



Question 1 12 points

Choose the most appropriate data structure for each of the following tasks.

- | | | | | | |
|---------------|--------------|---------------------|----------------|----------------|-----------|
| A. LinkedList | B. ArrayList | C. DoubleLinkedList | D. LinkedQueue | E. AVL | F. Stack |
| G. BT. | H. BST. | I. LinkedPQueue. | J. BPlusTree. | K. HeapPQueue. | L. Graph. |

- ✓ 1. An application that analyzes the communication patterns on a online social network. L
- ✓ 2. An application in a restaurant that keeps track of all tables assigned to every waiter. D
- ✓ 3. An algorithm that decides on the order of surgical procedures at a hospital. I, X
- ✓ 4. An application that keeps track of people that have been vaccinated and those who have not. J, X
- ✓ 5. A video play list that allows forward and backward navigation between the videos. C
- ✓ 6. An algorithm that receives a character and returns its ASCII code. E, X

Question 2 12 points

Write the method `public static int ls(List<Boolean> l)` which takes as input a non-empty list `l` of Booleans and returns the length of the longest contiguous sequence of `true` in `l`.Example 1. If $l = \{0, 1, 1, 0, 0, 1, 1, 1, 1, 0, 1\}$, `ls(l)` returns 4.

```
1 public static int ls(List<Boolean> l) {
2     int maxL = ...;
3     ...
4     while (true) {
5         ...
6         while (true) {
7             if (...)
8                 ...
9             else
10                ...
11            if (...)
12                break;
13            else
14                ...
15        }
16        if (...)
17            ...
18        if (...)
19            break;
20        else
21            ...
22    }
23    return maxL;
24 }
```

1. Line 2:

- (A) `int maxL = -1;`
- (B) `int maxL = 1;`
- (C) `int maxL = l.retrieve();`
- (D) `int maxL = 0;`
- (E) None

2. Line 3:

- (A) `l.insert(true);`
- (B) `l.update(false);`
- (C) `l.findNext();`
- (D) `maxL = l.length();`
- (E) None L.findFirst

3. Line 5:

- ☐ (A) int cpt = 1;
- ☐ (B) int cpt = -1;
- ☒ (C) int cpt = 0;
- ☐ (D) int cpt = l.retrieve();
- ☐ (E) None

4. Line 7:

- ☒ (A) if (l.retrieve() > cpt)
- ☐ (B) if (l.retrieve() > maxL)
- ☒ (C) if (l.retrieve())
- ☐ (D) if (l.last())
- ☒ (E) None *maxL retrieve = 1*

5. Line 8:

- ☒ (A) maxL++;
- ☐ (B) cpt--;
- ☒ (C) cpt++;
- ☐ (D) l.findFirst();
- ☐ (E) None

6. Line 10:

- ☐ (A) continue;
- ☒ (B) break;
- ☐ (C) l.findFirst();
- ☒ (D) l.findNext();
- ☐ (E) None

7. Line 11:

- ☐ (A) if (l.retrieve())
- ☒ (B) if (l.last())
- ☐ (C) if (maxL == 0)
- ☐ (D) if (cpt == 0)
- ☐ (E) None

8. Line 14:

- ☐ (A) maxL = 0;
- ☐ (B) maxL = cpt;
- ☐ (C) l.findFirst();
- ☒ (D) l.findNext();
- ☐ (E) None

9. Line 16:

- ☐ (A) if (cpt == maxL)
- ☐ (B) if (cpt < maxL)
- ☒ (C) if (cpt > maxL)
- ☐ (D) if (cpt == 0)
- ☐ (E) None

10. Line 17:

- ☐ (A) cpt = maxL;
- ☐ (B) maxL++;
- ☒ (C) maxL = cpt;
- ☐ (D) cpt++;
- ☐ (E) None

11. Line 18:

- ☐ (A) if (maxL < 0)
- ☒ (B) if (l.last())
- ☐ (C) if (maxL > 0)
- ☐ (D) if (l.retrieve())
- ☐ (E) None

12. Line 21:

- ☐ (A) l.findFirst();
- ☒ (B) l.findNext();
- ☐ (C) cpt++;
- ☐ (D) maxL++;
- ☐ (E) None

Question 3 14 points

- (a) Write a constructor for the class `BT` which builds the tree from its array representation `a`. The root element is at position 1, and the left and right children of a node at position `i` are located at positions `2*i` and `2*i+1` respectively. If a position contains `null`, then the corresponding node does not exist.


```

1 public BT(T[] a) {
2     current = root = ...;
3 }
4 private BTNode<T> rfa(T[] a, int i) {
5     if (...)
6         return ...;
7     BTNode<T> nnode = ...;
8     ...
9     ...
10    ...
11 }

```

1. Line 2:

- ☐ (A) rfa(a, 0)
☐ (B) null
☐ (C) rfa;
☒ (D) rfa(a, 1)
☐ (E) None

2. Line 5:

- ☐ (A) a[i] < 0
☒ (B) i >= a.length || a[i] == null
☐ (C) i < a.length && a[i] < 0
☐ (D) a[i] != null && i > a.length
☐ (E) None

3. Line 6:

- ☐ (A) current;
☒ (B) null
☐ (C) i
☐ (D) a[i/2]
☐ (E) None

4. Line 7:

- ☐ (A) root;
☐ (B) a[i];
☐ (C) new BTNode<T>(i)
☒ (D) new BTNode<T>(a[i])
☐ (E) None

5. Line 8:

- ☐ (A) current = current->left;
☐ (B) current.left = rfa(a, i);
☒ (C) nnode.left = rfa(a, i*2);
☐ (D) current.right = rfa(a, i*2);
☐ (E) None

6. Line 9:

- ☐ (A) current = current->right;
☒ (B) nnode.right = rfa(a, i*2+1);
☐ (C) current.right = rfa(a, i*2+1);
☐ (D) current.left = rfa(a, i*2+1);
☐ (E) None

7. Line 10:

- ☐ (A) return current;
☐ (B) return current.rfa();
☐ (C) return null;
☒ (D) return nnode;
☐ (E) None

(b) Repeat the same question as user of BT.

```

1 public static <T> BT<T> fa(T[] a) {
2     BT<T> bt = new BT<T>();
3     if (...) {
4         ...
5         ...
6     }
7     return bt;
8 }
9 private static <T> void rfa(BT<T> bt, T[] a,
10    int i) {
11     if (...) {
12         ...
13         ...
14     }

```

```

15     if (...) {
16         ...
17         ...
18         ...
19     }
20 }

```

1. Line 3:

- ☒ (A) if (a.length > 1 && a[1] != null){
☐ (B) if (a.length > 0 || a[0] != null){
☐ (C) if (a.length > 0 && a[0] != null){
☐ (D) if (a.length > 1){

(E) None.

$$64-1=63$$

2. Suppose T is a binary tree with 63 nodes and height 6. Which of the following is surely true?

- (A) T is a complete tree. (B) T is a min-heap. (C) T is a max-heap. (D) T is not a heap.
(E) None.

3. Given a complete binary tree T and its array representation A , which traversal algorithm visits the nodes of T in the same order as they have in the array A ?

- (A) Pre-order (B) In-order (C) Post-order (D) BFS (E) DFS (F) None

4. Given the heap $H = [3, 10, 8, 12, 11, 20, 14]$, what is H after inserting 5 and then 6?

- (A) 3, 4, 5, 8, 6, 12, 10, 11, 14, 20 (B) 3, 5, 8, 6, 11, 20, 14, 10, 12 (C) 3, 5, 6, 8, 10, 11, 12, 14, 20
(D) 3, 5, 6, 10, 11, 8, 14, 20, 12 (E) None

5. Given the heap $H = [3, 10, 8, 12, 11, 20, 14]$, what is H after deleting one key?

- (A) 8, 14, 12, 10, 11, 20 (B) 8, 10, 11, 12, 14, 20 (C) 8, 10, 12, 11, 20, 14 (D) 8, 10, 14, 12, 11, 20
(E) None

6. Given the heap $H = [3, 6, 8, 9, 23, 10, 13, 11]$, what is H after deleting two keys:

- (A) 3, 6, 8, 9, 23, 10. (B) 8, 9, 23, 10, 13, 11. (C) 8, 9, 10, 11, 13, 23. (D) 8, 9, 10, 11, 23, 13.
(E) None

7. Given two heaps, $H_1 = [6, 10, 18, 20, 22, 24, 30]$ and $H_2 = [8, 12, 14, 19, 23]$, what is the result of merging them along with the key 13?

- (A) 6, 10, 8, 13, 18, 12, 14, 20, 22, 24, 30, 19, 23 (B) 6, 8, 12, 14, 13, 18, 19, 23, 20, 22, 24, 30
(C) 6, 8, 12, 13, 14, 18, 19, 20, 22, 23, 24, 30 (D) 6, 8, 14, 10, 23, 12, 20, 22, 24, 30, 18, 19 (E) None

Question 5 14 points

(a) (4 points)

Remark 1. In what follows the height of tree is the number of levels in the tree. Hence, an empty tree has height 0, whereas a tree with 1 node has height 1.

Choose the most appropriate answer:

1. The maximum depth of an AVL tree with 5 nodes is:

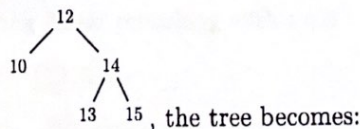
- (A) 1. (B) 2. (C) 3. (D) 4. (E) 5.

2. The worst case run time for insert in AVL tree is:

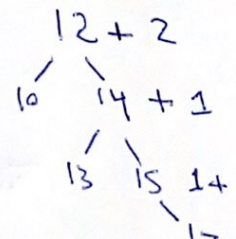
- (A) $O(1)$. (B) $O(n)$. (C) $O(\log n)$. (D) $O(n \log n)$ (E) $O(n^2)$.

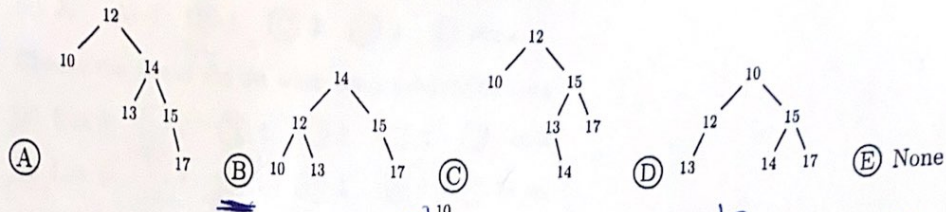
(b) (10 points) Choose the correct result in each of the following cases (follow the the convention of replacing with the smallest key in the right sub-tree when necessary):

1. After inserting the key 17 in the AVL

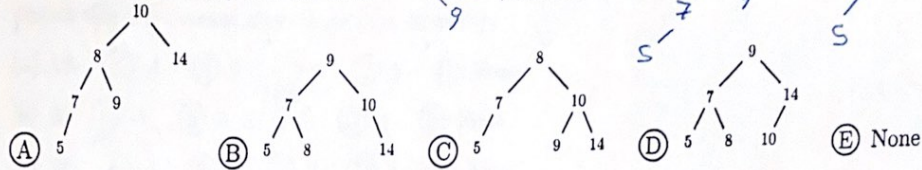


14

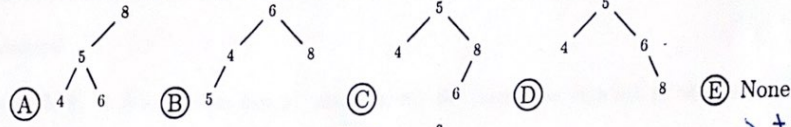




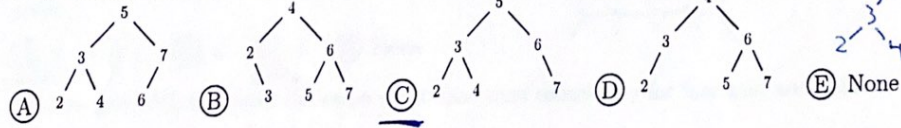
2. After inserting the key 9 in the AVL tree, the tree becomes:



3. After deleting the key 9 from the AVL tree, the tree becomes:



4. After deleting the key 1 from the AVL tree, the tree becomes:



Question 6 14 points

(a) (4 points) Choose the most appropriate answer:

1. The insert operation in a hash table using external chaining has a worst case run time:

- (A) $O(1)$ (B) $O(\log n)$ (C) $O(n)$ (D) $O(n \log n)$ (E) None

2. Consider the following hash function: The product of the rightmost digit and the leftmost digit

: 17×11 then apply $\% 11$ to the result. Which of the following couples of keys cause a collision?

- (A) 9 and 101 (B) 162 and 370 (C) 523 and 259 (D) All of the above. (E) None of the above.
- $17 \times 11 = 187$
 $187 \% 11 = 7$
 $9 \times 1 = 9$
 $101 \times 1 = 101$
 $162 \times 2 = 324$
 $370 \times 0 = 0$
 $523 \times 3 = 1569$
 $259 \times 9 = 2331$
 $1569 \% 11 = 7$
 $2331 \% 11 = 7$

(b) (10 points) Use the hash function $H(key) = key \% 6$ to store the sequence of keys 16, 5, 22, 18, 31 in a hash table of size 6. Use the following collision resolution strategies:

1. Choose the number of probes when using linear rehashing with $c=1$:

- (a) 5: (A) 0 (B) 1 (C) 2 (D) 3 (E) None

- (b) 22: (A) 0 (B) 1 (C) 2 (D) 3 (E) None

16 5 22 18 31

4	5	6	0	1			
1	1	3	1	1			

- (c) 31: (A) 0 (B) 1 (C) 2 (D) 3 (E) None
2. Choose the size of the list when using external chaining.
- (a) List 0: (A) 0 (B) 1 (C) 2 (D) 3 (E) None
- (b) List 2: (A) 0 (B) 1 (C) 2 (D) 3 (E) None
- (c) List 4: (A) 0 (B) 1 (C) 2 (D) 3 (E) None

16	5	22	18	31
4	5	4	0	1

3. Choose the index of the next element when using coalesced chaining with cell size 2 and address region size 4 (-1 means there is no next element):

- (a) 16: (A) -1 (B) 0 (C) 3 (D) 5 (E) None
- (b) 5: (A) -1 (B) 0 (C) 3 (D) 4 (E) None
- (c) 22: (A) -1 (B) 1 (C) 2 (D) 4 (E) None
- (d) 18: (A) -1 (B) 1 (C) 5 (D) 6 (E) None

16	5	22	18	31
4	5	4	0	1
3	-1	-1	-1	-1

clap = ~~3~~ 3, 2

Question 7 14 points

(a) (6 points)

Remark 2. A B+ tree has two parameters, m : the maximum number of children and l : the maximum number of elements in a leaf node.

Choose the most appropriate answer:

1. Consider a B+-tree in which the $m = l = 5$. What is the minimum number of keys in any non-root node :

- (A) 2 (B) 1 (C) 4 (D) 3 (E) None



2. B+-tree and AVL tree have the same worst case time complexity for insertion and deletion:

- (A) True (B) False

3. The best case running time for checking if a key exists in a B+-tree is:

- (A) $O(1)$ (B) $O(\log n)$ (C) $O(n)$ (D) $O(n \log n)$ (E) None

- (b) (8 points) Choose the correct result in each of the following cases (when possible, always borrow and transfer to the left). The order of the tree is $m = 3$:

1. After inserting the key 7 in the B+ tree

3	5	9
---	---	---

, the root of tree becomes:

- (A)

5	7
---	---

 (B)

7	
---	--

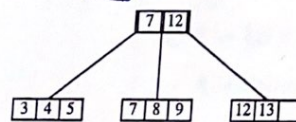
 (C)

5	
---	--

 (D)

9	
---	--

 (E) None



2. After inserting the key 10 in the B+ tree

3	4	5
---	---	---

, the root of the tree becomes:

- (A)

7	10
---	----

 (B)

7	9
---	---

 (C)

8	12
---	----

 (D)

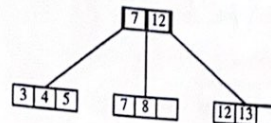
7	12
---	----

 (E) None



7 8 9 10 11 12

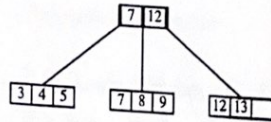
3. After deleting the key 12 from the B+ tree becomes:



, the root of the tree

- (A) [7 | 13] (B) [7] (C) [13] (D) [7 | 8] (E) None

4. After deleting the key 12 from the B+ tree becomes:



, the root of the tree

- (A) [7] (B) [5] (C) [7 | 9] (D) [7 | 13] (E) None

Question 8

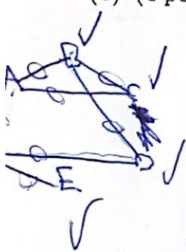
- (a) (2 points) Choose the most appropriate answer:

1. What is the maximum length of any simple path in a graph containing 100 nodes?
(A) 10. (B) 100. (C) 99. (D) 101. (E) None.

2. Which of the following is true for DFS and BFS graph traversal?

- (A) Both DFS and BFS use Stack. (B) DFS uses Queue and BFS uses Stack. (C) BFS uses Queue. (D) DFS uses Stack. (E) C and D.

- (b) (4 points) Given the following graph adjacency list, answer the questions below.



A	→ B → C → F
B	→ A → C → D
C	→ A → B
D	→ B → F
E	→ F
F	→ A → D → E

- (b) The number of edges in the graph is 7.

I

- (c) In the graph, (A, B, D, F, A) is a cycle.

I

- (d) The neighbors of node C are A, B, and D.

I

3. Which of the following is true for this graph?

- (A) The graph is a tree. (B) The graph is acyclic. (C) The graph is disconnected. (D) The graph is connected. (E) None.

4. The DFS traversal of this graph starting from A is (insert neighbors in the data structure in increasing alphabetic order):

- (A) A, C, B, E, D, F. (B) A, F, D, E, C, B. (C) A, F, B, C, E, D. (D) A, F, E, D, C, B. (E) None.

1. Which of the following sequences are paths in this graph? Answer by T (true) or F (false).

- (a) (F, B, D, A) F
(b) (A, B, D, F, A) T
(c) (A, B, C, F, D) F
(d) (F, A, B, C) T

2. Answer by T (true) or F (false).

- (a) The graph is a tree. F



A F E D C B



A F E D C B