# **Interpretable Machine Learning**

# SHAP (SHapley Additive exPlanation) Values



#### Learning goals

- Get an intuition of additive feature attributions
- Understand the concept of Kernel SHAP
- Ability to interpret SHAP plots
- Global SHAP methods

**Question:** How much does a feature *j* contribute to the prediction of a single observation.

Idea: Use Shapley values from cooperative game theory

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#### **Procedure:**

- ullet Compare "reduced prediction function" of feature coalition S with  $S \cup \{j\}$
- ullet Iterate over possible coalitions to calculate the marginal contribution of feature j to sample  ${f x}$

$$\phi_j = \frac{1}{p!} \sum_{\tau \in \Pi} \hat{f}_{S_j^{\tau} \cup \{j\}}(\mathbf{x}_{S_j^{\tau} \cup \{j\}}) - \hat{f}_{S_j^{\tau}}(\mathbf{x}_{S_j^{\tau}})$$
marginal contribution of feature  $j$ 

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marginal contribution of feature  $j$ 

#### Remember:

- $\hat{f}$  is the prediction function, p denotes the number of features
- Non-existent features in a coalition are replaced by values of random feature values
- Recall  $S_j^{\tau}$  defines the coalition as the set of players before player j in order  $\tau = (\tau^{(1)}, \dots, \tau^{(p)})$   $\tau^{(1)} | \dots | \tau^{(|S|)} | \tau^{(|S|+1)} | \tau^{(|S|+2)} | \dots | \tau^{(p)}$

$$S_i^{\tau}$$
: Players before player  $j$  player  $j$  Players after player  $j$ 

#### Example:

- Train a random forest on bike sharing data only using features humidity (hum), temperature (temp) and windspeed (ws)
- Calculate Shapley value for an observation **x** with  $\hat{f}(\mathbf{x}) = 2573$
- Mean prediction is  $\mathbb{E}(\hat{t}) = 4515$

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### **Exact Shapley calculation for humidity:**

S	$\mathcal{S} \cup \{j\}$	$\hat{f}_{\mathcal{S}}$	$\hat{f}_{\mathcal{S} \cup \{j\}}$	weight
Ø	hum	4515	4635	2/6
temp	temp, hum	3087	3060	1/6
ws	ws, hum	4359	4450	1/6
temp, ws	hum, temp, ws	2623	2573	2/6

$$\phi_{\textit{hum}} = \frac{2}{6}(4635 - 4515) + \frac{1}{6}(3060 - 3087) + \frac{1}{6}(4450 - 4359) + \frac{2}{6}(2573 - 2623) = 34$$

#### FROM SHAPLEY TO SHAP

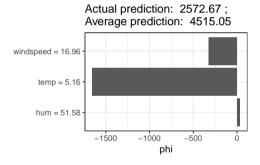
**Example continued**: Same calculation can be done for temperature and windspeed:

- $\phi_{temp} = ... = -1654$
- $\phi_{ws} = \ldots = -323$

**Remember**: Shapley values explain the difference between actual and average prediction:

$$2573 - 4515 = 34 - 1654 - 323 = -1942$$
  $\hat{f}(\mathbf{x}) - \mathbb{E}(\hat{f}) = \phi_{hum} + \phi_{temp} + \phi_{ws}$ 

$$\hat{f}(\mathbf{x}) = \underbrace{\mathbb{E}(\hat{f})}_{\phi_{tr}} + \phi_{hum} + \phi_{temp} + \phi_{ws}$$



# SHAP DEFINITION Lundberg et al. 2017

**Aim**: Find an additive combination that explains the prediction of an observation  $\mathbf{x}$  by computing the contribution of each feature to the prediction using a (more efficient) estimation procedure.

#### **Definition**

- Define simplified (binary) coalition feature space  $\mathbf{Z}' \in \{0,1\}^{K \times p}$  with K rows and p columns
- Rows are referred to as  $\mathbf{z}'^{(k)} = \{z_1'^{(k)}, \dots, z_p'^{(k)}\}$  with  $k \in \{1, \dots, K\}$  (indexes k-th coalition)
- Columns are referred to as  $\mathbf{z}_j$  with  $j \in \{1, \dots, p\}$  being the index of the original feature

#### Example:

Coalition	$\mathbf{z}^{\prime(k)}$	hum	temp	ws
Ø	$z'^{(1)}$	0	0	0
hum	$z'^{(2)}$	1	0	0
temp	$z'^{(3)}$ $z'^{(4)}$	0	1	0
ws	$z'^{(4)}$	0	0	1
hum, temp	<b>z</b> ′ <sup>(5)</sup>	1	1	0
temp, ws	<b>z</b> ′ <sup>(6)</sup>	0	1	1
hum, ws	$z'^{(7)}$ $z'^{(8)}$	1	0	1
hum, temp, ws	<b>z</b> ′ <sup>(8)</sup>	1	1	1

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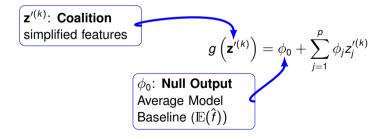
$$\mathbf{z}'^{(k)}$$
: Coalition simplified features  $g\left(\mathbf{z}'^{(k)}\right) = \phi_0 + \sum_{i=1}^p \phi_i z_j'^{(k)}$ 

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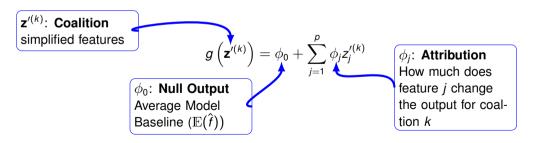


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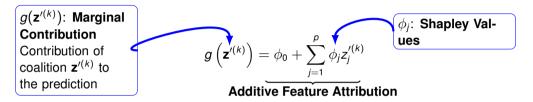
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#### **Problem**

How do we estimate the Shapley values  $\phi_j$ ?

**Definition:** A kernel-based, model-agnostic method to compute Shapley values via local surrogate models (e.g. linear model)

- Sample coalitions
- Transfer coalitions into feature space & get predictions by applying ML model
- Compute weights through kernel
- Fit a weighted linear model
- Return Shapley values

#### Step 1: Sample coalitions

• Sample K coalitions from the simplified feature space

$$\mathbf{z}^{\prime(k)} \in \{0,1\}^p, \quad k \in \{1,\ldots,K\}$$

• For our simple example, we have in total  $2^p = 2^3 = 8$  coalitions (without sampling)

Coalition	$\mathbf{z}^{\prime(k)}$	hum	temp	ws
Ø	<b>z</b> ′ <sup>(1)</sup>	0	0	0
hum	$z'^{(2)}$	1	0	0
temp	$z'^{(3)}$	0	1	0
ws	<b>z</b> ′(4)	0	0	1
hum, temp	$z'^{(5)}$	1	1	0
temp, ws	$z'^{(6)}$	0	1	1
hum, ws	$z'^{(7)}$	1	0	1
hum, temp, ws	<b>z</b> ′ <sup>(8)</sup>	1	1	1

#### Step 2: Transfer Coalitions into feature space & get predictions by applying ML model

- $\mathbf{z}^{\prime(k)}$  is 1 if features are part of the k-th coalition, 0 if they are absent
- To calculate predictions for these coalitions, we need to define a function which maps the binary feature space back to the original feature space

	_					
Coalition	$\mathbf{z}^{\prime(k)}$	hum	temp	ws	<b>x</b> <sup>coalition</sup> hum temp ws	
Ø	$z'^{(1)}$	0	0	0	$\mathbf{x}^{\{\varnothing\}}$ Ø Ø Ø	_
hum	$z'^{(2)}$	1	0	0	<b>x</b> <sup>{hum}</sup> 51.6 ∅ ∅	
temp	$z'^{(3)}$	0	1	0	$\mathbf{x}^{\{temp\}}$ $\varnothing$ 5.1 $\varnothing$	
ws	$z'^{(4)}$	0	0	1	$\mathbf{x}^{\{ws\}}$ $\varnothing$ $\varnothing$ 17.0	
hum, temp	$z'^{(5)}$	1	1	0	<b>x</b> <sup>{hum,temp}</sup> 51.6 5.1 ∅	
temp, ws	$z'^{(6)}$	0	1	1	$\mathbf{x}^{\{temp,ws\}}$ $\varnothing$ 5.1 17.0	
hum, ws	$z'^{(7)}$	1	0	1	$\mathbf{x}^{\{hum,ws\}}$ 51.6 $\varnothing$ 17.0	
hum, temp, ws	<b>z</b> ′ <sup>(8)</sup>	1	1	1	<b>x</b> <sup>{hum,temp,ws}</sup>   51.6  5.1  17.0	

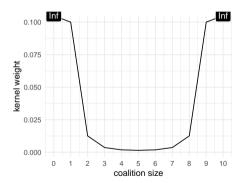
#### Step 2: Transfer Coalitions into feature space & get predictions by applying ML model

- Define  $h_x(\mathbf{z}'^{(k)}) = \mathbf{z}^{(k)}$  where  $h_x: \{0,1\}^p \to \mathbb{R}^p$  maps 1's to feature values of observation  $\mathbf{x}$  for features part of the k-th coalition and 0's to feature values of a randomly sampled observation for features absent in the k-th coalition (feature values are permuted multiple times)
- Predict with ML model on this dataset  $\hat{f}$ :  $\hat{f}\left(h_{x}\left(\mathbf{z}^{\prime\left(k\right)}\right)\right)$

	_				$h_{x}(\mathbf{z}^{\prime(k)})$			<b>&gt;</b>	
Coalition	$\mathbf{z}^{\prime(k)}$	hum	temp	ws	$\mathbf{z}^{(k)}$	) hum	temp	ws	$\hat{f}\left(h_{x}\left(\mathbf{z}^{\prime\left(k\right)}\right)\right)$
Ø	<b>z</b> ′ <sup>(1)</sup>	0	0	0	<b>z</b> (1	64.3	28.0	14.5	6211
hum	$z'^{(2)}$	1	0	0	<b>z</b> (2	51.6	28.0	14.5	5586
temp	$z'^{(3)}$	0	1	0	$\mathbf{z}^{(3}$	64.3	5.1	14.5	3295
ws	$z'^{(4)}$	0	0	1	$z^{(4}$	64.3	28.0	17.0	5762
hum, temp	<b>z</b> ′ <sup>(5)</sup>	1	1	0	<b>z</b> (5	51.6	5.1	14.5	2616
temp, ws	<b>z</b> ′ <sup>(6)</sup>	0	1	1	<b>z</b> (6	64.3	5.1	17.0	2900
hum, ws	<b>z</b> ′ <sup>(7)</sup>	1	0	1	<b>z</b> <sup>(7</sup>	51.6	28.0	17.0	5411
hum, temp, ws	<b>z</b> ′ <sup>(8)</sup>	1	1	1	<b>z</b> (8	51.6	5.1	17.0	2573

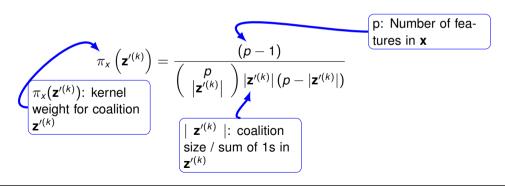
#### Step 3: Compute weights through Kernel

**Intuition**: We learn most about individual features if we can study their effects in isolation or at maximal interaction: Small coalitions (few 1's) and large coalitions (i.e. many 1's) get the largest weights



Step 3: Compute weights through Kernel See Shapley\_kernel\_proof.pdf

**Intuition**: We learn most about individual features if we can study their effects in isolation or at maximal interaction: Small coalitions (few 1's) and large coalitions (i.e. many 1's) get the largest weights



#### Step 3: Compute weights through Kernel

**Purpose**: to include this knowledge in the local surrogate model (linear regression), we calculate weights for each coalition which are the observations of the linear regression

$$\pi_{x}(\mathbf{z}') = \frac{(p-1)}{\binom{p}{|\mathbf{z}'|}|\mathbf{z}'|(p-|\mathbf{z}'|)} \rightsquigarrow \pi_{x}(\mathbf{z}' = (1,0,0)) = \frac{(3-1)}{\binom{3}{1}|1(3-1)} = \frac{1}{3}$$

Coalition	$\mathbf{z}^{\prime(k)}$	hum	temp	ws	weight
Ø	$z'^{(1)}$	0	0	0	$\infty$
hum	$z'^{(2)}$	1	0	0	0.33
temp	$z'^{(3)}$	0	1	0	0.33
ws	$z'^{(4)}$	0	0	1	0.33
hum, temp	$z'^{(5)}$	1	1	0	0.33
temp, ws	$z'^{(6)}$	0	1	1	0.33
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hum, temp, ws	<b>z</b> ′ <sup>(8)</sup>	1	1	1	$\infty$

weights for empty and full set are infinity and not used as observations for the linear regression instead constraints are used such that properties (local accuracy and missingness) are satisfied

### Step 4: Fit a weighted linear model

**Aim**: Estimate a weighted linear model with Shapley values being the coefficients  $\phi_i$ 

$$g\left(\mathbf{z}^{\prime(k)}\right) = \phi_0 + \sum_{i=1}^{p} \phi_i z_j^{\prime(k)}$$

and minimize by WLS using the weights  $\pi_{x}$  of step 3

$$L\left(\hat{f},g,\pi_{x}\right) = \sum_{k=1}^{K} \left[\hat{f}\left(h_{x}\left(\mathbf{z}^{\prime(k)}\right)\right) - g\left(\mathbf{z}^{\prime(k)}\right)\right]^{2} \pi_{x}\left(\mathbf{z}^{\prime(k)}\right)$$

with  $\phi_0 = \mathbb{E}(\hat{f})$  and  $\phi_p = \hat{f}(x) - \sum_{j=0}^{p-1} \phi_j$  we receive a p-1 dimensional linear regression problem

#### Step 4: Fit a weighted linear model

**Aim**: Estimate a weighted linear model with Shapley values being the coefficients  $\phi_i$ 

$$g\left(\mathbf{z}^{\prime(k)}\right) = \phi_0 + \sum_{i=1}^{p} \phi_i z_i^{\prime(k)} \leadsto g\left(\mathbf{z}^{\prime(k)}\right) = 4515 + 34 \cdot z_1^{\prime(k)} - 1654 \cdot z_2^{\prime(k)} - 323 \cdot z_3^{\prime(k)}$$

$\mathbf{z}'^{(k)}$	hum	temp	ws	weight	Î
$z'^{(2)}$	1	0	0	0.33	4635
$\mathbf{z}'^{(3)}$	0	1	0	0.33	3087
$\mathbf{z}'^{(4)}$	0	0	1	0.33	4359
$z'^{(5)}$	1	1	0	0.33	3060
$z'^{(6)}$	0	1	1	0.33	2623
$z'^{(7)}$	1	0	1	0.33	4450
	_	input	_		output

#### Step 5: Return SHAP values

Intuition: Estimated Kernel SHAP values are equivalent to Shapley values

$$g(\mathbf{z}^{\prime(8)}) = \hat{f}(h_{x}(\mathbf{z}^{\prime(8)})) = 4515 + 34 \cdot 1 - 1654 \cdot 1 - 323 \cdot 1 = \underbrace{\mathbb{E}(\hat{f})}_{\text{hum}} + \phi_{\text{hum}} + \phi_{\text{temp}} + \phi_{\text{ws}} = \hat{f}(\mathbf{x}) = 2573$$



#### **Local Accuracy**

$$f(\mathbf{x}) = g(\mathbf{x}') = \phi_0 + \sum_{j=1}^{p} \phi_j x_j'$$

**Intuition:** If the coalition includes all features  $(\mathbf{x}' \in \{1\}^p)$ , the attributions  $\phi_j$  and the null output  $\phi_0$  sum up to the original model output  $f(\mathbf{x})$ 

Local accuracy corresponds to the **axiom of efficiency** in Shapley game theory

#### **Local Accuracy**

$$f(\mathbf{x}) = g(\mathbf{x}') = \phi_0 + \sum_{j=1}^p \phi_j x_j'$$

Missingness

$$x'_j = 0 \Longrightarrow \phi_j = 0$$

Intution: A missing feature gets an attribution of zero

## **Local Accuracy**

$$f(\mathbf{x}) = g(\mathbf{x}') = \phi_0 + \sum_{j=1}^p \phi_j x_j'$$

#### Missingness

$$x_j' = 0 \Longrightarrow \phi_j = 0$$

#### Consistency

$$\hat{f}_{x}\left(\mathbf{z}^{\prime(k)}
ight)=\hat{f}\left(h_{x}\left(\mathbf{z}^{\prime(k)}
ight)
ight)$$
 and  $\mathbf{z}_{-i}^{\prime(k)}$  denote setting  $z_{i}^{\prime(k)}=0$  . For any two models  $\hat{f}$  and  $\hat{f}'$ , if

$$\hat{f}_{x}'\left(\mathbf{z}^{\prime(k)}\right) - \hat{f}_{x}'\left(\mathbf{z}_{-j}^{\prime(k)}\right) \geq \hat{f}_{x}\left(\mathbf{z}^{\prime(k)}\right) - \hat{f}_{x}\left(\mathbf{z}_{-j}^{\prime(k)}\right)$$

for all inputs  $\mathbf{z}^{\prime(k)} \in \{0,1\}^p$ , then

$$\phi_{j}\left(\hat{f}',\mathbf{x}
ight)\geq\phi_{j}(\hat{f},\mathbf{x})$$

#### **Local Accuracy**

$$f(\mathbf{x}) = g(\mathbf{x}') = \phi_0 + \sum_{j=1}^{p} \phi_j x_j'$$

#### Missingness

$$x'_j = 0 \Longrightarrow \phi_j = 0$$

#### Consistency

$$\hat{f}_{x}'\left(\mathbf{z}^{\prime(k)}\right) - \hat{f}_{x}'\left(\mathbf{z}_{-j}^{\prime(k)}\right) \geq \hat{f}_{x}\left(\mathbf{z}^{\prime(k)}\right) - \hat{f}_{x}\left(\mathbf{z}_{-j}^{\prime(k)}\right) \Longrightarrow \phi_{j}\left(\hat{f}', \mathbf{x}\right) \geq \phi_{j}(\hat{f}, \mathbf{x})$$

**Intution:** If a model changes so that the marginal contribution of a feature value increases or stays the same, the Shapley value also increases or stays the same

From consistency the Shapley axioms of additivity, dummy and symmetry follow

# GLOBAL SHAP Lundberg et al. 2018

#### Idea:

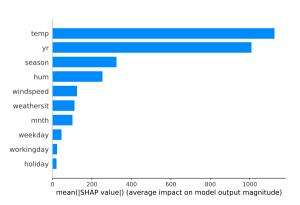
- Run SHAP for every observation and thereby get a matrix of Shapley values
- The matrix has one row per data observation and one column per feature
- We can interpret the model globally by analyzing the Shapley values in this matrix

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \dots & \phi_{1p} \\ \phi_{21} & \phi_{22} & \phi_{23} & \dots & \phi_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \phi_{n3} & \dots & \phi_{np} \end{bmatrix}$$

#### FEATURE IMPORTANCE

**Idea:** Average the absolute Shapley values of each feature over all observations. This corresponds to calculating averages column by column in  $\Phi$ 

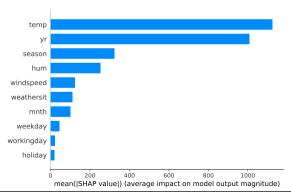
$$I_j = \frac{1}{n} \sum_{i=1}^n \left| \phi_j^{(i)} \right|$$



## **FEATURE IMPORTANCE**

#### Interpretation:

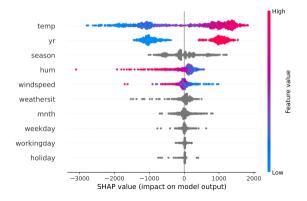
- The features temperature and year have by far the highest influence on the model's prediction
- Compared to Shapley values, no effect direction is provided, but instead a feature ranking similar to PFI
- However, Shapley FI is based on the model's predictions only while PFI is based on the model's performance (loss)



#### **SUMMARY PLOT**

Combines feature importance with feature effects

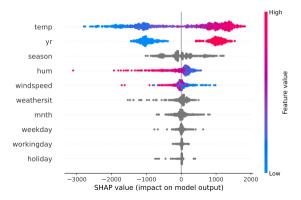
- Each point is a Shapley value for a feature and an observation
- The color represents the value of the feature from low to high
- Overlapping points are jittered in y-axis direction



### **SUMMARY PLOT**

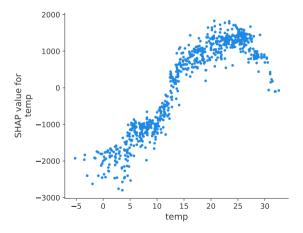
#### Interpretation:

- Low temperatures have a negative impact while high temperatures lead to more bike rentals
- Year: two point clouds for 2011 and 2012 (other categorical features are gray)
- A high humidity has a huge, negative impact on the bike rental, while low humidity has a rather minor positive impact on bike rentals



## **DEPENDENCE PLOT**

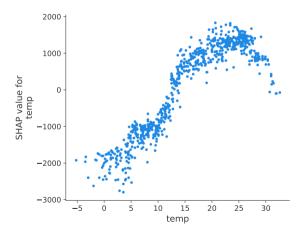
- Visualize the marginal contribution of a feature similar to the PDP
- Plot a point with the feature value on the x-axis and the corresponding Shapley value on the y-axis



# **DEPENDENCE PLOT**

#### Interpretation:

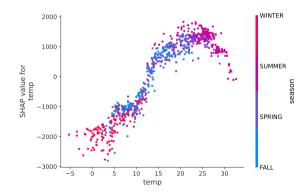
- Increasing temperatures induce increasing bike rentals until 25°C
- If it gets too hot, the bike rentals decrease



# **DEPENDENCE PLOT**

#### Interpretation:

- We can colour the observations by a second feature to detect interactions
- Visibly the temperatures interaction with the season is very strong



# **DISCUSSION**

#### **Advantages**

- All the advantages of Shapley values
- Unify the field of interpretable machine learning in the class of additive feature attribution methods
- Has a fast implementation for tree-based models
- Various global interpretation methods

#### **Disadvantages**

- Disadvantages of Shapley values also apply to SHAP
- KernelSHAP is slow (TreeSHAP can be used as a faster alternative for tree-based models
   Lundberg et al 2018 and for an intuitive explanation → see Sukumar: TreeSHAP
- KernelSHAP ignores feature dependence