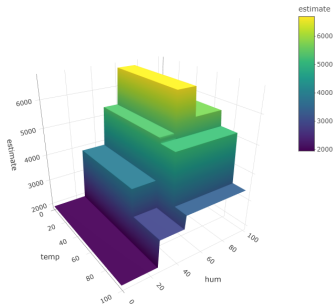


Interpretable Machine Learning

Rule-based Models



Learning goals

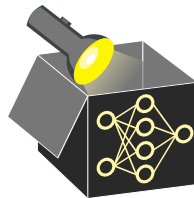
- Decision trees
- RuleFit
- Decision rules

DECISION TREES

► Breiman et al. (1984)

Idea of decision trees: Partition data into subsets based on cut-off values in features (found by minimizing a split criterion via greedy search) and predict constant mean c_m in leaf node \mathcal{R}_m :

$$\hat{f}(x) = \sum_{m=1}^M c_m \mathbb{1}_{\{x \in \mathcal{R}_m\}}$$



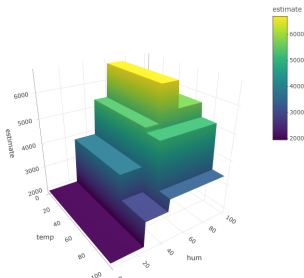
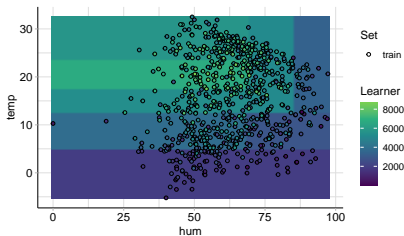
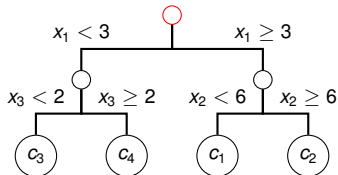
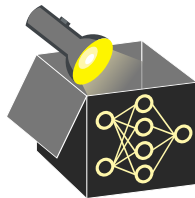
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- Applicable to regression and classification
- Able to model interactions and non-linear effects
- Able to handle mixed feature spaces and missing values



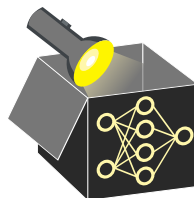
INTERPRETATION

- Directly by following the tree structure (i.e., sequence of decision rules)
- Importance of x_j : Aggregate “improvement in split criterion” over all splits where x_j was involved
 \rightsquigarrow e.g., variance for regression or Gini index for classification

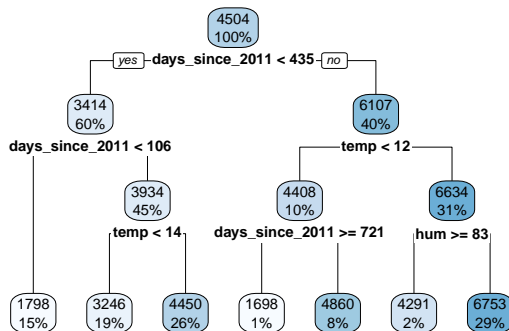


DECISION TREES - EXAMPLE

- Fit decision tree with tree depth of 3 on bike data
- E.g., mean prediction for the first 105 days since 2011 is 1798
 \rightsquigarrow Applies to $\hat{=}$ 15% of the data (leftmost branch)
- `days_since_2011`: highest feature importance (explains most of variance)



Feature	Importance
<code>days_since_2011</code>	79.53
<code>temp</code>	17.55
<code>hum</code>	2.92



UNBIASED RECURSIVE PARTITIONING

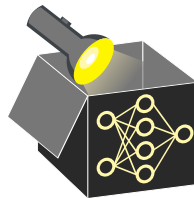
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Problems with CART (Classification and Regression Trees):

- ❶ Selection bias towards high-cardinal/continuous features
- ❷ Does not consider significant improvements when splitting (\rightsquigarrow overfitting)



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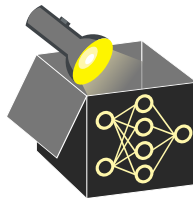
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Unbiased recursive partitioning via conditional inference trees (`ctree`) or model-based recursive partitioning (`mob`):

- ❶ Separate selection of **feature used for splitting** and **split point**
- ❷ Hypothesis test as stopping criteria

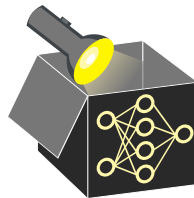


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Problems with CART (Classification and Regression Trees):

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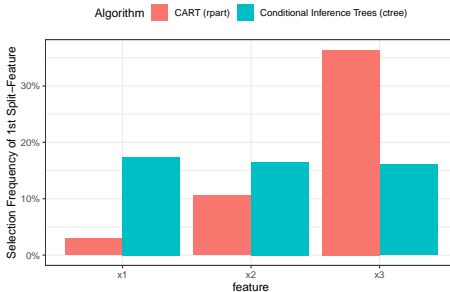
- 1 Separate selection of **feature used for splitting** and **split point**
- 2 Hypothesis test as stopping criteria

Example (selection bias):

Simulate data ($n = 200$) with $Y \sim N(0, 1)$ and 3 features of different cardinality independent from Y (repeat 500 times):

- $X_1 \sim \text{Binom}(n, \frac{1}{2})$
- $X_2 \sim M(n, (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}))$
- $X_3 \sim M(n, (\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}))$

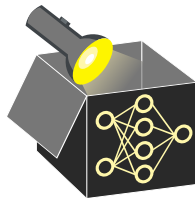
Which feature is selected in the first split?



UNBIASED RECURSIVE PARTITIONING

Differences to CART:

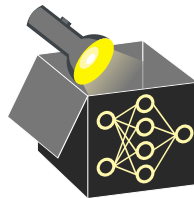
- Two-step approach (1. find most significant split feature, 2. find best split point)
- Significance of split (p-value) given in each node
- Parametric model can be fitted in leave nodes
- `ctree` and `mob` differ in hypothesis test used for selecting the split feature



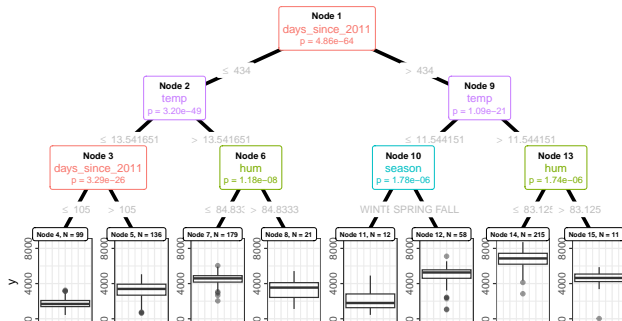
UNBIASED RECURSIVE PARTITIONING

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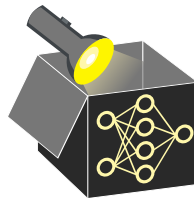
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Example (`ctree`): Bike data (constant model in final nodes)



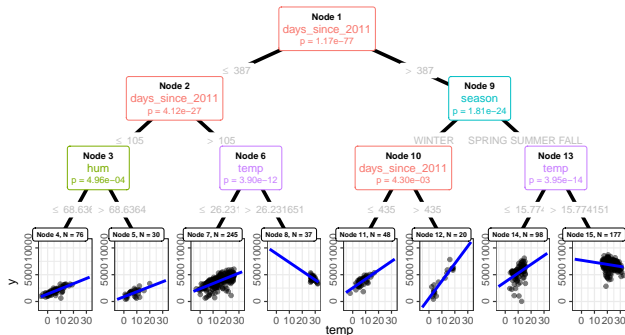
UNBIASED RECURSIVE PARTITIONING



Differences to CART:

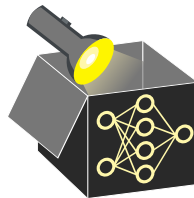
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Example (`mob`): Bike data (linear model with `temp` in final nodes)



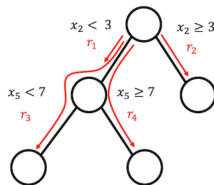
Train error (MSE): 758,844.0 (`ctree`), 742,244.4 (`mob`)

OTHER RULE-BASED MODELS

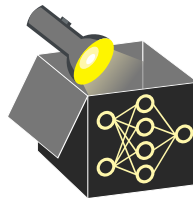


RuleFit ▶ Friedman and Popescu 2008

- Combination of LM and decision trees
- Allows for feature interactions and non-linearities

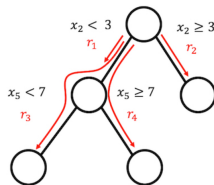


OTHER RULE-BASED MODELS



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Decision Rules ► Holte 1993

- Simple “if – then” statements - very intuitive and easy-to-interpret
- Most methods work only for classification and categorical feat.

IF size=small THEN value=low

IF size=medium THEN value=medium

IF size=big THEN value=high

► Molnar 2022