Math 151 – Python Lab 6

Directions: Use Python to solve each problem. (Template link)

1. Find the values of r for which $y = e^{rx}$ is a solution to the following differential equations:
   (a) $2y'' + y' - y = 0$
   (b) $y'' + 6y' + 10y = 0$
   (c) Note the solutions in (b) are complex. Compute $y'' + 6y' + 10y$ when $y = e^{-3x}(\cos(x) + \sin(x))$. What can you conclude based on your answers to b) and c)?

2. Given the vector $\langle e^{2\sin(t)}, e^{\cos(t)} \rangle$:
   (a) Find a vector equation for the tangent line at the point where $t = \dfrac{\pi}{6}$ (Give your answer in both exact and decimal approximation)
   (b) Find the points on the graph where the tangent line is:
      i. horizontal
      ii. vertical
   (c) Sketch the graph of the vector function on $t \in [0, 2\pi]$ and all tangent lines found in parts (a) and (b).

3. $\left(-\left(\dfrac{x^2+y^2}{4}\right) + 2x - 2\right)^2 = 5\left(x^2+y^2\right)$ is a variation of a curve called the **Limaçon**.
   (a) Plot the graph of the equation using **plot_implicit** with $x \in (-5, 20)$ and $y \in (-15, 15)$.
   (b) Find $\dfrac{dy}{dx}$.
   (c) Find the x and y-coordinates where the graph of the equation has vertical tangent lines.
   (d) Use the extend command to re-plot the equation with the vertical tangents found in part (c) (which should be done parametrically).

4. Given $y = \dfrac{x^{1/5}\sqrt{x^3+1}}{(2-7x)^4}$:
   (a) Use logarithmic differentiation to find $\dfrac{dy}{dx}$. (NOTE: The logarithm step can be done using **expand_log**).
   (b) Find $\dfrac{dy}{dx}$ by differentiating directly.
   (c) Simplify or factor your answers to parts (a) and (b) to show they are equivalent.