ECE 108 - Assignment 9

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Question 3a

If you want to test all possible configurations of devices connected to the bus, how many configurations satisfy the criteria below?

- All slots are identical – that is, a configuration is dependent upon only which devices are connected to the bus, independent of which slot a device is in.
- Empty slots are allowed.
- **Multiple** devices of the **same category** from **different manufacturers** may be used.
- **At most one** instance of the same device (**same category** and **same manufacturer**) may be used.
- Multiple **different categories** from the **same manufacturer** may be used.

Solution

To find the number of configurations that satisfy the criteria:

Each unique device (specific category and manufacturer combination) has two possibilities:

- Include it once
- Don't include it

Since we can choose independently for each unique device, the total number of configurations is:

$$2^n \tag{1}$$

Where n = total number of unique devices

For example, with 3 categories and 4 manufacturers, there would be 12 unique devices, giving $2^{12} = 4,096$ possible configurations.

Question 3b

If you want to test all possible configurations of devices connected to the bus, how many configurations satisfy the criteria below?

- All slots are identical
- Each slot has a device (no empty slots)
- **At most one** device from **each category** may be used.
- Multiple **different categories** from the **same manufacturer** may be used.

Solution

To find the number of configurations that satisfy the criteria:

For each category, we have the following options:

- Choose one device from any manufacturer
- Don't use any device from this category

The number of choices for each category is $(m + 1)$, where m is the number of manufacturers.

Using the multiplication principle of counting, the total number of possible configurations is:

$$(m + 1)^c \tag{2}$$

Where:

- m = number of manufacturers
- c = number of categories

However, since empty slots are not allowed (each slot must have a device), we must exclude the configuration where no devices are selected.

Therefore, the final answer is:

$$(m + 1)^c - 1 \tag{3}$$

For example, with 3 manufacturers and 4 categories, we would have $(3+1)^4 - 1 = 4^4 - 1 = 256 - 1 = 255$ possible configurations.

Question 3c

If we modify Q3b to allow empty slots, the problem does not fit any of our standard techniques. Explain why.

Solution

The modified problem doesn't fit standard combinatorial techniques because it creates a complex relationship between two factors:

1. The selection of devices (at most one per category)
2. The arrangement of these devices in slots (with some slots potentially empty)

This creates a dependency that's difficult to model with basic formulas:

- The number of ways to arrange devices depends on how many we select
- The number of valid device selections depends on how many slots we have

Unlike the previous version where we could simply count device selections, we now need to consider both selection and arrangement simultaneously. This makes the problem more complex and requires a different approach to solve.