Bayesian Statistics HW9

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Problem 1. Generate random numbers form a Student t_v distribution $p(x) \propto (1 + x^2/v)^{-(v+1)/2}$ where $v = 5$. Use the Metropolis-Hastings algorithm (5000 iterations) with the proposal distribution $X_{prop}\ N(X^{(t)}, \sigma^2)$. Your code/output must include 'acceptance probability (i.e., # of accepted iterations / # of total iterations)'. Make sure that your acceptance probability is within (0.25, 0.5).

```
x <- NULL; accept_prob <- NULL;
x[1] <- 1     # starting value
sd <- c(1:10) # standard deviaions
```

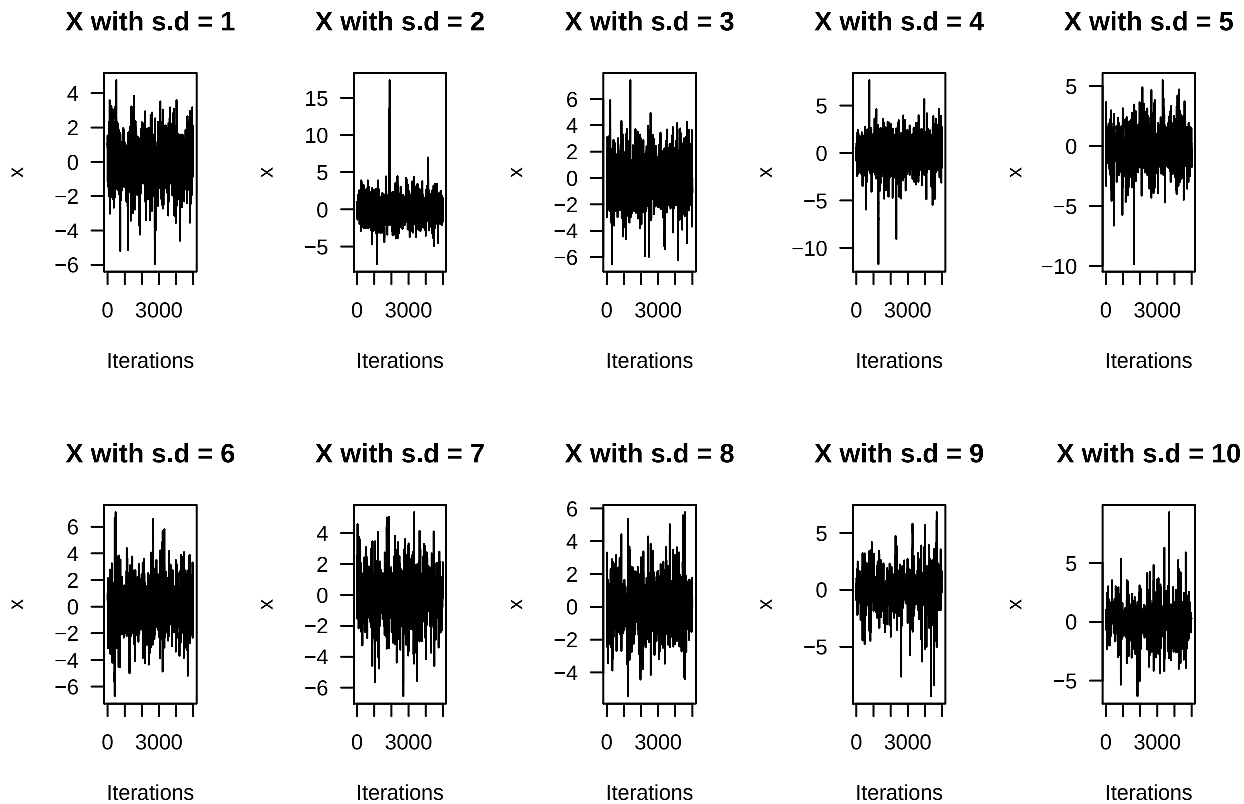
Step 1. Choose a starting value and set the standard deviations $\sigma = 1, ..., 10$.

```
par(mfrow=c(2,5))

for(j in 1:10){
accept_iter <- 0

    for(i in 2:5000){
    x_cur <- x[i-1]          # current value of x
    x_prop <- rnorm(1,x_cur,sd[j])  # Jumping distribution

    num <- (1 + x_prop^2/5)^(-6/2)  # numerator
    den <- (1 + x_cur^2/5)^(-6/2)   # denominator
    a <- min(num/den, 1)  # acceptance ratio

      if(runif(1,0,1) > a){
        x[i] <- x_cur    # if runif > a, not accept
        accept_iter <- accept_iter + 0} else{
        x[i] <- x_prop  # if runif <= a, accept
        accept_iter <- accept_iter + 1
        }
    }

accept_prob[j] <- accept_iter/5000
print(paste("acceptance probability with s.d. = ", sd[j], "is", accept_prob[j]))
plot(x, xlab="Iterations", ylab=expression(x), main=paste("X with s.d =", sd[j]), type="l", las=1)
}
```

Step 2. Compute the acceptance probability and Plot samples by drawing 5000 θ^* samples.

```
## [1] "acceptance probability with s.d. =  1 is 0.7238"
```

```
## [1] "acceptance probability with s.d. =  2 is 0.5326"
## [1] "acceptance probability with s.d. =  3 is 0.4194"
## [1] "acceptance probability with s.d. =  4 is 0.3402"
## [1] "acceptance probability with s.d. =  5 is 0.2544"
## [1] "acceptance probability with s.d. =  6 is 0.2472"
## [1] "acceptance probability with s.d. =  7 is 0.2074"
## [1] "acceptance probability with s.d. =  8 is 0.1836"
## [1] "acceptance probability with s.d. =  9 is 0.1684"
## [1] "acceptance probability with s.d. =  10 is 0.1378"
```



Interpretation:

Acceptance probability is within (0.25, 0.5) when the standard deviation is from 3 to 5.

There is a tendency that acceptance probability is decresing when the standard devation is incresing.

The plots show that x values oscillate.

2. Generate random numbers from the following target distribution $p(x) \propto x^{-2.5} e^{-2/x}$. With two different proposal distributions (χ_2^2 vs $\chi_1^2 0$), run the Metropolis-Hastings algorithm to draw samples. Check acceptance probabilities and plot samples(i.e., trace plots) (additional settings: 5000 total iterations, 2500 burn-in, thinning every 5th iterations).

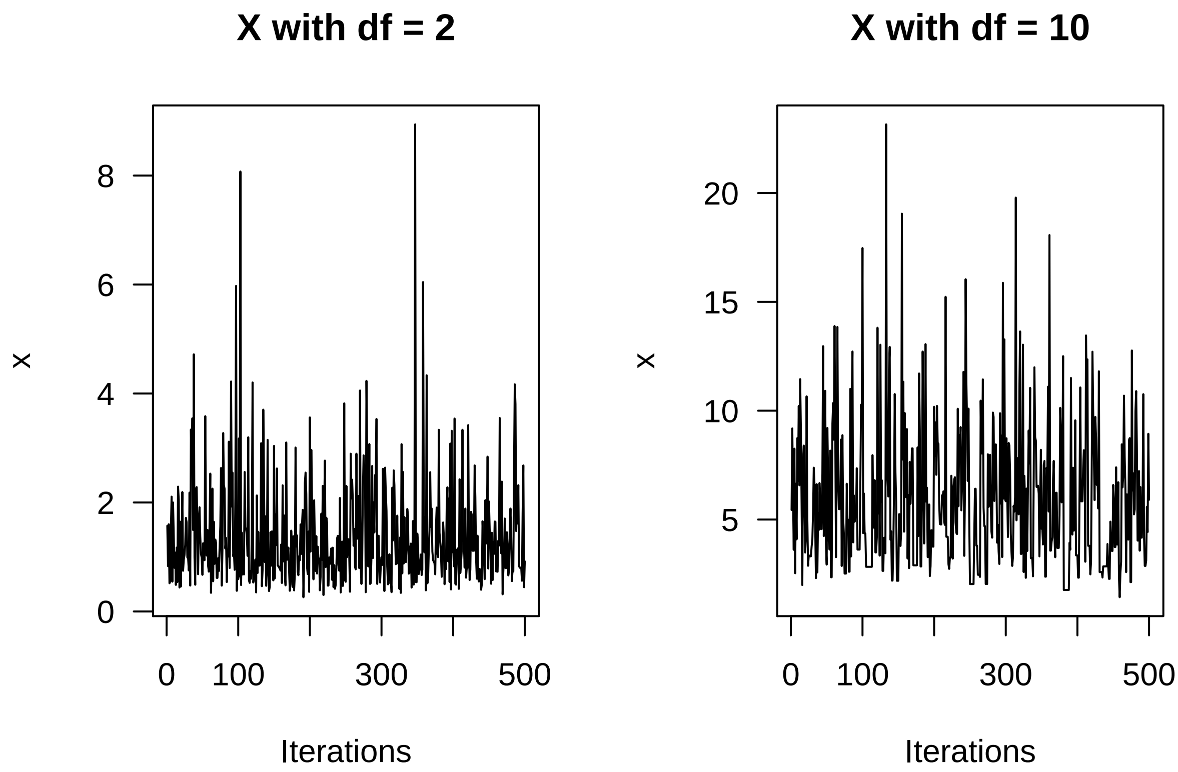
```r
x <- NULL; accept_prob <- NULL; accept_iter <- NULL;
x[1] <- 1        # starting variable
df <- c(2, 10)  # degrees of freedom
```

Step 1. Choose a starting value and set the degrees of freedom 2 and 10.

```r
par(mfrow=c(1,2))

for(j in 1:2){
  for(i in 2:5000){
    x_cur <- x[i-1]       # current value of x
    x_prop <- rchisq(1,df[j]) # Jumping distribution

    num <- x_prop^(-2.5)*exp(-2/x_prop) # numerator
    den <- x_cur^(-2.5)*exp(-2/x_cur)   # denominator
    a <- min(num/den, 1)  # acceptance ratio

    if(runif(1,0,1) > a){
      x[i] <- x_cur   # if runif > a, not accept
      accept_iter[i] <- 0} else{
      x[i] <- x_prop  # if runif <= a, accept
      accept_iter[i] <- 1
      }
  }

  # thinning
  for(i in 1:5000){
    if(i%%5!=0){
      x[i] <- NA
      accept_iter[i] <- NA} else{
      x[i] <- x[i]
      accept_iter[i] <- accept_iter[i]
      }
  }

  # burn-in
  x <- na.omit(x[-c(1:2500)])
  accept_iter <- na.omit(accept_iter[-c(1:2500)])
  accept_prob[j] <- sum(accept_iter)/length(accept_iter)

  print(paste("acceptance probability with df = ", df[j], "is", accept_prob[j]))
  plot(x, xlab="Iterations", ylab=expression(x), main=paste("X with df =", df[j]), type="l", las=1)
}
```

Step 2. Compute the acceptance probability and Plot samples by drawing 5000 θ^* samples. (additional settings: 2500 burn-in and thinning every 5th iterations.)

```
## [1] "acceptance probability with df =  2 is 0.592"

## [1] "acceptance probability with df =  10 is 0.44"
```



Interpretation:

Acceptance probability is close to 1 when the standard deviation is 2.

The plots show that x values oscillate.

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