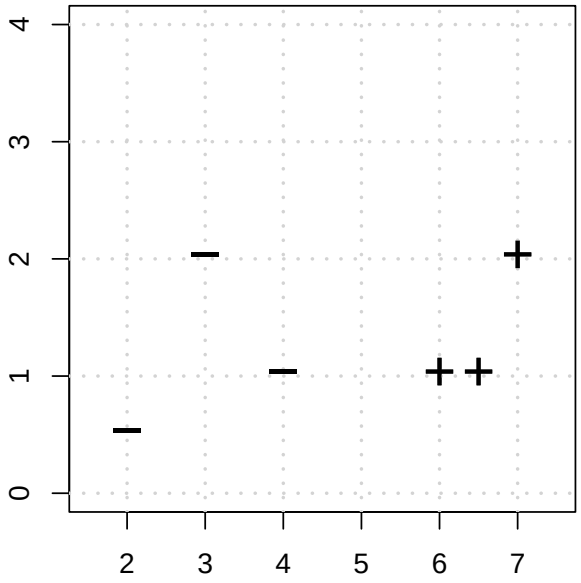
Exercise 1: Hard Margin Classifier

The primal optimization problem for the two-class hard margin SVM classification is given by

$$\begin{aligned} \min_{\boldsymbol{\theta}, \theta_0} \quad & \frac{1}{2}||\boldsymbol{\theta}||^2 \\ \text{s.t. :} \quad & y^{(i)} \left(\left\langle \boldsymbol{\theta}, \mathbf{x}^{(i)} \right\rangle + \theta_0 \right) \geq 1 \end{aligned}$$



(a) Calculate the following quantities:

- γ
- $\|\boldsymbol{\theta}\|$
- $\boldsymbol{\theta}$
- θ_0
- Determine which points are support vectors.

(b) Calculate the quantities in (a) after applying the following changes:

- All points are rotated by 45 degrees counterclockwise.

- All points are shifted by 2 to the right (in the x-axis).

- One SV moves closer to the separating hyperplane $(6, 1) \rightarrow (5.5, 1)$.

- One SV $(6, 1)$ is removed from the dataset.