In-EVM Mina State Verification Circuit Description

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1 Introduction

WIP

High level description according to RfP[1]

1. Computing several hash values from the data of the proof. This involves using the Poseidon hash function with 55 full rounds both over $\mathbb{F}_p$ and $\mathbb{F}_q$ with round constants and MDS matrix specified for $\mathbb{F}_p$[2] and for $\mathbb{F}_q$[3].
2. Checking arithmetic equations.
3. Performing one multi-scalar multiplication (MSM) of size $2n_2 + 4 + (2 + 25) = 63$, for which some of the bases are fixed and some are variable.
4. For each $i \in \{1, 2\}$, performing a multi-scalar multiplication over $\mathbb{G}_i$ of size 2^{n_i} with a fixed array of bases, and with scalars that can be very efficiently computed from the proof.

Note that for MSM in Step 4:

$$\sum_{i=0}^{2^{n_k}-1} s_i \cdot G_i = H$$
$$s_i \coloneqq \prod_{\substack{0 \leq j \leq n_k \\ \text{bits}(i)[j]=1}} \phi(c_j),$$

where:

- $\phi \colon \{0,1\}^{128} \to \mathbb{F}$ is defined as `to_field` in the implementation[4].
- Given an integer $i < 2^{n_k}$, bits(i) is defined as the little-endian bit array of length n representing the binary expansion of i.
- $G_0, ..., G_{2^{n_k-1}} \in \mathbb{G}_k$ is a fixed sequence of group elements[5].
- $c_0, ..., c_{n_k-1} \in \{0,1\}^{128}$ is a sequence of challenges.

We use the same 15-wires PLONK circuits that are designed for Mina.[6]

2 Preliminaries

WIP

[1] https://hackmd.io/u_2Ygx8XS5Ss1aObgOFjkA
[2] https://github.com/o1-labs/proof-systems/blob/master/oracle/src/pasta/fp.rs
[3] https://github.com/o1-labs/proof-systems/blob/master/oracle/src/pasta/fq.rs
[4] https://github.com/o1-labs/proof-systems/blob/49f81edc9c86e5907d26ea791fa083640ad0ef3e/oracle/src/sponge.rs#L33
[5] https://github.com/o1-labs/proof-systems/blob/master/dlog/commitment/src/srs.rs#L70
[6] https://o1-labs.github.io/mina-book/specs/15_wires/15_wires.html

2.1 Pasta Curves

Let $n_1 = 17$, $n_2 = 16$. Pasta curves parameters:

- $p = 2^{254} + 45560315531419706090280762371685220353$
- $q = 2^{254} + 45560315531506369815346746415080538113$
- Pallas:

$$\mathbb{G}_1 = \{(x, y) \in \mathbb{F}_p | y^2 = x^3 + 5\}$$
$$|\mathbb{G}_1| = q$$

- Vesta:

$$\mathbb{G}_2 = \{(x, y) \in \mathbb{F}_q | y^2 = x^3 + 5\}$$
$$|\mathbb{G}_2| = p$$

2.2 Verification Algorithm

Notations:

$N_{\texttt{wires}}$	Number of wires ('advice columns')
$N_{\texttt{perm}}$	Number of wires that are included in the permutation argument
$N_{\texttt{prev}}$	Number of previous challenges
$S_{\sigma_i}(\mathrm{X})$	Permutation polynomials for $0 \leq i < N_{\texttt{perm}}$
$pub(X)$	Public input polynomial
$w_i(X)$	Witness polynomials for $0 \leq i < N_{\texttt{wires}}$
$\eta_i(X)$	Previous challenges polynomials for $0 \leq i < N_{\texttt{prev}}$
ω	n-th root of unity

Proof π constains (here $\mathbb{F}_r$ is a scalar field of $\mathbb{G}$):

- Commitments:
  - Witness polynomials: $w_{0,\texttt{comm}}, ..., w_{N_{\texttt{wires}},\texttt{comm}} \in \mathbb{G}$
  - Permutation polynomial: $z_{\texttt{comm}} \in \mathbb{G}$
  - Quotinent polynomial: $t_{\texttt{comm}} = (t_{1,\texttt{comm}}, t_{2,\texttt{comm}}, ..., t_{N_{\texttt{perm}},\texttt{comm}}) \in (\mathbb{G}^{N_{\texttt{perm}}} \times \mathbb{G})$
- Evaluations:
  - $w_0(\zeta), ..., w_{N_{\texttt{wires}}}(\zeta) \in \mathbb{F}_r$
  - $w_0(\zeta\omega), ..., w_{N_{\texttt{wires}}}(\zeta\omega) \in \mathbb{F}_r$
  - $z(\zeta), z(\zeta\omega) \in \mathbb{F}_r$
  - $S_{\sigma_0}(\zeta), ..., S_{\sigma_{N_{\texttt{perm}}}}(\zeta) \in \mathbb{F}_r$
  - $S_{\sigma_0}(\zeta\omega), ..., S_{\sigma_{N_{\texttt{perm}}}}(\zeta\omega) \in \mathbb{F}_r$
  - $\bar{L}(\zeta\omega) \in \mathbb{F}_r$
- Opening proof o_π:
  - $(L_i, R_i) \in \mathbb{G} \times \mathbb{G}$ for $0 \leq i <$ `lr_rounds`
  - $\delta, \hat{G} \in \mathbb{G}$
  - $z_1, z_2 \in \mathbb{F}_r$
- previous challenges:
  - $\{\eta_i(\xi_j)\}_j, \eta_{i,\texttt{comm}}$, for $0 \leq i <$ `prev`

Denote multi-scalar multiplication $\sum_{s_i \in \mathbf{s}, G_i \in \mathbf{G}} [s_i]G_i$ by $\texttt{MSM}(\mathbf{s}, \mathbf{G})$ for $l_\mathbf{s} = l_\mathbf{G}$ where $l_\mathbf{s} = |\mathbf{s}|$, $l_\mathbf{G} = |\mathbf{G}|$. If $l_\mathbf{s} < l_\mathbf{G}$, then we use only first $l_\mathbf{s}$ elements of $\mathbf{G}$

Remark: For simplicity, we do not use distinct proofs index i for each element in the algorithm below. For instance, we write $pub_{\texttt{comm}}$ instead of $pub_{i,\texttt{comm}}$.

Algorithm 1 Verification

Input: $\pi_0, \ldots, \pi_{\texttt{batch\_size}}$ (see 2.2)

Output: `acc` or `rej`

1. for each π_i:

   1.1 $pub_{\texttt{comm}} = \texttt{MSM}(\mathbf{L}, \texttt{pub}) \in \mathbb{G}$, where $\mathbf{L}$ is Lagrange bases vector

   1.2 $\texttt{random\_oracle}(p_{\texttt{comm}}, \pi_i)$:

   1.2.1 $H_{\mathbb{F}_q}.\texttt{absorb}(pub_{\texttt{comm}}||w_{0,\texttt{comm}}||...||w_{N_{\text{wires}},\texttt{comm}})$

   1.2.2 $\beta, \gamma = H_{\mathbb{F}_q}.\texttt{squeeze}()$

   1.2.3 $H_{\mathbb{F}_q}.\texttt{absorb}(z_{\texttt{comm}})$

   1.2.4 $\alpha = \phi(H_{\mathbb{F}_q}.\texttt{squeeze}())$

   1.2.5 $H_{\mathbb{F}_q}.\texttt{absorb}(t_{1,\texttt{comm}}||...||t_{N_{\text{perm}},\texttt{comm}}||...||\infty||)$

   1.2.6 $\zeta = \phi(H_{\mathbb{F}_q}.\texttt{squeeze}())$

   1.2.7 Transfrorm $H_{\mathbb{F}_q}$ to $H_{\mathbb{F}_r}$

   1.2.8 $H_{\mathbb{F}_r}.\texttt{absorb}(pub(\zeta)||w_0(\zeta)||...||w_{N_{\text{wires}}}(\zeta)||S_0(\zeta)||...||S_{N_{\text{perm}}}(\zeta))$

   1.2.9 $H_{\mathbb{F}_r}.\texttt{absorb}(pub(\zeta\omega)||w_0(\zeta\omega)||...||w_{N_{\text{wires}}}(\zeta\omega)||S_0(\zeta\omega)||...||S_{N_{\text{perm}}}(\zeta\omega))$

   1.2.10 $H_{\mathbb{F}_r}.\texttt{absorb}(\bar{L}(\zeta\omega))$

   1.2.11 $v = \phi(H_{\mathbb{F}_r}.\texttt{squeeze}())$

   1.2.12 $u = \phi(H_{\mathbb{F}_r}.\texttt{squeeze}())$

   1.2.13 Compute evaluation of $\eta_i(\zeta), \eta_i(\zeta\omega)$ for $0 \leq i < N_{\texttt{prev}}$

   1.2.14 Compute evaluation of $\bar{L}(\zeta)$

   1.3 $\mathbf{f}_{\text{base}} \coloneqq \{S_{\sigma_{N_{\text{perm}}-1},\texttt{comm}}, \texttt{gate}_{\text{mult},\texttt{comm}}, w_{0,\texttt{comm}}, w_{1,\texttt{comm}}, w_{2,\texttt{comm}}, q_{\text{const},\texttt{comm}}, \texttt{gate}_{\text{psdn},\texttt{comm}}, \texttt{gate}_{\text{rc},\texttt{comm}}, \texttt{gate}_{\text{ec\_add},\texttt{comm}}, \texttt{gate}_{\text{ec\_dbl},\texttt{comm}}, \texttt{gate}_{\text{ec\_endo},\texttt{comm}}, \texttt{gate}_{\text{ec\_vbase},\texttt{comm}}\}$

   1.4 $s_{\text{perm}} \coloneqq (w_0(\zeta) + \gamma + \beta \cdot S_{\sigma_0}(\zeta)) \cdot \ldots \cdot (w_5(\zeta) + \gamma + \beta \cdot S_{\sigma_{N_{\text{perm}}}}(\zeta))$

   1.5 $\mathbf{f}_{\text{scalars}} \coloneqq \{-z(\zeta\omega) \cdot \beta \cdot \alpha_0 \cdot zkp(\zeta) \cdot s_{\text{perm}}, w_0(\zeta) \cdot w_1(\zeta), w_0(\zeta), w_1(\zeta), 1$ $s_{\text{psdn}}, s_{\text{rc}}, s_{\text{ec\_add}}, s_{\text{ec\_dbl}}, s_{\text{ec\_endo}}, s_{\text{ec\_vbase}}\}$

   1.6 $f_{\texttt{comm}} = \text{MSM}(\mathbf{f}_{\text{base}}, \mathbf{f}_{\text{scalars}})$

   1.7 $\bar{L}_{\texttt{comm}} = f_{\texttt{comm}} - t_{\texttt{comm}} \cdot (\zeta^n - 1)$

   1.8 $\mathbf{PE}$ is a set of elements of the form $(f_{\texttt{comm}}, f(\zeta), f(\zeta\omega))$ for the following polynomials: $\eta_0, \ldots, \eta_{N_{\texttt{prev}}}, pub, w_0, \ldots, w_{N_{\texttt{wires}}}, z, S_{\sigma_0}, \ldots, S_{\sigma_{N_{\texttt{perm}}}}, \bar{L}$

   1.9 $\mathcal{P}_i = \{H_{\mathbb{F}_q}, \zeta, v, u, \mathbf{PE}, o_{\pi_i}\}$

2. $\texttt{final\_check}(\mathcal{P}_0, \ldots, \mathcal{P}_{\texttt{batch\_size}})$

Algorithm 2 Final Check

Input: $\pi_0, \dots, \pi_{\texttt{batch\_size}}$, where $\pi_i = \{H_{i,\mathbb{F}_q}, \zeta_i, \zeta_i\omega, v_i, u_i, \mathbf{PE}_i, o_{\pi_i}\}$
Output: `acc` or `rej`

1. $\rho_1 \to \mathbb{F}_r$
2. $\rho_2 \to \mathbb{F}_r$
3. $r_0 = r'_0 = 1$
4. for $0 \le i <$ `batch_size`:
   - 4.1 $cip_i = \texttt{combined\_inner\_product}(\zeta_i, \zeta_i\omega, v_i, u_i, \mathbf{PE}_i)$
   - 4.2 $H_{i,\mathbb{F}_q}.\texttt{absorb}(cip_i - 2^{255})$
   - 4.3 $U_i = (H_{i,\mathbb{F}_q}.\texttt{squeeze}()).\texttt{to\_group}()$
   - 4.4 Calculate opening challenges $\xi_{i,j}$ from o_{π_i}
   - 4.5 $h_i(X) \coloneqq \prod_{k=0}^{\log(d+1)-1}(1 + \xi_{\log(d+1)-k} X^{2^k})$, where $d =$ `lr_rounds`
   - 4.6 $b_i = h_i(\zeta) + u_i \cdot h_i(\zeta\omega)$
   - 4.7 $C_i = \sum\limits_j v_i^j (\sum\limits_k r_i^k f_{j,\texttt{comm}}))$, where $f_{j,\texttt{comm}}$ from $\mathbf{PE}_i$.
   - 4.8 $Q_i = \sum(\xi_{i,j} \cdot L_{i,j} + \xi_{i,j}^{-1} \cdot R_j) + cip_i \cdot U_i + C_i$
   - 4.9 $c_i = \phi(H_{i,\mathbb{F}_q}.\texttt{squeeze}())$
   - 4.10 $r_i = r_{i-1} \cdot \rho_1$
   - 4.11 $r'_i = r'_{i-1} \cdot \rho_2$
   - 4.12 Check $\hat{G}_i = < s, G >$, where s is set of $h(X)$ coefficients.
     Remark: This check can be done inside the `MSM` below using r'_i.
5. $\texttt{res} = \sum\limits_i r^i (c_i Q_i + delta_i - (z_{i,1}(\hat{G}_i + b_i U_i) + z_{i,2} H))$
6. `return res` $== 0$

Algorithm 3 Combined Inner Product

Input: $\xi, r, f_0(\zeta_1), \dots, f_k(\zeta_1), f_0(\zeta_2), \dots, f_k(\zeta_2)$
Output: s

1. $s = \sum\limits_{i=0}^{k} \xi^i \cdot (f_i(\zeta_1) + r \cdot f_i(\zeta_2))$

3 Elliptic Curve Arithmetic

WIP

3.1 Addition

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	x_1	y_1	x_2	y_2	x_3	y_3	r	...	...	...	...	...	...	...	...

Constraints:

- $(x_2 - x_1) \cdot (y_3 + y_1) - (y_1 - y_2) \cdot (x_1 - x_3)$
- $(x_1 + x_2 + x_3) \cdot (x_1 - x_3) \cdot (x_1 - x_3) - (y_3 + y_1) \cdot (y_3 + y_1)$
- $(x_2 - x_1) \cdot r = 1$

3.2 Doubling and Tripling

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	x_1	y_1	x_2	y_2	x_3	y_3	r_1	r_2	...	...	...	...	...	...	...

Constraints:

- Doubling:
  - $4 \cdot y_1^2 \cdot (x_2 + 2 \cdot x_1) = 9 \cdot x_1^4$
  - $2 \cdot y_1 \cdot (y_2 + y_1) = (3 \cdot x_1^2) \cdot (x_1 - x_2)$
  - $y_1 \cdot r_1 = 1$
- Addition (for tripling):
  - $(x_2 - x_1) \cdot (y_3 + y_1) - (y_1 - y_2) \cdot (x_1 - x_3)$
  - $(x_1 + x_2 + x_3) \cdot (x_1 - x_3) \cdot (x_1 - x_3) - (y_3 + y_1) \cdot (y_3 + y_1)$
  - $(x_2 - x_1) \cdot r_2 = 1$

3.3 Variable Base Scalar Multiplication

For $S = [r]T$, where $r = 2^n + k$ and $k = [k_n ... k_0]$, $k_i \in \{0, 1\}$: [7]

1. $S = [2]T$
2. for i from $n - 1$ to 0:

   2.1 $Q = k_{i+1} \,?\, T : -T$

   2.2 $R = S + Q$

   2.3 $S = R + S$
3. $S = k_0 \,?\, S - T : S$

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	x_T	y_T	x_S	y_S	x_P	y_P	$n = 0$	x_R	y_R	s_1	s_2	b_1	s_3	s_4	b_2
$i+1$	s_5	b_3	x_S	y_S	x_P	y_P	n	x_R	y_R	x_V	y_V	s_1	b_1	s_3	b_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$i+100$	x_T	y_T	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_2	b_1	s_3	s_4	b_2
$i+101$	s_5	b_3	x_S	y_S	x_P	y_P	n	x_R	y_R	x_V	y_V	s_1	b_1	s_3	b_2

Constraints for $i + z$, where $z \mod 2 = 0$:

- $b_1 \cdot (b_1 - 1) = 0$
- $b_2 \cdot (b_2 - 1) = 0$
- $(x_P - x_T) \cdot s_1 = y_P - (2b_1 - 1) \cdot y_T$
- $s_1^2 - s_2^2 = x_T - x_R$
- $(2 \cdot x_P + x_T - s_1^2) \cdot (s_1 + s_2) = 2y_P$
- $(x_P - x_R) \cdot s_2 = y_R + y_P$
- $(x_R - x_T) \cdot s_3 = y_R - (2b_2 - 1) \cdot y_T$
- $s_3^2 - s_4^2 = x_T - x_S$
- $(2 \cdot x_R + x_T - s_3^2) \cdot (s_3 + s_4) = 2 \cdot y_R$
- $(x_R - x_S) \cdot s_4 = y_S + y_R$
- $n = 32 \cdot \texttt{next}(n) + 16 \cdot b_1 + 8 \cdot b_2 + 4 \cdot \texttt{next}(b_1) + 2 \cdot \texttt{next}(b_2) + \texttt{next}(b_3)$

Constraints for $i + z$, where $z \mod 2 = 1$:

[7] Using the results from https://arxiv.org/pdf/math/0208038.pdf

- $b_1 \cdot (b_1 - 1) = 0$
- $b_2 \cdot (b_2 - 1) = 0$
- $b_3 \cdot (b_3 - 1) = 0$
- $(x_P - x_T) \cdot s_1 = y_P - (2b_1 - 1) \cdot y_T$
- $(2 \cdot x_P + x_T - s_1^2) \cdot ((x_P - x_R) \cdot s_1 + y_R + y_P) = (x_P - x_R) \cdot 2y_P$
- $(y_R + y_P)^2 = (x_P - x_R)^2 \cdot (s_1^2 - x_T + x_R)$
- $(x_T - x_R) \cdot s_3 = (2b_2 - 1) \cdot y_T - y_R$
- $(2x_R - s_3^2 + x_T) \cdot ((x_R - x_V) \cdot s_3 + y_V + y_R) = (x_R - x_V) \cdot 2y_R$
- $(y_V + y_R)^2 = (x_R - x_V)^2 \cdot (s_3^2 - x_T + x_V)$
- $(x_T - x_V) \cdot s_5 = (2b_3 - 1) \cdot y_T - y_V$
- $(2x_V - s_5^2 + x_T) \cdot ((x_V - x_S) \cdot s_5 + y_S + y_V) = (x_V - x_S) \cdot 2y_V$
- $(y_S + y_V)^2 = (x_V - x_S)^2 \cdot (s_5^2 - x_T + x_S)$

3.4 Variable Base Endo-Scalar Multiplication

For $S = [r]T$, where $r = [r_n...r_0]$ and $r_i \in \{0, 1\}$: [8]

1. $S = [2](\phi(T) + T)$
2. for i from $\frac{\lambda}{2} - 1$ to 0:

    2.1 $Q = r_{2i+1}\ ?\ \phi([2r_{2i} - 1]T) : [2r_{2i} - 1]T$

    2.2 $R = S + Q$

    2.3 $S = R + S$

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	x_T	y_T	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_3	b_1	b_2	b_3	b_4
$i+1$	s_5	b_3	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_3	b_1	b_2	b_3	b_4
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$i+62$	x_T	y_T	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_3	b_1	b_2	b_3	b_4
$i+63$	s_5	b_3	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_3	b_1	b_2	b_3	b_4

Constraints:

- $b_1 \cdot (b_1 - 1) = 0$
- $b_2 \cdot (b_2 - 1) = 0$
- $b_3 \cdot (b_3 - 1) = 0$
- $b_4 \cdot (b_4 - 1) = 0$
- $((1 + (\texttt{endo} - 1) \cdot b_2) \cdot x_T - x_P) \cdot s_1 = (2 \cdot b_1 - 1) \cdot y_T - y_P$
- $(2 \cdot x_P - s_1^2 + (1 + (\texttt{endo} - 1) \cdot b_2) \cdot x_T) \cdot ((x_P - x_R) \cdot s_1 + y_R + y_P) = (x_P - x_R) \cdot 2 \cdot y_P$
- $(y_R + yP)^2 = (xP - x_R)^2 \cdot (s_1^2 - (1 + (\texttt{endo} - 1) \cdot b_2) \cdot x_T + x_R)$
- $((1 + (\texttt{endo} - 1) \cdot b_2) \cdot x_T - x_R) \cdot s_3 = (2 \cdot b_3 - 1) \cdot y_T - y_R$
- $(2 \cdot x_R - s_3^2 + (1 + (\texttt{endo} - 1) \cdot b_4) \cdot x_T) \cdot ((x_R - x_S) \cdot s_3 + y_S + y_R) = (x_R - x_S) \cdot 2 \cdot y_R$
- $(y_S + y_R)^2 = (x_R - x_S)^2 \cdot (s_3^2 - (1 + (\texttt{endo} - 1) \cdot b_4) \cdot x_T + x_S)$
- $n = 16 \cdot \texttt{next}(n) + 8 \cdot b_1 + 4 \cdot b_2 + 2 \cdot b_3 + b_4$

3.5 Fixed-base scalar multiplication circuit

We precompute all values $w(B, s, k) = (k_i + 2) \cdot 8^s B$, where $k_i \in \{0, ..7\}$, $s \in \{0, .., 83\}$ and $w(B, s, k) = (k_i \cdot 8^s - \sum_{j=0}^{84} 8^{j+1}) \cdot B$, where $k_i \in \{0, ..7\}$, $s = 84$.

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	b_0	b_1	b_2	b_3	b_4	b_5	u_0	u_1	v_0	v_1	x_1	y_1	x_2	y_2	acc
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$i+42$	b_0	b_1	b_2	u_0	v_0	x_w	y_w	α	β	γ	δ	λ	$-$	$-$	b

[8] Using the results from https://eprint.iacr.org/2019/1021.pdf

Define the following functions:

1. $\phi_1 : (x_1, x_2, x_3, x_4) \mapsto$
$x_3 \cdot (-u'_0 \cdot x_2 \cdot x_1 + u'_0 \cdot x_1 + u'_0 \cdot x_2 - u'_0 + u'_2 \cdot x_1 \cdot x_2 - u'_2 \cdot x_2 + u'_4 \cdot x_1 \cdot x_2 - u'_4 \cdot x_2 - u'_6 \cdot x_1 \cdot x_2 + u'_1 \cdot x_2 \cdot x_1 - u'_1 \cdot x_1 - u'_1 \cdot x_2 + u'_1 - u'_3 \cdot x_1 \cdot x_2 + u'_3 \cdot x_2 - u'_5 \cdot x_1 \cdot x_2 + u'_5 \cdot x_2 + u'_7 \cdot x_1 \cdot x_2) - (x_4 - u'_0 \cdot x_2 \cdot x_1 + u'_0 \cdot x_1 + u'_0 \cdot x_2 - u'_0 + u'_2 \cdot x_1 \cdot x_2 - u'_2 \cdot x_2 + u'_4 \cdot x_1 \cdot x_2 - u'_4 \cdot x_2 - u'_6 \cdot x_1 \cdot x_2)$

2. $\phi_2 : (x_1, x_2, x_3, x_4) \mapsto$
$x_3 \cdot (-v'_0 \cdot x_2 \cdot x_1 + v'_0 \cdot x_1 + v'_0 \cdot x_2 - v'_0 + v'_2 \cdot x_1 \cdot x_2 - v'_2 \cdot x_2 + v'_4 \cdot x_1 \cdot x_2 - v'_4 \cdot x_2 - v'_6 \cdot x_1 \cdot x_2 + v'_1 \cdot x_2 \cdot x_1 - v'_1 \cdot x_1 - v'_1 \cdot x_2 + v'_1 - v'_3 \cdot x_1 \cdot x_2 + v'_3 \cdot x_2 - v'_5 \cdot x_1 \cdot x_2 + v'_5 \cdot x_2 + v'_7 \cdot x_1 \cdot x_2) - (x_4 - v'_0 \cdot x_2 \cdot x_1 + v'_0 \cdot x_1 + v'_0 \cdot x_2 - v'_0 + v'_2 \cdot x_1 \cdot x_2 - v'_2 \cdot x_2 + v'_4 \cdot x_1 \cdot x_2 - v'_4 \cdot x_2 - v'_6 \cdot x_1 \cdot x_2)$

Constraints:

- For $i + 0$:
  - $b_i \cdot (b_i - 1) = 0$, where $i \in \{0, .., 5\}$
  - $\phi_1(b_0, b_1, b_2, u_0) = 0$, where $(u'_i, v'_i) = w(B, 0, i)$
  - $\phi_1(b_3, b_4, b_5, u_1) = 0$, where $(u'_i, v'_i) = w(B, 1, i)$
  - $\phi_2(b_0, b_1, b_2, v_0) = 0$, where $(u'_i, v'_i) = w(B, 0, i)$
  - $\phi_2(b_3, b_4, b_5, v_1) = 0$, where $(u'_i, v'_i) = w(B, 1, i)$
  - $acc = b_0 + b_1 \cdot 2 + b_2 \cdot 2^2 + b_3 \cdot 2^3 + b_4 \cdot 2^4 + b_5 \cdot 2^5$
  - $(x_1, y_1) = (u_0, v_0)$
  - $(x_2, y_2) = (x_1, y_1) + (u_1, v_1)$ incomplete addition, where $x_1 \neq u_1$
- For $i + z$, $z \in 1, ..., 41$:
  - $b_i \cdot (b_i - 1) = 0$, where $i \in \{0, .., 5\}$
  - $\phi_1(b_0, b_1, b_2, u_0) = 0$, where $(u'_i, v'_i) = w(B, z \cdot 2, i)$
  - $\phi_1(b_3, b_4, b_5, u_1) = 0$, where $(u'_i, v'_i) = w(B, z \cdot 2 + 1, i)$
  - $\phi_2(b_0, b_1, b_2, v_0) = 0$, where $(u'_i, v'_i) = w(B, z \cdot 2, i)$
  - $\phi_2(b_3, b_4, b_5, v_1) = 0$, where $(u'_i, v'_i) = w(B, z \cdot 2 + 1, i)$
  - $acc = b_0 + b_1 \cdot 2 + b_2 \cdot 2^2 + b_3 \cdot 2^3 + b_4 \cdot 2^4 + b_5 \cdot 2^5 + acc_{prev} \cdot 2^6$
  - $(x_1, y_1) = (u_0, v_0) + (x_2, y_2)_{prev}$ incomplete addition, where $u_0 \neq x_2$
  - $(x_2, y_2) = (x_1, y_1) + (u_1, v_1)$ incomplete addition, where $x_1 \neq u_1$
- For $i + 42$:
  - $b_i \cdot (b_i - 1) = 0$, where $i \in \{0, .., 2\}$
  - $\phi_1(b_0, b_1, b_2, u_0) = 0$, where $(u'_i, v'_i) = w(B, 84, i)$
  - $\phi_2(b_0, b_1, b_2, v_0) = 0$, where $(u'_i, v'_i) = w(B, 84, i)$
  - $b = b_0 + b_1 \cdot 2 + b_2 \cdot 2^2 + acc_{prev} \cdot 2^3$
  - $(x_w, y_w) = (u_0, v_0) + (x_2, y_2)_{prev}$ complete addition from Orchard

4 Multi-Scalar Multiplication Circuit

WIP

Input: $G_0, ..., G_{k-1} \in \mathbb{G}, s_0, ..., s_{k-1} \in \mathbb{F}_r$, where $\mathbb{F}_r$ is scalar field of $\mathbb{G}$.

Output: $S = \sum\limits_{i=0}^{k} s_i \cdot G_i$

4.1 Naive Algorithm

Using endomorphism:

1. $A = \infty$
2. for j from 0 to $k - 1$:

   2.1 $r := s_j$, $T := G_j$

   2.2 $S = [2](\phi(T) + T)$

   2.3 for i from $\frac{\lambda}{2} - 1$ to 0:

2.3.1 $Q = r_{2i+1} \mathbin{?} \phi([2r_{2i} - 1]T) : [2r_{2i} - 1]T$
2.3.2 $R = S + Q$
2.3.3 $S = R + S$

2.4 $A = A + S$

$$\text{rows} \approx k \cdot (\texttt{sm\_rows} + 1 + 2) \approx 67k,$$

where `sm_rows` is the number of rows in the scalar multiplication circuit.

Without endomorphism:

1. $A = \infty$
2. for j from 0 to $k - 1$:

   2.1 $r \coloneqq s_j$, $T \coloneqq G_j$

   2.2 $S = [2]T$

   2.3 for i from $n - 1$ to 0:

   2.3.1 $Q = k_{i+1} \mathbin{?} T : -T$
   2.3.2 $R = S + Q$
   2.3.3 $S = R + S$

   2.4 $S = k_0 \mathbin{?} S - T : S$

   2.5 $A = A + S$

$$\text{rows} \approx k \cdot (\texttt{sm\_rows} + 1 + 1) \approx 105k,$$

where `sm_rows` is the number of rows in the scalar multiplication circuit.

4.2 Simultanious Doubling

Using endomorphism:

1. $A = \sum_{j=0}^{k} [2](\phi(G_j) + G_j)$
2. for i from $\frac{\lambda}{2} - 1$ to 0:

   2.1 for j from 0 to $k - 1$:

   2.1.1 $r \coloneqq s_j$, $T \coloneqq G_j$
   2.1.2 $Q = r_{2i+1} \mathbin{?} \phi([2r_{2i} - 1]T) : [2r_{2i} - 1]T$
   2.1.3 $A = A + Q$

   2.2 if $i \neq 0$:

   2.2.1 $A = 2 \cdot A$

$$\text{rows} \approx \frac{\lambda}{2} \cdot (k \cdot \texttt{add\_rows} + \texttt{dbl\_rows}) + 2k \approx 64 \cdot (k + 1) \approx 66k + 64,$$

where

- `add_rows` is the number of rows in the addition circuit.
- `dbl_rows` is the number of rows in the doubling circuit.

Without endomorphism:

1. $A = \sum_{j=0}^{k} [2]G_j$
2. for i from $n-1$ to 0:

   2.1 for j from 0 to $k-1$:

   2.1.1 $r := s_j$, $T := G_j$

   2.1.2 $Q = k_{i+1} \,?\, T : -T$

   2.1.3 $A = A + Q$

   2.2 if $i \neq 0$:

   2.2.1 $A = 2 \cdot A$

3. $A = A + \sum_{j=0}^{k} [1 - s_{j,0}]G_j$

$$\text{rows} \approx \tfrac{2}{5}n \cdot (k \cdot \texttt{add\_rows} + \texttt{dbl\_rows}) + k \approx 103 \cdot (k+1) + 2k \approx 104k + 103,$$

where

- `add_rows` is the number of rows in the addition circuit.
- `dbl_rows` is the number of rows in the doubling circuit.

5 Poseidon Circuit

WIP

Mina uses Poseidon hash with width = 3. Therefore, each permutation state is represented by 3 elements and each row contains 5 states.

Denote i-th permutation state by $T_i = (T_{i,0}, T_{i,1}, T_{i,2})$.

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	$T_{0,0}$	$T_{0,1}$	$T_{0,2}$	$T_{4,0}$	$T_{4,1}$	$T_{4,2}$	$T_{1,0}$	$T_{1,1}$	$T_{1,2}$	$T_{2,0}$	$T_{2,1}$	$T_{2,2}$	$T_{3,0}$	$T_{3,1}$	$T_{3,2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$i+10$	$T_{50,0}$	$T_{50,1}$	$T_{50,2}$	$T_{54,0}$	$T_{54,1}$	$T_{54,2}$	$T_{51,0}$	$T_{51,1}$	$T_{51,2}$	$T_{52,0}$	$T_{52,1}$	$T_{52,2}$	$T_{53,0}$	$T_{53,1}$	$T_{53,2}$
$i+11$	$T_{55,0}$	$T_{55,1}$	$T_{55,2}$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

State change constraints:

$$\texttt{STATE}(i+1) = \texttt{STATE}(i)^{\alpha} \cdot \texttt{MDS} + \texttt{RC}$$

Denote the index of the first state in the row by `start` (e.g. `start` = 50 for 10-th row). We can expand the previous formula to:

- For i from `start` to `start` + 5:
  - $T_{i+1,0} = T_{i,0}^5 \cdot \texttt{MDS}[0][0] + T_{i,1}^5 \cdot \texttt{MDS}[0][2] + T_{i,2}^5 \cdot \texttt{MDS}[0][2] + \texttt{RC}_{i+1,0}$
  - $T_{i+1,1} = T_{i,0}^5 \cdot \texttt{MDS}[1][0] + T_{i,1}^5 \cdot \texttt{MDS}[1][2] + T_{i,2}^5 \cdot \texttt{MDS}[1][2] + \texttt{RC}_{i+1,1}$
  - $T_{i+1,2} = T_{i,0}^5 \cdot \texttt{MDS}[2][2] + T_{i,1}^5 \cdot \texttt{MDS}[2][2] + T_{i,2}^5 \cdot \texttt{MDS}[2][2] + \texttt{RC}_{i+1,2}$

Notice that the constraints above include the state from the next row (`start` + 5).

6 Other Circuits

WIP

6.1 Combined Inner Product

$$\sum_{i=0}^{k} \xi^i \cdot (f_i(\zeta_1) + r \cdot f_i(\zeta_2))$$

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$i+0$	f_1	f_1'	f_2	f_2'	$\mathtt{acc}$	ξ	$\xi_{\mathtt{acc}}$	s_1	s_2	$\xi'_{\mathtt{acc}}$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$i+1$	f_3	f_3'	f_4	f_4'	$\mathtt{acc}$	r	$\xi_{\mathtt{acc}}$	s_1	s_2	$\xi'_{\mathtt{acc}}$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$i+\lceil\frac{k}{2}\rceil-1$	f_{k-3}	f'_{k-3}	f_{k-2}	f'_{k-2}	$\mathtt{acc}$	ξ	$\xi_{\mathtt{acc}}$	s_1	s_2	$\xi'_{\mathtt{acc}}$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$i+\lceil\frac{k}{2}\rceil$	f_{k-1}	f'_{k-1}	f_k	f'_k	$\mathtt{acc}$	r	$\xi_{\mathtt{acc}}$	s_1	s_2	$\xi'_{\mathtt{acc}}$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$

Constraints for $i+z$, where $z \mod 2 = 0$:

- $(w_0 + w_1 \cdot \mathtt{next}(w_5)) \cdot w_6 = w_7$
- $(w_2 + w_3 \cdot \mathtt{next}(w_5)) \cdot w_9 = w_8$
- $w_5 \cdot w_6 = w_9$
- $w_5 \cdot w_9 = \mathtt{next}(w_9)$
- $w_5 \cdot \mathtt{next}(w_9) = \mathtt{next}(w_5)$
- $w_4 + w_7 + w_8 + \mathtt{next}(w_7) + \mathtt{next}(w_8) = \mathtt{next}(w_4)$

Constraints for $i+z$, where $z \mod 2 = 1$:

- $(w_0 + w_1 \cdot w_5) \cdot w_9 = w_7$
- $(w_2 + w_3 \cdot w_5) \cdot w_6 = w_8$

References