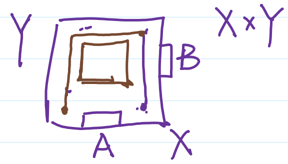
July 28

Wednesday, July 28, 2021 2:10 PM

#1 $X \times Y / A \times B$

X / A can be disconnected

Y / B



Y/B is disconnected

#2

	Compact	Non compact
Sim. Conn	....	...
Non Sim Conn	...	...

#4. $Y \subset X$. $r|_Y = Id_Y$. $r(X) \subseteq Y$

want to show $\overline{Y} = Y$.

$\forall y \in \overline{Y} \setminus Y$, want to show $y \in Y$ ($\overline{Y} \setminus Y = \phi$, done)

Hausdorff. $\exists$ a sequence $y_n \in Y$ s.t. $y_n \to y$

(Step 1 find U_1 containing y. pick $y_1 \neq y$, $y_1 \in U_1$

Step 2. let $U_2 \subset U_1$ containing y.

let $y_2 \neq y$, $y_2 \neq y_1$.

... etc)

$y_n \to y$ (Net) J is a net

$\lim_{\alpha \in J} y_\alpha \to y$.

Set

$r(y) = \lim_{n\to\infty} r(y_n) = \lim_{n\to\infty} y_n = y \in Y$.

since $y_n \in Y$, $r(y_n) = y_n$

$y \in Y$. $Y \supseteq \bar{Y}$ $\square$.

b) $\psi: M \times M \to M$

$(a, b) \to a \cdot b$

$\psi(a, b) = e$ (id)

c) $f(x,y) = y^2 - (x-1)(x-a)x$, $f = 0$, find what values of a make 0 be a regular value

df is a 1×2 matrix, $p \in M_a = f^{-1}(0)$

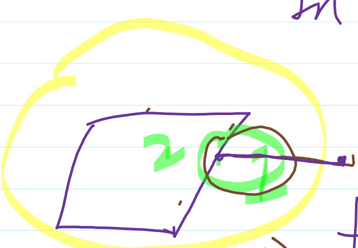
$df|_p = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$ $df|_p$ is non-sing.

iff $\frac{\partial f}{\partial x} \neq 0$ or $\frac{\partial f}{\partial y} \neq 0$

① If $\frac{\partial f}{\partial x} = 0$, $a = \{$set 1$\}$

② If $\frac{\partial f}{\partial y} = 0$, $a = \{$set 2$)$

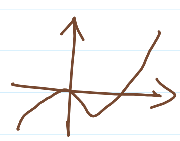
In general $a \neq \{$set 1$\} \cup \{$set 2$\}$



if 0 is critical

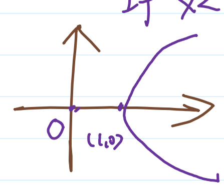
$f^{-1}(0)$

$\begin{cases} a \neq 0 & \text{, at } a=0 \quad y^2 = x^2(x-1) \\ a \neq 1 & \text{at } a=1 \quad y^2 = x(x-1)^2 \end{cases}$



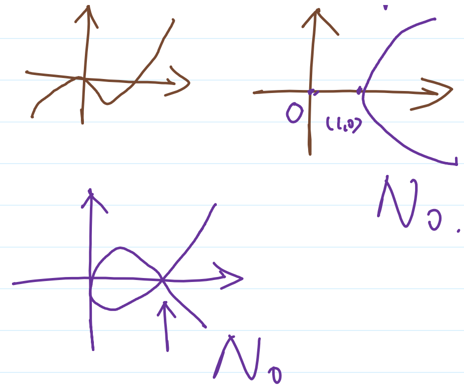
$y^2 = x^2(x-1)$

If $x<1$



No.

$$\begin{cases} a \neq 0 & , \text{ at } a=0 \quad y^2 = x^2(x-1) \\ a \neq 1 & \text{ at } a=1 \quad y^2 = x(x-1)^2 \end{cases}$$



b) $$\int_M d(\alpha \wedge \beta) = \int_{\partial M} \alpha \wedge \beta$$

$$= \int_{N_0} \alpha \wedge \beta + \int_{N_1} \alpha \wedge \beta \qquad \text{(Stoke's)}$$

$$= \int_{N_0} \underbrace{i_0^*(\alpha)}_{=0} \wedge i_0^*(\beta) + \int_{N_2} i_1^*(\alpha) \wedge \underbrace{i_1^*(\beta)}_{=0}$$

$$= 0$$

$$\Rightarrow \int_M d(\alpha \wedge \beta) = \int_M d\alpha \wedge \beta + \int_M (-1)^p \alpha \wedge d\beta = 0$$

$$\int_M d\alpha \wedge \beta = (-1)^{p+1} \int_M \alpha \wedge d\beta$$

c) $\omega - c\alpha$ is closed.

want to show (find a 'c', s.t.)

$\omega - c\alpha$ is exact.

$$\int_{S^1} \alpha = 2\pi \neq 0 \Rightarrow \alpha \text{ not exact}$$

$$\int_{S^1} (\omega - c\alpha) \overset{?}{=} 0$$

$$= \int_{S^1} \omega - 2\pi c \overset{?}{=} 0 \Rightarrow \text{Define } c = \frac{1}{2\pi} \int_{S^1} \omega$$

then $\int_{S^1} (\omega - c\alpha) = 0$, moreover $\omega - c\alpha$ is closed

$\int_\gamma (\omega - c\alpha) = 0$ for any γ in $\mathbb{R}^2 / \{0\}$ where γ closed

γ is random. $\Rightarrow$ $\omega - c\alpha$ is exact.

#7. $D = \ker \omega_1 \cap \ker \omega_2$.

If exists α, β s.t.

$$d\omega_1 = \alpha \wedge \omega_1 + \beta \wedge \omega_2$$

$\alpha = \sum_{i=1}^{n} \alpha_i \omega_i$

Can you find α_i's β_i's

$d\omega_1 (\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_j})$ $\frac{n(n-1)}{2}$ many eqns.