ECE-108 Assignment 10: Polymer Arrangements

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1 Polymers

Information

You have a friend in mechanical engineering who is developing a composite material by layering 8 different polymers named A, B, C, D, E, F, G, and H. Due to chemical compatibility issued, the material must satisfy some constraints about the layers. Your friend is trying to calculate how many different arrangements of the layers can be constructed under various constraints. Your friend knows that you are master of composite combinatorics, and so has asked you to help.

1. Each polymer is used exactly once
2. Polymer H shall be the top layer

Q1a: Number of Possible Composite Materials

How many different arrangements of the polymers are possible if you take into account constraints 1 and 2 above and:

3. Polymers A and B must be adjacent

Q1a Solution

Since H is fixed at the top position, we need to arrange the other 7 polymers (A through G). We can treat A and B as a single unit (either AB or BA) since they must be adjacent to each other. This gives us 6 units to arrange: (A,B), C, D, E, F, and G.

Number of arrangements:

- Ways to arrange A and B within their unit: $2! = 2$ (AB or BA)
- Ways to arrange 6 units: $6! = 720$
- Total: $2 \times 6! = 2 \times 720 = 1,440$

Answer: $1,440$ different arrangements are possible.

Q1b: Different Constraint

How many different arrangements of the polymers are possible if you take into account constraints 1 and 2 above and:

4. Polymer A shall not be adjacent to polymer B or polymer A shall not be adjacent to polymer C.

Q1b Solution

This constraint means "not(A adjacent to B) OR not(A adjacent to C)". This is equivalent to "not(A adjacent to both B AND C)".

- Total arrangements with H fixed at the top: $7! = 5,040$
- We need to subtract arrangements where A is adjacent to both B and C
- For A to be adjacent to both B and C, they must form a sequence like ABC, ACB, BAC, or CAB
- These 3 elements can be treated as a single unit with internal arrangements
- Internal arrangements of A,B,C: $3! = 6$ ways to arrange A,B,C
- But only 2 of these have A adjacent to both B and C: BAC and CAB
- So we have 2 ways to make a unit where A is adjacent to both B and C
- Ways to arrange the 5 units (the A,B,C unit plus D,E,F,G): $5! = 120$
- Arrangements to exclude: $2 \times 5! = 2 \times 120 = 240$

Total $= 7! - 240 = 5,040 - 240 = 4,800$

Answer: $4,800$ different arrangements are possible.

Q1c: Yet Another Different Constraint

How many different arrangements of the polymers are possible if you take into account constraints 1 and 2 above and:

5. Polymer A shall be adjacent to polymer B or polymer C. (NOTE: This is *not* the negation of the constraint from Q1b.)

Q1c Solution

For this constraint, A must be adjacent to at least one of B or C.

- Total arrangements with H fixed at the top: $7! = 5,040$
- We need to subtract arrangements where A is not adjacent to either B or C

- Total arrangements: $7! = 5,040$
- Complement: Arrangements where A is not adjacent to either B or C
- From our calculations, there are $1,440$ such arrangements
- Result: $7! - 1,440 = 5,040 - 1,440 = 3,600$

Answer: $3,600$ different arrangements are possible.