

In-EVM Mina State Verification Circuit Description

Cherniaeva Alisa

a.cherniaeva@nil.foundation

=nil; Crypto3 (<https://crypto3.nil.foundation>)

Shirobokov Ilia

i.shirobokov@nil.foundation

=nil; Crypto3 (<https://crypto3.nil.foundation>)

October 23, 2021

1 Introduction

WIP

High level description according to RfP¹

1. Computing several hash values from the data of the proof. This involves using the Poseidon hash function with 63 full rounds both over \mathbb{F}_p and \mathbb{F}_q with round constants and MDS matrix specified for \mathbb{F}_p ² and for \mathbb{F}_q ³.
2. Checking arithmetic equations.
3. Performing one multi-scalar multiplication (MSM) of size $2n_2 + 4 + (2 + 25) = 63$, for which some of the bases are fixed and some are variable.
4. For each $i \in \{1, 2\}$, performing a multi-scalar multiplication over \mathbb{G}_i of size 2^{n_i} with a fixed array of bases, and with scalars that can be very efficiently computed from the proof.

Note that for MSM in Step 4:

$$\sum_{i=0}^{2^{n_k}-1} s_i \cdot G_i = H$$
$$s_i := \prod_{\substack{0 \leq j \leq n_k \\ \text{bits}(i)[j]=1}} \phi(c_j),$$

where:

- $\phi: \{0, 1\}^{128} \rightarrow \mathbb{F}$ is defined as `to_field` in the implementation⁴.
- Given an integer $i < 2^{n_k}$, $\text{bits}(i)$ is defined as the little-endian bit array of length n representing the binary expansion of i .
- $G_0, \dots, G_{2^{n_k}-1} \in \mathbb{G}_k$ is a fixed sequence of group elements⁵.
- $c_0, \dots, c_{n_k-1} \in \{0, 1\}^{128}$ is a sequence of challenges.

We use the same 15-wires PLONK circuits that are designed for Mina.⁶

2 Preliminaries

WIP

¹https://hackmd.io/u_2Ygx8XS5Ss1a0bg0FjkA

²<https://github.com/o1-labs/proof-systems/blob/master/oracle/src/pasta/fp.rs>

³<https://github.com/o1-labs/proof-systems/blob/master/oracle/src/pasta/fq.rs>

⁴<https://github.com/o1-labs/proof-systems/blob/49f81edc9c86e5907d26ea791fa083640ad0ef3e/oracle/src/sponge.rs#L33>

⁵<https://github.com/o1-labs/proof-systems/blob/master/dlog/commitment/src/srs.rs#L70>

⁶https://o1-labs.github.io/mina-book/specs/15_wires/15_wires.html

2.1 Pasta Curves

Let $n_1 = 17$, $n_2 = 16$. Pasta curves parameters:

- $p = 2^{254} + 45560315531419706090280762371685220353$
- $q = 2^{254} + 45560315531506369815346746415080538113$
- Pallas:

$$\mathbb{G}_1 = \{(x, y) \in \mathbb{F}_p \mid y^2 = x^3 + 5\}$$

$$|\mathbb{G}_1| = q$$

- Vesta:

$$\mathbb{G}_2 = \{(x, y) \in \mathbb{F}_q \mid y^2 = x^3 + 5\}$$

$$|\mathbb{G}_2| = p$$

2.2 Verification Algorithm

Proof state (here \mathbb{F}_r is a scalar field of \mathbb{G}):

- DLog Commitments:
 - $l_{comm}, r_{comm}, o_{comm}, z_{comm} \in \mathbb{G}$
 - $t_{comm} = (t_{comm,1}, t_{comm,2}) \in (\mathbb{G}^5 \times \mathbb{G})$
- Openings:
 - $(L_i, R_i) \in \mathbb{G} \times \mathbb{G}$ for $0 \leq i < \text{lr_rounds}$
 - $\delta, SG \in \mathbb{G}$
 - $z_1, z_2 \in \mathbb{F}_r$
- Polynomial Evaluations a, b , for $i = \{1, 2\}$:
 - $l_i, r_i, o_i, z_i, f_i \in \mathbb{F}_r$
 - $t_i \in \mathbb{F}_r^5$
 - $\sigma_{1_i}, \sigma_{2_i} \in \mathbb{F}_r$
- $w \in \mathbb{F}_r^{sw}$ - witness
- previous challenges:
 - $(c_i, p_i) \in (\mathbb{F}_r \times \mathbb{G})$ for $0 \leq i < \text{prev}$

Let g_r, g_q are generators of \mathbb{F}_r and \mathbb{F}_q accordingly.

Verification algorithm:

1. for each \mathcal{P} :

- 1.1 $p_{comm} = \text{MSM}(\text{lgr_comm}, \text{proof}, \text{public}) \in \mathbb{G}$ // public input verification
- 1.2 $\text{ORACLES} \rightarrow \{\text{digest}, (\beta, \gamma, \alpha', \alpha, \zeta, v, u, \zeta', v', u'), \alpha_2, (\text{pub}_1, \text{pub}_2), \text{evlp}, \text{polys}, \zeta_1, \text{combined inner product}\}$:
 - 1.2.1 $H_{\mathbb{F}_q}.\text{absorb}(p_{comm} || l_{comm} || r_{comm} || o_{comm})$
 - 1.2.2 $\beta = H_{\mathbb{F}_q}.\text{squeeze}()$
 - 1.2.3 $\gamma = H_{\mathbb{F}_q}.\text{squeeze}()$
 - 1.2.4 $H_{\mathbb{F}_q}.\text{absorb}(z_{comm})$
 - 1.2.5 $\alpha' = H_{\mathbb{F}_q}.\text{squeeze}()$
 - 1.2.6 $\alpha = \phi(\alpha', \text{endo_r})$
 - 1.2.7 $H_{\mathbb{F}_q}.\text{absorb}(t_{comm,1} || \infty || \dots || \infty || t_{comm,2})$
 - 1.2.8 $\zeta' = H_{\mathbb{F}_q}.\text{squeeze}()$
 - 1.2.9 $\zeta = \phi(\zeta', \text{endo_r})$
 - 1.2.10 $\text{digest} = H_{\mathbb{F}_q}.\text{digest}()$
 - 1.2.11 $\zeta_1 = \zeta^n$
 - 1.2.12 $\zeta_w = \zeta * g_r$
 - 1.2.13 $\alpha_2 = [\alpha^2, \dots, \alpha^{19}]$
 - 1.2.14 compute Lagrange base evaluation denominators

- 1.2.15 evaluate public input polynomials (return pub_1, pub_2)
- 1.2.16 $H_{\mathbb{F}_r}.absorb(pub_1 || pub_2)$
- 1.2.17 $v' = H_{\mathbb{F}_r}.squeeze()$
- 1.2.18 $v = \phi(v', endo_r)$
- 1.2.19 $u' = H_{\mathbb{F}_r}.squeeze()$
- 1.2.20 $u = \phi(u', endo_r)$
- 1.2.21 $elvp = \zeta^{mp1}, \zeta_{\omega}^{mp1}$
- 1.2.22 `prev_chal_evals`
- 1.2.23 inner product calculations
- 1.3 arithmetic operations:
 - 1.3.1 polynomial evaluation over a, b (proof evaluations)
 - 1.3.2 polynomial evaluation over `zkpm` at ζ
 - 1.3.3 `perm_scalars`
- 1.4 $f_{comm} = \text{MSM}(p, s)$
- 1.5 linearization polynomial evaluation consistency:
- 2. `srs.verify`:
 - 2.1 ...
 - 2.2 MSM:

$$\sum_i r^i (c_i Q_i + \text{delta}_i - (z_{1,i}(G_i + b_i U_i) + z_{2,i} H))$$

3 Elliptic Curve Arithmetic

WIP

3.1 Addition

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	x_1	y_1	x_2	y_2	x_3	y_3	r	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

Constraints:

- $(x_2 - x_1) \cdot (y_3 + y_1) - (y_1 - y_2) \cdot (x_1 - x_3)$
- $(x_1 + x_2 + x_3) \cdot (x_1 - x_3) \cdot (x_1 - x_3) - (y_3 + y_1) \cdot (y_3 + y_1)$
- $(x_2 - x_1) \cdot r = 1$

3.2 Doubling and Tripling

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	x_1	y_1	x_2	y_2	x_3	y_3	r_1	r_2	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

Constraints:

- Doubling:
 - $4 \cdot y_1^2 \cdot (x_2 + 2 \cdot x_1) = 9 \cdot x_1^4$
 - $2 \cdot y_1 \cdot (y_2 + y_1) = (3 \cdot x_1^2) \cdot (x_1 - x_2)$
 - $y_1 \cdot r_1 = 1$
- Addition (for tripling):
 - $(x_2 - x_1) \cdot (y_3 + y_1) - (y_1 - y_2) \cdot (x_1 - x_3)$
 - $(x_1 + x_2 + x_3) \cdot (x_1 - x_3) \cdot (x_1 - x_3) - (y_3 + y_1) \cdot (y_3 + y_1)$
 - $(x_2 - x_1) \cdot r_2 = 1$

3.3 Variable Base Scalar Multiplication

For $S = [r]T$, where $r = 2^n + k$ and $k = [k_n \dots k_0]$, $k_i \in \{0, 1\}$: ⁷

1. $S = [2]T$
2. for i from $n - 1$ to 0:
 - 2.1 $Q = k_{i+1} ? T : -T$
 - 2.2 $R = S + Q$
 - 2.3 $S = R + S$
3. $S = k_0 ? S - T : S$

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	x_T	y_T	x_S	y_S	x_P	y_P	$n = 0$	x_R	y_R	s_1	s_2	b_1	s_3	s_4	b_2
$i + 1$	s_5	b_3	x_S	y_S	x_P	y_P	n	x_R	y_R	x_V	y_V	s_1	b_1	s_3	b_2
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
$i + 100$	x_T	y_T	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_2	b_1	s_3	s_4	b_2
$i + 101$	s_5	b_3	x_S	y_S	x_P	y_P	n	x_R	y_R	x_V	y_V	s_1	b_1	s_3	b_2

Constraints for $i + z$, where $z \bmod 2 = 0$:

- $b_1 \cdot (b_1 - 1) = 0$
- $b_2 \cdot (b_2 - 1) = 0$
- $(x_P - x_T) \cdot s_1 = y_P - (2b_1 - 1) \cdot y_T$
- $s_1^2 - s_2^2 = x_T - x_R$
- $(2 \cdot x_P + x_T - s_1^2) \cdot (s_1 + s_2) = 2y_P$
- $(x_P - x_R) \cdot s_2 = y_R + y_P$
- $(x_R - x_T) \cdot s_3 = y_R - (2b_2 - 1) \cdot y_T$
- $s_3^2 - s_4^2 = x_T - x_S$
- $(2 \cdot x_R + x_T - s_3^2) \cdot (s_3 + s_4) = 2 \cdot y_R$
- $(x_R - x_S) \cdot s_4 = y_S + y_R$
- $n = 32 \cdot \text{next}(n) + 16 \cdot b_1 + 8 \cdot b_2 + 4 \cdot \text{next}(b_1) + 2 \cdot \text{next}(b_2) + \text{next}(b_3)$

Constraints for $i + z$, where $z \bmod 2 = 1$:

- $b_1 \cdot (b_1 - 1) = 0$
- $b_2 \cdot (b_2 - 1) = 0$
- $b_3 \cdot (b_3 - 1) = 0$
- $(x_P - x_T) \cdot s_1 = y_P - (2b_1 - 1) \cdot y_T$
- $(2 \cdot x_P + x_T - s_1^2) \cdot ((x_P - x_R) \cdot s_1 + y_R + y_P) = (x_P - x_R) \cdot 2y_P$
- $(y_R + y_P)^2 = (x_P - x_R)^2 \cdot (s_1^2 - x_T + x_R)$
- $(x_T - x_R) \cdot s_3 = (2b_2 - 1) \cdot y_T - y_R$
- $(2x_R - s_3^2 + x_T) \cdot ((x_R - x_V) \cdot s_3 + y_V + y_R) = (x_R - x_V) \cdot 2y_R$
- $(y_V + y_R)^2 = (x_R - x_V)^2 \cdot (s_3^2 - x_T + x_V)$
- $(x_T - x_V) \cdot s_5 = (2b_3 - 1) \cdot y_T - y_V$
- $(2x_V - s_5^2 + x_T) \cdot ((x_V - x_S) \cdot s_5 + y_S + y_V) = (x_V - x_S) \cdot 2y_V$
- $(y_S + y_V)^2 = (x_V - x_S)^2 \cdot (s_5^2 - x_T + x_S)$

3.4 Variable Base Endo-Scalar Multiplication

For $S = [r]T$, where $r = [r_n \dots r_0]$ and $r_i \in \{0, 1\}$: ⁸

1. $S = [2](\phi(T) + T)$
2. for i from $\frac{\lambda}{2} - 1$ to 0:

⁷Using the results from <https://arxiv.org/pdf/math/0208038.pdf>

⁸Using the results from <https://eprint.iacr.org/2019/1021.pdf>

$$2.1 \quad Q = r_{2i+1} ? \phi([2r_{2i} - 1]T) : [2r_{2i} - 1]T$$

$$2.2 \quad R = S + Q$$

$$2.3 \quad S = R + S$$

Row	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
i	x_T	y_T	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_3	b_1	b_2	b_3	b_4
$i + 1$	s_5	b_3	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_3	b_1	b_2	b_3	b_4
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
$i + 62$	x_T	y_T	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_3	b_1	b_2	b_3	b_4
$i + 63$	s_5	b_3	x_S	y_S	x_P	y_P	n	x_R	y_R	s_1	s_3	b_1	b_2	b_3	b_4

Constraints:

- $b_1 \cdot (b_1 - 1) = 0$
- $b_2 \cdot (b_2 - 1) = 0$
- $b_3 \cdot (b_3 - 1) = 0$
- $b_4 \cdot (b_4 - 1) = 0$
- $((1 + (\text{endo} - 1) \cdot b_2) \cdot x_T - x_P) \cdot s_1 = (2 \cdot b_1 - 1) \cdot y_T - y_P$
- $(2 \cdot x_P - s_1^2 + (1 + (\text{endo} - 1) \cdot b_2) \cdot x_T) \cdot ((x_P - x_R) \cdot s_1 + y_R + y_P) = (x_P - x_R) \cdot 2 \cdot y_P$
- $(y_R + y_P)^2 = (x_P - x_R)^2 \cdot (s_1^2 - (1 + (\text{endo} - 1) \cdot b_2) \cdot x_T + x_R)$
- $((1 + (\text{endo} - 1) \cdot b_2) \cdot x_T - x_R) \cdot s_3 = (2 \cdot b_3 - 1) \cdot y_T - y_R$
- $(2 \cdot x_R - s_3^2 + (1 + (\text{endo} - 1) \cdot b_4) \cdot x_T) \cdot ((x_R - x_S) \cdot s_3 + y_S + y_R) = (x_R - x_S) \cdot 2 \cdot y_R$
- $(y_S + y_R)^2 = (x_R - x_S)^2 \cdot (s_3^2 - (1 + (\text{endo} - 1) \cdot b_4) \cdot x_T + x_S)$
- $n = 16 \cdot \text{next}(n) + 8 \cdot b_1 + 4 \cdot b_2 + 2 \cdot b_3 + b_4$

4 Multi-Scalar Multiplication Circuit

WIP

Input: $G_0, \dots, G_{k-1} \in \mathbb{G}, s_0, \dots, s_{k-1} \in \mathbb{F}_r$, where \mathbb{F}_r is scalar field of \mathbb{G} .

Output: $S = \sum_{i=0}^k s_i \cdot G_i$

4.1 Naive Algorithm

Using endomorphism:

$$1. \quad A = \infty$$

$$2. \quad \text{for } j \text{ from } 0 \text{ to } k - 1:$$

$$2.1 \quad r := s_j, T := G_j$$

$$2.2 \quad S = [2](\phi(T) + T)$$

$$2.3 \quad \text{for } i \text{ from } \frac{\lambda}{2} - 1 \text{ to } 0:$$

$$2.3.1 \quad Q = r_{2i+1} ? \phi([2r_{2i} - 1]T) : [2r_{2i} - 1]T$$

$$2.3.2 \quad R = S + Q$$

$$2.3.3 \quad S = R + S$$

$$2.4 \quad A = A + S$$

$$\text{rows} \approx k \cdot (\text{sm_rows} + 1 + 2) \approx 67k,$$

where **sm_rows** is the number of rows in the scalar multiplication circuit.

Without endomorphism:

1. $A = \infty$
2. for j from 0 to $k - 1$:
 - 2.1 $r := s_j, T := G_j$
 - 2.2 $S = [2]T$
 - 2.3 for i from $n - 1$ to 0:
 - 2.3.1 $Q = k_{i+1} ? T : -T$
 - 2.3.2 $R = S + Q$
 - 2.3.3 $S = R + S$
 - 2.4 $S = k_0 ? S - T : S$
 - 2.5 $A = A + S$

$$\text{rows} \approx k \cdot (\text{sm_rows} + 1 + 1) \approx 105k,$$

where **sm_rows** is the number of rows in the scalar multiplication circuit.

4.2 Simultaneous Doubling**Using endomorphism:**

1. $A = \sum_{j=0}^k [2](\phi(G_j) + G_j)$
2. for i from $\frac{\lambda}{2} - 1$ to 0:
 - 2.1 for j from 0 to $k - 1$:
 - 2.1.1 $r := s_j, T := G_j$
 - 2.1.2 $Q = r_{2i+1} ? \phi([2r_{2i} - 1]T) : [2r_{2i} - 1]T$
 - 2.1.3 $A = A + Q$
 - 2.2 if $i \neq 0$:
 - 2.2.1 $A = 2 \cdot A$

$$\text{rows} \approx \frac{\lambda}{2} \cdot (k \cdot \text{add_rows} + \text{dbl_rows}) + 2k \approx 64 \cdot (k + 1) \approx 66k + 64,$$

where

- **add_rows** is the number of rows in the addition circuit.
- **dbl_rows** is the number of rows in the doubling circuit.

Without endomorphism:

1. $A = \sum_{j=0}^k [2]G_j$
2. for i from $n - 1$ to 0:
 - 2.1 for j from 0 to $k - 1$:
 - 2.1.1 $r := s_j, T := G_j$
 - 2.1.2 $Q = k_{i+1} ? T : -T$
 - 2.1.3 $A = A + Q$
 - 2.2 if $i \neq 0$:
 - 2.2.1 $A = 2 \cdot A$
3. $A = A + \sum_{j=0}^k [1 - s_{j,0}]G_j$

$$\text{rows} \approx \frac{2}{5}n \cdot (k \cdot \text{add_rows} + \text{dbl_rows}) + k \approx 103 \cdot (k + 1) + 2k \approx 104k + 103,$$

where

- **add_rows** is the number of rows in the addition circuit.
- **dbl_rows** is the number of rows in the doubling circuit.

5 Poseidon Circuit

WIP

Constraints:

- $\text{STATE}(i + 1) = \text{STATE}(i)^\alpha \cdot \text{MDS} + \text{RC}$

5.1 \mathbb{F}_p

5.2 \mathbb{F}_q

6 Other Circuits

WIP

7 Bringing it all together

WIP

References