Problem_8_1

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[33]:
```
import pandas as pd
import networkx as nx
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy as np
from collections import Counter
import random
```

1 Assignment 8

1.1 Problem 8-1 European Power Grid

[3]:
```
!wget https://zenodo.org/record/47317/files/gridkit_euorpe.zip --no-clobber
```

```
--2022-01-16 19:17:44--
https://zenodo.org/record/47317/files/gridkit_euorpe.zip
Resolving zenodo.org (zenodo.org)… 137.138.76.77
Connecting to zenodo.org (zenodo.org)|137.138.76.77|:443… connected.
HTTP request sent, awaiting response… 200 OK
Length: 1805506 (1.7M) [application/octet-stream]
Saving to: ‘gridkit_euorpe.zip’

gridkit_euorpe.zip  100%[===================>]   1.72M  2.48MB/s    in 0.7s

2022-01-16 19:17:46 (2.48 MB/s) - ‘gridkit_euorpe.zip’ saved [1805506/1805506]
```

[6]:
```
!unzip *.zip
```

```
Archive:  gridkit_euorpe.zip
  inflating: gridkit_europe-highvoltage-links.csv
```

```
  inflating: gridkit_europe-highvoltage-vertices.csv
 gridkit_euorpe.zip                     gridkit_europe-highvoltage-vertices.csv
'gridkit_euorpe.zip?download=1'         sample_data
 gridkit_europe-highvoltage-links.csv
```

[7]:
```
!ls
```

```
gridkit_euorpe.zip                    gridkit_europe-highvoltage-vertices.csv
gridkit_europe-highvoltage-links.csv  sample_data
```

[11]:
```
data = pd.read_csv('./gridkit_europe-highvoltage-links.csv', sep=',')

display(data)
```

```
        l_id  …                                        wkt_srid_4326
0      22139  …  SRID=4326;LINESTRING(7.61602141237887 47.23259…
1      65908  …  SRID=4326;LINESTRING(26.6449735029238 50.29628…
2      67370  …  SRID=4326;LINESTRING(18.1844471738857 40.01608…
3       3868  …  SRID=4326;LINESTRING(10.2724738664462 49.88060…
4       4982  …  SRID=4326;LINESTRING(3.80033661099728 45.04510…
…        …   …                                                   …
18799  33852  …  SRID=4326;LINESTRING(34.891839086949 48.400953…
18800  12859  …  SRID=4326;LINESTRING(13.0725880043994 46.33610…
18801  17974  …  SRID=4326;LINESTRING(22.9977678819448 63.71031…
18802  69707  …  SRID=4326;LINESTRING(5.34304391214955 43.44674…
18803  28512  …  SRID=4326;LINESTRING(10.3699302949897 45.62925…

[18804 rows x 17 columns]
```

create network

[12]:
```
G = nx.from_pandas_edgelist(data, source='v_id_1',target='v_id_2')
print(nx.info(G))
# convert to undirected graph
G = G.to_undirected()
```

```
Graph with 13844 nodes and 17277 edges
```

[13]:
```
G = nx.Graph(G) # multigraph to graph
print(nx.info(G))
```

```
Graph with 13844 nodes and 17277 edges
```

1. Molloy-Reed criterion: Calculate κ

[17]:
```
# average degree <k>
degrees = [val for (node, val) in G.degree()]
avg_k = sum(degrees)/len(degrees)
print(f"<k> = {avg_k}")
```

```
<k> = 2.4959549263218723
```

[23]:
```python
# second moment of degree <k^2>
avg_k2 = sum([k**2 for k in degrees])/len(degrees)
print(f"<k^2> = {avg_k2}")
```

```
<k^2> = 7.888182606183184
```

[22]:
```python
# Molloy-Reed criterion
kappa = avg_k2/avg_k
print(f"Molloy-Reed: kappa = {kappa}")
```

```
Molloy-Reed: kappa = 3.1603866411992825
```

$\kappa > 2$, i.e. the Molloy-Reed criterion is satisfied and a giant component is expected in the network.

2. Absolute and relative size of the largest component:

[26]:
```python
largest_cc = max(nx.connected_components(G), key=len)
print(f"absolute size of largest component {len(largest_cc)}")
print(f"relative size of largest component {len(largest_cc)/G.
↪number_of_nodes()}")
```

```
absolute size of largest component 13478
relative size of largest component 0.9735625541750939
```

Yes, the largest component is almost of the same size as the network and can therefore be regarded as the giant component.

3. Compute f_c and f_c^{ER} for the network's degree distribution.

[28]:
```python
fc = 1-1/(kappa-1)
fc_ER = 1-1/avg_k
print(f"fc = {fc} < fc_ER = {fc_ER}")
```

```
fc = 0.5371198928332217 < fc_ER = 0.5993517393065926
```

The critical threshold of the network is slightly smaller than what would be expected by an Erdős–Rényi model with the same $\langle k \rangle$, the network is therefore slightly less robust against random failure.

4.

(i) maximum component size and the (ii) average component size as a function of f [0,1].

[57]:
```python
def get_component_sizes(G, n_samples=10, f_samples=21):
  fractions = np.linspace(0,1,f_samples)
  G = G.copy()
  N = G.number_of_nodes()
  Gc_0 = len(max(nx.connected_components(G), key=len))
  Gc_max = np.zeros((len(fractions),n_samples))
  Gc_avg = np.zeros((len(fractions),n_samples))
  for i in range(n_samples):
```

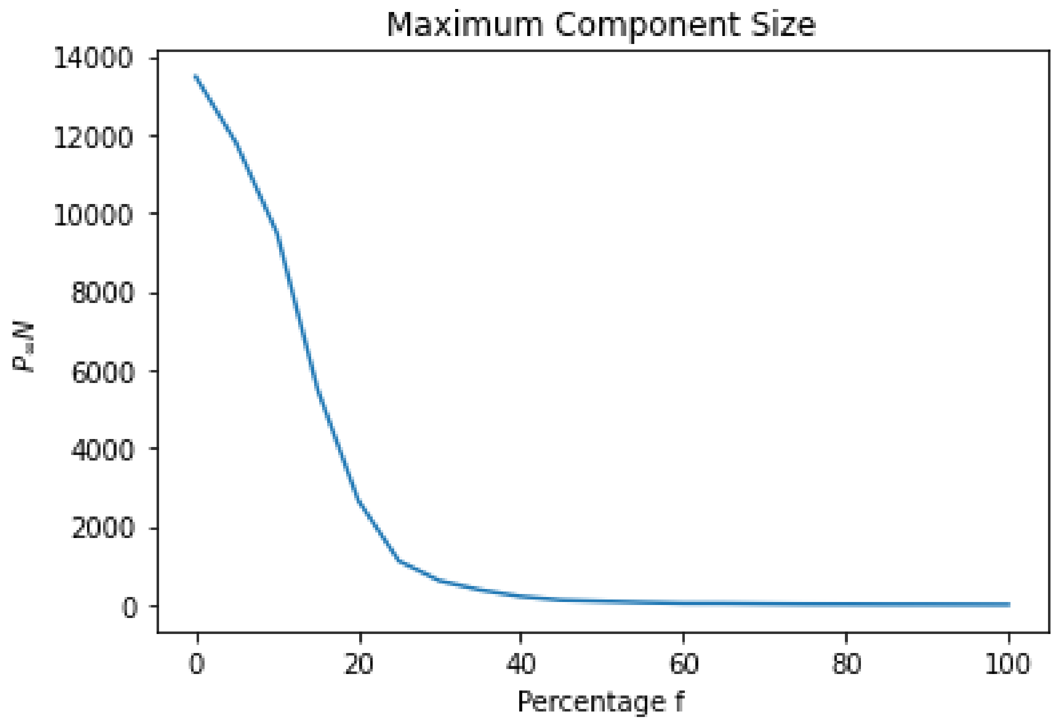
```
    for j, f in enumerate(fractions):
      G_ = G.copy()
      nodes = [n for n in G_.nodes]
      # remove f*N nodes
      if f > 0.0:
        rnd_nodes = random.sample(nodes,round(f*N))
        for n in rnd_nodes:
          G_.remove_node(n)
      cc = [len(c) for c in sorted(nx.connected_components(G_), key=len,␣
↪reverse=True)]
      largest_cc = max(cc) if len(cc)>0 else 0
      Gc_max[j,i] = largest_cc
      Gc_avg[j,i] = sum(cc)/len(cc) if len(cc)>0 else 0
  Gc_max = np.average(Gc_max,axis=1)
  Gc_avg = np.average(Gc_avg,axis=1)
  return Gc_max, Gc_avg, fractions
```

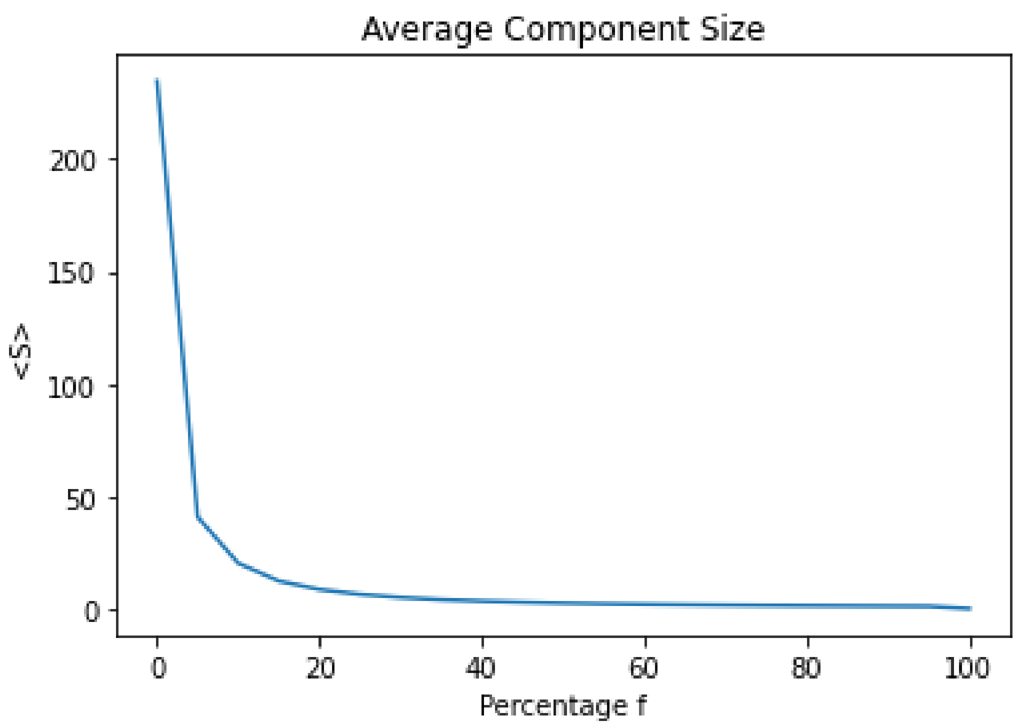
[58]:
```
Gc_max, Gc_avg, fractions = get_component_sizes(G, n_samples=10)
```

[60]:
```
plt.figure()
plt.plot(fractions*100, Gc_max)
plt.ylabel("$P_{\infty}N$")
plt.xlabel("Percentage f")
plt.title("Maximum Component Size")

plt.figure()
plt.plot(fractions*100, Gc_avg)
plt.ylabel("<S>")
plt.xlabel("Percentage f")
plt.title("Average Component Size")
```

[60]:
```
Text(0.5, 1.0, 'Average Component Size')
```


Maximum Component Size
14000
12000
10000
8000
6000
4000
2000
0
$P_\infty N$
0
20
40
60
80
100
Percentage f


Average Component Size
200
150
100
50
0
<S>
0
20
40
60
80
100
Percentage f

Both maximum and average component size are rapidly decreasing, even before a fraction fc or fc_ER is removed, indicating that the network is very prone to random attacks. (Note: When calculating the average component size we have also included the largest (giant) component)

[ ]:

```

```