ASSIGN - 7

MAB-103

PARTH
23116073

1). (i) $\int_0^{1.2} \log(1+x^2)\,dx$, $n=6$ $\quad h = \frac{1.2-0}{6} = 0.2$

a) Trapezoidal :

$I = \frac{(0.2)}{2}\left[(0 + 0.891998) + 2(0.0392207 + 0.1484200 + 0.307484 + 0.494696 + \cancel{0.2189141}\ 0.693147)\right]$

$= \boxed{0.42579}$

b) Simpson's 1/3rd

$I = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+ \dots + y_{n-1}) + 2(y_2+y_4+ \dots y_{n-2})\right]$

$= \frac{(0.2)}{3}\left[(0+0.891998) + 4(0.0392207 + 0.307484 + 0.693147) + 2(0.1484200 + 0.494696)\right]$

$= 0.422509$

c) Simpson 3/8

$I = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4 + \dots y_{n-1}) + 2(y_3+y_6+y_{\dots n-3})\right]$

$= \frac{3(0.2)}{8}\left[(0+0.891998) + 2(0.307484) + 3(0.0392207 + 0.1484200 + 0.494096 + 0.693147)\right]$

$= 0.4225062$

(ii) $\int_0^1 \cos(x^2)\,dx$, $n=12$ $\Rightarrow h = \frac{1-0}{12} = 0.083333$

(i) Trapezoidal: $\frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2 \cdots + y_{n-1})\right]$

$= \frac{(0.083333)}{2}\left[(1+0.540302) + 2(0.999975 + 0.9999614 + 0.998047 + 0.9938335 + 0.984967 + 0.968912 + 0.942661 + 0.902849 + 0.845924 + 0.7684093 + 0.66725595\right.$

$= 0.904670$

(ii) Simpson's $1/3^{rd}$ rule

$\frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+\cdots+y_{n-2}) + 4(y_1+y_3+y_5+\cdots+y_{n-1})\right]$

$= \frac{(1/12)}{3}\left[(1+0.540302) + 2(0.9999614 + 0.9938335 + 0.968912 + 0.902849 + 0.7684093) + 4(0.999975 + 0.998047 + 0.984967 + 0.942661 + 0.845924 + 0.66725595)\right]$

$= \frac{1}{36}[\quad 0.90454311$

(iii) Simpson's $3/8^{th}$

$\frac{3h}{8}\left[(y_0+y_n) + 2(y_3+y_6+\cdots y_{n-3}) + 3(y_1+y_2+y_4+y_5+y_7\cdots)\right]$

$= \frac{3(1/12)}{8}\left[(1+0.540302) + 2(0.998047 + 0.968912 + 0.845924) + 3(0.999975 + 0.9999614 + 0.9938335 + 0.984967 + 0.942661 + 0.902849 + 0.7684093 + 0.66725595\right.$

$= 0.904524$

iii) $\int_0^{0.5} (\tan^{-1} x)^2$, $n = 18$ $\quad h = \frac{0.5 - 0}{18}$

a) Trapezoidal

$$I = \frac{h}{2}\left[f(0) + f(0.5) + 2\left(f(1/36) + f(2/36) + \cdots + f(17/36) \right) \right]$$

$$= \frac{1}{2 \times 36} \times 2.738923378 = 0.0380406024l$$

$$= 0.038041$$

b) Simpson : $\frac{h}{3}\left[f(0) + f(0.5) + 2\left(f(2/36) + f(4/36) + \cdots + f(16/36) \right) + 4\left(f(1/36) + \cdots + f(17/36) \right) \right]$

$$= \frac{1}{3 \times 36} \times 4.103231948 = 0.03799288841$$

$$= 0.037993$$

c) Simpson :

$$\frac{3h}{8}\left[f(0) + f(0.5) + 3\left(f(1/36) + f(2/36) + \cdots + f(4/36) \right) + 2\left(f(3/36) + f(6/36) + \cdots \right) \right]$$

$$= \frac{3}{8} \times \frac{1}{36} \times 3.647315971$$

$$= 0.037993$$

iv) $\int_{\pi/4}^{\pi/2} \frac{\sin x}{x} \cdot dx$, $n = 6$ $\quad h = \frac{\pi/2 - \pi/4}{6} = \frac{\pi}{24}$

Trapezoidal:

$$I = \frac{h}{2}\left[f(\tfrac{\pi}{4}) + f(\pi/2) + 2\left(f(\tfrac{\pi}{4} + \tfrac{\pi}{24}) + \cdots + f(\tfrac{\pi}{4} + \tfrac{5\pi}{24}) \right) \right]$$

$$= \frac{1}{2} \times \frac{\pi}{24} \times 9.343930387 = 0.61155 88138$$

$$= 0.611559$$

(v) $\int_{0.4}^{1.6} \frac{x}{\sin x} dx$, $n=12$; $h = \frac{1.6-0.4}{12} = 0.1$

Simpson's 1/3th rule.

$I = \frac{h}{3}\left[\cancel{f(0)+f(n)+2(f(} y_0+y_n + 2(y_2+y_4+\cdots+y_{n-2}) + 4(y_1+y_3+\cdots+y_{n-1})\right]$

$= \frac{0.1}{3} \times 44.23929171 = 1.474643057$

$= 1.474643$

(vi) $\int_{0.125}^{0.875} \frac{\cos x}{\sqrt{x}} \cdot dx$, $n=18$ $h = \frac{0.875-0.125}{18} = \frac{1}{24}$.

Simpson's 3/8th rule:

$I = \frac{3h}{8}\left[\cancel{f(0)+} y_0+y_n + \cancel{3}^{2}(y_3+y_6+\cdots+y_{n-3}) + 3(y_1+y_2+y_4+\cdots+y_{n-1})\right]$

$= \frac{3}{8} \times \frac{1}{24} \times 65.70596598 = 1.026655718$

$\simeq 1.026656$

2.7 $\int_0^1 \frac{dx}{1+x^2}$, $n=10$ $h = \frac{1-0}{10} = 0.1$

Simpson's $\frac{1}{3}$

$I = \frac{h}{3}\left[y_0+y_n+2(y_2+y_4+y_6+\cdots+y_{n-2}) + 4(y_1+y_3+y_5+\cdots y_{n-1})\right]$

$= \frac{(0.1)}{3}\left[1+\frac{1}{2}+2(0.961538+0.862068+0.735294+0.609756+) + 4(0.990099+0.917431+0.8+0.671140+0.552486)\right]$

$= 0.785397$

$\cancel{\int_0^1 \frac{dx}{1+x^2}}$ $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \Rightarrow \int_0^1 \frac{dx}{1+x^2} = \frac{\pi}{4} = I$

$\Rightarrow \pi = 4 \times I = \underline{3.141588}$

3~~8~~ a) Trapezoidy

$$I = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots y_{n-1})\right]$$

$$= \frac{0.1}{2} \times 15.69962994 = 0.7849814972$$

$$= 0.784981$$

$$\pi = 4 \times I = \underline{3.139926}$$

3.7 $\displaystyle\int_0^{1.2} e^{-x^2}\, dx$ $\qquad h = 0.2 \qquad n = \frac{1.2 - 0}{0.2} = 6$

$$I = \frac{h}{2}\left[y_0 + y_6 + 2(y_1 + y_2 + \cdots y_5)\right]$$

$$= \frac{(0.2)}{2}\left[1 + \cancel{2.31952 \times 10^{-16}}\ 0.236927 + 2(0.960789 + 0.852143 + 0.697676 + 0.5272924 + 0.367879)\right]$$

$$= \underline{0.804848}$$

Error in Trapezoidal rule

$$E = -\frac{h^3}{12}\left[y_0'' + y_{0.2}'' + y_{0.4}'' + \cdots + y_1''\right]$$

$$= -\frac{(0.2)^3}{12}\left[-4.28624224\right] = -0.002858$$

& $|\text{Max error}| \le \underline{0.008}$

4.7 $\displaystyle\int_0^1 \frac{dx}{1+x^2}$

$\cancel{I = \frac{h}{3} \int y\, dx}$

Simpson's 1/3 rule.

$$|E| \leq \frac{h^4}{180}(b-a)\, y^{(iv)}(x)$$

$$\frac{h^4}{180}(b-a)\, y^{(iv)}(x) = 5\times10^{-6}$$

$$h^4 = \frac{5\times10^{-6}\times180}{1\times y^{(iv)}(x)}$$

$$y(x) = \frac{1}{1+x^2} \Rightarrow y^4(x) = \frac{24(5x^4-10x^2+1)}{(x^2+1)^5}$$

Max. value of $y^4(x) = 24$

$$h^4 = \frac{5\times10^{-6}\times180}{24}$$

$$= 3.75\times10^{-5}$$

$$h = 0.0782542$$

$$\cancel{\frac{(b-a)^5}{n^4\times180}\times24 = 5\times10^{-6}}$$

$$\cancel{n^4 = \frac{24\times10^6}{180\times5}} =$$

$$n = \frac{b-a}{h} = \frac{1}{0.0782542} = 12.77 \simeq 13.$$

5.7 $\displaystyle\int_0^{1.2} (\sin x - \ln(1+x) + e^x)\,dx$, $n = \frac{1.2}{0.1} = 12$

$$I = \frac{3h}{8}\left[y_0 + y_{12} + 2(y_3+y_6+\cdots y_9) + 3(y_1+y_2+y_4+\cdots y_{11})\right]$$

$$= \frac{3(0.1)}{8}\Big[1 + 2.463698 + 2(1.383014 + 1.916757 + 2.6010761) + 3(1.109694 + 1.237750 + 1.544770 + 1.722681 + 2.127342 + 2.355110\,3 + 2.866605 + 3.1534360)\Big]$$

$= 2.4231588\cancel{8}39$

$= \underline{2.42816}$

exact : $2.4231529\,75 \Rightarrow \underline{2.42315}$.

6.7 a) $\displaystyle\int_{0.2}^{1.5} e^{-x^2}.dx$

$y = \dfrac{2x}{b-a} - \left(\dfrac{a+b}{b-a}\right) \Rightarrow y = \dfrac{2x}{1.3} - \left(\dfrac{1.7}{1.3}\right)$

$x = \dfrac{1.3y}{2} + \dfrac{1.7}{2} \Rightarrow dx = 0.65\,dy$

$\therefore I = 0.65 \displaystyle\int_{-1}^{1} e^{-(0.65y+0.85)^2} dy \Rightarrow f(y) = e^{-(0.65+0.85)^2}.(0.65)$

$n=2 : \quad I = 1.f(-1/\sqrt{3}) + 1.f(1/\sqrt{3})$

$= 0.663693$

$n=3 : \quad I = 0.55555\, f(-0.77460) + 0.88889\,f(0) + 0.55555.\,f(0.77460)$

$= 0.658599$

b) $\displaystyle\int_0^{\pi} .\sin(x^2)\,dx$

$x = \dfrac{(b-a)y}{2} + \left(\dfrac{a+b}{2}\right) \Rightarrow x = \dfrac{\pi y}{2} + \dfrac{\pi}{2}$

$dx = \dfrac{\pi\, dy}{2}.$

$\therefore I = \dfrac{\pi}{2}\displaystyle\int_{-1}^{1} \sin\left(\left(\tfrac{\pi}{2}(y+1)\right)^2\right) dy \Rightarrow f = \dfrac{\pi}{2}.\sin\left(\left(\tfrac{\pi}{2}(y+1)\right)^2\right)$

$n=2 : \quad I = f(-1/\sqrt{3}) + f(1/\sqrt{3})$

$= 0.444406$

$n=3 : \quad I = \cancel{1.8}\; 0.55555\, f(-0.77460) + 0.55555\, f(0.77460) + 0.88889\,f(0)$

$= 1.85035383\mathcal{8}$

$\simeq 1.850354$

c) $\displaystyle\int_{-4}^{4} \frac{dx}{1+x^2}$ $\qquad x = \left(\frac{b-a}{2}\right) y + \left(\frac{a+b}{2}\right)$

$= \cancel{\;}\, 4y \Rightarrow dx = 4.dy$

$I = 4 \displaystyle\int_{-1}^{1} \frac{dy}{1 + \cancel{4y}\, 16y^2} \Rightarrow f(y) = \frac{4}{1+16y^2}$

$n=2: \quad I = f(-1/\sqrt{3}) + f(1/\sqrt{3})$

$= 1.263158$

$n = 3: \quad \cancel{I = 3.974840}$

$I = 0.55555\,(0.3773555) + 0.88889(4) + 0.55555\,(0.3773555)$

$= 3.974839$

d) $\displaystyle\int_{0}^{\pi/2} \frac{dx}{\sqrt{(1-0.25 \sin^2 x)}}$ $\qquad x = \dfrac{(b-a)y + (a+b)}{2.}$

$= \dfrac{\pi/2\, y + \pi/2}{2} = \frac{\pi}{4}(y+1)$

$I = \dfrac{\pi}{4} \displaystyle\int_{-1}^{1} \frac{dy}{\sqrt{1 - \frac{\sin^2(\pi/4(y+1))}{4}}}$ $\qquad dx = \pi dy/4$

$\Rightarrow f(y) = \dfrac{\pi}{2\sqrt{4 - \sin^2(\pi/4(y+1))}}$

$n=2: \quad I = f(-1/\sqrt{3}) + f(1/\sqrt{3})$

$= 1.7037853 3$

$n=3: \quad I = 0.55555 f(-0.77460) + 0.55555 f(0.77460) + 0.88889 f(0)$

$= 1.635651$