

Exercise 1: Entropy in Binary Classification

Suppose we are facing a binary classification learning problem with the feature space $\mathcal{X} = \{0, 1\}$ and label space $\mathcal{Y} = \{-1, 1\}$. Let us assume that the relationship between the features and labels is specified by

$$y = 2 \max\{x, \epsilon\} - 1$$

where $\mathbb{P}_x = \text{Ber}(\theta_x)$, $\mathbb{P}_\epsilon = \text{Ber}(\theta_\epsilon)$, and x and ϵ are independent.

(a) What is $H(\epsilon)$?

(b) What is $H(y|x)$?

(c) What is $H(y, x)$?

(d) Now if $\epsilon = \psi(x)$ for some deterministic function $\psi : \{0, 1\} \rightarrow \{0, 1\}$. What are the answers to the previous questions in this case?