Math 151 – Python Lab 8

Directions: Use Python to solve each problem. (Template link)

1. Given $f(x) = \dfrac{1}{40}\left(x^6 + 2x^5 - 16x^4 - 20x^3 + 64x^2 - 36x + 72\right)$:

   (a) Plot f on the domain $x \in [-10, 10]$, $y \in [-10, 10]$. In a print command, indicate how many local extrema and how many inflection points there appear to be.

   (b) Find $f'(x)$ and the approximate critical values (real values only). Plot f' in the window $x \in [-5, 5]$, $y \in [-5, 5]$ to determine the intervals where f is increasing and decreasing (If intervals are not clear from the graph, test numbers around the critical values to determine the sign of f').

   (c) Find $f''(x)$ and the possible inflection values of f (real values only). Plot f'' using the same plot window as b) to determine the intervals where f is concave up and concave down (If intervals are not clear from the graph, test numbers around the critical values to determine the sign of f'').

   (d) How many local extrema and inflection points actually exist? Plot f twice, each in a different domain and range to show ALL extrema and inflection points.

2. Given the family of functions $g(x) = x^3 e^{bx^2}$:

   (a) Plot g for $b \in [-3, -2, -1, 0, 1, 2, 3]$ on the same axes. Use a plot window of $x \in [-3, 3]$, $y \in [-1, 1]$.

   (b) Notice there are two main shapes to the graph. Find the critical values of f (in terms of b). In a print statement, indicate the values of b for which these critical values are real.

   (c) In a print statement, explain what happens to the critical values as $b \to -\infty$. Show this by plotting the function with $b = -100$

   (d) Find the inflection points of f in terms of the parameter b. In a print statement, indicate the values of b for which all inflection points are real.

   (e) Find the value of b for which the critical values are ± 1. Find the values of b for which the inflection points include ± 1 (should be two different b-values for the inflection points). Plot all three functions on the same axes.

(Problems continued on next page...)

3. Verify whether the functions below satisfy the hypotheses of the Mean Value Theorem on the given interval, then find the number c that satisfies the conclusion. (HINT: If the solve command doesn't work, plot the derivative and $y = \dfrac{f(b) - f(a)}{b - a}$, then use **nsolve** to make a guess.)

   (a) $f(x) = \ln(5 - x), [1, 4]$

   (b) $g(x) = (x - 5)^{-5}, [0, 8]$

   (c) $h(x) = 8x^2 \cos(4x), \left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$