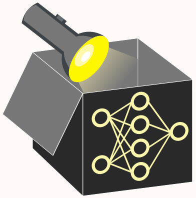
# Interpretable Machine Learning

## Interaction

**Learning goals**

- Feature interactions

# FEATURE INTERACTIONS



- While feature dependencies concern data distribution, feature interactions may occur in structure of both model or DGP (e.g., functional relationship between $X$ and $\hat{f}(X)$ or $X$ and $Y = f(X)$)
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- No. of potential interactions increases exponentially with no. of features
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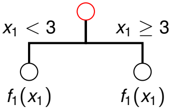
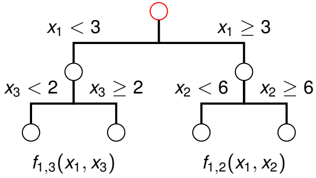
- Interactions: A feature's effect on the prediction depends on other features
  $\rightsquigarrow$ Example: $\hat{f}(\mathbf{x}) = x_1 x_2 \Rightarrow$ Effect of $x_1$ on $\hat{f}$ depends on $x_2$ and vice versa



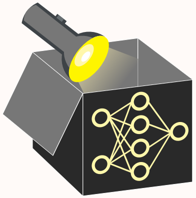

No interaction




Interactions: $x_1$ and $x_3$,
$x_1$ and $x_2$
No interactions: $x_2$ and $x_3$

# FEATURE INTERACTIONS

▸ Friedman and Popescu (2008)



**Definition:** A function $f(\mathbf{x})$ contains an interaction between $x_j$ and $x_k$ if a difference in $f(\mathbf{x})$-values due to changes in $x_j$ will also depend on $x_k$, i.e.:

$$\mathbb{E}\left[\frac{\partial^2 f(\mathbf{x})}{\partial x_j \partial x_k}\right]^2 > 0$$

$\Rightarrow$ If $x_j$ and $x_k$ do not interact, $f(\mathbf{x})$ is sum of 2 functions, each independent of $x_j$, $x_k$:

$$f(\mathbf{x}) = f_{-j}(x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_p) + f_{-k}(x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_p)$$

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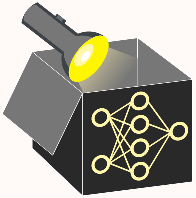

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Example ($f(\mathbf{x})$ not separable):

$$f(\mathbf{x}) = x_1 + x_2 + x_1 \cdot x_2$$

$$\mathbb{E}\left[\frac{\partial^2 (x_1 + x_2 + x_1 \cdot x_2)}{\partial x_1 \partial x_2}\right]^2 = \mathbb{E}\left[\frac{\partial (1 + x_2)}{\partial x_2}\right]^2 = 1 > 0$$

$\Rightarrow$ interaction between $x_1$ and $x_2$

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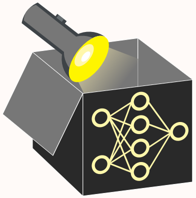

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$\Rightarrow$ interaction between $x_1$ and $x_2$

Example ($f(\mathbf{x})$ separable):

$$\begin{aligned} f(\mathbf{x}) &= x_1 + x_2 + \log(x_1 \cdot x_2) \\ &= x_1 + x_2 + \log(x_1) + \log(x_2) \\ &= f_1(x_1) + f_2(x_2), \text{ with} \end{aligned}$$

$f_1(x_1) = x_1 + \log(x_1)$ and
$f_2(x_2) = x_2 + \log(x_2)$

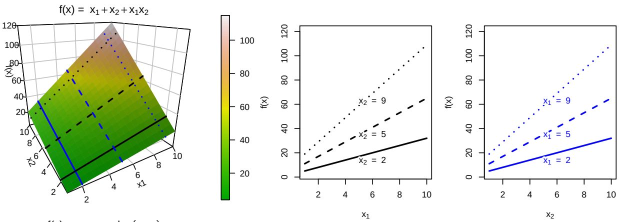
$\Rightarrow$ no interactions, also $\mathbb{E}\left[\frac{\partial^2 f(\mathbf{x})}{\partial x_1 \partial x_2}\right]^2 = 0$

# FEATURE INTERACTIONS

Interaction:

- Effect of $x_1$ on $f(\mathbf{x})$ varies for different $x_2$ values (and vice versa)

$\Rightarrow$ Different slopes




No interaction:

- Effect of $x_1$ on $f(\mathbf{x})$ stays the same for different $x_2$ values (and vice versa)

$\Rightarrow$ Parallel lines at different horizontal (blue) or vertical (black) slices