Exercise 1: Multiclass Classification with 0-1-Loss

Assume that the feature space is $\mathcal{X} = \{1, \ldots, 10\}$ and we are facing a multiclass classification learning problem with 10 classes, i.e., $\mathcal{Y} = \{1, \ldots, 10\}$. We consider the 0-1-loss:

$$L_{0-1}(y, h(\mathbf{x})) = \mathbb{1}_{\{y \neq h(\mathbf{x})\}},$$

where h is an element of $\mathcal{H} = \{h : \mathcal{X} \to \mathcal{Y}\}$. Further, suppose that $p_x \sim \text{Unif}\{1, \ldots, 10\}$ and that $p_{y|x=x} \sim \text{Unif}\{1, \ldots, x\}$.

(a) What is $h^*(\mathbf{x})$?

(b) What is its theoretical risk? (Hint: $\sum_{i=1}^{10} \frac{1}{i} = \frac{7381}{2520}$)

(c) What is $\bar{h}$, i.e., the optimal constant model in terms of the theoretical risk we derived on Exercise Sheet 02?

(d) What is its risk?