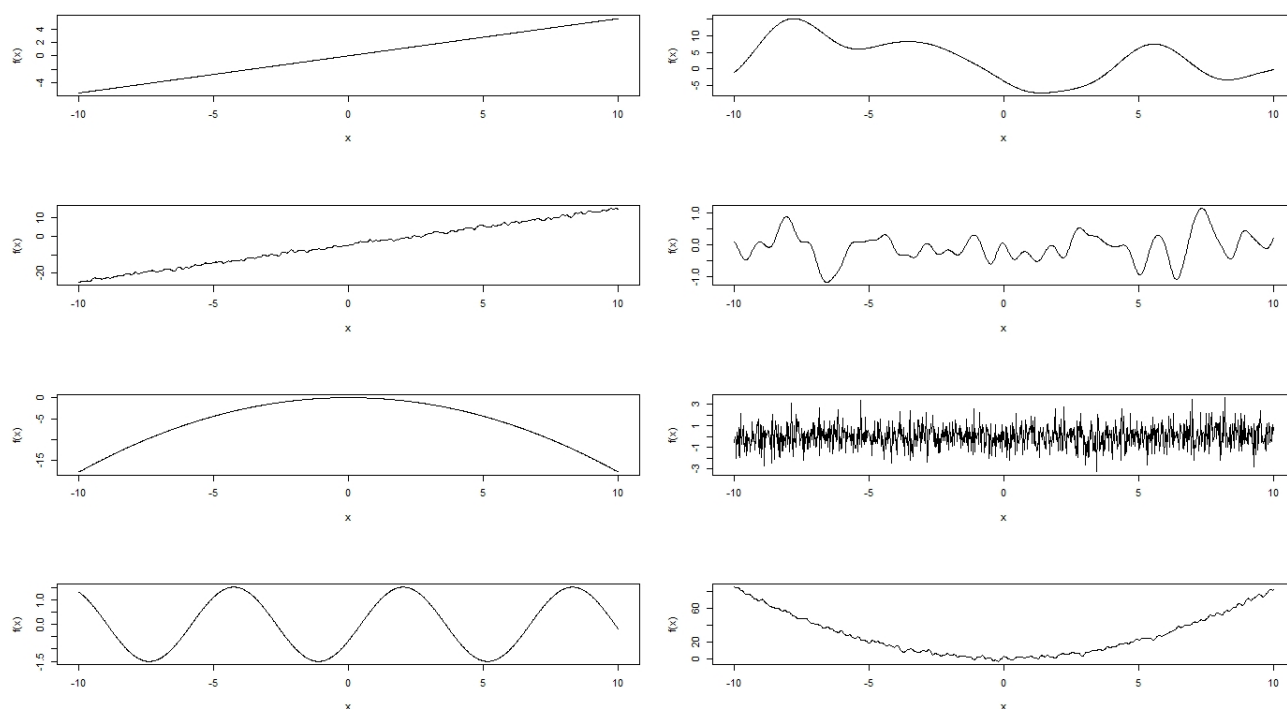


### Exercise 1: Gaussian Processes - Covariance Function

Consider  $\mathcal{X} = \mathbb{R}$ . The following plot shows eight functions

$$\begin{aligned} f : \mathcal{X} &\rightarrow \mathbb{R} \\ x &\mapsto f(x) \end{aligned}$$

randomly drawn from eight different Gaussian processes, each of which has a mean function of zero.



The covariance functions are one of those listed below. Indicate which of the functions above is most likely to have been drawn from the Gaussian process  $\mathcal{GP}(0, k(x, x'))$  with that covariance function.

- (a)  $k(x, x') = \mathbb{1}_{[x=x']}$ .
- (b)  $k(x, x') = x \cdot x'$ .
- (c)  $k(x, x') = 0.5 \cdot x^2 \cdot (x')^2$ .
- (d)  $k(x, x') = 0.5^2 \cdot \exp\left(-\frac{(x-x')^2}{0.5^2}\right)$ .
- (e)  $k(x, x') = \cos(x - x')$ .

(f)  $k(x, x') = 8^2 \cdot \exp\left(-\frac{(x-x')^2}{5}\right).$

(g)  $k(x, x') = 25 + 25 \cdot x \cdot x' + 0.25 \cdot \exp\left(-\frac{(x-x')^2}{0.1^2}\right).$

(h)  $k(x, x') = 2 \cdot x^2 \cdot (x')^2 + 2 \cdot \exp\left(-\frac{(x-x')^2}{0.1^2}\right).$