In-EVM Mina State Verification
Proof System Description

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1 Introduction

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To prove Mina blockchain's state on the Ethereum Virtual Machine, we use Redshift SNARK[1]. RedShift is a transparent SNARK that uses PLONK[2] proof system but replaces the commitment scheme. The authors utilize FRI[3] protocol to obtain transparency for the PLONK system.

However, FRI cannot be straightforwardly used with the PLONK system. To achieve the required security level without huge overheads, the authors introduce *list polynomial commitment* scheme as a part of the protocol. For more details, we refer the reader to [1].

The original RedShift protocol utilizes the classic PLONK[2] system. To provide better performance, we generilize the original protocol for use with PLONK with custom gates [4], [5] and lookup arguments [6], [7].

2 RedShift Protocol

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Notations:

$N_{\texttt{wires}}$	Number of wires (‘advice columns‘)
$N_{\texttt{perm}}$	Number of wires that are included in the permutation argument
$N_{\texttt{sel}}$	Number of selectors used in the circuit
$N_{\texttt{const}}$	Number of constant columns
$\mathbf{f}_i$	Witness polynomials, $0 \leq i < N_{\texttt{wires}}$
$\mathbf{f}_{c_i}$	Constant-related polynomials, $0 \leq i < N_{\texttt{const}}$
$\mathbf{gate}_i$	Gate polynomials, $0 \leq i < N_{\texttt{sel}}$
$\sigma(\text{col} : i, \text{row} : j) = (\text{col} : i', \text{row} : j')$	Permutation over the table

For details on polynomial commitment scheme and polynomial evaluation scheme, we refer the reader to [1].

Preprocessing:

1. $\mathcal{L}' = (\mathbf{q}_0, ..., \mathbf{q}_{N_{\mathtt{sel}}})$
2. Let ω be a 2^k root of unity
3. Let δ be a T root of unity, where $T \cdot 2^S + 1 = p$ with T odd and $k \leq S$
4. Compute $N_{\mathtt{perm}}$ permutation polynomials $S_{\sigma_i}(X)$ such that $S_{\sigma_i}(\omega^j) = \delta^{i'} \cdot \omega^{j'}$
5. Compute $N_{\mathtt{perm}}$ identity permutation polynomials: $S_{id_i}(X)$ such that $S_{id_i}(\omega^j) = \delta^i \cdot \omega^j$
6. Let $H = \{\omega^0, ..., \omega^n\}$ be a cyclic subgroup of $\mathbb{F}^*$
7. Let $Z(X) = \prod a \in H^*(X - a)$

Protocol (Prover):

1. Choose masking polynomials:

$$h_i(X) \leftarrow \mathbb{F}_{<k}[X] \text{ for } 0 \leq i < N_{\mathtt{wires}}$$

   Remark: For details on choice of k, we refer the reader to [1].

2. Define new witness polynomials:

$$f_i(X) = \mathbf{f}_i(X) + h_i(X)Z(X) \text{ for } 0 \leq i < N_{\mathtt{wires}}$$

3. Send commitments to f_i to $\mathbf{V}$
4. Get $\beta, \gamma \leftarrow \mathbb{F}$ from $\mathbf{V}$
5. For $0 \leq i < N_{\mathtt{perm}}$

$$\begin{aligned} p_i &= f_i + \beta \cdot S_{id_i} + \gamma \\ q_i &= f_i + \beta \cdot S_{\sigma_i} + \gamma \end{aligned}$$

6. Define:

$$\begin{aligned} p'(X) &= \prod_{0 \leq i < N_{\mathtt{perm}}} p_i(X) \in \mathbb{F}_{<N_{\mathtt{perm}} \cdot n}[X] \\ q'(X) &= \prod_{0 \leq i < N_{\mathtt{perm}}} q_i(X) \in \mathbb{F}_{<N_{\mathtt{perm}} \cdot n}[X] \end{aligned}$$

7. Compute $P(X), Q(X) \in \mathbb{F}_{<n+1}[X]$, such that:

$$\begin{aligned} P(\omega) = Q(\omega) &= 1 \\ P(\omega^i) &= \prod_{1 \leq j < i} p'(\omega^i) \text{ for } i \in 2, \ldots, n+1 \\ Q(\omega^i) &= \prod_{1 \leq j < i} q'(\omega^i) \text{ for } i \in 2, \ldots, n+1 \end{aligned}$$

8. Compute and send commitments to P and Q to $\mathbf{V}$
9. Get $\alpha_0, \ldots, \alpha_5 \leftarrow \mathbb{F}$ from $\mathbf{V}$
10. Define polynomials ($F_0, \ldots, F_4$ - copy-satisfability):

$$\begin{aligned} F_0(X) &= L_1(X)(P(X) - 1) \\ F_1(X) &= L_1(X)(Q(X) - 1) \\ F_2(X) &= P(X)p'(X) - P(X\omega) \\ F_3(X) &= Q(X)q'(X) - Q(X\omega) \\ F_4(X) &= L_n(X)(P(X\omega) - Q(X\omega)) \\ F_5(X) &= \sum_{0 \leq i < N_{\mathtt{sel}}} (\mathbf{q}_i(X) \cdot \mathtt{gate}_i(X)) + \sum_{0 \leq i < N_{\mathtt{const}}} (\mathbf{f}_{c_i}(X)) + PI(X) \end{aligned}$$

11. Compute:

$$F(X) = \sum_{i=0}^{5} \alpha_i F_i(X)$$
$$T(X) = \frac{F(X)}{Z(X)}$$

12. Split $T(X)$ into separate polynomials $T_0(X), ..., T_{N_{\texttt{perm}}}(X)$
13. Send commitments to $T_0(X), ..., T_{N_{\texttt{perm}}}(X)$ to $\mathbf{V}$
14. Get $y \leftarrow \mathbb{F}/H$ from $\mathbf{V}$
15. Run evaluation scheme with the committed polynomials and y.
Remark: Depending on the circuit, evaluation can be done also on $y\omega, y\omega^{-1}$.
16. Send proof π to $\mathbf{V}$

2.1 Non-Interactive Verification

1. Let $f_{0,\texttt{comm}}, \ldots, f_{N_{\texttt{wires}},\texttt{comm}}$ be commitments to $f_0(X), \ldots, f_{N_{\texttt{wires}}}(X)$
2. transcript = setup_values$||f_{0,\texttt{comm}}|| \ldots ||f_{N_{\texttt{wires}},\texttt{comm}}$
3. $\beta, \gamma = H(\text{transcript})$
4. Let $P_{\texttt{comm}}, Q_{\texttt{comm}}$ be commitments to $P(X), Q(X)$
5. transcript = transcript$||P_{\texttt{comm}}||Q_{\texttt{comm}}$
6. $\alpha_0, \ldots, \alpha_5 = H(\text{transcript})$
7. Let $T_{0,\texttt{comm}}, ..., T_{N_{\texttt{perm,comm}}}$ be commitments to $T_0(X), ..., T_{N_{\texttt{perm}}}(X)$
8. transcript = transcript$||T_{0,\texttt{comm}}||...||T_{N_{\texttt{perm,comm}}}$
9. $y = H_{\mathbb{F}/H}(\text{transcript})$
10. Run evaluation scheme verification with the committed polynomials and y to get values $f_i(y), P(y), P(y\omega), Q(y), Q(y\omega), T_j(y)$.
Remark: Depending on the circuit, evaluation can be done also on $f_i(y\omega), f_i(y\omega^{-1})$ for some i.
11. Calculate:

$$F_0(y) = L_1(y)(P(y) - 1)$$
$$F_1(y) = L_1(y)(Q(y) - 1)$$
$$p'(y) = \prod p_i(y) = \prod f_i(y) + \beta \cdot S_{id_i}(y) + \gamma$$
$$F_2(y) = P(y)p'(y) - P(y\omega)$$
$$q'(y) = \prod q_i(y) = \prod f_i(y) + \beta \cdot S_{\sigma_i}(y) + \gamma$$
$$F_3(y) = Q(y)q'(y) - Q(y\omega)$$
$$F_4(y) = L_n(y)(P(y\omega) - Q(y\omega))$$
$$F_5(y) = \sum_{0 \le i < N_{\texttt{sel}}} (\mathbf{q}_i(y) \cdot \texttt{gate}_i(y)) + \sum_{0 \le i < N_{\texttt{const}}} (\mathbf{f}_{c_i}(y)) + PI(y)$$
$$T(y) = \sum_{0 \le j < N_{\texttt{perm}+1}} y^{n \cdot j} T_j(y)$$

12. Check the identity:

$$\sum_{i=0}^{5} \alpha_i F_i(y) = Z(y)T(y)$$

3 Optimizations

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3.1 Batched FRI

Instead of check each commitment individualy, we can aggregate them for FRI. For polynomials $f_0, \dots, f_k$:

1. Get θ from transcript
2. $f = f_0 \cdot \theta^{k-1} + \dots + f_k$
3. Run FRI over f, using oracles to $f_0, \dots, f_k$

Thus, we can run only one FRI instance for all commited polynomials.
See [1] for details.

3.2 Hash By Column

Instead of committing each of the polynomials, we can use the same Merkle tree for several polynomials. It decreases the number of Merkle tree paths that need to be provided by the prover.
See [8], [1] for details.

3.3 Hash By Subset

On the each $i+1$ FRI round, the prover should send all elements from a coset $H \in D^{(i)}$. Each Merkle leaf is able to contain the whole coset instead of separate values.
See [8] for details. Similar approach is described in [1]. However, the authors of [1] use more values per leaf, that leads to better performance.

References

1. Kattis A., Panarin K., Vlasov A. RedShift: Transparent SNARKs from List Polynomial Commitment IOPs. Cryptology ePrint Archive, Report 2019/1400. 2019. https://ia.cr/2019/1400.
2. Gabizon A., Williamson Z. J., Ciobotaru O. PLONK: Permutations over Lagrange-bases for Oecumenical Noninteractive arguments of Knowledge. Cryptology ePrint Archive, Report 2019/953. 2019. https://ia.cr/2019/953.
3. Fast Reed-Solomon interactive oracle proofs of proximity / E. Ben-Sasson, I. Bentov, Y. Horesh et al. // 45th international colloquium on automata, languages, and programming (icalp 2018) / Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik. 2018.
4. Gabizon A., Williamson Z. J. Proposal: The Turbo-PLONK program syntax for specifying SNARK programs. https://docs.zkproof.org/pages/standards/accepted-workshop3/proposal-turbo_plonk.pdf.
5. PLONKish Arithmetization - The halo2 book. https://zcash.github.io/halo2/concepts/arithmetization.html.
6. Gabizon A., Williamson Z. J. plookup: A simplified polynomial protocol for lookup tables. Cryptology ePrint Archive, Report 2020/315. 2020. https://ia.cr/2020/315.
7. Lookup argument - The halo2 book. https://zcash.github.io/halo2/design/proving-system/lookup.html.
8. Chiesa A., Ojha D., Spooner N. Fractal: Post-Quantum and Transparent Recursive Proofs from Holography. Cryptology ePrint Archive, Report 2019/1076. 2019. https://ia.cr/2019/1076.