ECE-108 Assignment 10: Printed Circuit Boards

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1 Printed Circuit Boards

Information

You work for a company that makes engine control units (ECU) for cars.

- The ECE consists of a printed circuit board (PCB) with 20 chips on it.
- The defect rate (probability that an ECU does not work correctly) is 3%.
- A defect is the result of a problem in manufacturing and affects an individual ECU. In comparison, a design bug affects all ECUs. This question is concerned only with defects, not bugs.
- All possible defects are independent.

Q2a: Probability of 10 Good ECUs

If you randomly sample 10 ECUs from the production line, what is the probability that all 10 of the ECUs work correctly (do not have a defect)?

Q2a Solution

Given information:

- The defect rate (probability that an ECU doesn't work) is 3%.
- We need the probability that all 10 randomly selected ECUs work correctly.

Solution:

- Probability that one ECU works correctly = 1 - 0.03 = 0.97 (97%)
- For all 10 ECUs to work correctly, each must work independently
- Probability = $(0.97)^{10} \approx 0.7374$ or about 73.74%

Answer: The probability that all 10 ECUs work correctly is 73.74%.

Q2b: Component Defect Rate

An ECU works correctly iff the PCB and all of the chips work correctly. Assuming that the PCB and all of the chips have the same defect rate, calculate the defect rate for the PCB.

Q2b Solution

Given information:

- An ECU consists of a PCB with 20 chips
- The ECU defect rate is 3%
- The ECU works correctly if and only if the PCB and all 20 chips work correctly
- All components (PCB and chips) have the same defect rate

Let's call the defect rate of an individual component (PCB or chip) as p.
Solution:

- Probability that a component works correctly $= 1 - p$
- For the ECU to work, all 21 components (1 PCB + 20 chips) must work
- Probability ECU works $= (1-p)^{21}$
- Given: Probability ECU works $= 0.97$

Thus:

$$(1-p)^{21} = 0.97 \tag{1}$$
$$1 - p = (0.97)^{1/21} \tag{2}$$
$$1 - p \approx 0.9986 \tag{3}$$
$$p \approx 0.0014 \tag{4}$$

Answer: The defect rate for each component (including the PCB) is approximately 0.14%.