

② Suppose n activities apply for using a common resource. Activity a_i ($1 \leq i \leq n$) has a starting time $S[i]$ and a finish time $F[i]$ such that $0 < S[i] < F[i]$. Two activities a_i and a_j ($1 \leq i, j \leq n$) are compatible if intervals $[S[i], F[i])$ and $[S[j], F[j])$ do not overlap. We assume the activities have been sorted such that $S[1] \leq S[2] \leq \dots \leq S[n]$.

A Design an $\mathcal{O}(n^2)$ dynamic programming algorithm to find a set of compatible activities such that the total amount of time the resource is used by these compatible activities is maximized. You need to define the sub-problems, establish the inductive formula and show the initial conditions. Pseudocode is not required.

B Apply your algorithm to the following set of activities:

i	1	2	3	4	5	6	7	8	9	10	11
S[i]	2	3	5	6	7	9	10	12	13	14	16
F[i]	6	5	7	10	8	13	16	14	14	18	20

②(A)

Algorithm 1 A dynamic programming algorithm usable to solve the activity problem above in $\mathcal{O}(n^2)$ time. In this algorithm...

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1: function MAXACTIVITIES(S[1..n], F[1..n])
2:    $M \leftarrow P \leftarrow \emptyset$ 
3:   INITIALIZE( $M, P$ )
4:   for  $i$  from 1 to  $n$  do                                      $\triangleright$  Locate max for  $m_i$ 
5:      $max \leftarrow M[i]$ 
6:     for  $j$  from 1 to  $i$  do
7:       if  $max < M[j]$  and  $F[j] < S[i]$  then
8:          $max \leftarrow M[j]$ 
9:          $P[i] \leftarrow j$ 
10:      end if
11:    end for
12:  end for
13:   $max \leftarrow 1$                                               $\triangleright$  Find global maximum
14:  for  $i$  from 2 to  $n$  do
15:    if  $M[max] < M[i]$  then
16:       $max \leftarrow i$ 
17:    end if
18:  end for
19:  return  $max, P$ 
20: end function

```
