

**Exercise 1: Risk Minimizers for the L2-Loss**

Assume that the feature space is  $\mathcal{X} = [-10, 10]$  and the label space is  $\mathcal{Y} = \mathbb{R}$ . We consider the L2-loss:

$$L(y, f(\mathbf{x})) = (y - f(\mathbf{x}))^2.$$

Assume that  $p_x \sim \text{Unif}(-10, 10)$  and that  $p_{y|x=x} \sim \mathcal{N}(a + bx, 1)$  for some  $a, b \in \mathbb{R}$ .

(a) What is  $f^*$ ?

(b) What is its risk?

(c) What is  $f_c^*$  (the optimal constant model in terms of the theoretical risk)?

(d) What is its risk? (Hint:  $\mathbb{E}_x(x^2) = 100/3$ .)