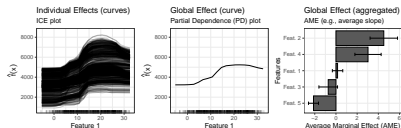


Interpretable Machine Learning

Individual Conditional Expectation (ICE) Plot



Learning goals

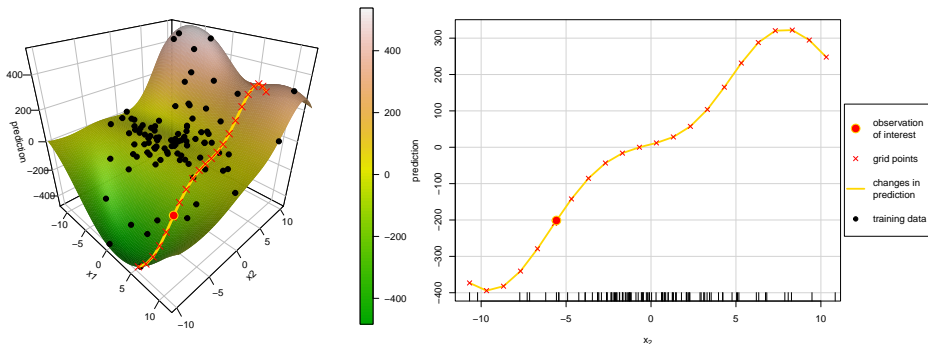
- ICE curves as local effect method
- How to sample grid points for ICE curves

MOTIVATION

Question: How does changing values of a single feature of an observation affect model prediction?

Idea: Change values of observation and feature of interest, and visualize how prediction changes

Example: Prediction surface of a model (left), select observation and visualize changes in prediction for different values of x_2 while keeping x_1 fixed \Rightarrow **local interpretation**



INDIVIDUAL CONDITIONAL EXPECTATION (ICE)

► Goldstein et. al (2013)

Partition each observation \mathbf{x} into \mathbf{x}_S (features of interest) and remaining features \mathbf{x}_{-S} .

↪ In practice, \mathbf{x}_S consists of one or two features (i.e., $|S| \leq 2$ and $-S = S^c$).

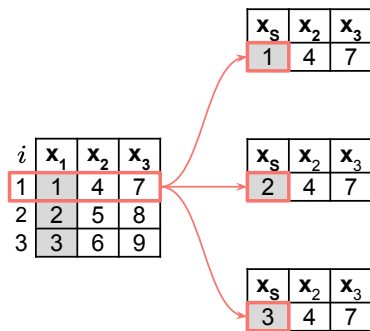
ICE curves visualize how prediction of i -th observation changes after varying its feature values indexed by S using grid points \mathbf{x}_S^* while keeping all values in $-S$ fixed:

$$\hat{f}_S^{(i)}(\mathbf{x}_S^*) \text{ vs. } \mathbf{x}_S^*$$

where $\hat{f}_S^{(i)}(\mathbf{x}_S^*) = \hat{f}(\mathbf{x}_S^*, \mathbf{x}_{-S}^{(i)})$ is prediction of i -th observation in which original feature value $\mathbf{x}_S^{(i)}$ was replaced by \mathbf{x}_S^* .

	\mathbf{x}_S		\mathbf{x}_{-S}
i	\mathbf{x}_1	\mathbf{x}_2	\mathbf{x}_3
1	1	4	7
2	2	5	8
3	3	6	9

ICE CURVES - ILLUSTRATION



1. Step - Grid points:

Sample grid values $\mathbf{x}_s^{*(1)}, \dots, \mathbf{x}_s^{*(g)}$ along feature of interest \mathbf{x}_s and replace vector $\mathbf{x}^{(i)}$ in data with grid
 \Rightarrow Creates new artificial points for the i -th observation (here: $\mathbf{x}_s^* = x_1^* \in \{1, 2, 3\}$ is a scalar)

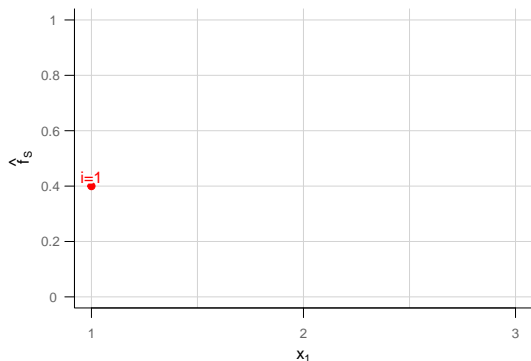
ICE CURVES - ILLUSTRATION

i	x_1	x_2	x_3
1	1	4	7
2	2	5	8
3	3	6	9

x_s	x_2	x_3	\hat{f}
1	4	7	0.4

x_s	x_2	x_3	\hat{f}
2	4	7	0.6

x_s	x_2	x_3	\hat{f}
3	4	7	0.7



2. Step - Predict and visualize:

For each artificially created data point of i -th observation, plot prediction $\hat{f}_S^{(i)}(\mathbf{x}_S^*)$ vs. grid values \mathbf{x}_S^* :

$$\hat{f}_1^{(i)}(x_1^*) = \hat{f}(x_1^*, \mathbf{x}_{2,3}^{(i)}) \text{ vs. } x_1^* \in \{1, 2, 3\}$$

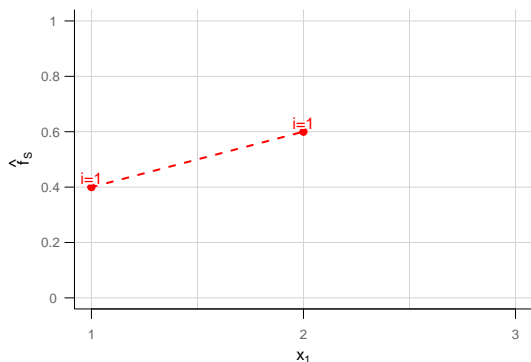
ICE CURVES - ILLUSTRATION

i	\mathbf{x}_1	\mathbf{x}_2	\mathbf{x}_3	
	1	4	7	
1	1	4	7	
2	2	5	8	
3	3	6	9	

\mathbf{x}_S	\mathbf{x}_2	\mathbf{x}_3	\hat{f}
1	4	7	0.4

\mathbf{x}_S	\mathbf{x}_2	\mathbf{x}_3	\hat{f}
2	4	7	0.6

\mathbf{x}_S	\mathbf{x}_2	\mathbf{x}_3	\hat{f}
3	4	7	0.7



2. Step - Predict and visualize:

For each artificially created data point of i -th observation, plot prediction $\hat{f}_S^{(i)}(\mathbf{x}_S^*)$ vs. grid values \mathbf{x}_S^* :

$$\hat{f}_1^{(i)}(x_1^*) = \hat{f}(x_1^*, \mathbf{x}_{2,3}^{(i)}) \text{ vs. } x_1^* \in \{1, 2, 3\}$$

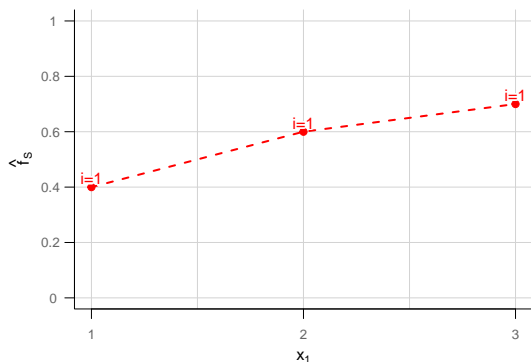
ICE CURVES - ILLUSTRATION

i	\mathbf{x}_1	\mathbf{x}_2	\mathbf{x}_3
	1	4	7
1	1	4	7
2	2	5	8
3	3	6	9

\mathbf{x}_S	\mathbf{x}_2	\mathbf{x}_3	\hat{f}
1	4	7	0.4

\mathbf{x}_S	\mathbf{x}_2	\mathbf{x}_3	\hat{f}
2	4	7	0.6

\mathbf{x}_S	\mathbf{x}_2	\mathbf{x}_3	\hat{f}
3	4	7	0.7

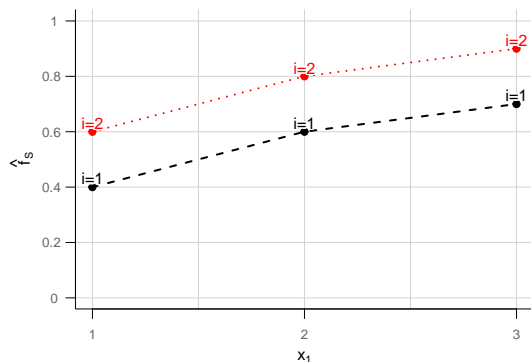
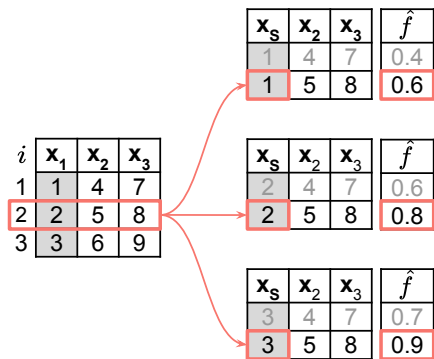


2. Step - Predict and visualize:

For each artificially created data point of i -th observation, plot prediction $\hat{f}_S^{(i)}(\mathbf{x}_S^*)$ vs. grid values \mathbf{x}_S^* :

$$\hat{f}_1^{(i)}(x_1^*) = \hat{f}(x_1^*, \mathbf{x}_{2,3}^{(i)}) \text{ vs. } x_1^* \in \{1, 2, 3\}$$

ICE CURVES - ILLUSTRATION



3. Step - Repeat for other observations:

ICE curve for $i = 2$ connects all predictions at grid values associated to i -th observation.

ICE CURVES - ILLUSTRATION

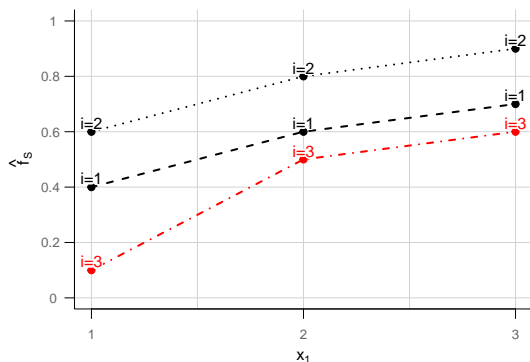
i	x_1	x_2	x_3	
	1	4	7	
	2	5	8	
	3	6	9	

x_s	x_2	x_3	\hat{f}
1	4	7	0.4
1	5	8	0.6
1	6	9	0.1

x_s	x_2	x_3	\hat{f}
2	4	7	0.6
2	5	8	0.8
2	6	9	0.5

x_s	x_2	x_3	\hat{f}
3	4	7	0.7
3	5	8	0.9
3	6	9	0.6

Red boxes highlight the rows for $i=1, 2, 3$ in the second and third tables, and the row for $i=3$ in the first table. Red arrows point from the first table's row $i=3$ to the corresponding rows in the second and third tables.



3. Step - Repeat for other observations:

ICE curve for $i = 3$ connects all predictions at grid values associated to i -th observation.

COMMENTS ON GRID VALUES

- Plotting ICE curves involves generating grid values \mathbf{x}_S^* that are visualized on the x-axis
- Common choices for grid values are
 - equidistant grid values within feature range
 - randomly sampled values or quantile values of observed feature values
- Except equidistant grid, the other two options preserve (approximately) the marginal distribution of feature of interest \Rightarrow Avoids unrealistic feature values for distributions with outliers

Grid points for X_S (red) for highlighted observation (blue)

