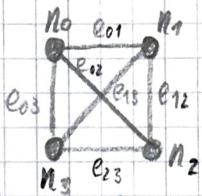
Problem 1-3 Difficulty of an Exhaustive Search

Number of walks that need to be checked for an Eulerian trail for:

1. complete graph with $N=4$ nodes:

sketch:



average degree $\langle k \rangle = N-1$ ($k = \langle k \rangle$ $\forall$ nodes $n_0, \dots, n_3$)

total number of links $L = 6 = \frac{\langle k \rangle \cdot N}{2}$

Procedure for checking for Eulerian trails as described in the problem description (exhaustive search):

(a) start at any of the nodes $n_{i'}$ ($\underbrace{4}_{=N}$ possibilities)

(b) traverse an edge $e_{i'i} = (n_{i'}, n_i)$ ($i \neq i'$)

($k = \langle k \rangle = 3$ possibilities)

(c) repeat (b) $l = L$ times so that a walk of length $l = L$ is performed ($L=6$ times)

$\Rightarrow$ # walks checked $= N \cdot \langle k \rangle^L = 4 \cdot 3^6 = \underline{\underline{2916}}$

$= N \cdot k^L$

Assignment 1 In practice, this number could be reduced, for example by remembering the previous ~~path~~ edge that was taken (i.e. the edge "where you came from") and not taking that ~~path~~ edge again. Then, only $N \cdot \langle k \rangle \cdot (\langle k \rangle - 1)^{L-1}$ walks would need to be checked. (At the start, still $\langle k \rangle$ possibilities remain.)

2. regular graph with N nodes, each node with k links:
We can follow the same procedure as in described in part 1.:

(a) start at any of the N nodes $n_{i'}$ ($\rightarrow N$ possibilities)
(b) traverse an edge $e_1 = (n_{i'}, n_i)$ starting from node n_i $(i \neq i')$ ($\rightarrow k$ possibilities)
(c) repeat (b) $l = L = \frac{k \cdot N}{2}$ times (where $\underbrace{\langle k \rangle}_{=k} = \frac{2L}{N}$)

$\Rightarrow$ # walks checked (regular graph)

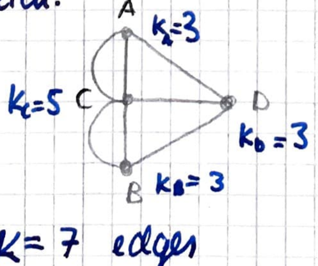
$$= N \cdot k^{L} = \underline{N \cdot k^{\frac{kN}{2}}} = \underline{\underline{N \langle k \rangle^{\frac{\langle k \rangle N}{2}}}}$$

Estimate for number of walks of length K:

walks of length K checked $= N \cdot \langle k \rangle^{K}$

3 the bridges of Königsberg:

sketch:



average degree:

$$\langle k \rangle = \frac{2 \cdot K}{N} = \frac{2 \cdot 7}{4} = \frac{7}{2}$$

$$\left(= \frac{1}{N} \sum_i k_i = \frac{3+3+3+5}{4} \right)$$

$l = K = 7$ edges
$N = 4$ nodes

$\rightarrow$ approximate number of walks:

walks of length K checked $= N \cdot \langle k \rangle^{K}$

$$= 4 \cdot \left(\frac{7}{2}\right)^{7} \approx 25736$$

$$\approx \underline{\underline{2.6 \times 10^{4}}}$$

4. historical part of Venice with 428 bridges:

$L = 428$ bridges or edges

$\rightarrow N = \frac{2L}{\langle k \rangle} = \frac{2 \cdot 428}{2} = 428$ "nodes"

$\rightarrow$ approximate number of walks checked

$= N \cdot \langle k \rangle^{L} = 428 \cdot 2^{428} \approx \underline{\underline{3 \times 10^{131}}}$

(using previously shown formula)