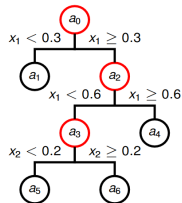


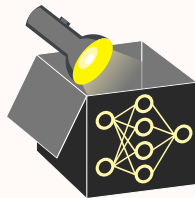
# Interpretable Machine Learning

## Rule-based Models



### Learning goals

- Decision trees
- RuleFit
- Decision rules

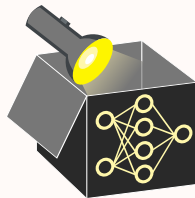


# DECISION TREES

► Breiman et al. (1984)

**Idea of decision trees:** Partition data into subsets based on cut-off values in features (found by minimizing a split criterion via greedy search) and predict constant mean  $c_m$  in leaf node  $\mathcal{R}_m$ :

$$\hat{f}(x) = \sum_{m=1}^M c_m \mathbb{1}_{\{x \in \mathcal{R}_m\}}$$



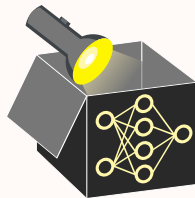
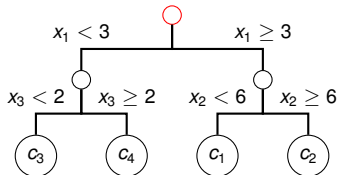
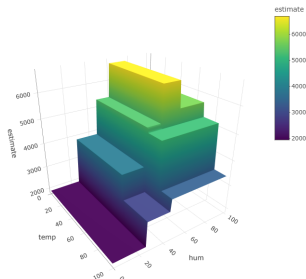
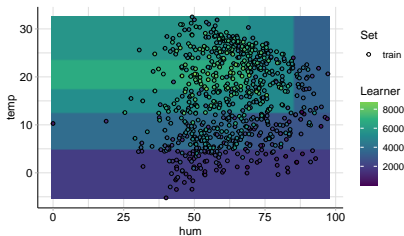
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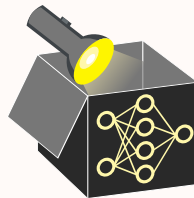
$$\hat{f}(x) = \sum_{m=1}^M c_m \mathbb{1}_{\{x \in \mathcal{R}_m\}}$$

- Applicable to regression and classification
- Able to model interactions and non-linear effects
- Able to handle mixed feature spaces and missing values



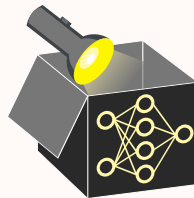
# INTERPRETATION

- Directly by following the tree structure (i.e., sequence of decision rules)
- Importance of  $x_j$ : Aggregate “improvement in split criterion” over all splits where  $x_j$  was involved  
     $\rightsquigarrow$  e.g., variance for regression or Gini index for classification

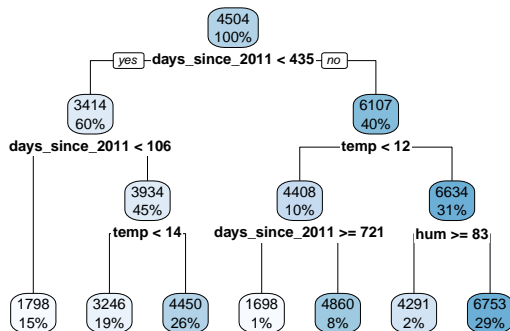


# DECISION TREES - EXAMPLE

- Fit decision tree with tree depth of 3 on bike data
- E.g., mean prediction for the first 105 days since 2011 is 1798 (applies to  $\hat{=}$ 15% of the data)
- `days_since_2011`: highest feature importance (explains most of variance)



Feature	Importance
<code>days_since_2011</code>	79.53
<code>temp</code>	17.55
<code>hum</code>	2.92



# UNBIASED RECURSIVE PARTITIONING

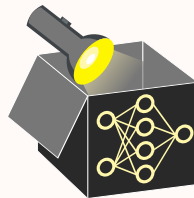
► Hothorn et al. (2006)

► Zeileis et al. (2008)

► Strobl et al. (2007)

**Problems** with CART (Classification and Regression Trees):

- ❶ Selection bias towards high-cardinal/continuous features
- ❷ Does not consider significant improvements when splitting ( $\rightsquigarrow$  overfitting)



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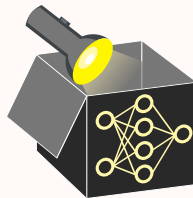
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- ❷ Hypothesis test as stopping criteria



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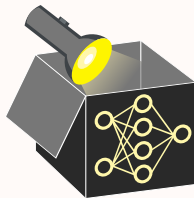
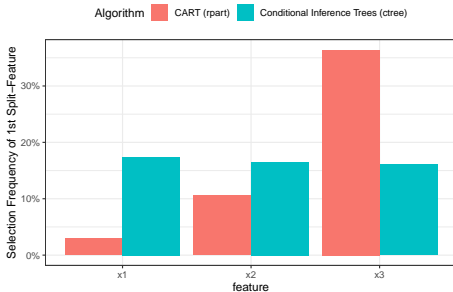
- 1 Separate selection of **feature used for splitting** and **split point**
- 2 Hypothesis test as stopping criteria

**Example (selection bias):**

Simulate data ( $n = 200$ ) with  $Y \sim N(0, 1)$  and 3 features of different cardinality independent from  $Y$  (repeat 500 times):

- $X_1 \sim \text{Binom}(n, \frac{1}{2})$
- $X_2 \sim M(n, (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}))$
- $X_3 \sim M(n, (\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}))$

Which feature is selected in the first split?

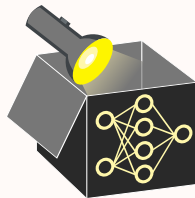




# UNBIASED RECURSIVE PARTITIONING

Differences to CART:

- Two-step approach (1. find most significant split feature, 2. find best split point)
- Significance of split (p-value) given in each node
- Parametric model can be fitted in leave nodes
- `ctree` and `mob` differ in hypothesis test used for selecting the split feature

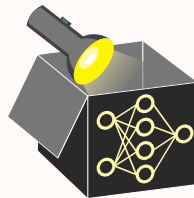
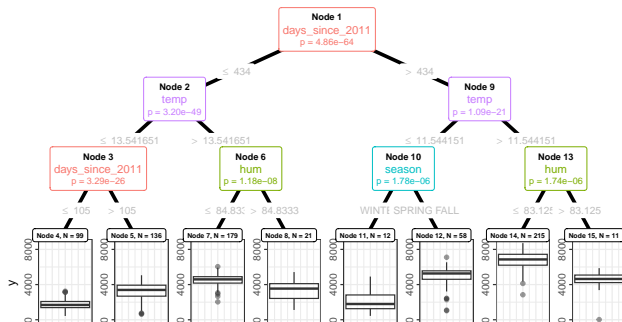


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**Example** (`ctree`): Bike data (constant model in final nodes)

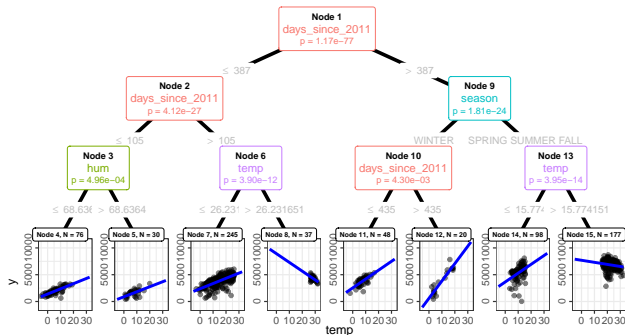


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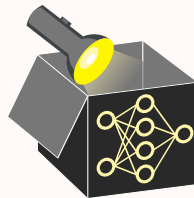
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**Example** (`mob`): Bike data (linear model with `temp` in final nodes)



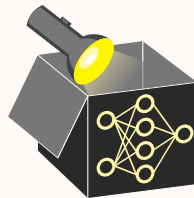
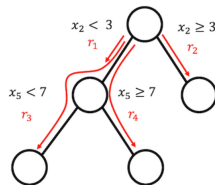
Train error (MSE): 758,844.0 (`ctree`), 742,244.4 (`mob`)



# OTHER RULE-BASED MODELS

## RuleFit ▶ Friedman and Popescu 2008

- Combination of LM and decision trees
- Allows for feature interactions and non-linearities



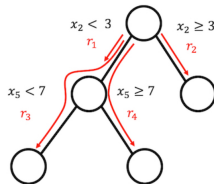
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## Decision Rules ► Holte 1993

- Simple “if – then” statements - very intuitive and easy-to-interpret
- Most methods work only for classification and categorical feat.



IF size=small THEN value=low

IF size=medium THEN value=medium

IF size=big THEN value=high

► Molnar 2022

