# **Interpretable Machine Learning**

# **Leave One Covariate Out (LOCO)**

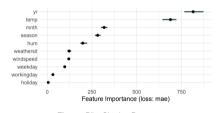


Figure: Bike Sharing Dataset

#### Learning goals

- Definition of LOCO
- Interpretation of LOCO

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• learn model on dataset  $\mathcal{D}_{\text{train }-i}$  where feature  $x_i$  was removed, i.e.  $\hat{f}_{-i} = \mathcal{I}(\mathcal{D}_{\text{train }-i})$ 

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- **2** compute the difference in local  $L_1$  loss for each element in  $\mathcal{D}_{test}$ , i.e.

$$\Delta_j^{(i)} = \left| y^{(i)} - \hat{f}_{-j}(x_{-j}^{(i)}) \right| - \left| y^{(i)} - \hat{f}(x^{(i)}) \right| \text{ with } i \in \mathcal{D}_{\mathsf{test}}$$

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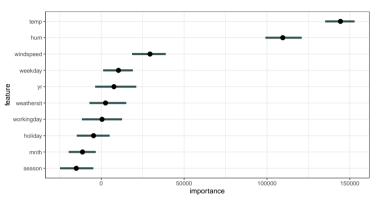
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The method can be generalized to other loss functions and aggregations. If we use mean instead of median we can rewrite LOCO as

$$\mathsf{LOCO}_j = \mathcal{R}_{\mathsf{emp}}(\hat{t}_{-j}) - \mathcal{R}_{\mathsf{emp}}(\hat{t}).$$

## **BIKE SHARING EXAMPLE**



**Figure:** A random forest with default hyperparameters was fit on 70% of the bike sharing data (training set) to optimize MSE. Then LOCO was computed for all features on the test data. The temperature is the most important feature. Without access to temp, the MSE increases by approx. 140,000.

**Interpretation:** LOCO estimates the generalization error of the learner on a reduced dataset  $\mathcal{D}_{-j}$ .

Can we get insight into whether the ...

- feature  $x_i$  is causal for the prediction  $\hat{y}$ ?
  - In general, no also because we refit the model (counterexample on the next slide)
- $\bullet$  feature  $x_i$  contains prediction-relevant information?
  - In general, no (counterexample on the next slide)
- $\bullet$  model requires access to  $x_i$  to achieve its prediction performance?
  - ullet Approximately, it provides insight into whether the *learner* requires access to  $x_j$

### **Example:** Sample 1000 observations with

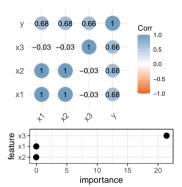
- $x_1, x_3 \sim N(0, 5)$
- $x_2 = x_1 + \epsilon_2$  with  $\epsilon_2 \sim N(0, 0.1)$
- $y = x_2 + x_3 + \epsilon$  with  $\epsilon \sim N(0,2)$
- $\Rightarrow$  Fitting a LM yields  $\hat{f}(x) = -0.02 1.02x_1 + 2.05x_2 + 0.98x_3$

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Top: Correlation matrix

Bottom: LOCO importance of LM fitted on 70% of the data computed on 30% remaining observations

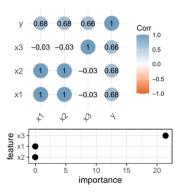


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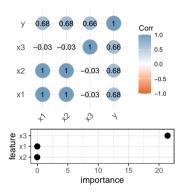
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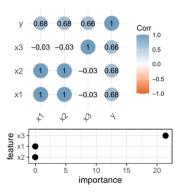
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- $\Rightarrow$  We can get insight into (3):  $\textit{x}_2$  and  $\textit{x}_1$  highly correlated with LOCO<sub>1</sub> = LOCO<sub>2</sub>  $\approx$  0
  - $\rightsquigarrow x_2$  and  $x_1$  can take each others place if one of them is left out (not the case for  $x_3$ )

## **PROS AND CONS**

#### Pros:

- Requires (only?) one refitting step per feature for evaluation
- Easy to implement
- Testing framework available in Lei et al. (2018)

#### Cons:

- Does not provide insight into a specific model, but rather a learner on a specific dataset
- Model training is a random process, so estimates can be noisy (which is problematic for inference about model and data)
- ullet Requires re-fitting the learner for each feature o computationally intensive compared to PFI