

# Bayesian Statistics HW13

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## Problem 1.

```
set.seed(123)

y <- c(28,8,-3,7,-1,1,18,12)
sigma <- c(15,10,16,11,9,11,10,18)
J <- length(y)

eps <- 0.6
L <- 10
n.iter <- 5000

a <- array(0,dim=c(1,J))
```

Step 1. Set the starting values of position & momentum.

```
mom <- array(NA,dim=c(n.iter,J))
pos <- array(NA,dim=c(n.iter,J))
mu <- 10
tau <- 2
M <- diag(1:8)
for (j in 1:J){
  mom[1,j] <- rnorm(1,0,sqrt(M[j,j]))
  pos[1,j] <- rnorm(1,mu,tau)
}
```

Step 2. Declare the functions of gradient and posterior distribution.

```
# gradient of log posterior density
gradient <- function(x){
  f <- NULL
  for(j in 1:8){
    f[j] <- (-(x[j]-mu)/tau^2) + ((y[j]-x[j])/sigma[j])
  }
  f
}

# posterior distribution
posterior <- function(x){
  f <- 1/sqrt(2*pi*(sigma[j]^2+tau^2))*exp(-1/(2*(sigma[j]^2+tau^2))
    *(x-((y[j]+mu)/(1/sigma[j]^2+1/tau^2)))^2)
  return(f)
}
```

```
}
```

### Step 3. Run a HMC Markov chain (5000 iterations).

```
for (i in 2:n.iter){

  for (k in 1:L){
    # leapfrog steps
    mom1 <- mom[i-1,] + 0.5*eps*gradient(pos[i-1,])
    pos1 <- pos[i-1,] + eps*solve(M)%*% mom1
    mom2 <- mom1 + 0.5*eps*gradient(pos1)

    pos_mom <- matrix(c(pos1, mom2), ncol=2)
  }

  # acceptance rate
  pos_prop <- pos_mom[seq_len(J)]
  mom_prop <- pos_mom[-seq_len(J)]

  pos_cur <- pos[i-1,]
  mom_cur <- mom[i-1,]

  for (j in 1:J){
    num <- posterior(pos_prop[j]) * dnorm(mom_prop[j],0,sqrt(M[j,j]))
    den <- posterior(pos_cur[j]) * dnorm(mom_cur[j],0,sqrt(M[j,j]))

    r <- num/den

    if(runif(1) < min(r,1)){
      pos[i,j] <- pos_prop[j]
      mom[i,j] <- mom_prop[j]; a[,j] <- a[,j]+1
    }else{
      pos[i,j] <- pos_cur[j]
      mom[i,j] <- mom_cur[j]
    }
  }
}
```

### Acceptance probability.

```
a / 5000
```

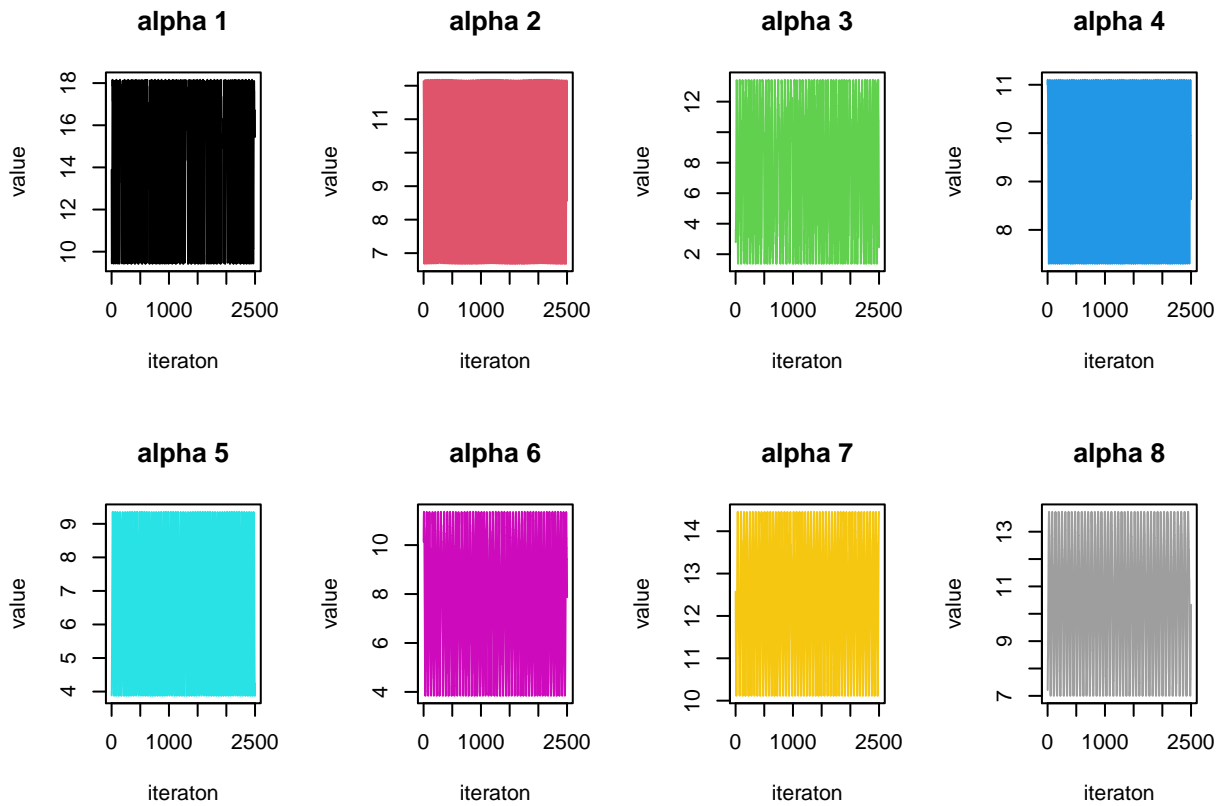
```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8]
## [1,] 0.5694 0.8614 0.6376 0.951 0.9226 0.8896 0.9122 0.9286
```

### Burn-in.

```
pos_burn <- pos[-c(1:2500),]
```

Trace plots of  $\alpha_j, j = 1, \dots, 8$ .

```
par(mfrow=c(2,4))
for(j in 1:J){
  plot(pos_burn[,j], type="l", col=j, main=paste("alpha",j), xlab="iteraton", ylab="value")
}
```



Comment :

It seems that parameters does not converge. It might result from the problem of parameters itself.