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Complex Network Analysis 7

Assignment 7: "Assortativity and Robustness"

Problem 7-2 Molloy-Reed Criterion

$$p_k = \delta_{k,1} p_1 + \delta_{k,2} p_2 + \delta_{k,3} p_3 \qquad \begin{cases} \delta_{k,1} = 3 & \text{if } k=1 \\ \delta_{k,2} = 2 & \text{if } k=2 \\ \delta_{k,3} = 2 & \text{if } k=3 \end{cases}$$

1. $$\langle k \rangle = \sum_k k\, p_k = \underbrace{3}_{\delta_{k,1}} \cdot p_1 \cdot \underbrace{1}_{k} + \underbrace{2}_{\delta_{k,2}} \cdot p_2 \cdot \underbrace{2}_{k} + \underbrace{2}_{\delta_{k,3}} \cdot p_3 \cdot \underbrace{3}_{k} = 3p_1 + 4p_2 + 6p_3$$

$$\langle k^2 \rangle = \sum_k k^2 p_k = 3 \cdot p_1 \cdot 1^2 + 2 \cdot p_2 \cdot 2^2 + 2 \cdot p_3 \cdot 9 = 3p_1 + 8p_2 + 18p_3$$

2. Molloy-Reed Criterion: A giant component exists in a network if $\frac{\langle k^2 \rangle}{\langle k \rangle} > 2$.

$$\rightarrow \frac{\langle k^2 \rangle}{\langle k \rangle} \overset{!}{>} 2 \iff \langle k^2 \rangle > 2\langle k \rangle$$

$$\iff 3p_1 + 8p_2 + 18p_3 > 2(3p_1 + 4p_2 + 6p_3) = 6p_1 + 8p_2 + 12p_3$$

$$\iff 3p_1 + 18p_3 > 6p_1 + 12p_3 \qquad |-3p_1, -12p_3$$

$$\iff 6p_3 > 3p_1 \qquad |:3$$

$$\iff \underline{p_1 < 2p_3}$$

(This is different from the result given on the problem sheet: $p_1 \overset{?}{<} 3p_3$)

3. There should be less nodes with degree 1 than degree 3 to make sure that there are ~~no~~ only a few 'loose ends' and that the network is not fragmented into small subnetworks, otherwise no giant ~~networ~~ component could form.