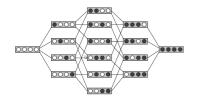
Supervised Learning

Wrapper methods



Learning goals

- Understand how wrapper methods work
- Understand how they could help in feature selection
- Know their advantages and disadvantages



INTRODUCTION

- Wrapper methods emerged from the idea that different sets of features can be optimal for different classification learners.
- Given a set of features, we can use the classifier itself to assess their quality.
- We could just evaluate on the test set or use resampling techniques to achieve this.
- A wrapper is nothing else than a discrete search strategy for S, where the cross-validated test error of a learner as a function of S is now the objective criterion.



INTRODUCTION

There are a lot of varieties of wrappers. To begin with we have to determine the following components:

- A set of starting values
- Operators to create new points out of the given ones
- A termination criterion



INTRODUCTION

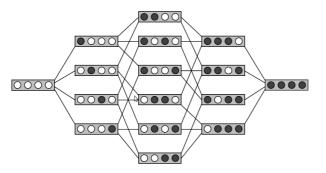


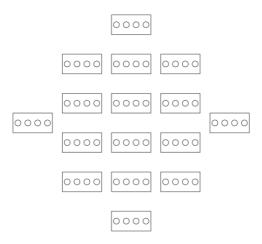


Figure: Space of all feature sets for 4 features. The indicated relationships between the sets insinuate a greedy search strategy which either adds or removes a feature.

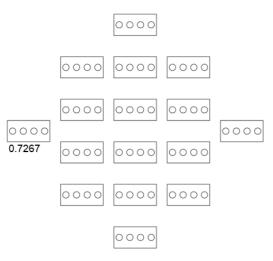
- Let $S \subset \{1, ..., p\}$, where $\{1, ..., p\}$ is an index set of all features.
- Start with the empty feature set $S = \emptyset$.
- For a given set S, generate all $S_j = S \cup \{j\}$ with $j \notin S$.
- Evaluate the classifier on all S_j and use the best S_j .
- Iterate over this procedure.
- Terminate if:
 - the performance measure no longer shows relevant improvement,
 - a maximum number of features is used, or
 - a given performance value is reached.



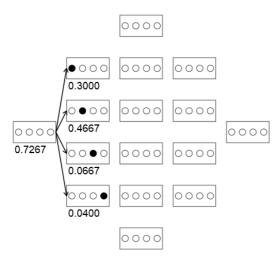
Example for greedy forward search on iris data:



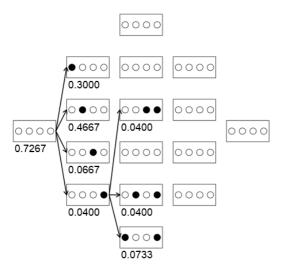




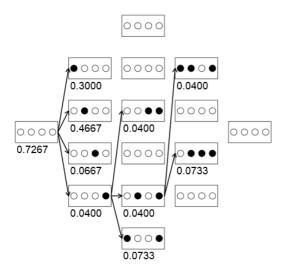




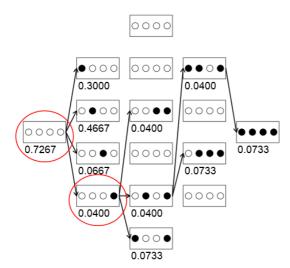














GREEDY BACKWARD SEARCH

- Start with the full index set of features $S = \{1, ..., p\}$.
- For a given set S generate all $S_j = S \setminus \{j\}$ with $j \in S$.
- Evaluate the classifier on all S_i and use the best S_i .
- Iterate over this procedure.
- Terminate if:
 - the performance drops drastically, or
 - a given performance value is undershot.



EXTENSIONS

- Eliminate or add several features at once to increase speed.
- Allow alternating forward and backward search.
- Randomly create candidate feature sets in each iteration.
- Continue search based on the set of features where an improvement is present.
- Use improvements of earlier iterations.



EXTENSIONS

Algorithm A simple 1+1 genetic algorithm

- 1: Start with a random set of features *S* (bit vector *b*).
- 2: repeat
- 3: Flip a couple of bits in b with probability p.
- 4: Generate set S' and bit vector b'.
- 5: Measure the classifier's performance on S'.
- 6: If S' performs better than S, update $S \leftarrow S'$, otherwise $S \leftarrow S$.
- 7: **until** One of the following conditions is met:
 - A given performance value is reached.
 - Budget is exhausted.



WRAPPERS

Advantages:

- Can be combined with every learner.
- Can be combined with every performance measure.
- Optimizes the desired criterion directly.

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Disadvantages:

- Evaluating the target function is expensive.
- Does not scale well if number of features becomes large.
- Does not use much structure or available information from our model.