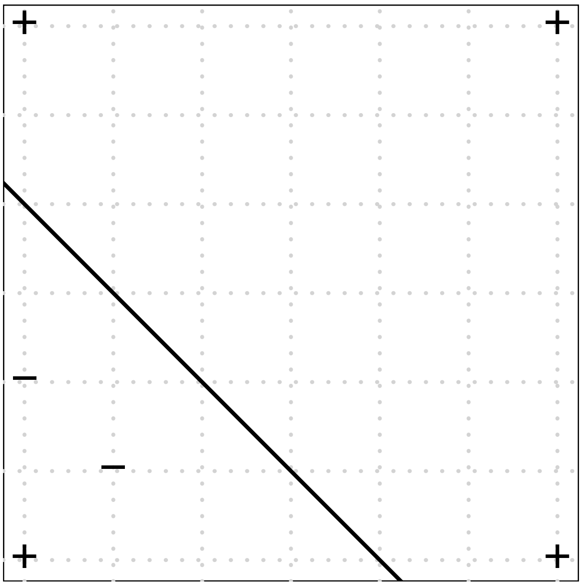
Solution 1: SVM - Support Vectors and Separating Hyperplane

(a) The hyperplane is given by:

$$\theta_1 x_1^{(i)} + \theta_2 x_2^{(i)} + \theta_0 = 0 \tag{1}$$

Plugging in the values for the θs and solving for x_2, we get the decision boundary as function of x_1:

$$x_2 = -x_1 + 2 \tag{2}$$

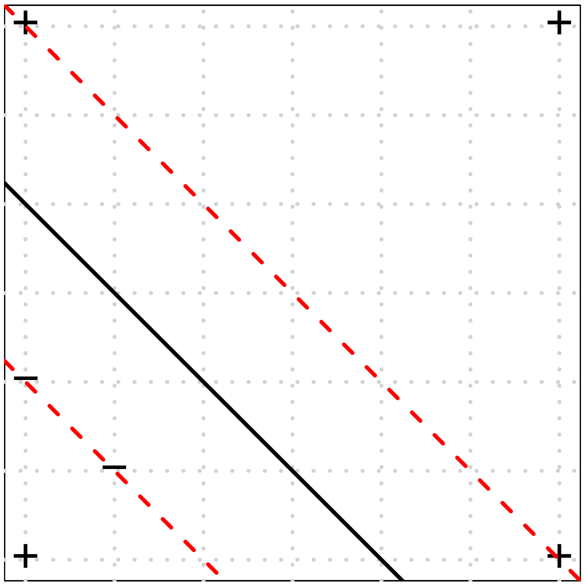


(b) To determine which points are support vectors, we will use the constraint:

$$y^{(i)} \left(x^{(i)} \hat{\theta} + \hat{\theta}_0 \right) \geq 1 - \zeta^{(i)} \tag{3}$$

$$\begin{cases} (0,0): \ 1\,(0+0-2) = -2 \geq 1 - \zeta^{(1)} \longrightarrow \zeta^{(1)} \geq 3 \\ (0.5,0.5): \ -1\,(0.5+0.5-2) = 1 \geq 1 - \zeta^{(2)} \longrightarrow \zeta^{(2)} \geq 0 \\ (0,1): \ -1\,(0+1-2) = 1 \geq 1 - \zeta^{(3)} \longrightarrow \zeta^{(3)} \geq 0 \\ (0,3): \ 1\,(0+3-2) = 1 \geq 1 - \zeta^{(4)} \longrightarrow \zeta^{(4)} \geq 0 \\ (3,0): \ 1\,(3+0-2) = 1 \geq 1 - \zeta^{(5)} \longrightarrow \zeta^{(5)} \geq 0 \\ (3,3): \ 1\,(3+3-2) = 4 \geq 1 - \zeta^{(6)} \longrightarrow \zeta^{(6)} \geq -3 \end{cases} \tag{4}$$

$(0.5, 0.5), (0, 1), (0, 3), (3, 0)$ are support vectors with slack value of $\zeta^{(i)} = 0$ as they lie on the margin hyperplanes. $(0, 0)$ is also a support vector with slack value of $\zeta^{(i)} = 3$.



(c) Using $\mathbf{x}^{(i)} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$:

$$d(f, \mathbf{x}^{(i)}) = \frac{y^{(i)} f(\mathbf{x}^{(i)})}{\|\theta\|_2} = \frac{-1(0.5 + 0.5 - 2)}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

The distance is the same for all non-margin-violating support vectors.

(d) Some alternatives are:

- Convert the $(0, 0)$ into a negative class.
- Move the $(0, 0)$ to $(2, 2)$.
- Delete $(0, 0)$.