ASSIGNMENT-06

MAB-103

23116073

PARTH

1>

x	0.62	0.68	0.70	0.73	0.75
$f(x)$	0.660 4918	0.7336304	0.758537	0.7965858	0.8223167

Newton's divided differe formula : $y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \cdots$

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.62	0.6604918				
		1.2189766			
0.68	0.7336304		0.358605		
		1.247665		0.2057	
0.70	0.7585837		0.381232		0.080984
		1.2667266		0.216228	
0.73	0.7965858		0.396368		
		1.286545			
0.75	0.8223167				

$$y(0.72) = 0.6604918 + (0.1)(1.2189766) + (0.1)(0.04)(0.358605) + (0.1)(0.04)(0.02)(0.2057) + (0.1)(0.04)(0.02)(-0.01)(0.080984)$$

$$= \boxed{0.7838405}$$

2>

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.0	0.3639				
		-0.381			
1.1	0.3258		0.1933		
		-0.323		-0.0641	
1.3	0.2612		0.16125		0.0082
		-0.2585		-0.05918	
1.5	0.2095		0.13166		
		-0.219			
1.6	0.1876				

$$y(1.25) = 0.3639 + (0.25)(-0.381) + (0.25)(0.15)(0.1933) + (0.25)(0.15)(-0.05)(-0.0641) + (0.25)(0.15)(-0.05)(-0.25)(0.0082)$$

$$= \boxed{0.2760}$$

3.)

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
300	2.4771			
		0.00145		
304	2.4829		0.00001	
		0.0014		0.000001 42857
305	2.4843		0	
		0.0014		
307	2.4871			

$\Rightarrow f(310) = 2.4771 + (10)(0.00145) + (10)(6)(0.00001) + (10)(6)(5)(0.000001\,42857)$

$= \boxed{2.498028}$

4.)

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.1	2.0091				
		0.0242			
1.2	2.0333		0.0117		
		0.0359		−0.0025	
1.3	2.0692		0.0092		0.0006
		0.0451		−0.0019	
1.4	2.1143		0.0073		
		0.0524			
1.5	2.1667				

a

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{3!}\Delta^3 y_0 + \frac{4p^3-18p^2+22p-6}{4!}\Delta^4 y_0\right]$$

$x = 1.1 \Rightarrow p = 0$

b

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4}\right]$$

$$= \frac{1}{(0.1)}\left[2.0091 - \frac{0.0117}{2} + \frac{(-0.0025)}{3} - \frac{0.0006}{4}\right]$$

$$f'(1.1) = 0.17366667 \simeq 0.1737$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\frac{2}{2!}\Delta^2 y_0 + \frac{6p-6}{3!}\Delta^3 y_0 + \frac{12p^2-36p+22}{4!}\Delta^4 y_0\right]$$

$x = 1.1 \Rightarrow p = 0$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0\right]$$

$$f''(1.1) = \frac{1}{(0.1)^2}\left[0.0117 - (-0.0025) + \frac{11}{12}(0.0006)\right]$$

$$= 1.4750$$

5.)

x	f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1.5708				
		0.0030			
5	1.5738		0.0060		
		0.0090		0.0003	
10	1.5828		0.0063		0.0000
		0.0153		0.0003	
15	1.5981		0.0066		
		0.0219			
20	1.6200				

a) $x = 0$

$$f'(0) = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3}\right] = \frac{1}{5}\left[1.5708\right]$$

$$= \frac{1}{5}\left[0.0030 - \frac{(0.0060)}{2} + \frac{(0.0003)}{3}\right] = \boxed{0.00002}$$

$$f''(0) = \frac{1}{h^2}\left[\Delta^2 y_0 + \Delta^3 y_0\right]$$

$$= \frac{1}{25}\left[0.0060 - 0.0003\right] = \boxed{0.00023}$$

b) $x = 3 \Rightarrow p = \frac{3-0}{5} = 0.6$

$$f'(3) = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{3!}\Delta^3 y_0\right]$$

$$= \frac{1}{5}\left[(0.0030) + \left(\frac{1.2-1}{2!}\right)(0.0060) + \left(\frac{3(0.6)^2 - 6(0.6)+2}{3!}\right)(0.0003)\right]$$

~~$= 0.0002352 = \boxed{0.0002}$~~

$$= \boxed{0.0007148}$$

$$f''(3) = \frac{1}{h^2}\left[\frac{2}{2!}\Delta^2 y_0 + \frac{6p-6}{3!}\Delta^3 y_0\right]$$

$$= \frac{1}{25}\left[0.0060 + \left(\frac{6(0.6-1)}{3!}\right)(0.0003)\right]$$

$$= 0.0002352 \simeq \boxed{0.0002}$$

c) $x = 18$

$$f'(18) = \frac{1}{h}\left[\nabla y_n + \left(\frac{2p+1}{2}\right)\nabla^2 y_n + \left(\frac{3p^2+6p+2}{3!}\right)\nabla^3 y_n\right]$$

$$p = \frac{18-20}{5} = \underline{\underline{-0.4}}.$$

$$f'(18) = \frac{1}{5}\left[0.0219 + \left(\frac{2(-0.4)+1}{2}\right)(0.0066) + \frac{0.0003}{75}\right]$$

$$= 0.0045128 \simeq \boxed{0.0045}$$

$$f''(18) = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{(6p+6)}{3!}\nabla^3 y_n + \frac{6p^2+18p+1}{12}\cdot\nabla^4 y_n\right]$$

$$= \frac{1}{25}\left[(0.0066) + \left(\frac{6(-0.4+1)}{3!}\right)(0.0003)\right]$$

$$= 0.0002568 \simeq \boxed{0.0003}$$

d) $x = 20$

$$f'(20) = \frac{1}{h}\left[\nabla y_n + \frac{\Delta^2 y_n}{2} + \frac{\nabla^3 y_n}{3}\right]$$

$$= \frac{1}{5}\left[0.0219 + \frac{0.0066}{2} + \frac{0.0003}{3}\right] = \boxed{0.00506}$$

$$f''(20) = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{6}{3!}\nabla^3 y_n\right] = \frac{1}{25}\left[0.0066 + \frac{6}{6}(0.0003)\right]$$

$$= \boxed{0.0002760}$$

6.)

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.4	1.08107				
		0.10439 Δy_{-2}			
0.6	1.18546		0.04758 $\Delta^2 y_{-2}$		
		0.15197 Δy_{-1}		0.0061 $\Delta^3 y_{-2}$	
0.8	1.33743		0.05368 $\Delta^2 y_{-1}$		0.00215 $\Delta^4 y_{-2}$
		0.20565 Δy_0		0.00825 $\Delta^3 y_{-1}$	
1.0	1.54308		0.06193 $\Delta^2 y_0$		
		0.26758 Δy_1			
1.2	1.81066				

- $f'(0.8) = \frac{1}{h}\left[\frac{\Delta y_{-1} + \Delta y_0}{2} + p\Delta^2 y_{-1} + \left(\frac{3p^2-1}{6}\right)\left(\frac{\Delta^3 y_{-2} + \Delta^3 y_{-1}}{2}\right) + \left(\frac{2p^3-p}{12}\right)\Delta^4 y_{-2}\right]$

$= \frac{1}{0.2}\left[\frac{0.15197 + 0.20565}{2} + 0 + \left(\frac{-1}{6}\right)\left(\frac{0.0061 + 0.00825}{2}\right) + 0\right]$

$= 0.8880708 \simeq \boxed{0.8881}$

- $f''(0.8) = \frac{1}{h^2}\left[\Delta^2 y_{-1} - \frac{\Delta^4 y_{-2}}{12}\right] = \frac{1}{(0.2)^2}\left[0.05368 - \frac{0.00215}{12}\right]$

$= 1.3375208 \simeq \boxed{1.3375}$

$p = \frac{0.82 - 0.8}{0.2} = 0.1$

- $f'(0.82) = \frac{1}{(0.2)}\left[\frac{0.15197 + 0.20565}{2} + (0.1)(0.05368) + \left(\frac{3(0.1)^2-1}{6}\right)\left(\frac{0.0061 + 0.00825}{2}\right) + \frac{2(0.1)^3 - 0.1}{12} \times 0.00215\right]$

$= \boxed{0.9150}$

- $f''(0.82) = \frac{1}{h^2}\left[\Delta^2 y_{-1} + \frac{p}{2}\left(\Delta^3 y_{-1} + \Delta^3 y_{-2}\right) + \frac{(12p^2-2)}{41}\Delta^4 y_{-2}\right]$

$= \frac{1}{(0.2)^2}\left[0.04758 + \frac{(0.1)}{2}\left(0.0061 + 0.00825\right) + \frac{12(0.1)^2 - 2}{41}(0.00215)\right]$

$$f''(0.82) = 1.355727 \simeq \boxed{1.3557}$$

7.)

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.2	0.4660				
		0.0158			
1.3	0.4818		−0.0048		
		0.011		−0.0002	
1.4	0.4928		−0.005		0.0002
		0.006		0	
1.5	0.4988		−0.005		
		0.001			
1.6	0.4998				

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \left(\frac{2p+1}{2}\right)\nabla^2 y_n + \left(\frac{3p^2+6p+2}{3}\right)\nabla^3 y_n + \left(\frac{4p^3+18p^2+22p+6}{4!}\right)\Delta^4 y_n\right]$$

$$0 = \frac{1}{(0.1)}\left[(0.001) + \left(\frac{2p+1}{2}\right)(-0.005) + 0 + \left(\frac{4p^3+18p^2+22p+6}{4!}\right)(0.0002)\right]$$

$$0 = (0.01) + \left(\frac{2p+1}{2}\right)(-0.05) + \ldots$$

(upto 2D) so neglected.

$$\frac{1}{5} = \frac{2p+1}{2}$$

$$2p = 0.4 - 1 = -0.6$$

$$(p = -0.3)$$

$$x = x_n + ph$$

$$= 1.6 + (-0.3)(0.1) = \boxed{1.57}$$

$$y = f(1.57) = y_0 + p\cdot\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$$

$$= 0.4660 + (-0.3)(0.0158) + \frac{(-0.3)(-1.3)}{2}(-0.0048)$$

$$= 0.500025$$

$$\simeq \boxed{0.5000}$$

8.)

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0.2	2.10022						
		-0.11292					
0.4	1.98730		0.03502				
		-0.07790		0.00026			
0.6	1.90940		0.03522		-0.00009		
		-0.04268		0.00011		0.00018	
0.8	1.86672		0.03533		0.00009		-0.00016
		-0.00735		0.000020		0.00002	
1.0	1.85937		0.03535		0.00011		
		0.028		-0.00009			
1.2	1.88737		0.03526				
		0.06326					
1.4	1.95063						

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2p-1)}{2!}\Delta^2 y_0 + \frac{(3p^2-6p+2)}{3!}\Delta^3 y_0\right]$$

$$0 = \frac{1}{(0.2)}\left[(-0.11292) + \frac{(2p-1)}{2}(0.03502) + \frac{(3p^2-6p+2)}{6}(0.0020)\right]$$

$\Rightarrow p = -1.292115504$

$x = x_n + ph = 1.2 + (-1.292)(0.2) = 0.94$

$x = x_0 + ph = 0.2 + (3.548500)(0.2) = 0.8717$

$x_0 = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{6}\nabla^3 y_n$

$= 1.88737$

$$x = x_n + ph$$

$$= 1.2 + (-1.29211\,5504)(0.2)$$

$$= 0.94$$

$$x_{min} = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{6!}\nabla^3 y_n$$

$$= 1.88737 + (-1.29)(0.028) + \underbrace{(-1.29)(-0.29)(0.03535)}_{2}$$

$$+ \frac{(-1.29)(-0.29)(0.71)(0.0002)}{6} + \frac{(-1.29)(-0.29)(0.71)(1.71)(0.0009)}{24}$$

$$= 1.85786\,13205$$

$$x_{min} = \boxed{1.85786}$$