Random graph $G(N,p)$ $\quad p = \frac{a}{N^z} \quad a>0,\ z\geq 0$

$$\langle k \rangle = p(N-1)$$

$$= \frac{a}{N^z}(N-1) = \frac{a}{N^{z-1}} - \frac{a}{N^z}$$

- $a = 1/2,\ z = 1$

giant component $\rightarrow \langle k \rangle \geq 1$

$$\langle k \rangle \overset{N\to\infty}{=} 1/2 \text{ ; no giant comp.}$$

- $a = 2,\ z = 1$

$$\langle k \rangle \overset{N\to\infty}{=} 2 \quad \text{giant comp.}$$

- $a > 0,\ z = 2$

$$\langle k \rangle = \frac{a}{N} - \frac{a}{N^2} \overset{N\to\infty}{=} 0 \quad \text{no giant comp.}$$

- $a > 0 \quad z = 0.5$

$$\langle k \rangle = a\sqrt{N} - \frac{a}{\sqrt{N}} \overset{N\to\infty}{=} \infty \quad \text{giant component}$$

2.

$$\langle u \rangle = \frac{a}{N^{z-1}} - \frac{a}{N^{z}} \overset{N\to\infty}{=} \begin{cases} \infty & z=0 \\ \infty & z\geq 0,\ z<1 \\ a & z=1 \\ 0 & z>0 \end{cases}$$

3. Critical when $\langle u \rangle = 1$

$\longrightarrow$ $z \geq 0,\ z<1$ Super-critical

$z = 1$ critical if $a=0$

$z > 1$ Sub-critical