a-b)^2 = a^2 - 2ab + b^2$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
, $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
 , $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

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Exercises

1) Find the value of α such that

$$\sum_{k=1}^{n} (k + \alpha) = \frac{n^2}{2} : (n \ge 1)$$

2) Find the value of α such that

$$\sum_{k=1}^{4} (\alpha k + 1) = 14$$

3) Find the sum

$$\sum_{k=1}^{n} \left(\frac{1}{2} - k\right) =$$

4) Find the sum

$$\sum_{k=1}^{n} (k+3) =$$

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5) Find the sum

$$\sum_{k=1}^{4} k(k^2 - 1) =$$

6) Find the limit

$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{2k}{n^2}$$

7) Find the limit

$$\lim_{n\to\infty}\sum_{k=1}^n\frac{5k}{n^2}$$

8) Find the limit

$$\lim_{n\to\infty}\sum_{k=1}^n\frac{(6k+2)}{n^2}$$

9) Find the limit

$$\lim_{n\to\infty}\sum_{k=1}^n\frac{(k-1)^2}{n^3}$$

Norm of Partition:

Let $P = \{x_0, x_1, x_2, ..., x_n\}$ in the interval $[a,b] \Rightarrow$ $[x_0, x_1], [x_1, x_2], [x_2, x_3], ..., [x_{k-1}, x_k]$ are sub-intervals of [a,b] $\Delta x_k = x_k - x_{k-1} \qquad : k = 1, 2, 3, ..., n$ $||P|| = max \{\Delta x_1, \Delta x_2, \Delta x_3, ..., \Delta x_n\}$

||P|| called a norm of partition P of the interval[a,b].

Riemann Sum:

Let f(x) be defined on the closed interval [a, b],

Let
$$P = \{x_0, x_1, x_2, ..., x_n\}$$
 be a partition of $[a, b]$,
Let $w_k \in [x_{k-1}, x_k]$, $k = 1, 2, 3, ..., n$.

Riemann sum of f(x) for a partition P is

$$R_p = f(w_1)\Delta x_1 + f(w_2)\Delta x_2 + f(w_3)\Delta x_3 + \dots + f(w_n)\Delta x_n$$

$$R_p = \sum_{k=1}^n f(w_k) \Delta x_k$$

For a regular partition of the interval [a, b], $\Delta x = \frac{b-a}{n} \Rightarrow$

$$R_p = \sum_{k=1}^n f(w_k) \Delta x$$

Area under the graph of a function:

If $f(x) \ge 0 : x \in [a, b] \Rightarrow$ Area under the graph of a function f(x)

$$Area = A = \lim_{\|P\| \to 0} R_p = \lim_{n \to \infty} \sum_{k=1}^{n} f(w_k) \Delta x$$

Definite Integral:

If f(x) is a continuous function on [a,b]

$$\int_{a}^{b} f(x)dx = \lim_{\|P\| \to 0} R_{p} = \lim_{n \to \infty} \sum_{k=1}^{n} f(w_{k}) \Delta x$$

If
$$f(x) \ge 0 : x \in [a, b] \Rightarrow$$

$$\int_{a}^{b} f(x)dx = A \text{ , where A is the Area under the graph of a function } f(x)$$

Math 106 (Riemann sum) - Malek Zein AL-Abdin 10) Find the norm of the partition $P = \{0,0.3,0.8,1\}$ of the interval [0,1]?

11) Find the Riemann sum R_p for the function f(x) = 2x + 1 on the partition

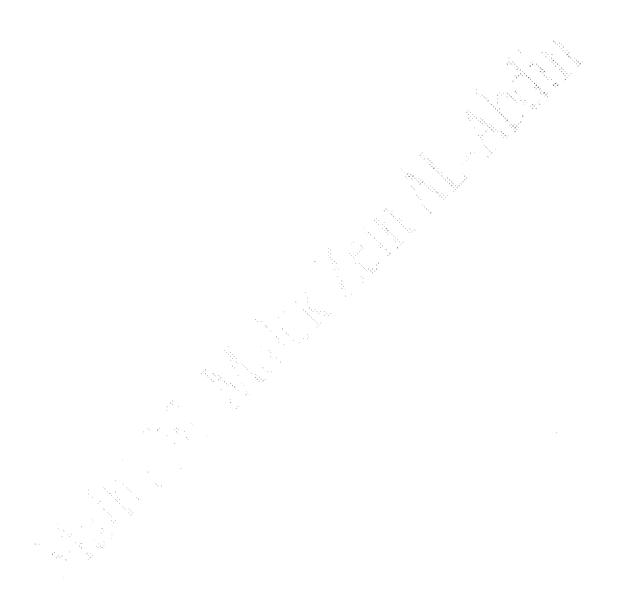
 $P=\{-1,0,2,3,4.5,5\}$ of the interval [-1,5] by choosing on each subinterval of P:

- (a) The left-hand end point, $w_k = x_{k-1}$
- (b) The right-hand end point, $w_k = x_k$
- (c) The mid-point, $w_k = \frac{x_{k-1} + x_k}{2}$



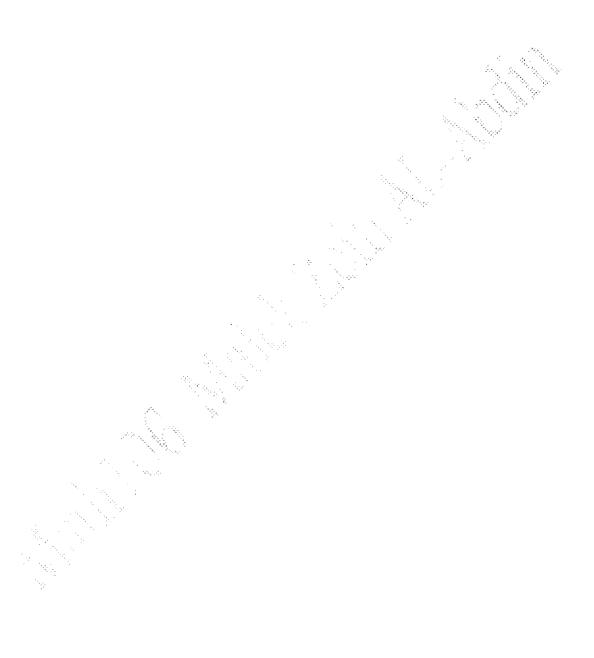
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- 12) Find the area under the curve of the function f(x) = 3x + 1, on the interval [1,3] using Riemann sum R_p (regular partition) by choosing on each subinterval of P:
- (a) The left-hand end point , $w_k = x_{k-1}$
- (b) The right-hand end point , $w_k = x_k$
- (c) The mid-point, $w_k = \frac{x_{k-1} + x_k}{2}$



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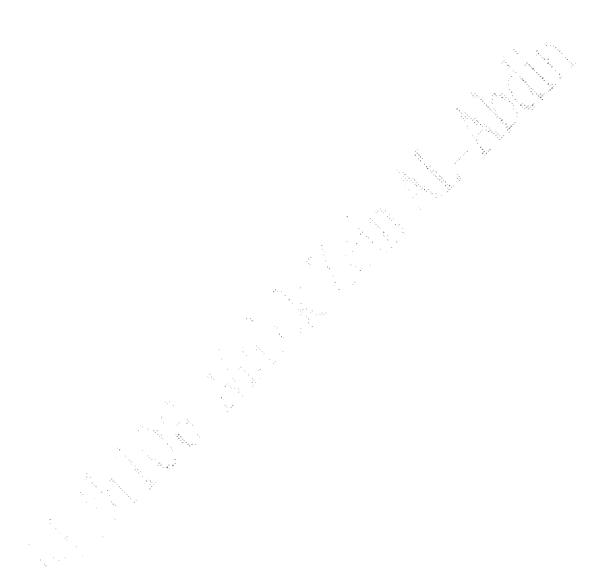
$$\int_{0}^{6} (3-x)dx$$



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$$\int_{-1}^{2} (9 - x^2) dx$$





15) Find the definite integral representing $\lim_{n\to\infty}\sum_{k=1}^n\sqrt{3+x}~\Delta x$ using regular partition of the interval [1,2]