

```

1 import UW
2
3 Var (p q : Prop)
4
5 -- Proof of commutativity of AND
6 -- Demonstrates AndElim Asm and AndElim Goal proof rules
7 Def and_commute := Proof
8   ⊢ p ∧ q → q ∧ p
9   By ImplElim Goal
10    Asm 1: p ∧ q
11    ⊢ q ∧ p
12    By AndElim Asm 1
13    Asm 2: p
14    Asm 3: q
15    By AndElim Goal
16    { Subproof 1: "lhs"
17      ⊢ q
18      QED Asm 3
19      Subproof 2: "rhs"
20      ⊢ p
21      QED Asm 2
22    }
23
24 -- Alternative proof of commutativity of AND.
25 -- Here we swap the order of AndElim Goal and AndElim Asm.
26 -- The proof works, but is longer than the first option.
27 -- By doing AndElim Goal first, we have to do AndElim Asm
28 -- in both of the subproofs that result from the AndElim Goal.
29 Def and_commute2 := Proof
30   ⊢ p ∧ q → q ∧ p
31   By ImplElim Goal
32   Asm 1: p ∧ q
33   ⊢ q ∧ p
34   By AndElim Goal
35   { Subproof 1: "lhs"
36     ⊢ q
37     By AndElim Asm 1
38     Asm 1/1: p
39     Asm 1/2: q
40     QED Asm 1/2
41     Subproof 2: "rhs"
42     ⊢ p
43     By AndElim Asm 1
44     Asm 2/1: p
45     Asm 2/2: q
46     QED Asm 2/1
47   }
48
49 -- Proof of commutativity of OR.
50 -- It follows a similar pattern to commutativity of AND.
51 Def or_commute := Proof
52   ⊢ p ∨ q → q ∨ p
53   By ImplElim Goal

```

```

54   Asm 1: p ∨ q
55   ⊢ q ∨ p
56   By OrElim Asm 1
57   { Subproof 1: "lhs"
58       Asm 1/1: p
59       By OrElimRight Goal
60       ⊢ p
61       QED Asm 1/1
62   Subproof 2: "rhs"
63       Asm 2/1: q
64       By OrElimLeft Goal
65       ⊢ q
66       QED Asm 2/1
67   }
68
69 -- Alternative proof *attempt* for commutativity of OR.
70 -- This proof doesn't work.
71 -- When we do OrElim Goal, we have to choose which goal ('p' or 'q') we want
72 -- to prove. But, we don't know which goal we want to prove until after
73 -- we do OrElim Asm.
74 Def or_commute := Proof
75   ⊢ p ∨ q → q ∨ p
76   By ImplElim Goal
77   Asm 1: p ∨ q
78   ⊢ q ∨ p
79   By OrElimLeft Goal
80   ⊢ q
81   By OrElim Asm 1
82   { Subproof 1: "lhs"
83       Asm 1/1: p
84       -- stuck: have asm for p,
85       -- but need to prove q
86   }
87
88 /- Lesson learned: In the and_commute proof,
89 - the order in which we did AndElim Asm vs
90 - AndElim Goal affected the length of the proof,
91 - but both orders worked (led to a successful proof).
92 - In the or_commute proof, to complete the proof,
93 - we had to do OrElim Asm before OrElim Goal.
94 -/

```