

(F.3) Given a sequence of n ($n > 2$) real numbers in an array, $A[1]$, $A[2]$, $A[3]$, ..., $A[n]$, we wish to find two numbers $A[i]$ and $A[j]$, where $i < j$, such that $A[i] \leq A[j]$ and their distance is the largest. That is,

$$j - i = \max\{v - u \mid 1 \leq u < v \leq n \text{ and } A[u] \leq A[v]\}$$

If no such pair, report $-\infty$. Please design an $\mathcal{O}(n \cdot \lg n)$ algorithm to solve this problem. You can use any method, including divide and conquer, greedy or dynamic programming.

The following should be noted about the algorithm below (in addition to the statements in the problem):

1. Function SORTANDTRACKINDICES(A , I) returns array A sorted in non-decreasing order ($a_1 \leq a_2 \leq a_3 \leq \dots \leq a_n$) as well as an array, I , which maps each a_i to its original position, I_i .

Algorithm 1 Greedy solution to the problem above. Make sure to take note of the assumptions above.

```
1: function MAXDIST(A, n)
2:    $A, I \leftarrow \text{SORTANDTRACKINDICES}(A)$ 
3:    $i_{max} \leftarrow j_{max} \leftarrow I[n]$ 
4:    $i \leftarrow j \leftarrow I[n]$ 
5:
6:   for  $k$  from  $n - 1$  to  $1$  do
7:     if  $j < I[k]$  then
8:        $i \leftarrow I[k]$ 
9:        $j \leftarrow I[k]$ 
10:    else
11:      if  $i > I[k]$  then
12:         $i \leftarrow I[k]$ 
13:      end if
14:    end if
15:    if  $j - i > j_{max} - i_{max}$  then
16:       $i_{max} \leftarrow i$ 
17:       $j_{max} \leftarrow j$ 
18:    end if
19:  end for
20:
21:  if  $i_{max}$  equals  $j_{max}$  then
22:    return  $-\infty$ 
23:  else
24:    return  $i$  and  $j$ 
25:  end if
26:
27: end function
```
