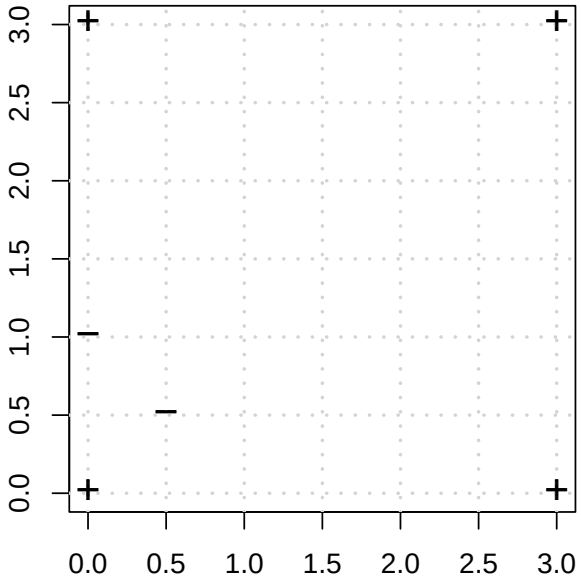
Exercise 1: SVM - Support Vectors and Separating Hyperplane

The primal optimization problem for the two-class soft margin SVM classification is given by

$$\begin{aligned} \min_{\theta,\theta_0,\zeta^{(i)}} \quad & \frac{1}{2}||\theta||^2 + C\sum_{i=1}^{n}\zeta^{(i)} \\ \text{s.t. :} \quad & y^{(i)}(\theta^\top \mathbf{x}^{(i)} + \theta_0) \geq 1 - \zeta^{(i)}, \\ & \zeta^{(i)} \geq 0, \quad \forall i = 1, \ldots, n. \end{aligned}$$



(a) Add the decision boundary to the figure for $\hat{\theta} = (1, 1)^T, \hat{\theta}_0 = -2$. (NB: This is the approximate optimum for $C = 10$)

(b) Identify the coordinates of the support vector(s) and compute the values of their slack variables $\zeta^{(i)}$.

(c) Compute the Euclidean distance of the non-margin-violating support vector(s) (i.e. support vectors that are located on the margin hyperplanes) to the decision boundary.

(d) What needs to be changed in the plot such that a hard margin SVM results into the same decision boundary?