Math 151 – Python Lab 7

Directions: Use Python to solve each problem. (Template link)

1. Given $f(x) = \begin{cases} 8 - x^2 & \text{if } x < 0 \\ 5e^{-((x-2)/2)^2} + x & \text{if } x \geq 0 \end{cases}$

   (a) Find the critical values of the function.

   (b) Find the absolute extrema of f on the interval $[-5, 5]$.

   (c) Use the method to find the absolute extrema on $[-10, 10]$.

   (d) Plot f on the interval $[-10, 10]$. Choose a **ylim** that allows you to see the graph accurately.

2. When a foreign object lodged in the trachea forces a person to cough, the velocity of the air stream is related to the radius of the trachea by the equation:

$$v(r) = k(r_0 - r)r^2$$

   where r_0 is the normal radius of the trachea and k is a positive constant.

   (a) Determine the value of r in the interval $\left[\frac{1}{2}r_0, r_0\right]$ at which v has an absolute maximum.

   (b) What is the absolute maximum value of v on the same interval?

   (c) Let $r_0 = 0.65$ inches and $k = 15000$. What is the maximum value of the function and where does it occur?

   (d) Sketch the graph of v with the conditions from part (c) on the interval $[0, r_0]$

3. Given $f(x) = \arctan(x)$ and $g(x) = \text{arccot}(x)$:

   (a) Find the derivative of $f(x)$ and $g(x)$ separately, then simplify the derivative of $f(x)+g(x)$.

   (b) In a print command, explain what this tells you about $f(x) + g(x)$.

   (c) Plot $f(x) + g(x)$ and, in a print statement, specifically define the simplified function. (Hint: consider using a piecewise function)

   (d) In a print statement, explain why this makes sense for $x > 0$. (HINT: consider using a cofunction identity)