# Calculus

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# Contents

# 1 Integrals

## 1.1 Line Integrals

## 1.2 Surface Integrals

As with line integrals, integrals over surfaces can involve vector and scalar fields. There are four kinds of suface integrals, namely

$$\int_S \phi dS, \quad \int_S \phi d\mathbf{S}, \quad \int_S \mathbf{a} \cdot d\mathbf{S} \text{ and } \int_S \mathbf{a} \times d\mathbf{S}, \tag{1.1}$$

where $d\mathbf{S} = \hat{\mathbf{n}} dS$ is the infinitesimal vector area element. The direction of $\hat{\mathbf{n}}$ is conventionally assumed to be directed outwards from the nclosed volume if the surface is closed; or given by the right-hand rule if the surface is open and spans some perimeter curve $C$.

We start with the first and simplest surface integral involving only scalars $\int_S \phi dS$. In Cartesian coordinates,

$$I = \int_S f(x, y) dS = \iint_S f(x, y) dx dy \tag{1.2}$$

where one or two integral signs are used depending on whether $dS = dxdy$ is written explicitly.

Refering to fig. 1.1, we can see that the integral can be evaluated two different ways.

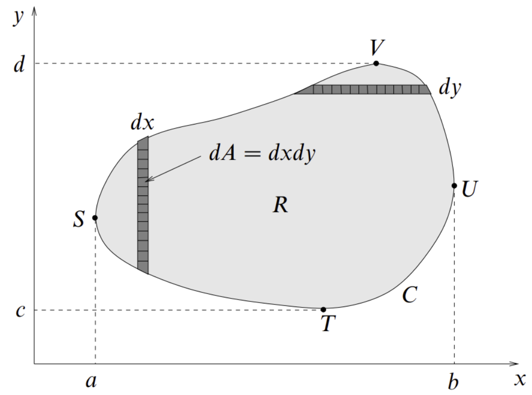


Figure 1.1

The first is to sum up all the horizontal strips, then

$$I = \int_{c}^{d} \int_{x_1(y)}^{x_2(y)} f(x, y) dx dy, \tag{1.3}$$

where $x_1(y)$ and $x_2(y)$ are the equations of the curves $TSV$ and $TUV$ respectively.

For a specific range $y \to y + dy$ , the inner integral calculates the contribution to $I$ by the horizontal strip located from $y$ to $y + dy$. The outer integral then sums up the contributions of these horizontal strips.

The second way is to sum up all the vertical strips, then

$$I = \int_{a}^{b} \int_{y_1(x)}^{y_2(x)} f(x, y) dy dx, \tag{1.4}$$

where $y_1(x)$ and $y_2(x)$ are the equations of the curves $STU$ and $SVU$ respectively.