Math 151 – Python Lab 3

Directions: Use Python to solve each problem. (Template link)

1. Verify that the conditions of the Intermediate Value Theorem are satisfied on the indicated interval.

$$f(x) = x^5 + 4x^3 - 2x^2 + 8x - 1 \text{ on } [0, 1]$$

   (a) Use the Intermediate Value Theorem to show there is a root to the function on the interval.

   (b) Find the root of the function inside the interval.

2. Given $f(x) = \begin{cases} 2x - 3 & \text{if } 0 \leq x \leq 3 \\ 4x - x^2 & \text{if } 3 < x \leq 4 \\ \dfrac{x^2 - 6x + 8}{x - 4} & \text{if } 4 < x < 5 \\ e^{(x-4)\ln(3)} & \text{if } x \geq 5 \end{cases}$

   (a) Find the left and right hand limits of f at all "break points" to determine whether f is continuous at these points or not. If f is not continuous, state whether it is left or right continuous at those points.

   (b) Graph the function on the domain $[0, 6]$ to confirm your answer to part (a).

3. Logistic Growth Models for a population follow the form $P(t) = \dfrac{KP_0}{P_0 + (K - P_0)e^{-rt}}$ where r is the intrinsic growth rate, P_0 is the starting amount, and t is time. K is a special coefficient that we will investigate here.

   To learn more about the value of K, let's keep P_0 fixed at 10 and r fixed at 0.1. Find the limit as t goes to $+\infty$ in each case, then graph the functions to verify your answer.

   (a) $K = 1000$

   (b) $K = 1$

   (c) $K = 15$

   Based on your observations, what happens to the population size as t goes to infinity? What can we infer about the population given the value K?