Assignment 4

Shuhan Xiao (kg410 3160697)
Klaus Kades (fw448 3408463)
Lucas-Raphael Müller (al413 3205638)
Melanie Schellenberg (gh400 3146390)

Problem 4-2:

$q_k = \frac{1}{C_2} p_k \cdot k$

$p_k = C_1 k^{-\gamma}$

1. $1 = \int_{k_{min}}^{k_{max}} p_k \, dk = \int_{k_{min}}^{k_{max}} C_1 k^{-\gamma} \, dk$ || normalization condition

$$= C \frac{1}{1-\gamma} k^{1-\gamma} \Big|_{k_{min}}^{k_{max}} = \frac{C}{1-\gamma} \left(k_{max}^{1-\gamma} - k_{min}^{1-\gamma} \right)$$

$$\frac{1-\gamma}{k_{max}^{1-\gamma} - k_{min}^{1-\gamma}} = C_1$$

$$1 = \int_{k_{min}}^{k_{max}} q_k \, dk = \int_{k_{min}}^{k_{max}} \frac{C_1}{C_2} k^{1-\gamma} \, dk \quad \| \text{ normalization constraint}$$

$$= \frac{C_1}{C_2} \frac{1}{2-\gamma} k^{2-\gamma} \Big|_{k_{min}}^{k_{max}} = \frac{C_1}{C_2} \frac{1}{2-\gamma} \left(k_{max}^{2-\gamma} - k_{min}^{2-\gamma} \right)$$

$$C_2 = \frac{C_1}{2-\gamma} \left(k_{max}^{2-\gamma} - k_{min}^{2-\gamma} \right) = \frac{1-\gamma}{2-\gamma} \frac{k_{max}^{2-\gamma} - k_{min}^{2-\gamma}}{k_{max}^{1-\gamma} - k_{min}^{1-\gamma}}$$

2. $\langle k_{neigh} \rangle = \int_{k_{min}}^{k_{max}} q_k \, k \, dk$

$$= \int_{k_{min}}^{k_{max}} \frac{C_1}{C_2} k^{2-\gamma} \, dk$$

$$= \frac{C_1}{C_2} \frac{1}{3-\gamma} \left(k_{max}^{3-\gamma} - k_{min}^{3-\gamma} \right)$$

$$= \frac{2-\gamma}{3-\gamma} \frac{1}{\left(k_{max}^{2-\gamma} - k_{min}^{2-\gamma}\right)} \left(k_{max}^{3-\gamma} - k_{min}^{3-\gamma}\right)$$

$$= \frac{2-\gamma}{3-\gamma} \frac{k_{max}^{3-\gamma} - k_{min}^{3-\gamma}}{k_{max}^{2-\gamma} - k_{min}^{2-\gamma}}$$

3. $$\langle k \rangle = \int_{k_{min}}^{k_{max}} q_k \, k \, dk = \int_{k_{min}}^{k_{max}} C_1 \, k^{1-\gamma} \, dk$$

$$= \frac{C_1}{2-\gamma} \left(k_{max}^{2-\gamma} - k_{min}^{2-\gamma}\right)$$

$$= \frac{1-\gamma}{2-\gamma} \frac{k_{max}^{2-\gamma} - k_{min}^{2-\gamma}}{k_{max}^{1-\gamma} - k_{min}^{1-\gamma}}$$

with $k_{max} = 1000$ $k_{min} = 1$ $\gamma = 2.3$

$$\langle k \rangle = \frac{+1.3}{+0.3} \frac{1000^{-0.3} - 1}{1000^{-1.3} - 1} = 3.79$$

$$\langle k_{neigh} \rangle = \frac{-0.3}{0.7} \frac{1000^{0.7} - 1}{1000^{-0.3} - 1} = 61,23$$

4. It is more likely to be connected to a node of a high degree. This phenomenon is called the friendship paradox.

↳ People with more friends are more likely to be in one's own friend group. Hence, nodes of high degree are more likely to be the neighbour of a randomly chosen node.