Math 151 – Python Lab 9

Directions: Use Python to solve each problem. (Template link)

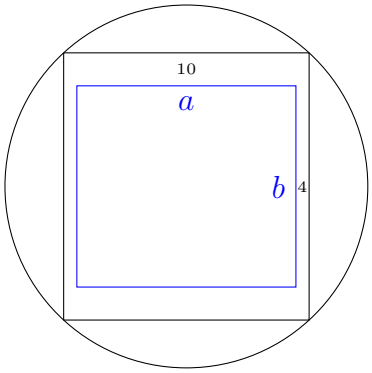
1. Given $y = \left(1 + \dfrac{26}{x}\right)^x$:

   (a) By hand, rewrite $\ln y$ as a fraction $\dfrac{f(x)}{g(x)}$. Define f and g in Python.

   (b) Find the limits of f and g as $x \to \infty$

   (c) If the answers in part (b) allow for it, use L'Hôpital's Rule to compute $\lim\limits_{x\to\infty} y$ (print all intermediate computations as well).

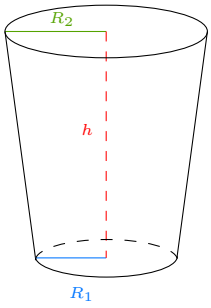
   (d) Evaluate the limit directly using Python to verify your answer to part (c).

2. 

   (a) Suppose a photograph with a width of 42 inches and a height of 50 inches is placed in the blue frame of the round billboard shown. The margins between the rectangular frame and the picture are 10 inches at the top and bottom and 4 inches on the sides. What would the radius of the billboard be that would fit a photograph of such dimensions?

   (b) A billboard with radius 55 inches is designed like the figure to the left. Determine the dimensions, a and b, of the largest picture that could fit in the frame.

3. Buc-ee's wants to hire you to design cups for coffee. The cups will be made out of paper and shaped like a frustum of a cone (pictured below) with $R_2 = 2R_1$. They want the cups to hold 20 ounces (590 cm^3) and use as little material as possible.



$$V = \frac{1}{3}\pi h(R_1^2 + R_2^2 + R_1R_2)$$

$$SA = \pi(R_1 + R_2)\sqrt{(R_2 - R_1)^2 + h^2} + \pi(R_1^2 + R_2^2)$$

   (a) Plot the Optimization function on $R_1 \in (0, 5)$ with $y \in (300, 800)$

   (b) Determine R_1, R_2, and h such that minimal material is used. (May need to use **nsolve** with an estimate from part (a))

Problems continued on next page...

4. Given $f''(x) = \dfrac{5}{(x+1)^2}$:

   (a) If $f'(0) = 3$, and $f(0) = 9$, find $f'(x)$ and $f(x)$.
   (HINT: Python does NOT include the "+C" when you use integrate, so you have to put it in yourself, then use the initial conditions to find each C. This type of problem is called an initial value problem.)

   (b) If $f(1) = f(4) = 10$, find $f(x)$.
   (Notice this time you cannot solve for the C values until the end, so give them different names, like C1 and C2, then solve a system of equations for them. This type of problem is called a boundary value problem.)