King Saud University

College of Science

Department of Mathematics

106 Math Exercises

(2)

Change of Variables

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Change of Variables

Assume $u = g(x) \Rightarrow du = g'(x)dx$

$$\int f[\boldsymbol{g}(\boldsymbol{x})]\, \boldsymbol{g}'(\boldsymbol{x})\, dx = \int f(\boldsymbol{u})\, d\boldsymbol{u} = F(\boldsymbol{u}) + c = F\big(\boldsymbol{g}(\boldsymbol{x})\big) + c$$

Basic Rules of Integration:

$$\int 1\, du = u + c$$

$$\int k\, du = ku + c \ : k \in \mathbb{R}$$

$$\int u^n\, du = \frac{u^{n+1}}{n+1} + c \ : \ n \neq -1\, , n \in \mathbb{Q}$$

$$\int \sin u\, du = -\cos u + c$$

$$\int \cos u\, du = \sin u + c$$

$$\int sec^2 u\, du = \tan u + c$$

$$\int csc^2 u\, du = -\cot u + c$$

$$\int \sec u \tan u\, du = \sec u + c$$

$$\int \csc u \cot u\, du = -\csc u + c$$

$$\sec u = \frac{1}{\cos u} \quad , 1 + tan^2 u = sec^2 u$$

$$\csc u = \frac{1}{\sin u} \quad , 1 + cot^2 u = csc^2 u$$

1)

$$\int x(x^2+1)^{30}\ dx$$

2)

$$\int \frac{x}{\sqrt[3]{x^2+1}}dx$$

3)

$$\int (2x-1)^{20}\,dx$$

4) If

$$\int \sqrt{2x+3}\,dx = k(2x+3)^{3/2} + c$$

, then

(a) $k = \frac{1}{3}$ (b) $k = \frac{2}{3}$ (c) $k = -\frac{3}{2}$ (d) None of these

5)

$$\int \frac{(3+\frac{1}{x})^{3/2}}{x^2}\,dx$$

6)

$$\int \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}\,dx$$

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7) $$\int \frac{(1+\sqrt{x})^7}{\sqrt{x}}dx$$

8)

$$\int \frac{\sqrt{x}}{(x^{3/2} + 4)^3} dx$$

9)

$$\int x\sqrt{x-3}\ dx$$

10)

$$\int x\sqrt{x^2+3}\ dx$$

11)

$$\int \frac{x\ dx}{\sqrt{x^2+9}}$$

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12)

$$\int x^2\ \sqrt{x+1}\ dx$$

13)

$$\int (x-1)\ \sqrt[3]{x-3}\ \ dx$$

14)

$$\int \frac{\sin(\,1+\sqrt{x}\,)}{\sqrt{x}}\,dx$$

15)

$$\int x\sin(x^2+1)\,dx$$

16)

$$\int \frac{sec^2(\frac{1}{x}+2)}{x^2}\,dx$$

17)

$$\int \frac{\cos(\sqrt{x}+1)}{\sqrt{x}}\,dx$$

18)

$$\int \frac{csc^2(\sqrt{x})}{\sqrt{x}}\,dx$$

19)

$$\int \frac{sinx}{\sqrt{2+cosx}}\,dx$$

20)

$$\int sin^3x\ cosx\ dx$$

21)

$$\int 3tan^2x\ sec^2x\ dx$$

22)

$$\int sin2x\ \sqrt{1+sin^2x}\ dx$$

23)

$$\int \frac{\cos\left(\sqrt{\theta}\right)}{\sqrt{\theta}\ sin^2(\sqrt{\theta})}\, d\theta$$

24)

$$\int \frac{x^2}{\sqrt{x^3+1}}\, dx$$

25)

$$\int \frac{x^3}{\sqrt{x^2+16}}dx$$

26)

$$\int (1+\frac{2}{x^3})\sqrt{x-\frac{1}{x^2}}\,dx$$

27)

$$\int \frac{1}{x^2} \sec(3-\frac{1}{x}) \tan(3-\frac{1}{x})\, dx$$

28)

$$\int \frac{x^2}{cos^2(x^3)}\, dx$$

29)

$$\int \frac{xdx}{sin^2(x^2)}$$

30)

$$\int \frac{1}{cos^3 x \csc x} dx$$