ECE-108 Assignment 10: Analysis of Covid-19 Rapid Test

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1 Analysis of Covid-19 Rapid Test

Given information:

- The Covid-19 rapid test has:
  - Sensitivity = 78%
  - Specificity = 99%
- First scenario: Covid-19 actual prevalence = 10%
- Second scenario: Covid-19 actual prevalence = 1%

Q4a: Positive Results (10% prevalence)

What percentage of people test positive for Covid-19?

Solution

We need to calculate the percentage of people who test positive.
Total positive test results = True positives + False positives

$$\text{True positives} = \text{Sensitivity} \times \text{Prevalence} = 0.78 \times 0.10 = 0.078 \ (7.8\%) \tag{1}$$

$$\text{False positives} = (1 - \text{Specificity}) \times (1 - \text{Prevalence}) = 0.01 \times 0.90 = 0.009 \ (0.9\%) \tag{2}$$

$$\text{Total positive tests} = 0.078 + 0.009 = 0.087 \ (8.7\%) \tag{3}$$

Answer: 8.7% of people test positive for Covid-19.

Q4b: False Negatives (10% prevalence)

What percentage of people get a false negative result from the test?

Solution

False negatives occur when infected people test negative:

$$\text{False negative rate} = (1 - \text{Sensitivity}) \times \text{Prevalence} \tag{4}$$

$$= 0.22 \times 0.10 = 0.022 \ (2.2\%) \tag{5}$$

Answer: 2.2% of people get a false negative result from the test.

Q4c: Estimated Prevalence (10% prevalence)

Based on the test results, what is the estimated prevalence?

Solution

The estimated prevalence is the percentage of people who test positive:

Answer: The estimated prevalence is 8.7%.

Q4d: Analysis of Estimated Prevalence

Why does the estimated prevalence differ from the actual prevalence?

Solution

The estimated prevalence (8.7%) is lower than the actual prevalence (10%) because:

- The test's sensitivity isn't perfect (78%)
- This means 22% of actual positive cases are missed (false negatives)
- Even though there are some false positives, they don't fully compensate for the missed cases

Repeat for an actual prevalence of 1%

Q4e: Positive Results (1% prevalence)

What percentage of people test positive for Covid-19?

Solution

Total positive test results = True positives + False positives

$$\text{True positives} = \text{Sensitivity} \times \text{Prevalence} = 0.78 \times 0.01 = 0.0078 \ (0.78\%) \tag{6}$$

$$\text{False positives} = (1 - \text{Specificity}) \times (1 - \text{Prevalence}) = 0.01 \times 0.99 = 0.0099 \ (0.99\%) \tag{7}$$

$$\text{Total positive tests} = 0.0078 + 0.0099 = 0.0177 \ (1.77\%) \tag{8}$$

Answer: 1.77% of people test positive for Covid-19.

Q4f: False Negatives (1% prevalence)

What percentage of people get a false negative result from the test?

Solution

False negatives occur when infected people test negative:

$$\text{False negative rate} = (1 - \text{Sensitivity}) \times \text{Prevalence} \tag{9}$$

$$= 0.22 \times 0.01 = 0.0022 \ (0.22\%) \tag{10}$$

Answer: 0.22% of people get a false negative result from the test.

Q4g: Estimated Prevalence (1% prevalence)

Based on the test results, what is the estimated prevalence?

Solution

The estimated prevalence is the percentage of people who test positive:

Answer: The estimated prevalence is 1.77%.

Q4h: Analysis of Effect of Prevalence

Why does the actual prevalence affect the difference between the actual and estimated prevalence?

Solution

The actual prevalence affects the difference between actual and estimated prevalence because:

- At lower prevalence (1%), false positives become more significant relative to true positives
- This causes the estimated prevalence (1.77%) to be higher than the actual (1%)
- At higher prevalence (10%), false negatives become more significant
- This causes the estimated prevalence (8.7%) to be lower than the actual (10%)
- The balance point depends on the test's sensitivity and specificity values