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1 Exercise 1

```
In [39]: import numpy as np
         import pylab as plt
         %matplotlib inline
```

1.1 1.1 data creation

Given the likelihoods, priors and the fact that the data desities sum up to one, one can derive the posteriors. The cumulative distributions (cdf) result from integrating the posteriors. To perform inverse sampling the cdfs are inverted and for each data point a cdf value is derived from a unifor distribution which is then used to find the corresponding feature. Wheather the data point is to be of class zero or one is sampled from a uniform distribution with value 0.5.

```
In [168]: def createData(N):
              #sampling a label and a cdf value
              classLabels = np.random.uniform(0, 1, N)
              cumulatives = np.random.uniform(0, 1, N)
              #initializing data points
              y = np.zeros(N)
              x = np.zeros(N)
              #deriving the feature for a randomly sampled label
              for i in range(N):
                  if classLabels[i] > 0.5: #label is one
                      y[i] = 1
                      x[i] = np.sqrt(cumulatives[i])
                  else: #y is zero and already initialized as such
                      x[i] = 1 - np.sqrt(1 - cumulatives[i])
              return x, y

          features, labels = createData(20)
          #meanOfOnes = features[labels==1].mean()
          #meanOfZeros = features[labels==0].mean()
          #print(meanOfOnes)
          #print(meanOfZeros)
          #plt.scatter(features, labels)
          #plt.show()
```

1.2 1.2 classification by thresholding

```
In [54]: #function to classify an array of features according to the given rules
         def classify(rule, threshold, features):
             classifications = np.zeros(len(features))
             for i in range(len(features)):
                 if rule == 'A':
                     if features[i] > threshold:
                         classifications[i] = 1
                 if rule == 'B':
                     if features[i] <= threshold:
                         classifications[i] = 1
             return classifications

         #functino to determine the error rate of the classify function given an array of labe
         #containing the true labels
         def errorRate(rule, threshold, features, labels):
             #classify data set according to rule
             classifications = classify(rule, threshold, features)
             #determine error
             mistakes = 0
             for i in range(len(features)):
                 if labels[i] != classifications[i]:
                     mistakes += 1
             return mistakes/len(features)
```

```
In [158]: #function to perform a classification experiment from generated data given a single
          #the size of the data sets to be created and the number of repetitions.
          #It returns the average error and standard deviation of the number of repetitions as
          #used threshold.
          def experiment(threshold, size, repetitions, rule):
              errors = np.empty(repetitions)
              for rep in range(repetitions):
                  #generate data
                  features, labels = createData(size)
                  errors[rep] = errorRate(rule, threshold, features, labels)
              meanErr = np.mean(errors)
              std = np.std(errors)
              return meanErr, std, threshold


          #function to perform a series of experiments taking an array of thresholds, one of t
          #and the number of repetitions in each experiment as input. It returns a numpy array
          #dimensions threshold, size, experiments. Additionally it returns the innput array o
          #and sizes as well as the used rule
          def test(thresholds, sizes, repetitions, rule):
              experiments = np.empty((len(thresholds), len(sizes), 3))
              for i, t in enumerate(thresholds):
                  for j, M in enumerate(sizes):
                      meanErr, std, threshold = experiment(t, M, repetitions, rule)
                      experiments[i, j, 0] = meanErr
                      experiments[i, j, 1] = std
                      experiments[i, j, 2] = t
              return experiments, thresholds, sizes, rule
```

```
In [159]: #function to plot the evolution of the mean error's standard deviation as a function
          #size for each threshold in a series of experiments performed by the test function.
          def plotStds(test):
              x = test[2]
              f, axes = plt.subplots(1, len(test[1]))
              for i in range(len(test[1])):
                  y = []
                  for j in range(len(test[2])):
                      y.append(test[0][i, j, 1])
                  axes[i].plot(x, y)
                  axes[i].set_ylim([np.min(y), np.max(y)])
                  axes[i].set_title(test[1][i])
              plt.tight_layout()
              plt.show()
              return
```

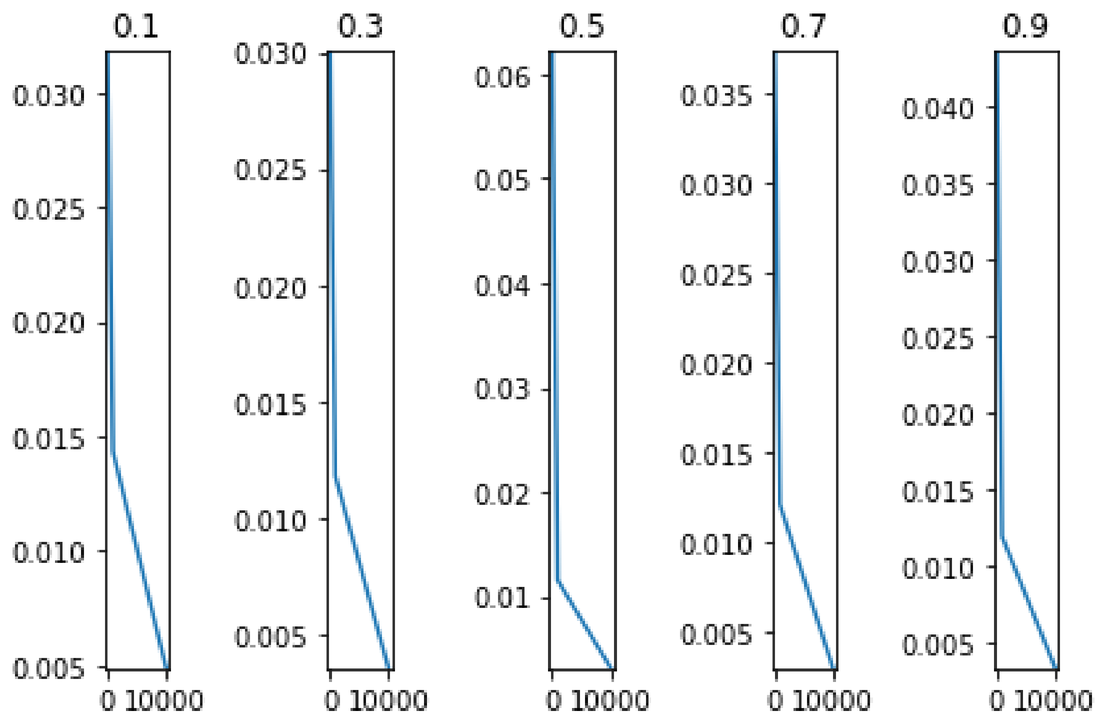
```
In [183]: #function to plot the mean error of a all experiemnts in a set generated by the test
          #Different data set sizes are not explicitily marked. Next to the experimental error
          #the theoretical expectacion is plotted (not a fit!)
          def plotErr(test):
              thresholds = test[1]
              sizes = test[2]
              rule = test[3]
              errTheo = np.empty(100)
              t = np.linspace(0, 1, 100)
              errReal = []
              tReal = []
              for i in range(len(thresholds)):
                  for j in range(len(test[2])):
                      errReal.append(test[0][i, j, 0])
                      tReal.append(test[0][i, j, 2])
              if rule == 'A':
                  errTheo = 1/4 + (t - 1/2)**2
              else:
                  errTheo = 3/4 - (t - 1/2)**2
              #print(errReal, tReal)
              plt.title('Rule ' + rule + ' errors')
              plt.xlabel('threshold')
              plt.ylabel('mean error rate')
              plt.plot(t, errTheo)
              plt.scatter(tReal, errReal)
              plt.show()
              return
```

1.2.1 Rule A

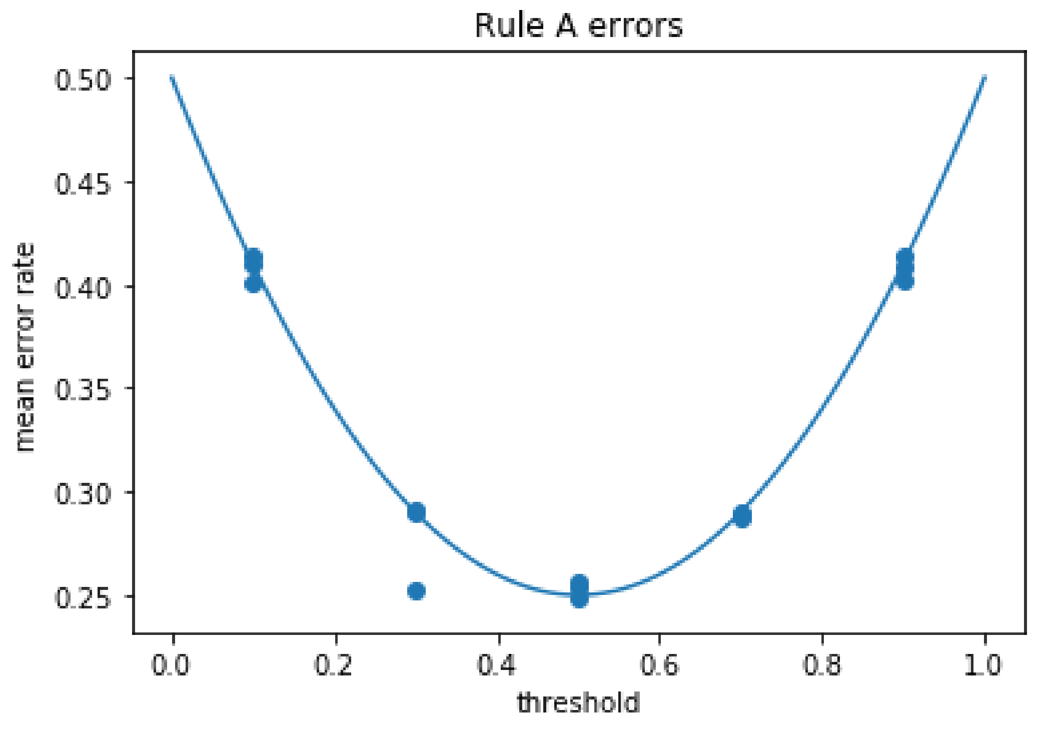
The test results are also printed. Columns are: average error, error's standard deviation and threshold from left to right. Lines are size of test sets as in the input (first dimension) and thresholds (second dimension).

```
In [184]: testA = test([0.1, 0.3, 0.5, 0.7, 0.9], [100, 1000, 10000], 10, 'A')
          print(testA[0])
          plotStds(testA)
```

```
[[[ 0.401       0.03176476  0.1       ]
  [ 0.414       0.0143248   0.1       ]
  [ 0.40977     0.00489634  0.1       ]]

 [[ 0.253       0.03001666  0.3       ]
  [ 0.2913      0.01179873  0.3       ]
  [ 0.28985     0.00355956  0.3       ]]

 [[ 0.256       0.06216108  0.5       ]
  [ 0.2536      0.01154296  0.5       ]
  [ 0.2487      0.00311288  0.5       ]]

 [[ 0.289       0.03726929  0.7       ]
  [ 0.287       0.0121326   0.7       ]
  [ 0.28975     0.00304212  0.7       ]]

 [[ 0.414       0.04363485  0.9       ]
  [ 0.4027      0.01195031  0.9       ]
  [ 0.40865     0.00335894  0.9       ]]]
```



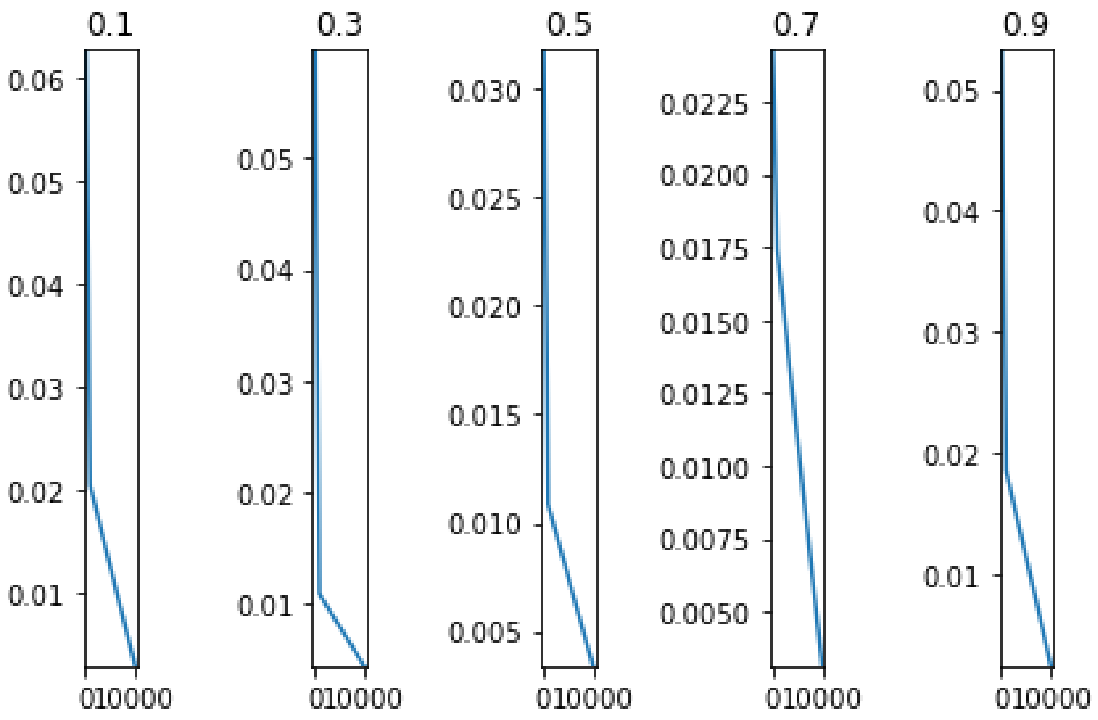
```
In [185]: plotErr(testA)
```



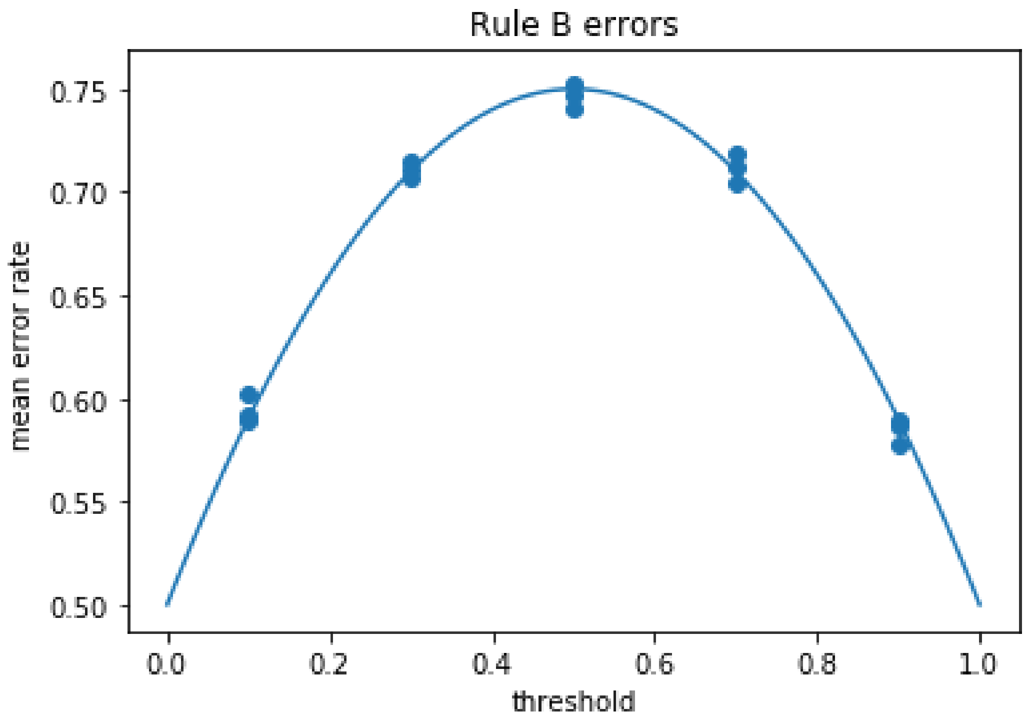
1.2.2 Rule B

```
In [186]: testB = test([0.1, 0.3, 0.5, 0.7, 0.9], [100, 1000, 10000], 10, 'B')
          print(testA[0])
          plotStds(testB)
```

```
[[[ 0.401       0.03176476  0.1       ]
  [ 0.414       0.0143248   0.1       ]
  [ 0.40977     0.00489634  0.1       ]]

 [[ 0.253       0.03001666  0.3       ]
  [ 0.2913      0.01179873  0.3       ]
  [ 0.28985     0.00355956  0.3       ]]

 [[ 0.256       0.06216108  0.5       ]
  [ 0.2536      0.01154296  0.5       ]
  [ 0.2487      0.00311288  0.5       ]]

 [[ 0.289       0.03726929  0.7       ]
  [ 0.287       0.0121326   0.7       ]
  [ 0.28975     0.00304212  0.7       ]]

 [[ 0.414       0.04363485  0.9       ]
  [ 0.4027      0.01195031  0.9       ]
  [ 0.40865     0.00335894  0.9       ]]]
```



```
In [187]: plotErr(testB)
```



1.3 1.3 NN Classifier

```
In [188]: #function to a two instance (always one per class) training set by inverse sampling.
          def nnTS():
              y = np.array([0,1])
              cumulatives = np.random.uniform(0, 1, 2)
              x = np.array([1 - np.sqrt(1 - cumulatives[0]), np.sqrt(cumulatives[1])])
              return [x, y]

          #function to classify the features given in varset according to the given trainset u
          #nearest neighbour classification
          def nnClassify(varset, trainset):
              guesses = np.empty(len(varset[0]))
              for i in range(len(varset[0])):
                  #measuring and comparing distances to the two training instances
                  d0 = np.absolute(trainset[0][0] - varset[0][i])
                  d1 = np.absolute(trainset[0][1] - varset[0][i])
                  if d0 <= d1: guesses[i] = 0
                  else: guesses[i] = 1
              return guesses

          #function to determine the error rate of a classification using nnClassify given a d
          #true labels and a classification
          def nnErr(varset, guesses):
              mistakes = 0
              for i in range(len(varset[1])):
                  if varset[1][i] != guesses[i]:
                      mistakes += 1
              return mistakes/len(varset[1])

          #function performing a given number of NN classifications and determining the averag
          #given also the size of the data sets to be generated.
          def testNNClassify(numberTS, varSetSize):
              errors = np.empty(numberTS)
              varSet = createData(varSetSize)
              for i in range(numberTS):
                  TS = nnTS()
                  guesses = nnClassify(varSet, TS)
                  errors[i] = nnErr(varSet, guesses)
              return np.mean(errors)

          print('Average NN classification error:', testNNClassify(100, 10000))
```

```
Average NN classification error: 0.365426
```