Problem 2-2

2-2: 1. $d_{max} = 3(\ell - 1)$ since each side consists of ~~...~~ N nodes and N-1 links

2-2: 2. It holds: $N = \ell^3$

the maximum number of links per node is 6, the minimum number is 3. Therefore $i \in \{3, 4, 5, 6\}$

$\Rightarrow$ $p_3 = \frac{8}{N}$, since the cube has 8 corners and only the corners have only 3 links

$p_4 = \frac{12(\ell-2)}{\ell^3}$, since the cube has 12 edges and only the edges have ~~only~~ 4 links

$p_5 = \frac{6(\ell-2)^2}{\ell^3}$, since the cube has 6 sides and only nodes on the surface have 5 links

$p_6 = \frac{(\ell-2)^3}{N} = 1 - (p_3 + p_4 + p_5)$, since only nodes inside the cube have 6 edges.

For $N \to \infty$ holds:

$p_3 = \lim_{N \to \infty} \frac{8}{N} = 0$ $\quad p_4 = \lim_{N \to \infty} \frac{12(\ell-2)}{N} = 0$

$p_5 = \lim_{N \to \infty} \frac{6(\ell-2)^3}{N} = 0$ $\quad p_6 = \lim_{N \to \infty} 1 - \overbrace{(p_3 + p_4 + p_5)}^{\to 0} = \underline{\underline{1}}$

Only $p_6$ converts towards ~~...~~ 1, which makes sense, since the proportion of the number of nodes inside the cube is bigger and bigger in comparison with the other nodes of the cube.

2-2: 3. ~~...~~ The clustering coefficient is defined as the proportion of links ~~between~~ between the direct neighbors of a node. In the case of the cube no direct link between the neighbors exists. ~~...~~ Therefore:

$c_i = 0$ and $\langle c \rangle = \frac{1}{n} \sum_{i=1}^{n} c_i = 0$

with $c_i = \frac{\#\text{links between neighbors}}{\left(\frac{k_i(k_i-1)}{2}\right)}$ with $k_i$ number of possible neighbors.

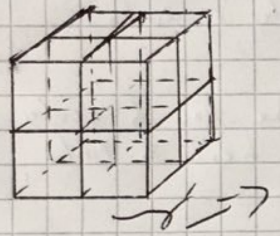
2-2: 4. If ~~x not~~ nodes x and y are connected for $d(x,y) \le \sqrt{3}$

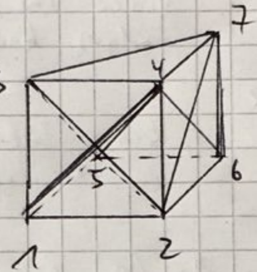
a node in the middle of the cube has

$$\underset{\text{(on the axis)}}{6} + \underset{\text{(on the surface)}}{3 \cdot 4} + \underset{\text{(in the volume)}}{8 \cdot 1} = 26 \text{ neighbors}$$

since we consider ~~las~~ ~~∞~~ only nodes in the middle of the cube are of interest.

To get the number of total ~~core~~ links between the 26 neighbours we start ~~by~~ ~~cond~~ considering only ~~on~~ a small cube:






→ We have a fully connected network with 7 nodes

→ $\frac{7 \cdot 6}{2} = \frac{42}{2} = 21$ links

or

$$\# \begin{matrix} 1-2 \\ 1-3 \\ 1-4 \\ 1-5 \\ 1-6 \\ 1-7 \end{matrix} + \# \begin{matrix} 2-3 \\ 2-4 \\ 2-5 \\ 2-6 \\ 2-7 \end{matrix} + \# \begin{matrix} 3-4 \\ 3-5 \\ 3-6 \\ 3\,7 \end{matrix} + \# \begin{matrix} 4-5 \\ 4-6 \\ 4-7 \end{matrix} + \# \begin{matrix} 5-6 \\ 5-7 \end{matrix} + \#\, 6-7$$

$$= 6 + 5 + 4 + 3 + 2 = 21$$

~~Each layer~~ we have 8 neighbor cubes, however, the following links are shared ~~neighbors~~ between neighboring cubes:

$$\# \begin{matrix} 1-3 & 4-3 & 6-7 \\ 1-5 & 4-7 & \\ 3-5 & 5-6 & \\ 3-7 & 5-7 & \end{matrix} = 9$$

$$\rightarrow \underset{\text{(node in the middle)}}{\langle C_i \rangle} = \frac{\#\text{ links between neighbors}}{\left(\frac{k_i(k_i-1)}{2}\right)} =$$

with $N \to \infty$

$$= \frac{2 \cdot \left(8 \cdot \left(21 - \frac{9}{2}\right)\right)}{26 \cdot 25} = \frac{2 \cdot 132}{650} = \underline{\underline{0{,}41}}$$