

Random graph $G(N, p)$ $p = \frac{a}{N^z}$ $a > 0, z \geq 0$

$$\langle k \rangle = p(N-1)$$

$$= \frac{a}{N^z}(N-1) = \frac{a}{N^{z-1}} - \frac{a}{N^z}$$

• $a = 1/2, z = 1$

giant component

$\rightarrow \langle k \rangle \geq 1$

$$\langle k \rangle \stackrel{N \rightarrow \infty}{=} 1/2; \text{ no giant comp.}$$

• $a = 2, z = 1$

$$\langle k \rangle \stackrel{N \rightarrow \infty}{=} 2 \quad \text{giant comp.}$$

• $a > 0, z = 2$

$$\langle k \rangle = \frac{a}{N} - \frac{a}{N^z} \stackrel{N \rightarrow \infty}{=} 0 \quad \text{no giant comp.}$$

• $a > 0, z = 0.5$

$$\langle k \rangle = a\sqrt{N} - \frac{a}{\sqrt{N}} \stackrel{N \rightarrow \infty}{=} \infty \quad \text{giant component}$$

2.

$$\langle h \rangle = \frac{a}{N^{z+1}} - \frac{a}{N^z} \stackrel{N \rightarrow \infty}{=} \begin{cases} \infty & z = 0 \\ \infty & z \geq 0, z < 1 \\ a & z = 1 \\ 0 & z > 1 \end{cases}$$

3. Critical when $\langle h \rangle = 1$

→ $z \geq 0, z < 1$ Super-critical
 $z = 1$ critical if $a = 1$
 $z > 1$ Sub-critical