Acceleration-Based Optimal Shift Point Algorithm

ACCRPMMonitor by Q.M. 114

1 Objective

Find the RPM at which shifting to the next gear provides better acceleration than staying in the current gear, optimizing lap times by maximizing acceleration throughout the power band.

2 Mathematical Formulation

2.1 Step 1: Acceleration Calculation

For each pair of consecutive telemetry data points $(i-1, i)$, calculate the instantaneous acceleration:

$$a_i = \frac{v_i - v_{i-1}}{t_i - t_{i-1}} \tag{1}$$

where:

- a_i = acceleration at point i (m/s^2)
- v_i = speed at point i (m/s)
- t_i = timestamp at point i (seconds)

Group acceleration values by RPM buckets (100 RPM intervals):

$$\text{RPM}_{\text{bucket}} = \left\lfloor \frac{\text{RPM}_{\text{avg}}}{100} \right\rfloor \times 100 \tag{2}$$

Average all acceleration values in each bucket to create the acceleration function:

$$A_{\text{gear}}(\text{RPM}) = \frac{1}{n} \sum_{i \in \text{bucket}} a_i \tag{3}$$

where n is the number of samples in the RPM bucket $[\text{RPM}, \text{RPM} + 100)$.
This gives us the acceleration function $A_{\text{gear}}(\text{RPM})$ for each gear.

2.2 Step 2: Gear Ratio Estimation

Find the overlapping speed range between gear N and gear $N+1$:

$$V_{\text{overlap}} = \left[\max\left(V_{\min}^{N}, V_{\min}^{N+1}\right), \min\left(V_{\max}^{N}, V_{\max}^{N+1}\right)\right] \tag{4}$$

Calculate the RPM-to-speed ratio for each gear in the overlap region:

$$R_N = \frac{1}{n_N} \sum_{i:v_i \in V_{\text{overlap}}, \text{gear}=N} \frac{\text{RPM}_i}{v_i} \tag{5}$$

$$R_{N+1} = \frac{1}{n_{N+1}} \sum_{i:v_i \in V_{\text{overlap}}, \text{gear}=N+1} \frac{\text{RPM}_i}{v_i} \tag{6}$$

The gear ratio between gear N and gear $N+1$ is:

$$\text{GR}_{N \to N+1} = \frac{R_N}{R_{N+1}} \tag{7}$$

2.3 Step 3: Crossover Point Detection with Significance Threshold

For each RPM level in gear N, predict the RPM after shifting to gear $N+1$:

$$\text{RPM}_{N+1} = \frac{\text{RPM}_N}{\text{GR}_{N \to N+1}} \tag{8}$$

Calculate the acceleration advantage ratio (relative improvement):

$$R_A(\text{RPM}_N) = \frac{A_{N+1}(\text{RPM}_{N+1}) - A_N(\text{RPM}_N)}{A_N(\text{RPM}_N)} \tag{9}$$

Define the minimum advantage threshold to avoid premature shifting:

$$\theta_{\min} = 0.05 \quad \text{(5\% better acceleration required)} \tag{10}$$

Find the optimal shift point where the advantage is both significant and maximized:

$$\text{RPM}_{\text{optimal}} = \arg\max_{\text{RPM}} \left\{R_A(\text{RPM}) \mid R_A(\text{RPM}) > \theta_{\min}\right\} \tag{11}$$

In other words: shift at the RPM where the next gear provides at least 5% better acceleration and this advantage is maximized. This prevents shifting too early when the advantage is marginal.

3 Algorithm Pseudocode

```
function FindOptimalShiftPoint(gear_N)
    // Step 1: Calculate acceleration curves
    A_N ← CalculateAccelerationCurve(gear_N)
    A_{N+1} ← CalculateAccelerationCurve(gear_{N+1})

    if InsufficientData(A_N) or InsufficientData(A_{N+1}) then
        return FallbackMaxSpeedMethod(gear_N)
    end if

    // Step 2: Estimate gear ratio
    GR ← EstimateGearRatio(gear_N, gear_{N+1})

    if GR ≤ 0 then
        return FallbackMaxSpeedMethod(gear_N)
    end if

    // Step 3: Find crossover point with significance threshold
    best_rpm ← null
    best_advantage_ratio ← 0
    θ_min ← 0.05    // Require 5% better acceleration

    for each rpm ∈ A_N.keys() do
        a_current ← A_N[rpm]
        rpm_next ← rpm/GR
        a_next ← A_{N+1}[ClosestRPM(rpm_next)]

        // Calculate relative advantage (percentage improvement)
        R_a ← (a_next − a_current)/a_current

        // Require significant advantage to avoid early shifting
        if R_a > θ_min and R_a > best_advantage_ratio then
            best_advantage_ratio ← R_a
            best_rpm ← rpm
        end if
    end for

    if best_rpm ≠ null then
        return best_rpm
    else
        return FallbackMaxSpeedMethod(gear_N)
    end if
end function
```

4 Example Calculation

Consider shifting from 2nd gear to 3rd gear:

4.1 Given Data

- At 6800 RPM in 2nd gear: $A_2(6800) = 2.5 \text{ m/s}^2$
- At 6500 RPM in 2nd gear: $A_2(6500) = 2.8 \text{ m/s}^2$
- At 6200 RPM in 2nd gear: $A_2(6200) = 3.0 \text{ m/s}^2$
- Gear ratio: $\text{GR}_{2\to3} = 1.31$
- Minimum advantage threshold: $\theta_{\min} = 0.05$ (5%)

4.2 Analysis at 6800 RPM

After shifting to 3rd gear:

$$\text{RPM}_3 = \frac{6800}{1.31} \approx 5191 \text{ RPM} \tag{12}$$

If $A_3(5191) = 3.1 \text{ m/s}^2$, then:

$$R_A(6800) = \frac{3.1 - 2.5}{2.5} = \frac{0.6}{2.5} = 0.24 = 24\% \tag{13}$$

Since $24\% > 5\%$, this is a valid shift point with significant advantage. **(SHIFT NOW!)**

4.3 Analysis at 6500 RPM

After shifting to 3rd gear:

$$\text{RPM}_3 = \frac{6500}{1.31} \approx 4962 \text{ RPM} \tag{14}$$

If $A_3(4962) = 2.4 \text{ m/s}^2$, then:

$$R_A(6500) = \frac{2.4 - 2.8}{2.8} = \frac{-0.4}{2.8} = -0.14 = -14\% \tag{15}$$

Since $-14\% < 5\%$, staying in 2nd gear is better. **(TOO EARLY, STAY IN GEAR)**

4.4 Analysis at 6200 RPM

After shifting to 3rd gear:

$$\text{RPM}_3 = \frac{6200}{1.31} \approx 4733 \text{ RPM} \tag{16}$$

If $A_3(4733) = 3.05 \text{ m/s}^2$, then:

$$R_A(6200) = \frac{3.05 - 3.0}{3.0} = \frac{0.05}{3.0} = 0.017 = 1.7\% \tag{17}$$

Since 1.7% $<$ 5%, the advantage is marginal and not significant. **(TOO EARLY, WAIT)**

This example shows how the 5% threshold prevents premature shifting when acceleration advantage is minimal.

5 Key Parameters

- **RPM Bucket Size**: 100 RPM (groups similar RPM levels)
- **Minimum Time Delta**: 0.01 seconds (10 ms, filters noise)
- **Maximum Time Delta**: 1.0 seconds (filters different laps)
- **RPM Match Tolerance**: $\pm$200 RPM (acceptable range for gear ratio lookup)
- **Minimum Data Points per Gear**: 50 full-throttle samples
- **Minimum Samples per Bucket**: 3 samples for statistical reliability
- **Full Throttle Threshold**: $\geq$ 95% throttle position
- **Minimum Speed Threshold**: $>$ 0 km/h (excludes stationary data)
- **Minimum Advantage Threshold** ($\theta_{\min}$): 0.05 (5% better acceleration required)

6 Fallback Strategy

If acceleration-based method fails (insufficient data, invalid gear ratio, etc.), use the maximum speed fallback:

$$\text{RPM}_{\text{optimal}} = \min \{\text{RPM}_i \mid v_i \geq 0.99 \times v_{\max}, \text{gear} = N\} \tag{18}$$

This finds the lowest RPM that achieved at least 99% of the maximum speed in that gear.

7 Benefits

1. **Physics-based**: Directly uses Newton's second law ($F = ma$)
2. **Vehicle-specific**: Adapts to each car's unique power curve
3. **Track-independent**: Works on any track configuration
4. **Self-optimizing**: More data yields better shift points
5. **Robust**: Graceful degradation with fallback strategy
6. **Optimal for lap times**: Maximizes acceleration = minimizes time

7. **Prevents premature shifting**: 5% threshold ensures significant advantage before shifting

8. **High-quality data**: Filters out invalid telemetry (speed = 0, throttle ¡ 95%)

8 Dynamic Audio Warning System

To provide real-time feedback, the system employs a dynamic beeping distance based on RPM rate of change:

8.1 RPM Rate Calculation

Track RPM history over a 200ms sliding window and calculate the rate of change:

$$\dot{\text{RPM}} = \frac{\text{RPM}_{\text{current}} - \text{RPM}_{\text{old}}}{t_{\text{current}} - t_{\text{old}}} \quad \text{(RPM/second)} \tag{19}$$

8.2 Dynamic Warning Distance

The beeping warning distance d_{beep} adapts to acceleration intensity:

$$d_{\text{beep}} = \begin{cases} 200 \text{ RPM} & \text{if } \dot{\text{RPM}} > 1500 \text{ RPM/s} \\ 150 \text{ RPM} & \text{if } \dot{\text{RPM}} > 1000 \text{ RPM/s} \\ 120 \text{ RPM} & \text{if } \dot{\text{RPM}} > 600 \text{ RPM/s} \\ 100 \text{ RPM} & \text{if } \dot{\text{RPM}} > 300 \text{ RPM/s} \\ 80 \text{ RPM} & \text{if } \dot{\text{RPM}} > 150 \text{ RPM/s} \\ 50 \text{ RPM} & \text{if } \dot{\text{RPM}} > 50 \text{ RPM/s} \\ 30 \text{ RPM} & \text{otherwise} \end{cases} \tag{20}$$

Beeping begins when:

$$\text{RPM}_{\text{current}} \geq \text{RPM}_{\text{optimal}} - d_{\text{beep}} \tag{21}$$

This approach provides earlier warning during rapid acceleration while avoiding premature beeping during steady-state driving.

9 Adaptive Learning Mode

The system supports continuous learning during normal driving:

1. **Real-time data collection**: Collects telemetry at 20 Hz during full-throttle acceleration

2. **Automatic filtering**: Rejects data when throttle < 95% or speed = 0

3. **Periodic updates**: Recalculates optimal shift points every 10 seconds
4. **Live adaptation**: Shift point thresholds update dynamically as new data is collected
5. **Persistent learning**: Optionally saves learned configuration for future sessions

This enables the system to continuously refine shift points as driving conditions and techniques evolve.