Math 151 – Python Lab 4

Directions: Use Python to solve each problem. (Template link)

1. Let $f(x) = \dfrac{2x+1}{x^2+2}$:

   (a) Find the equation of the tangent line at the point where $x = 2$.

   (b) Find the equation of the normal line at the point where $x = 2$.

   (c) Graph the function, tangent line, and normal line on the same axes with domain $[0, 3]$ and range $[-1, 2]$ (use **ylim** command).

2. Consider the function $y = \sin(x) - \dfrac{1}{1000}\sin(1000x)$

   (a) Plot the function on the interval $[-2\pi, 2\pi]$ and make an estimate of the slope of the graph near $x = 0$.

   (b) Plot the same function on the interval $[-0.25, 0.25]$ and, again, estimate the slope of the graph near $x = 0$.

   (c) Find the actual slope of the tangent line at $x = 0$.

   (d) Zoom in one more time and plot the same function on the interval $[-0.001, 0.001]$ to verify your answer to part (c).

3. If a cylindrical tank holds 100,000 gallons of water, which can be drained from the bottom of the tank in an hour, then Torricelli's Law gives the volume of water remaining in the tank after t minutes as:

$$V(t) = 100,000\left(1 - \frac{1}{60}t\right)^2$$

   (a) Find the average rate of change of the V from 0 to 10 seconds.

   (b) Find the instantaneous rate of change (IROC) of V with respect to t.

   (c) Find the IROC of V at $t = 10$.

   (d) Notice that part (a) and part (c) have different answers. What could a reason be for the difference in those values?

4. Given $f(x) = e^x(1 + x^2)$:

   (a) Find the first 8 derivatives of f.

   (b) In a print command, state the formula for the nth derivative of f.

   (c) Verify your formula by finding the $50th$ derivative of f.